Plasmonic-Dielectric Systems for High-Order Dispersionless Slow or Stopped Subwavelength Light

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A material platform of multilayered surface-plasmon-dielectric-polariton systems is introduced, along with a new physical mechanism enabling simultaneous cancellation of group-velocity and attenuation dispersion to extremely high orders for subwavelength light of any small positive, negative, or zero group velocity. These dispersion-free systems could have significant impact on the development of nanophotonics, e.g., in the design of efficient and very compact delay lines and active devices. The same dispersion-manipulation mechanism can be employed to tailor at will exotic slow-light dispersion relations.

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A fundamental quest of modern photonic technology is for the perfect slow-light guiding system, which would exhibit, over a large frequency bandwidth, subwavelength modes of very small group velocity and small attenuation, both *devoid of frequency dispersion*. Achieving all of these attributes *simultaneously* appears to be physically impossible. Dielectric structures [1] cannot support highly subwavelength light propagation, polaritons in dispersive (e.g., plasmonic [2]) materials typically suffer from high absorption losses, and, most importantly, all existing slow- or stopped-light systems [3–6] commonly suffer from modal dispersion. In fact, this has been proven as the major reason for the limitation on their achievable so-called "bandwidth-delay product" [7–9], motivating thus the recent invention of a few advanced dispersion-cancellation schemes, using coupled geometric [10] *or* gain-material [11] resonances or nonlinearities [12]. In this Letter, we demonstrate a new physical mechanism, employing a fine balance of both geometric *and* material dispersion, present in a *multilayered* axially uniform hybrid plasmonic-dielectric system that enables subwavelength surface-polaritonic modes—for small positive, negative, or zero group velocity—to exhibit cancellation, to unusually high orders, of the dispersion coefficients for both the group velocity and the attenuation simultaneously. Therefore, this is the first, to our knowledge, linear passive system in nature that essentially is dispersionless, breaks the bandwidth-delay product limitation, and displays extraordinarily high densities of states. With the last obstacle the attenuation losses, this material system approaches the ideal slow-light guiding system. Moreover, it can be tailored to a variety of novel intricate dispersion relations with multiple points of zero group velocity.

Surface-plasmon polaritons (SPPs) [2] are electromagnetic waves that propagate along the interface between a plasmonic (e.g., metallic) and a dielectric material of permittivities ϵ_p and ϵ. A SPP exists only for TM polarization (**H**—field parallel to the interface), and its $\omega(\mathbf{k})$ dispersion relation is $k \equiv |\mathbf{k}| = (\omega/c)\sqrt{\epsilon\, \epsilon_p(\omega)/[\epsilon + \epsilon_p(\omega)]}$, where $\mathbf{k}$ is in the plane of $2d$ translational symmetry. For example, assuming for now lossless materials and using the Drude model $\epsilon_p(\omega) = \epsilon_\infty - \omega_p^2/\omega^2$, where ϵ_∞ is the permittivity at high frequencies and ω_p is the bulk plasma frequency, the condition $\epsilon_p < -\epsilon < 0$ for the existence of SPPs leads to an upper frequency cutoff at $\omega_c(\epsilon) = \omega_p/\sqrt{\epsilon_\infty + \epsilon}$, to which the SPP asymptotes for large wave vectors k, as it is then tightly confined to the interface, while the SPP asymptotes to the light line $k = (\omega/c)\sqrt{\epsilon}$ for small wave vectors, as it extends far into the dielectric.

Instead of known extensions consisting of planar plasmonic layers [13], let us keep the plasmonic substrate semi-infinite and insert, between this and the dielectric, a planar dielectric layer of permittivity $\epsilon_1 > \epsilon$ and thickness d_1. For this structure, it was shown in [14] (and Fig. 1, curve C therein) that, for small k, the surface-plasmon-dielectric-polariton (SPDP) mode extends a lot into the ϵ-dielectric not "seeing" much the ϵ_1-dielectric layer, and thus it moves asymptotically to a SPP mode on a ϵ_p-ϵ interface, while, for large k, the SPDP mode is tightly confined on the ϵ_p-ϵ_1 interface, and thus it moves asymptotically to a SPP mode on such an interface; for a thin enough layer, since both of these SPPs have positive group velocity and $\epsilon_1 > \epsilon \Rightarrow \omega_c(\epsilon_1) < \omega_c(\epsilon)$, the limiting k regions are connected through a regime of negative group velocity, with two boundary points of zero group velocity (ZGV), one with negative curvature at a small $k_{n,1}$ and one with positive curvature at a large $k_{p,1} > k_{n,1}$.

