Delayed-Action Interaction and Spin-Orbit Coupling between Solitons

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We report on new fundamental phenomena in soliton interactions: delayed-action interaction and "spin"-orbit coupling upon collision between two-dimensional composite solitons carrying topological charges.

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Solitons have attracted much attention in the scientific community for almost four decades. This is because solitons are perhaps the only nonlinear waves that behave and interact like real particles. Soliton interactions are universal, displaying features common to all solitons, in spite of the diversity of the physical systems in which solitons are found [1]. The reason for this universality is the fact that solitons can be viewed as bound states of their own induced potential or, in nonlinear optics, as modes of their own induced waveguide [2]. The simplest case arises when the self-induced waveguide has only one mode populated, in which case the soliton is a "scalar soliton." In this vein, collisions of single-mode solitons can be viewed intuitively as interactions between guided modes of adjacent waveguides. The interaction outcome is determined by the relation between the collision angle and the (complementary) critical angle for total internal reflection in each waveguide, θ_c. Experimentally, collisions between scalar solitons were studied in detail, demonstrating elastic collisions between Kerr solitons [3], almost-elastic collisions between solitons in saturable nonlinearities interacting at angles above θ_c [4], and inelastic collisions that yield fusion [4–6], fission [6], annihilation [6], and spiraling [7].

More than a decade ago, multimode (or composite) solitons were proposed, in the temporal [8] and later on in the spatial domain [9]. The discovery of photorefractive solitons has eventually led to the experimental observation of composite solitons [10]. Theoretical and experimental papers on interactions of composite solitons followed, reporting on shape transformations upon collision [11], and on a bound state between two vector solitons [12]. In all of these studies the solitons were (1 + 1)D type. Recently, however, we have proposed the possibility of generating (2 + 1)D multimode composite solitons in which at least one component carries topological charge [13]. Consequently, two-dimensional dipole-type composite solitons were suggested [14] and observed [15].

Here, we study theoretically interactions between (2 + 1)D composite solitons carrying topological charges. The collision results in one of the most intriguing phenomena in soliton science: "delayed-action collision." It occurs when the interacting composite solitons carry opposite topological charges, and when the collision angle is slightly below θ_c. Upon collision, the two composite solitons fuse to form a *metastable bound state,* which survives as a single entity for large propagation distances (tens of diffraction lengths) until it breaks up and new vector solitons emerge. This interaction resembles the interaction between elementary particles in which the "mediator" (the fusion product) has a prolonged yet finite "lifetime" and eventually decays into new entities. The interaction between our solitons has similar features: the emerging (new) solitons diverge away from one another as free particles. Apart from the delayed-action interaction, the collision process between composite solitons gives rise to a series of other new phenomena. One such feature, which has no counterpart either with scalar solitons or with (1 + 1)D composite solitons, is "spin"-orbit coupling. It occurs for composite solitons carrying identical topological charges colliding at angles smaller than θ_c. The spin-orbit coupling is manifested in the transfer of angular momentum from spin [16] to orbital motion in which the solitons spiral around each other.

These interaction features should occur in any isotropic saturable self-focusing nonlinear media, yet in our study we consider two beams interacting in a medium with normalized nonlinear refractive index change $\delta n(I) = -1/(1 + I)$, where I is the total (time-average) intensity. The slowly varying envelope functions of the two interacting beams, $\psi_j, j = 0, 1$ satisfy two coupled normalized nonlinear Schrödinger (NLS) equations [13,14]:

$$i \frac{\partial \psi_j}{\partial z} + \frac{1}{2} \nabla^2 \psi_j + \delta n(I) \psi_j = 0, \tag{1}$$

where $I = \sum_{j=0}^{1} |\psi_j|^2$, and z is the propagation distance; $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. Equation (1) admits the conservation laws: power $E_j = \iint |\psi_j|^2 \, dx \, dy$; transverse momentum $\mathbf{P} = \iint \mathbf{p} \, dx \, dy = \frac{i}{2} \int (\sum_{j=0}^{1} \psi_j \nabla \psi_j^* - \text{c.c.}) \, dx \, dy$ and angular momentum $L\hat{z} = \iint \mathbf{r} \times \mathbf{p} \, dx \, dy$, where $\mathbf{p}$ is the transverse momentum density carried by the composite solitons and $\mathbf{r}$ is the transverse vector coordinate. Stationary solutions to Eq. (1) can be found in the form $\psi_j(r, \varphi, z) = u_j(r)\exp(i\mu_j z + im_j\varphi)$, $j = 0, 1$ with $m_0 = 0$, $m_1 = \pm 1$ being the solitons topological charges (spin) [16] and μ_j are the propagation constants. We denote such solutions by $(0, \pm 1)$. Their first mode has radial symmetry with maximum peak intensity at the origin, while the second mode is of the vortex type (doughnut). The stability analysis of these structures was done in [14,17] where it was shown that they are *weakly* linearly unstable [18], and that saturation leads to a *strong* stabilization effect.

