Cutoff solitons in axially uniform systems

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The optical response of axially uniform nonlinear photonic bandgap fibers is studied theoretically. We observe gap-soliton-like generation and associated bistability, similar to what is typically found in periodically modulated nonlinear structures. This response stems from the nature of the guided-mode dispersion relations, which involve a frequency cutoff at zero wave vector. In such systems, solutions with zero group velocities and minimal coupling to radiation modes come in naturally. We term such solitons "cutoff solitons"; they provide an interesting alternative to gap solitons in periodically index-modulated fibers for in-fiber all-optical signal processing.

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Gap solitons and optical switching[1–5] have been extensively studied in nonlinear dielectric structures with axial periodicity in their linear refractive properties. Corresponding experimental realizations include fiber Bragg gratings[6,7] and integrated multilayer heterostructures.[8] Such systems exhibit spectral gaps of high reflectivity for wave propagation along the axial direction. For intense light illumination at a frequency inside the gap they exhibit solitary wave solutions, called gap solitons, and introduce a strong power dependence to the transmissivity, in some cases resulting in a bistable response. Such systems are attractive for all-optical switching, logic-gate operation, memory, etc.

Because of the necessity of an axial spectral gap for their existence, gap solitons have always been studied in axially periodic systems. In this Letter we show, for what is believed to be the first time, that similar complex behavior is also possible in axially uniform photonic bandgap (PBG) fibers.[9,10] We show that in the presence of an optical Kerr-type nonlinearity, axially uniform PBG fibers exhibit gap-soliton-like generation and associated bistability. This nonlinear response is a direct consequence of the particular guided-mode dispersion relation, which involves a frequency cutoff at $k = 0$: tuning the input frequency below the cutoff provides for an effective axial spectral gap. This is particularly true because there are minimal radiation losses (we operate far from the light line), resulting in a distributed feedback mechanism similar to a Bragg reflector. Also, stationary solutions with very low group velocities (even 0) come in naturally at $k = 0$.[11] We term these solitons "cutoff solitons" to distinguish them from all other gap solitons, which have thus far been described only for axially periodic structures.

We study a two-dimensional (2D) embodiment of PBG fibers, described in Fig. 1(a). This system captures the most essential features of the three-dimensional (3D) fiber, including the guided-mode cutoff at $k = 0$ and the absence of a complete spectral gap. In Fig. 1(b) we plot the guided mode dispersion relation in the linear (low-intensity) limit, as calculated by the finite-difference time-domain (FDTD) method. Any small change in linear refractive index will result in an almost constant frequency shift of the dispersion relations.

For input frequencies below the cutoff $\omega < \omega_c$ there are no available guided states in the core. Assume an input port consisting of a similar fiber with a higher-index core n' and cutoff frequency $\omega_c{}' < \omega_c$. Low-intensity incident guided waves with $\omega_c{}' < \omega < \omega_c$ will decay exponentially in the axial direction and result in strong reflection. This is similar to waves that are incident upon a Bragg reflector and is in contrast to all other axially uniform fibers. At high input power we observe a wide range of nonlinear phenomena, such as bistability and self-pulsing, similar to those found in nonlinear axially periodic gratings. We study these

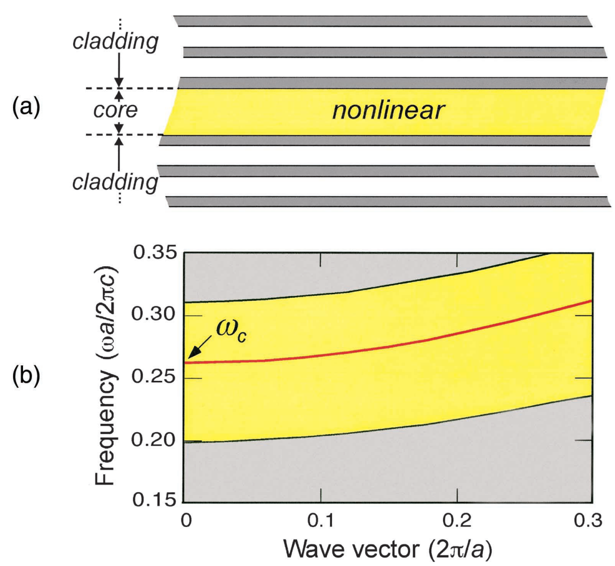

Fig. 1. (a) 2D simulation system: the cladding consists of alternating dielectric layers of high- (2.8) and low- (1.5) index with thicknesses of $0.3a$ and $0.7a$, where a is the period. The core diameter is $d = 1.2a$, and index $n' = n_0 + n_2|E|^2$, where $n_0 = 1.6$. (b) Linear dispersion relations calculated by the FDTD method. The cutoff frequency is $\omega_c = 0.26215$. The gray areas represent both the cladding and radiation modes.

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phenomena within the limit of small nonlinearities (which is the experimentally correct limit).