Let us further insert, between the plasmonic substrate and the ϵ_1-dielectric layer, another planar dielectric layer of permittivity $\epsilon_2 < \epsilon_1$ and thickness $d_2 < d_1$. This structure has never, to our knowledge, been analyzed before and behaves as follows: For small k, the behavior will not change, but, for large k, the SPDP mode must now move asymptotically to a SPP mode on a ϵ_p-ϵ_2 interface; since

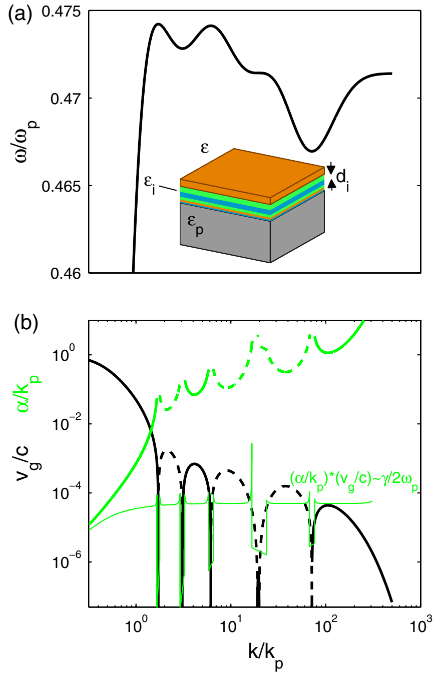

FIG. 1 (color online). (a) $\omega(k)$ dispersion diagram for the layered SPDP structure shown in the inset with $\epsilon = 1$, $\epsilon_i = \{4, 2, 6, 2, 4, 3.5\}$, $d_i/\lambda_p = \{5\times10^{-2}, 1.2\times10^{-2}, 6.5\times10^{-3}, 2.9\times10^{-3}, 2.8\times10^{-3}, 1.4\times10^{-3}\}$ (from top to bottom), and $\epsilon_\infty = 1$. (b) [Solid and dashed lines denote that the quantity plotted logarithmically is positive and negative, respectively] (Black line) $v_g(k)$ GVD diagram [namely, first derivative of (a)]: Note the apparent scaling $\max|v_g| \sim 1/k$. (Green line) $\alpha(k)$ AD diagram for small added loss $\gamma/\omega_p = 10^{-4}$: Note the apparent scaling $\min|\alpha| \sim k$ and that, away from the special ZGV points and for subwavelength wave vectors, $\alpha v_g \approx \gamma/2$ independently of k. $k_p = \omega_p/c = 2\pi/\lambda_p$.

$\epsilon_2 < \epsilon_1 \Rightarrow \omega_c(\epsilon_2) > \omega_c(\epsilon_1)$, the large-$k_{p,1}$ ZGV point will move to a smaller wave vector $k_{p,2} < k_{p,1}$, and two ZGV points will remain at $\sim k_{n,1}$ and $k_{p,2}$, provided d_2 is small enough so that $k_{n,1} < k_{p,2}$.

This process of inserting layers of thickness $\{d_i\}$ of various dielectrics $\{\epsilon_i\}$ onto the plasmonic substrate can be continued arbitrarily, providing ample degrees of freedom to create a large variety of dispersion relations. Using dielectric layers that are progressively thinner from top to bottom, one can create exotic dispersions with many controllable ZGV points, as illustrated above, where the enabling key for the dispersion manipulation is that, upon insertion of the ith layer, always $\omega(k \to \infty) = \omega_c(\epsilon_i)$. Such an example, calculated using a standard transfer-matrix method, is shown in Fig. 1. In general, by using layers not necessarily progressively thinner, even further interesting dispersions can be tailored. Given enough layers, the main restriction, imposed by the general nature of plasmonic systems, to what dispersion relations can be achieved is that the maximum attainable $|v_g|$ of the target relation has to decrease rapidly as k increases, with an apparent scaling $\max|v_g| \sim 1/k$ as seen from Fig. 1(b).

