# Comment on "Observation of the Inverse Doppler Effect"

Seddon and Bearpark present a creative and exciting observation of a reversed Doppler effect when an electromagnetic shock propagates through a transmission line (*1*). We find that the physical origin of this anomalous effect is fundamentally different from the one suggested by Seddon and Bearpark (that $v_{phase}v_{group} < 0$), but that the experimental results can be properly validated with the correct theory.

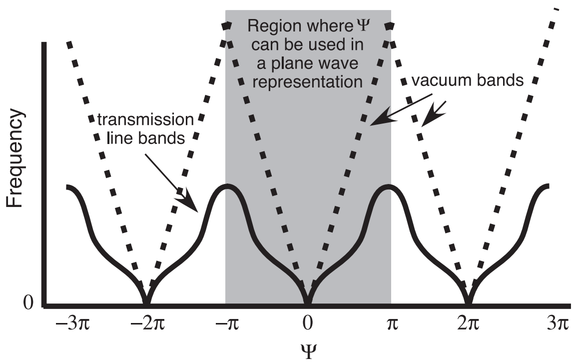


**Fig. 1.** Depicted is a periodic Brillouin zone schematic of the qualitatively similar dispersion relations for vacuum (dotted line) and the transmission line of the Seddon and Bearpark experiment (solid line). Both dispersion relations have an unphysical region where $v_{phase}v_{group} < 0$ outside the first Brillouin zone.

The system studied by Seddon and Bearpark falls into the general class of systems that involve a propagating shocklike excitation in a periodic medium, for which we have predicted reversed Doppler effects using a different theoretical framework (*2*). For this system, an extended Brillouin zone (BZ) scheme should be used, rather than the periodic BZ scheme considered by Seddon and Bearpark (*3*). In their analysis, a phase-matching condition $v_{shock} = v_{phase}$ leads to the conclusion that radiation emitted by the shock has a $\Psi_0$ (wave vector) value in the second BZ, where $v_{phase}v_{group} < 0$. Although the condition $v_{shock} = v_{phase}$ predicts the correct emission frequency $\omega_0$, the suggestion that this emitted radiation has a $\Psi$ value in the second BZ is not founded. The discretized nature of this system precludes unique measurement of $v_{phase}$ (assignment of $\Psi$ to a particular BZ) by measuring voltages or other quantities at points that are spatially periodically related.

Radiation well characterized by plane waves in the first band of periodic systems is poorly characterized by plane waves with wave-vector values outside the first BZ. Imposing a periodicity on the vacuum dispersion reveals a region similar to that of the Seddon and Bearpark transmission-line system, where $v_{phase}v_{group} < 0$, that is clearly unphysical (Fig. 1). Applied to vacuum, the analysis of Seddon and Bearpark [equation 1 in (*1*)] incorrectly predicts that a reversed Doppler shift can occur in that system. Away from the cutoff frequency, physical values of wave vector $\Psi$ in the experiment of Seddon and Bearpark fall within the first BZ, where $v_{phase}v_{group} > 0$.

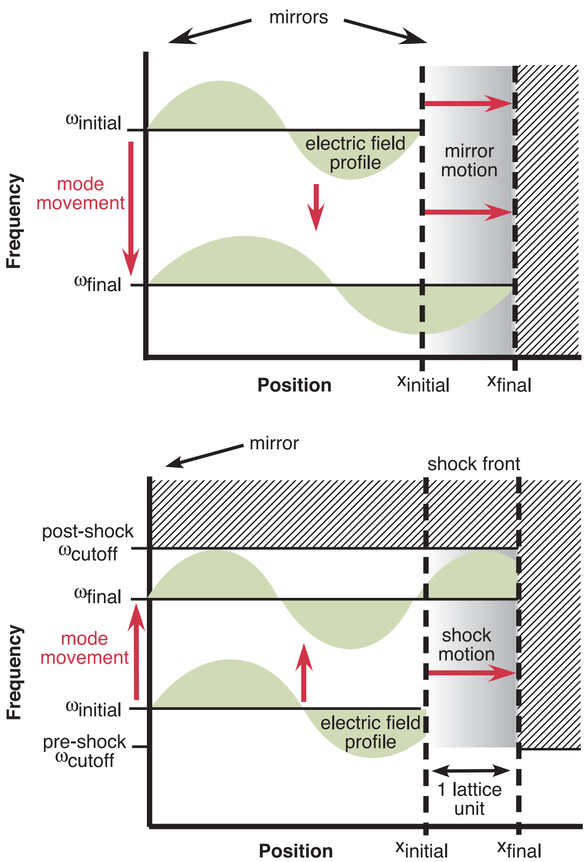


**Fig. 2.** Schematic frequency as a function of position for the normal Doppler shift from a moving metallic mirror (**top**) and the reversed Doppler shift in a transmission line (**bottom**). Radiation of frequency $\omega_{initial}$ is confined between a fixed mirror on the left and a moving mirror (top) or shock front (bottom) on the right side. In the top panel, as the right mirror slowly (adiabatically) moves to the right, the number of nodes of the radiation is preserved, giving rise to a frequency-lowering effect. A Doppler shift occurs upon each reflection of the radiation from the moving mirror. In this case, the Doppler shift is in the negative direction, which is the usual Doppler shift. In the periodic transmission line in the bottom panel, the cutoff frequency is increased from the preshock $\omega_{cutoff}$ to the postshock $\omega_{cutoff}$ as the shock propagates. As the shock propagates slowly (adiabatically) to the right through one lattice unit of the transmission line, an extra node is added to the field profile by the shift of the reflection phase of the shock front through $2\pi$. The addition of an extra node results in a frequency increase despite the increase in cavity length, providing an inverse Doppler shift.