To explore these effects in detail, we perform time-domain simulations for the system shown in Fig. 1(a), using the FDTD method. Any choice of refractive indices and structural parameters for the linear input and output fibers suffices as long as $\omega_c' < \omega_c$. For example, enlarging the core area would have the desired effect. For simplicity we just use a higher-index core of $n' = 1.9$. In Fig. 2 we plot the fiber's nonlinear response for two nonlinear-core lengths, $L = 5a$ and $L = 8a$. A cw excitation with $\omega_c' < \omega < \omega_c$ is the input for both cases. Fields and flux are monitored at the output while perfectly matched layer absorbing boundary conditions are used to simulate perfect absorption at the edges of the computational cell.

We obtain two types of response: (i) cw, observed in the smaller $L = 5a$ system, involving bistable switching between a cw low- and a cw high-transmission state [Figs. 2(a) and 2(b)], and (ii) pulsating, observed in the larger $L = 8a$ system, involving bistable switching between a stable low- and a pulsating high-transmission state [Figs. 2(c) and 2(d)]. Such a dual response is well known in grated nonlinear systems. The steady state is the result of the excitation of a stationary gap-soliton-like object in the structure. In Fig. 3(b) we plot the field intensity profile along the nonlinear core for the resonant transmission point. Although a similar output response could also be obtained in a simple nonlinear slab,[12] there are important differences. In a nonlinear slab low-intensity propagation is allowed, nonlinear feedback occurs entirely at the boundaries, and bistability results from an intensity-dependent frequency shift of the resonant transmission condition. In our case low-intensity propagation is prohibited [as shown in Fig. 3(b)], and thus nonlinear feedback is axially distributed throughout the nonlinear fiber. This is achieved because we operate below cutoff while having minimal coupling to radiation modes (possible at $k = 0$), which is equivalent to operating in a gap. Bistability results from gap-soliton-like excitations, which share similar physics with gap solitons in nonlinear Bragg gratings. Since they appear in an axially uniform system, however, we term them cutoff solitons to distinguish them.

The transition from a steady solution to a pulsating one is related to the cutoff soliton's becoming nonstationary.[5] The excited cutoff soliton then propagates along the core resulting in an output that consists of a periodic series of pulses. This alternative (convective) method of energy transfer, nevertheless, also results in high transmission. Overall, our axially uniform nonlinear system responds similarly to a nonlinear Bragg grating.

The key feature common to our system and a Bragg grating is the cutoff, or gap, of the dispersion relations, found only at $k = 0$ in our case. We create a one-dimensional (1D) model in which the dispersion relations are fit to quadratic forms that include the nonlinear shift $\delta\omega$:

$$\omega(|E|^2, k) = \omega_c + \alpha k^2 + \delta\omega(|E|^2),$$

$$\omega'(k) = \omega_c' + \alpha' k^2 \tag{1}$$

for the nonlinear and linear core regions, respectively. We write the field in the nonlinear core as the product of a normalized transverse distribution $F(x, y)$ and a slowly varying amplitude $A(z, t)$, $\mathbf{E}(r, t) = 1/2\{\mathbf{F}(x, y)[A(z, t)\exp(ik_0 z)]\exp(-i\omega_0 t) + \text{c.c.}\}$, where z is the axial direction. From first-order perturbation theory in small nonlinearities, we find[13–15]

$$\delta\omega(z) = -\frac{\omega_0}{4}\frac{\int n_2(x,y)n_0(x,y)(|\mathbf{F}\cdot\mathbf{F}|^2 + 2|\mathbf{F}|^4)\mathrm{d}x\mathrm{d}y}{\int n_0(x,y)^2|\mathbf{F}|^2\mathrm{d}x\mathrm{d}y} \times |A(z)|^2 \equiv -\gamma|A(z)|^2, \tag{2}$$

where the integral is performed over the total cross-sectional area. Equation (2) is the general 3D expression. The nonlinear coefficient γ is calculated with the FDTD method as $\gamma \cong 0.02 n_0 n_2|_{\max}$ at $\omega_0 = \omega_c$. It is a weakly increasing function