For a particularly important case, let us use a structure of the type shown in Fig. 1 as a starting point and consider a small group velocity v_{g0}. From Fig. 1(b), it is clear that the function $v_g(k) - v_{g0}$ has several roots, depending on the chosen v_{g0}. If we could now force N of these roots to all coincide at an intermediate wave vector k_0 and at frequency ω_0, by appropriate choice of the thicknesses $\{d_i\}$ (and/or the indices $\{\epsilon_i\}$), we would effectively have cancelled dispersion up to the Nth order for the SPDP mode. Specifically, the dispersion relation around (ω_0, k_0) would look, in two equivalent ways, like

$$k - k_0 = v_{g0}^{-1}(\omega - \omega_0) + \frac{c^{-1}\omega_0^{-N}}{(N+1)!} D_{k,N+1}(\omega - \omega_0)^{N+1}, \tag{1}$$

$$\omega - \omega_0 = v_{g0}(k - k_0) + \frac{c^{N+1}\omega_0^{-N}}{(N+1)!} D_{\omega,N+1}(k - k_0)^{N+1}, \tag{2}$$

where $D_{k,n} \equiv \omega_0^{n-1}\frac{\partial^{n-1}}{\partial\omega^{n-1}}\left(\frac{c\partial k(\omega_0)}{\partial\omega}\right)$ and $D_{\omega,n} \equiv \omega_0^{n-1}\frac{\partial^{n-1}}{\partial(ck)^{n-1}}\left(\frac{\partial\omega(k_0)}{c\partial k}\right)$ are the normalized dimensionless dispersion constants of nth order, and, when all are zero up to Nth order as in Eqs. (1) and (2), then $D_{\omega,N+1}/D_{k,N+1} = -(v_{g0}/c)^{N+2}$. Indeed, such an exceptionally high-order cancellation of group-velocity dispersion (GVD) for a slow subwavelength light mode is possible, not only for a planar system, discussed so far, but also for a linear waveguide. In the latter case, k denotes the conserved wave vector along the $1d$ translationally invariant propagation

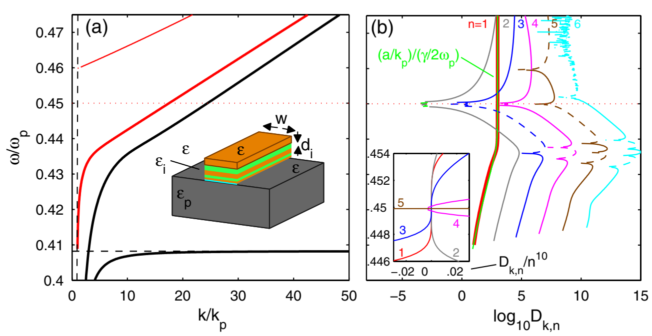

FIG. 2 (color online). (a) $\omega(k)$ dispersion diagram for the layered SPDP waveguide structure of the inset with $\epsilon = 5$, $\epsilon_i = \{2, 5, 2, 5, 1\}$, $d_i/\lambda_p = \{1.707\times10^{-3}, 8.083\times10^{-2}, 6.877\times10^{-4}, 5.096\times10^{-3}, 3.104\times10^{-4}\}$, $w/\lambda_p = 0.015$, $\epsilon_\infty = 1$, and $\gamma/\omega_p = 10^{-2}$: Note the extremely linear regime around $\omega_0/\omega_p = 0.45$ and $k_0/k_p = 17.5$. (b) (All lines except green) $\omega(D_{k,n})$ GVD diagrams [namely, derivatives of (a)]: Plotted logarithmically in the main box and linearly in the inset, note that GVD orders $n = 2, \ldots, 5$ are cancelled at ω_0 (dotted line), where $v_{g0}/c = D_{k,1}^{-1} = D_{\omega,1} = 10^{-3}$, $D_{k,6} = 3.27\times10^{10} \Rightarrow D_{\omega,6} = -3.27\times10^{-11}$, and thus the inset depicts the Eq. (1)-type behavior. (Green line) $\omega(\alpha)$ AD diagrams: Note that, for large enough k, $\alpha v_g \approx \gamma/2$ independently of ω and therefore $D_{\alpha,n-1} \approx D_{k,n}(\gamma/2\omega_p)$ implying that AD orders $n - 1 = 1, \ldots, 4$ are also suppressed at ω_0.

