Bright Spatial Solitons on a Partially Incoherent Background

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We present the first observation of incoherent antidark spatial solitons in noninstantaneous nonlinear media. This new class of soliton states involves bright solitons on a partially incoherent background of infinite extent. In the case where the nonlinearity is of the Kerr type, their existence is demonstrated analytically by means of an exact solution. Computer simulations and experiments indicate that these incoherent antidark solitons can propagate in a stable fashion provided that the spatial coherence of their background is reduced below the incoherent modulation instability threshold.

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Over the years, the very idea of a bright optical soliton that can exist on top of a nonvanishing background (antidark soliton) has been a topic of considerable interest [1–3]. Yet, the stability of antidark solitons has always been a critical issue. Thus far, antidark solitons were found to propagate in a stable fashion in physical systems, only when higher-order effects or nontrivial nonlinearities are involved [1,2]. Typically, however, these latter conditions are quite specific and rather difficult to meet. In the simple case where the underlying evolution equation is of the nonlinear Schrödinger type, the instability is triggered by the constant background (modulation instability) as a result of the self-focusing nonlinearity required to confine the bright part of this soliton beam.

From a different direction, the rapid progress in the area of incoherent solitons [4–12] has opened up a host of new possibilities that have no counterpart whatsoever in the coherent limit. These include, for example, gray fundamental dark solitons and associated phase memory effects [7–10], the existence of multimode asymmetric incoherent solitons [11], and soliton shape transformations upon collisions [12], just to mention a few. Along these lines, it is important to note that antidark vector solitons are also possible in a self-focusing environment as suggested in Ref. [13]. These involve a (coherent) dark soliton component incoherently coupled to a bright one. This idea was recently generalized in the case where the number of modes composing such a vector soliton is more than two [14]. Unfortunately, however, the background of these antidark vector states is again modulationally unstable [13], and, in fact, this instability was experimentally observed in biased photorefractive crystals [15]. Recently, however, it was shown theoretically that modulation instability (MI) can be totally suppressed provided that the degree of incoherence of the constant wave front involved exceeds a certain threshold [16]. The existence of a threshold for incoherent MI is a feature unique to incoherent systems, since such a threshold is totally absent in the coherent regime. In view of this latter result, it is natural to ask whether stable antidark solitons can be made possible by employing coherence control.

In this Letter we show both theoretically and experimentally that stable partially coherent antidark solitons can exist in noninstantaneous nonlinear media. The modal structure of these antidark states is analyzed in detail in the case where the nonlinearity is of the Kerr type. We find that these solitons *always* involve a set of discrete bound states as well as a continuum of odd and even radiation modes. Experimental results and computer simulations indicate that the instability affecting an antidark soliton can be totally eliminated by properly increasing the incoherence of its background beam above the threshold of incoherent MI. These antidark solitons are the only known solitons that have nonzero intensity everywhere in space yet they propagate in a stable fashion in a self-focusing medium.

To demonstrate the existence of such partially incoherent antidark solitons, let us assume, for simplicity, that the self-focusing nonlinearity is of the Kerr type. In this case, the refractive index varies linearly with the optical intensity I, i.e., $n^2 = n_0^2 + n_2 I$. Here n_0 is the linear part of the refractive index and n_2 is the nonlinear Kerr coefficient. As usual, we assume that the response time of the nonlinear material is much longer than the characteristic fluctuation time across the incoherent beam so as to avoid any speckle-induced filamentation instability. As a result,

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the material will see only the time averaged beam intensity. Thus the field envelope U of a planar incoherent beam propagating in this material evolves according to

$$i\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U}{\partial s^2} + \left(\frac{k_0^2 x_0^2 n_2 I}{2}\right)U = 0, \qquad (1)$$

where $k_0 = k/n_0$ is the free space wave vector and x_0 is associated with the spatial extent of the beam. In obtaining this equation, we have used normalized coordinates, i.e., $s = x/x_0$ and $\xi = z/kx_0^2$. Moreover, let us assume that the time averaged intensity profile of this antidark incoherent soliton is given by

$$I = I_0[1 + \varepsilon^2 \text{sech}^2(s)], \qquad (2)$$

where $I_0\varepsilon^2$ defines the peak intensity of the antidark soliton and I_0 is the background intensity of beam. By following a procedure similar to that outlined in Ref. [9], one can easily show that this new soliton structure involves a continuum of radiation modes as well as bound states. In general, the number of bound modes involved depends on initial parameters. To begin with, let us first assume that the soliton-induced waveguide can support only one bound mode. This requires that $x_0^2 = 2/[\varepsilon^2 k_0^2 n_2 I_0]$. In this case, the bound and the continuum radiation modes are given by

$$U_b = \text{sech}(s), \qquad (3)$$

$$U_{re} = Q\cos(Qs) - \tanh(s)\sin(Qs),$$
$$U_{ro} = Q\sin(Qs) + \tanh(s)\cos(Qs).$$