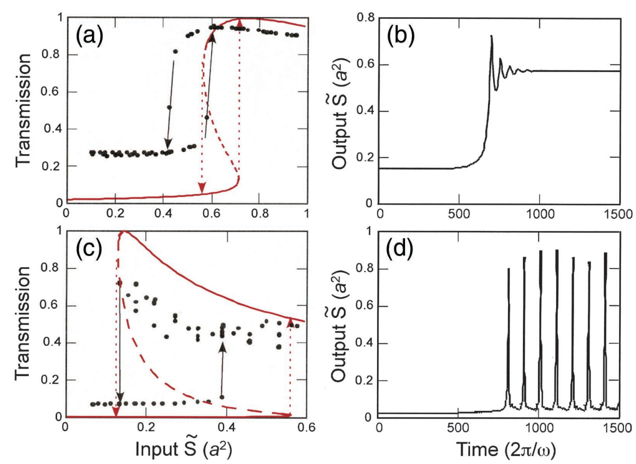

Fig. 2. (a) Transmission versus input flux $\tilde{S}$ for the $L = 5a$ system and $\omega a/2\pi c = 0.26$. The normalized flux $\tilde{S}$ is defined as $\tilde{S} = n_0 n_2 S$, where S is the electromagnetic flux through the fiber's cross section. Both 2D FDTD (filled circles) and the 1D model (solid and dashed curves for the stable and unstable solutions, respectively) results are shown. The unstable solutions (physically unobservable) of the 1D model are represented by the dotted lines. (b) Output flux during switch-up (path marked by the upward-pointing arrow) for the $L = 5a$ system. (c) and (d) Same as (a) and (b) but for the $L = 8a$ system.

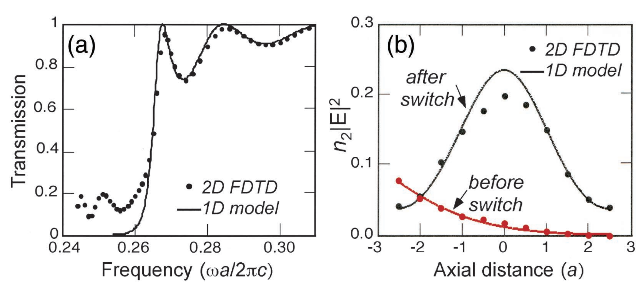

Fig. 3. (a) Linear transmission coefficient versus frequency for the $L = 5a$ system. (b) Normalized intensity (or local index change $\delta n = n_2|E|^2$) along the nonlinear core for the $L = 5a$ system.

of frequency, but for simplicity we assume it to be constant. The other parameters used are $\omega_c = 0.26215(2\pi c/a)[\omega_c' = 0.244(2\pi c/a)]$ and $\alpha = 0.564(ac/2\pi)[\alpha' = 0.463(ac/2\pi)]$.

We first calculate the predicted frequency dependence of the linear transmission coefficient for the $L = 5a$ system, obtained via a transfer matrix solution of the 1D model. This is plotted in Fig. 3(a) along with the full 2D FDTD data. Excellent agreement is found above the cutoff. Note that the 1D model does not show structure below the cutoff, because it ignores the small coupling with the cladding modes [gray areas in Fig. 1(b)]. This coupling could be further suppressed by reducing the index contrast between the different segments.

In Figs. 2(a) and 2(c) we plot the high-intensity predictions of the 1D model along with the full 2D nonlinear FDTD data. Considering the simplicity of the 1D model, the general agreement is surprisingly good. The differences in switching intensities can be attributed to neglecting the frequency dependence of γ, neglecting second-order corrections in γ, and to nonzero contributions from cladding modes. In Fig. 3(b) we plot the intensity along the nonlinear core for the $L = 5a$ system at two different points: at the peak of the upper transmission branch where a cutoff soliton has been excited and at the lower branch where the wave decays exponentially; the 1D model captures all the essential features of the 2D system's nonlinear response.

We next derive an analytic expression for slowly varying amplitude A. In the frequency domain the equation for $\tilde{A}(z, \omega - \omega_c)$ is $\partial^2\tilde{A}/\partial_z{}^2 + k^2\tilde{A} = 0$, where we expand around cutoff frequency ω_c, and k contains the nonlinear index change (the first-order terms vanish at cutoff). With the dispersion relations of Eqs. (1), $k^2 = (\omega - \omega_c - \delta\omega)/\alpha$ and transforming the slowly varying amplitude equation back into the time domain we arrive at the nonlinear Schrödinger equation[14]

$$\frac{\partial A}{\partial t} - i\alpha\frac{\partial^2 A}{\partial z^2} - i\gamma|A|^2 A = 0\,. \tag{4}$$

This is the same equation as the one derived for 1D nonlinear periodic systems[3] (with α as the curvature of the band edge). The main similarities between the two systems are the form of the dispersion relations and the existence of a gap. The latter is the marked difference between cutoff solitons and the usual nonlinear fiber solitons, along with that cutoff solitons can have any arbitrary group velocity, even zero. Any nonlinear system with similar dispersion relations is thus well described by Eq. (4). Such systems include metallic waveguides, multilayer stacks, PBG fibers, and photonic crystal linear-defect waveguides.

In conclusion, we have studied an axially uniform nonlinear system that exhibits gap-soliton-like formation and associated switching. The most important quality of this system is the special nature of the dispersion relations of the guided modes, involving a frequency cutoff at zero wave vector. In practical experimental setups these solitons will physically behave, in the vast majority of cases, the same as gap solitons. We term these solitons cutoff solitons to distinguish them from the usual gap solitons appearing in axially periodic systems. The ability to obtain gap-soliton-like formation without imposing axial periodicity leads to new design and fabrication opportunities for eventual experimental realization of all-optical devices.

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