direction of a guided mode, which employs effective-index guiding [14] for lateral confinement (see also [15–17]). Examples for such high-order dispersion cancellation are given in Fig. 2 for a linear waveguide with positive v_{g0}, calculated via the approximate (but qualitatively sufficient) effective-index method, and in Fig. 3 for a planar structure with zero v_{g0}, respectively. To determine $\{d_i\}$ and $\{\epsilon_i\}$ for desired ω_0, k_0, and v_{g0}, we solved a nonlinear system of N equations of the form $v_g(\omega, k, d_i, \epsilon_i) - v_{g0} = 0$, for N values of (ω, k) very close to (so practically all are approximately equal to) (ω_0, k_0). To our knowledge, there is no other system in nature whose photonic dispersion relation can be such a long straight-line segment and which is not a plane wave. Note also that in Fig. 2 guidance occurs, counterintuitively, in the region of lower average dielectric permittivity [14].

Using an optimal design of alternating higher- and lower-index layers, a minimal number of $\approx N$ layers are needed for an Nth order dispersion cancellation. In principle, one can keep adding pairs of layers to increase N, and whether this can be increased arbitrarily depends on how rapid a change in group velocity within the dispersion curve the plasmonic material system allows. We can argue that a larger N is easier to get as $|v_{g0}|$ gets smaller and k_0 increases, since, as seen from Fig. 1(b), $v_g(k) - v_{g0}$ can have more roots more easily. In practice, N does not need to be too large anyway, as shown in examples later.

In the operational regime of practical interest, the geometric dispersion enforced by the design is stronger than

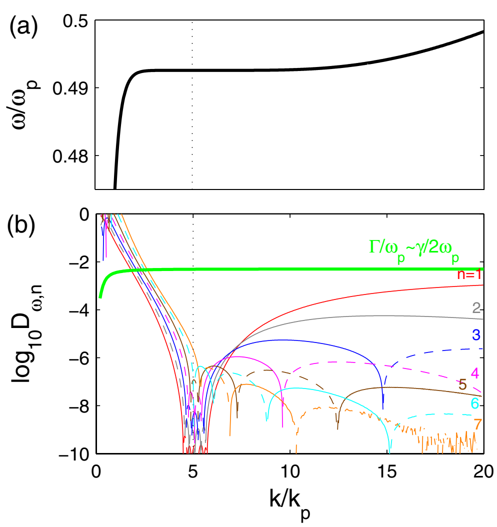

FIG. 3 (color online). (a) $\omega(k)$ dispersion diagram for a planar layered SPDP structure with $\epsilon = 1$, $\epsilon_i = \{5, 2, 5, 2, 5, 2\}$, $d_i/\lambda_p = \{1.634 \times 10^{-2}, 3.697 \times 10^{-2}, 1.367 \times 10^{-2}, 1.213 \times 10^{-2}, 6.340 \times 10^{-3}, 2.247 \times 10^{-3}\}$, $\epsilon_\infty = 1$, and $\gamma/\omega_p = 10^{-2}$: Note the extremely flat regime around $\omega_0/\omega_p = 0.492\,56$ and $k_0/k_p = 5$. (b) (All lines except green) $D_{\omega,n}(k)$ GVD diagrams [namely, derivatives of (a)]: Note that GVD orders $n = 2, \ldots, 6$ are cancelled at k_0 (dotted line), where $v_{g0}/c = D_{\omega,1} = 0$, $D_{\omega,7} = 2.14 \times 10^{-6}$. (Green line) $\Gamma(k)$ AD diagram: Note that, for large enough k, $\Gamma \approx \gamma/2$ independently of k.

the potential material frequency dispersion of the dielectric layers and wave vector dispersion of the plasmonic substrate (which typically appears only for very large wave vectors); therefore, these dispersions are not a problem and can be taken into account by this scheme.