We numerically study the interactions between such solitons by launching two vector solitons in the (x, z) plane, each of which is composed of a circular and a vortex component. We consider only single hump composite solitons which were shown numerically [13] to be stable against deviations of at least 2% in initial amplitudes, and at least 5% relative displacement of the components, all of these within a propagation distance in excess of 100 physical diffraction lengths. We simulate Eq. (1) with the initial condition given by $(j = 0, 1)$

$$\psi_j^{\text{initial}} = \sum_{s=\pm 1} \psi_j(\mathbf{r} + s\mathbf{r}_0; z = 0)e^{is\vartheta x}, \qquad (2)$$

where 2ϑ is the dimensionless collision angle, and $2|\mathbf{r}_0|$ is the separation between the centers of the two composite solitons. Our results are presented in dimensional units with the parameters of [19], where $\theta_c \simeq 0.66°$. To facilitate understanding, we first recall known results from interactions between scalar solitons ($\psi_1 \equiv 0$) [1]. When two scalar solitons with circular cross sections collide at angle $\theta > \theta_c$ then they pass through each other unaffected (maintaining their circular shape) [4], whereas they undergo fusion or fission [5,6] for $\theta < \theta_c$ (Fig. 1). This, in turn, serves as a definition of the critical angle θ_c [20].

Interaction between composite vector solitons exhibits many new features. Here, we describe in detail only the most important effects. In essence, the interaction between solitons always displays a dramatic change near the transition from $\theta > \theta_c$ to $\theta < \theta_c$. In the regime $\theta > \theta_c$, two pairs of composite solitons (either with identical or opposite topological charges) cross the trajectories of each other. But, in distinction from collisions between scalar solitons, the vortex components always transform upon collision into *rotating dipoles.* The dipoles of the emerging solitons corotate (counterrotate) for input solitons with identical (opposite) charges. Evidently, this shape transformation into rotating dipoles is *solely* due to the spin carried by the second mode and is *not* observed with scalar solitons. At $\theta < \theta_c$, the dynamics always include a change in the number of solitons. In this regime we observe the most intriguing new phenomena: delayed-action interaction and spin-orbit coupling.

Delayed-action interaction.—When two composite solitons of opposite spins [solitons of type $(0, 1)$ and $(0, -1)$] collide at angle $\theta \approx 0.37° < \theta_c$, then both vector constituents undergo a fusion process and form a resonant bound state that has a prolonged lifetime (propagation distance) of about 35 diffraction lengths. When this metastable bound state eventually disintegrates, it gives rise to new vector solitons that lie in a plane almost orthogonal to the initial plane (Fig. 2 and the schematic presentation in Fig. 3). This delayed-action process bears much resemblance to interactions between elementary particles where the intermediate (metastable) fusion product plays the role of a mediator. This phenomenon is observed here for the first time [21]. We find (numerically) that the generic delayed-action interaction is robust against deviation of at least 3% in initial amplitudes. Even at 3% perturbations, this metastable state still occurs, and survives for a distance of 20 diffraction lengths. Nevertheless, the details of the delayed-action interaction process and the emerging solitons critically depend upon the initial amplitudes. In the examples depicted in Figs. 2c and 2d, the input solitons differ by merely 3% in initial amplitudes, yet in one case two solitons emerge from the collision process, whereas in the other three solitons are emitted. Evidently, tiny changes in the input parameters completely change the details of the interaction.

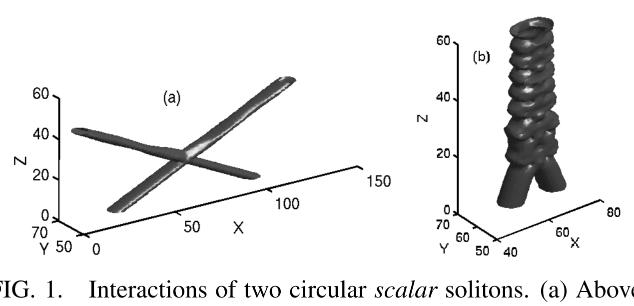

FIG. 1. Interactions of two circular *scalar* solitons. (a) Above the critical angle and (b) below the critical angle.