We have shown (*2*) that the phase of the reflection coefficient of the shock front is time dependent, unlike that of a normal moving reflecting surface assumed by Seddon and Bearpark in equation 1. This key feature is the actual origin of the inverse Doppler effect and explains how it can be observed in a region in which $v_{phase}v_{group} > 0$. The condition on the magnetic field at the shock-front location $x = x_0 + v_s t$ is

$$H_0 u_{k_0}(x_0 + v_s t) e^{i(k_0 v_s t - \omega_0 t)} + H_r u_{k_r}(x_0 + v_s t) e^{i(k_r v_s t - \omega_r t)} e^{i\frac{2\pi}{a} v_s t} = 0$$

where $u_{k_0}$ and $u_{k_r}$ are the periodic parts of the Bloch states for the incident and reflected radiation of wave vectors $k_0$ and $k_r$, respectively, and the phase term $e^{i\frac{2\pi}{a} v_s t}$ is the phase of the reflection coefficient at the shock-front leading edge (*2*). This reflection coefficient can have multiple reflection phase-frequency components in some regimes (*2*). When $k_r$ and $k_0$ are chosen in the first BZ so that $u_{k_0}$ and $u_{k_r}$ have no nodes, the approximation $u_{k_0} \approx u_{k_r}$ leads to

$$k_0 v_s - \omega_0 - k_r v_s + \omega_r - \frac{2\pi}{a} v_s = 0$$

which predicts results in direct agreement with the experimental observations. By contrast, when $k_0$ is (incorrectly) measured in the second Brillouin zone as in the analysis of

Seddon and Bearpark, $k_0' = k_0 - \frac{2\pi}{a}$, which gives $k_0' v_s - \omega_0 - k_r v_s + \omega_r = 0$

This equation is equivalent to equation 1 of Seddon and Bearpark and explains the good agreement achieved between their theory and their experimental data. The use of an unphysical phase velocity fortuitously cancels with the neglect of the time-dependent shock-wave reflection coefficient, producing the correct result.

A schematic depiction (Fig. 2) provides additional insight on the origin of the inverse Doppler effect in the Seddon and Bearpark system. First, we consider the origin of the normal Doppler shift (Fig. 2A); in this case, the right-hand mirror moves to the right slowly enough that the electromagnetic mode evolves adiabatically, so that the nodal structure of the mode is preserved and the frequency is lowered as the cavity length increases. A normal Doppler shift (with a negative sign) occurs each time the light reflects from the moving mirror. By contrast, in the transmission-line system of Seddon and Bearpark, which produces an inverse Doppler effect (Fig. 2B), the system has a cutoff frequency, $\omega_{cutoff}$. As the shock propagates to the right through one lattice unit, a node is added to the electric-field profile by the shift of the reflection phase of the shock front, and the frequency shifts up. The addition of a node tends to increase the frequency, and the increase of the cavity length tends to decrease the frequency, but the frequency-increasing effect has greater magnitude in this particular case. The Doppler shift has a positive sign, which is an inverse Doppler shift. Modes must move up in frequency because they all start out in the frequency range from zero to the preshock $\omega_{cutoff}$ and all end (after the shock has propagated through the entire transmission line) in the frequency range from 0 to the postshock $\omega_{cutoff}$, which is higher than the preshock $\omega_{cutoff}$. Physically, incident radiation resonantly couples into individual units of the transmission line as their frequencies move up through the incident radiation frequency. The radiation is re-emitted at a later time and at a higher frequency.

Although the theory of (*2*) and the theory of Seddon and Bearpark happen to predict the same results in the experimental conditions published by Seddon and Bearpark, differing results are predicted in other regimes of the Seddon and Bearpark system. For example, their analysis predicts that if radiation is emitted within the first BZ by altering the shock speed or by other means, no anomalous effect will occur because the first BZ has $v_{phase} v_{group} > 0$. However, our analysis predicts that an anomalous effect will still occur in this case because of the time-dependent phase of the shock-wave reflection coefficient. Our analysis also predicts that multiple frequencies may be reflected from the shock as the shock-front thickness is decreased, whereas the Seddon and Bearpark analysis provides no mechanism for more than one frequency to be emitted. These predictions can be tested within the computational model of Seddon and Bearpark and may also be realizable within their experiment.

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**References and Notes**

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