All materials in reality exhibit intrinsic absorption losses, which are usually quantified as losses-per-unit-distance α (also called "propagation losses") or losses-per-unit-time Γ (related to the "quality factors" $Q = \omega/2\Gamma$). The dispersion relation $F(\tilde{\omega}, \tilde{k}) = 0$ is analytic with respect to the generally complex (denoted with a tilde) $\tilde{\omega} = \omega - i\Gamma$ and $\tilde{k} = k + i\alpha$, so the Cauchy-Riemann equation $\frac{\partial(\mathrm{Im}\tilde{k})}{\partial(\mathrm{Im}\tilde{\omega})} = \frac{\partial(\mathrm{Re}\tilde{k})}{\partial(\mathrm{Re}\tilde{\omega})} = 1/v_g$ suggests that, for small loss, the two loss rates relate by $\alpha \approx \frac{\partial(\mathrm{Im}\tilde{k})}{\partial(\mathrm{Im}\tilde{\omega})}\Gamma \Rightarrow \alpha v_g \approx \Gamma$. Therefore, α is not a good measure in regimes close to ZGV points, where "propagation" lacks physical meaning and $\alpha \sim 1/v_g \to \infty$ [see Fig. 1(b)], but Γ is more appropriate. Using the appropriate loss measure, the GVD evolution [$\mathrm{Re}(\tilde{\omega}) - \mathrm{Re}(\tilde{k})$ relation] and the techniques for its manipulation remain unaffected in the presence of weak material absorption, its basic effect being just field attenuation. This holds also for our SPDP structures and GVD cancellation mechanism, as shown in Figs. 2 and 3, calculated using the simple Drude model for plasmonic materials $\tilde{\epsilon}_p(\tilde{\omega}) = \epsilon_\infty - \omega_p^2/(\tilde{\omega}^2 + i\gamma\tilde{\omega})$ with a typical small γ/ω_p ($\sim 10^{-2}$ at room temperature for metals).

In the subwavelength high-k regime of interest, the dispersion relation of a SPDP structure depends on $\tilde{\omega}$ only through $\tilde{\epsilon}_p(\tilde{\omega})$, namely, $F(\tilde{\omega}, \tilde{k}) = F(\tilde{\epsilon}_p(\tilde{\omega}), \tilde{k}) = 0$. Without loss, $F(\epsilon_p(\omega), k) = 0$. With loss added *only* to the plasmonic material, for small γ/ω_p in the Drude form $\tilde{\epsilon}_p(\tilde{\omega}) \approx \epsilon_p(\tilde{\omega} + i\gamma/2)$, therefore, seeking solutions with a real k, $F(\tilde{\epsilon}_p(\tilde{\omega}), k) = F(\epsilon_p(\tilde{\omega} + i\gamma/2), k) = 0$. Comparing with the lossless case, the pair $(\tilde{\omega} = \omega - i\gamma/2, k)$ is a solution of the lossy case. This implies that $\alpha v_g \approx \Gamma \approx \gamma/2$, namely, a constant, independent of frequency and wave vector [see part (b) of Figs. 1–3]. This has the remarkable implication that, at points (ω_0, k_0) where GVD has been cancelled to order N ($D_{k,n} = 0$ for $n = 2 \ldots N$), also the attenuation dispersion (AD) has been cancelled to order $N - 1$ [$D_{\alpha,n} \equiv \omega_0^n \frac{\partial^n}{\partial\omega^n}(\frac{c\alpha(\omega_0)}{\omega_p}) = \omega_0^n \frac{\partial^n}{\partial\omega^n}(\frac{c\partial k(\omega_0)}{\partial\omega}) \times (\gamma/2\omega_p) = D_{k,n+1}(\gamma/2\omega_p) = 0$ for $n = 1, \ldots, N - 1$], so the propagation loss rate can be written as

$$\alpha = \alpha_0 + \frac{c^{-1}\omega_0^{-N}\omega_p}{N!} D_{\alpha,N}(\omega - \omega_0)^N. \tag{3}$$

Since these surface-polaritonic modes exist inherently only for a single (TM) polarization, they do not suffer from polarization mode dispersion either. This triple simultaneous dispersion cancellation is a unique feature of layered SPDP material structures, unparalleled in nature.