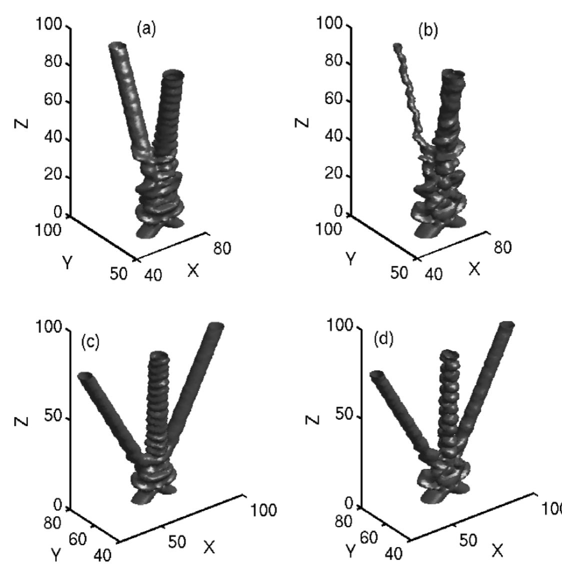

FIG. 2. Interactions of two composite solitons colliding at $\theta \approx 0.37° < \theta_c$, with $|\psi_0(0)|^2 = 4$ and $\max(|\psi_1|^2) = 0.64$. (a),(b) show the ψ_0,ψ_1 components. (c),(d) same as (a),(b) but with deviations of 3% in initial amplitudes.

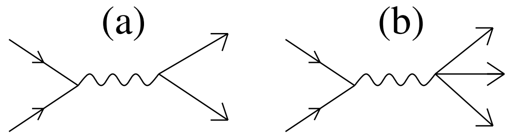

FIG. 3. Schematic representation of the two delayed-action examples shown in Fig. 2. Two vector solitons collide, a metastable bound state forms, and after many diffraction lengths, the bound state decays into new vector solitons. (a) corresponds to Figs. 2a and 2b, whereas (b) refers to Figs. 2c and 2d.

Spin-orbit coupling.—In this case, both input solitons are of the $(0, 1)$ type and the collision angle is below θ_c, e.g., $\theta \approx 0.12°$. The collision results in the production of three vector solitons which are completely decoupled from each other (Fig. 4). The resulting solitons propagate in a plane that is tilted by an angle 28.65°. In the outer two pairs ψ_0 and ψ_1 both occupy the lowest mode of their jointly induced waveguide. On the other hand, for the inner soliton components, ψ_0 and ψ_1 occupy self-consistently the first and second modes of their jointly induced potential. The inner composite soliton emerging from the collision develops complicated dynamics: its ψ_0 constituent initially shows strong pulsation that gradually evolved into rotation around its own center of mass (Fig. 4a). This dynamics relaxes after some distance. The emerging ψ_1 component of the middle fission product has the form of a dipole, and exhibits *stable spiraling* for all propagation distances in access (Figs. 4b and 5). The spiraling of the middle ψ_1 component occurs even though the initially launched solitons have trajectories in the same plane. The spiraling occurs because some of the angular momentum carried

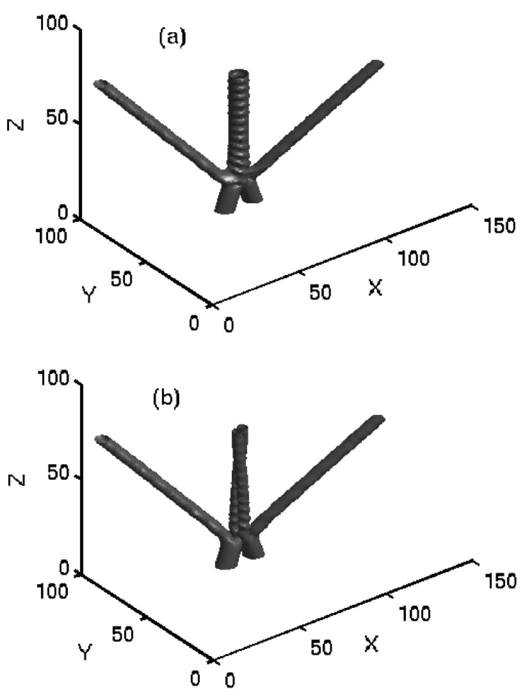

FIG. 4. As in Fig. 2 but for collision angle $\theta \approx 0.66° \approx \theta_c$.

FIG. 5. A magnified version of the dynamics displayed in Fig. 4b for a longer propagation distance.

by the spin $[\exp(im\varphi)]$ is transferred to orbital angular momentum. This is a clear manifestation of *spin-orbit* interaction between solitons.