This material system can be designed to support slow and subwavelength propagation of short pulses that do not suffer any type of distortion (phase or amplitude) as they travel. This is a very important feature required for the design of very compact and efficient optical delay lines. Their common characterization figure of merit, the bandwidth-delay product, has been shown to be fundamentally limited by dispersion [7–9]. Expressed as the number M of bits the line can store (the largest possible product of the bit rate $B = 1/T_B$ and the distortionless time delay $\tau = L/|v_{g0}|$), for Gaussian pulses on a line with only GVD as in Eq. (1), we find (using the method in [18])

$$M = \tau/T_B = \frac{S_{N+1}(\omega_0 T_B)^N}{|D_{k,N+1}|(|v_{g0}|/c)}, \tag{4}$$

where $S_{N+1} = \sqrt{N^N/(N+1)^{N+1}}/s_{N+1}/2^{2N+2}$, $s_{N+1} = \sqrt{(2N-1)!!/(N!)^2 - \mathrm{mod}(N+1,2)/(N!!)^2/2^N}$, and $N!!$ denotes the even or odd factorial. For pulses broad enough that $\omega_0 T_B \gg 1$, when N (the GVD cancellation order) is large, the N-polynomial increase in M can thus be tremendous. To test our proposed SPDP structures, note that AD can typically be ignored when $\gamma/\omega_p \ll 1$, so Eq. (4) predicts that the delay line of Fig. 2 could, e.g., hold $M \sim 1.2 \times 10^6$ undistorted bits of 50-optical-cycle pulses ($B \sim 1$ Tbit/s at $\lambda_0 \equiv 2\pi c/\omega_0 = 1.3\ \mu$m) with line length $L \sim 2.6 \times 10^5 \lambda_0$. This delay-line performance is orders of magnitude larger than any ever demonstrated before [8].

Furthermore, the presently proposed system could greatly enhance the performance of various active optical devices (nonlinear, electronic, thermal, etc.). The rate of the underlying interaction depends strongly on the number of the participating photonic states (Fermi's golden rule). Therefore, we define here, as the figure of merit for a photonic structure, the enhancement ratio M' of its number of states within a frequency bandwidth $\Delta\omega$ around ω_0 compared to those of a uniform medium with the same effective index at ω_0. Then, for a dispersion relation as in Eq. (2) with $v_{g0} = 0$, we find that the associated densities of single-polarization $1d$ and $2d$ states close to ω_0 (and k_0) interestingly have the *same* frequency dependence, which for $N > 1$ exhibits a new type of singularity (not classified by van Hove [19]) and for $N \gg 1$ approaches the non-integrable $|\omega - \omega_0|^{-1}$ as $\omega \to \omega_0$, and then within $\Delta\omega$

$$M' = \frac{\omega_0}{ck_0}\left[\frac{(N+1)!}{2|D_{\omega,N+1}|}\right]^{(1/N+1)}\left(\frac{\omega_0}{\Delta\omega}\right)^{(N/N+1)}. \tag{5}$$

For narrow interaction bandwidths $\Delta\omega/\omega_0 \ll 1$, when the system has a large N, the almost-linear enhancement in M' can thus be huge. To test our proposed SPDP structures, Eq. (5) predicts that, for the system of Fig. 3 and, e.g., for typical due-to-collisions homogeneously broadened atomic linewidths $\Delta\omega/\omega_0 \sim 10^{-8}$ [20], the radiative lifetime of the atoms would be reduced approximately by $M' \sim 1.4 \times 10^7$ compared to that in a uniform medium, so even a dipole-forbidden transition could have a lifetime ~70 ps instead of its typical ~1 ms [20]. This enhancement is orders of magnitude larger than for any other known translationally invariant material system.

The most significant practical problem of integrated plasmonic structures is loss. Although AD induced by plasmonic-material-type absorption losses was shown earlier in this Letter to be cancellable, unfortunately, the attenuation rate α_0 itself is strong: For typical $\gamma/\omega_p \approx 1/100$, then $Q \approx 100\omega/\omega_p$ and $\alpha \approx 10\log_{10}[(c/|v_g|)\pi/100]dB/\lambda_p$, where α increases prohibitively with k, since $\min|\alpha| \approx (\gamma/2)/\max|v_g| \sim k$ [as seen earlier and in Fig. 1(b)]. In the subwavelength regime, these intrinsic losses are expected to be primarily due to electron-scattering mechanisms.

The suggested material system and dispersion-manipulation method can be extended to the case of a continuous dielectric distribution rather than discrete layers and/or a layered or continuous plasmonic distribution. For GVD-AD cancellation, an oscillatory profile of the continuous distribution(s) would be required.

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