This generic behavior appears for all collisions with $\theta < \theta_c$ including $\theta = 0$ (when the solitons are launched in parallel). In other words, the spiraling behavior of the emerging inner ψ_1 dipole component appears also in the case of fully parallel-launched solitons. In the most counterintuitive manner we find that solitons launched in parallel with one another undergo fission, and one of the fission products exhibits spiraling. All this fascinating dynamics occurs just because the input solitons are carrying spin. Note that the emerging solitons, for any collision angle $\theta \lesssim \theta_c$, lie in a plane tilted with respect to the plane of incidence (x, z). The tilt angle increases with decreasing θ, and at $\theta = 0$ reaches 37°. If the colliding solitons have $m = -1$, then the plane of the emerging solitons is tilted in an opposite direction, and all the rotation dynamics presented above, including the spiraling, occurs in reverse (anticlockwise) direction. In summary of the last result, this collision process displays spin-orbit interaction: the initial angular momentum was carried solely by the topological charge (spin) of each soliton, yet upon collision, part of the angular momentum is transferred to orbital motion of the emerging solitons. As we increase the collision angle θ to an "intermediate" value, e.g., $\theta \approx 0.66° \approx \theta_c$, we observe a qualitative change in the interaction picture (Fig. 6).

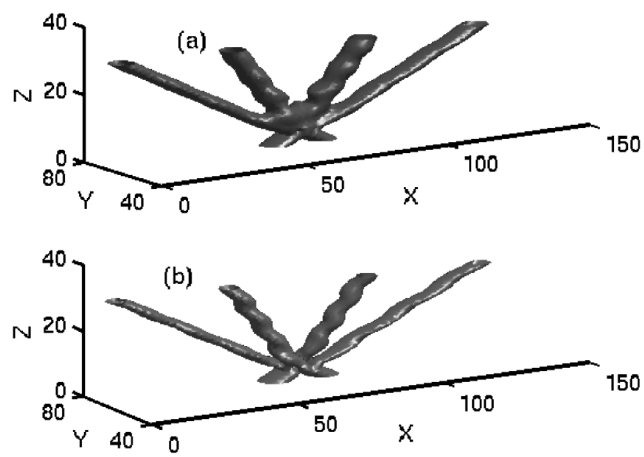

FIG. 6. As in Fig. 2 but for a collision angle $\theta \approx 0.12° < \theta_c$.

In this case, the two interacting vector solitons undergo a *fission* process which ends up in the creation of four pairs of components, in which each pair is self-trapped separately. The two outer pairs travel almost like free particles, whereas the inner pairs show strong pulsation and, after propagating at close proximity for some distance, they eventually escape away one from another. Although the initial paths (before collision) of the composite solitons were solely in the (x, z) plane, the emerging four soliton pairs do not lie in that plane anymore, but rather in a plane that is tilted clockwise with respect to (x, z) by an angle of 5.7° in the direction of the corotating topological charges. This tilt of the plane of propagation is contrary to the $\theta > \theta_c$ case in which the emerging soliton paths are *always* in the (x, z) plane.

Before closing, we revisit the particlelike quantities that characterize the solitons: power E_j of each component and total angular momentum L. We compute E_j and L in a local window around each soliton in each case discussed above. Consider, for example, the collision of Fig. 4. Before the collision takes place (up to $z = 10$), E_j and L decrease very slowly at a rate of 0.003% per diffraction length (set up by our numerical accuracy). In the second stage, during which the solitons strongly overlap and interact, E_j and L decrease at a higher rate (this is where radiation modes are excited) of 0.05% per diffraction length. After the collision has ended, only the inner spiraling dipole and its ψ_0 companion continue to lose power and angular momentum at almost the same rate as before the collision. The long term prospect of evolution of the inner solitons is that the dipole eventually coalesces to a circular shape and both components occupy the lowest mode of their jointly induced waveguide. It seems that this three-stage behavior is characteristic of all such interactions of composite solitons.

In conclusion, we presented fundamentally new phenomena that occur upon collision between (2 + 1)D composite solitons: delayed action collision and spin-orbit interaction. The implications to other fields of science are fascinating. For example, multicomponent Bose-Einstein condensates [22] are described by Gross-Pitaevskii equations that are almost identical to the NLS-type equations describing composite optical solitons.

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[19] The dimensional angle θ can be obtained from $\tan\theta = (\frac{2\Delta n_0}{n})^{1/2} \tan\vartheta$, where Δn_0 is the maximum physical index change. Typically, $\Delta n_0/n$ is of the order of 2×10^{-4} for 10 μm FWHM solitons at $\lambda \approx 0.5$ μm. The (x, y) coordinates are scaled by $\frac{\lambda}{2\pi}(\frac{n}{2\Delta n_0})^{1/2}$.

[20] The value of θ_c is slightly different for scalar and vector solitons, since the nonlinearity necessary to generate a scalar soliton is lower than that for a vector soliton.

[21] This delayed-action interaction might be related to a phenomenon called "excitation of soliton internal modes" [see Y. S. Kivshar, D. E. Pelinovsky, T. Cretegny, and M. Peyrard, Phys. Rev. Lett. **80**, 5032 (1998)], which has been predicted to occur for 1D kinks in the sine-Gordon equation and for a coupled set of discrete NLS. The internal modes of a soliton, however, do *not* lead to actual delayed-action phenomena but rather to large persistent oscillations of the soliton amplitudes.

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