Note that the strength of each mode involved $(c_b, \tilde{c}_{re,ro})$ varies randomly in time and the statistical cross correlation between different modes is zero [9]. Here, c_b represents the instantaneous occupancy of bound states, whereas $\tilde{c}_{re,ro}$ represents that of even and odd radiation modes. Furthermore, the radiation mode correlation function satisfies $\langle \tilde{c}_{ro,e}(Q)\tilde{c}^*_{ro,e}(Q')\rangle = D(Q)\delta(Q - Q')$ where $D(Q)$ is the radiation mode distribution function. Let us now assume for simplicity that $D(Q)$ is Boltzmann-like, i.e., $D(Q) = D_0\exp(-Q/Q_0)$ where Q_0 represents the width of this distribution. Under these conditions, it can then be shown that self-consistency leads to $\langle |c_b|^2\rangle = [I_0/(2Q_0^2 + 1)][1 + \varepsilon^2(2Q_0^2 + 1)]$ and $D_0 = I_0/[Q_0(2Q_0^2 + 1)]$. These last results lead to the following important conclusions. First, unlike the case of incoherent dark solitons [9], there is no upper limit as to the incoherence (Q_0) of the background beam associated with these antidark states. Moreover, the required nonlinear index change $(n_2 I_0)$ to support these solitons depends only on the spatial width x_0 and the contrast parameter ε^2, whereas it is independent of the degree of coherence of the background. Therefore, as we will see later, one can increase at will the incoherence of the background component of the antidark soliton so as to totally eliminate modulation instability. In addition, in this case, the bound state is always populated (since $\langle |c_b|^2\rangle > 0$) which implies that a first-order antidark soliton is primarily dominated by even modes. Therefore, in contrast to dark incoherent solitons, no initial phase manipulation is required to launch these states. Figure 1(a) shows the intensity profile of such an incoherent antidark soliton when $\varepsilon^2 = 2$ and $I_0 = 1$ a.u. Figure 1(b) depicts the correlation length of this incoherent antidark soliton when $Q_0 = 0.9$ and $x_0 \simeq 5.6$ μm (intensity FWHM is 10 μm). From this figure it is clear that the coherence length structure of such an antidark soliton is different from that associated with its incoherent dark and bright soliton counterparts [6,7,9]. In the antidark soliton case, the coherence increases at the center in a bell-like fashion as one may expect since this region is dominated by the bound mode.

Similarly, higher-order antidark solitons can be obtained by adjusting the beam parameters and the strength of nonlinearity. For example, if $x_0^2 = 6/[\varepsilon^2 k_0^2 n_2 I_0]$, the soliton-induced waveguide can support two bound modes. In this case, the two bound states and the radiation modes are given by

$$U_{b1} = \text{sech}^2(s), \qquad U_{b2} = \text{sech}(s)\tanh(s),$$
$$U_{r,e} = [1 + Q^2 - 3\tanh^2(s)]\cos(Qs) - 3Q\tanh(s)\sin(Qs), \qquad (4)$$
$$U_{r,o} = [1 + Q^2 - 3\tanh^2(s)]\sin(Qs) + 3Q\tanh(s)\cos(Qs).$$

Again, from self-consistency requirements one quickly finds the following results: $\langle |c_{b1}|^2\rangle = Q_0 D_0[(3 + 6Q_0^2) + 2\varepsilon^2(12Q_0^4 + 5Q_0^2 + 2)]$, $\langle |c_{b2}|^2\rangle = 2Q_0 D_0[(6 + 3Q_0^2) + \varepsilon^2(12Q_0^4 + 5Q_0^2 + 2)]$, and $D_0 = (I_0/2Q_0)(12Q_0^4 + 5Q_0^2 + 2)^{-1}$. As in the single mode case, the bound states are always populated. Figure 1(c) demonstrates the correlation length of such a second-order incoherent antidark soliton for the same parameters used before; i.e., $Q_0 = 0.9$, $\varepsilon^2 = 2$, and $x_0 \simeq 5.56$ μm while its intensity profile is the same as Fig. 1(a). The intensity profiles of the modes involved (bound modes and radiation mode belt) are shown in Fig. 2. Note that for this set of parameters the required nonlinear index change is 3 times higher than that of the first-order antidark soliton. Third-order, fourth-order, etc., partially incoherent antidark solitons can be similarly obtained for $k_0^2 x_0^2 \varepsilon^2 n_2 I_0 = 12, 20, \ldots$ [9].

In the special limit where $Q_0 \to 0$, the radiation mode belt collapses into a single radiation or background mode

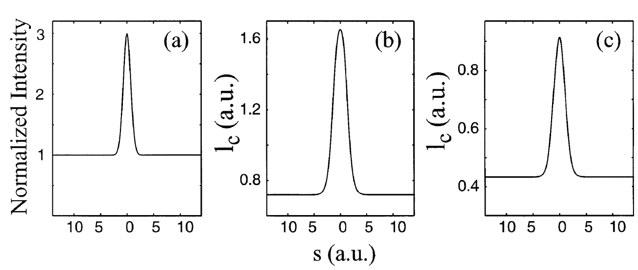

FIG. 1. (a) Intensity profile and correlation length curves of (b) a first-order and (c) a second-order incoherent antidark soliton when $x_0 \simeq 5.56$ μm, $I_0 = 1$, $\varepsilon^2 = 2$, and $Q_0 = 0.9$.

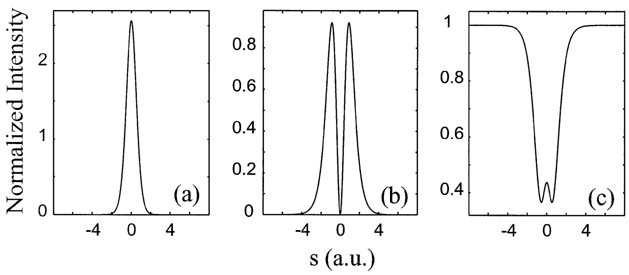

FIG. 2. Intensity profiles of (a) first bound mode, (b) second bound mode, and (c) total intensity of radiation mode belt in a second-order incoherent antidark soliton when $I_0 = 1$, $\varepsilon^2 = 2$, and $Q_0 = 0.9$. The total intensity of this antidark soliton is identical to that in Fig. 1(a).

(odd or even depending on the number of bound states) and as a result the background part of the antidark soliton becomes fully coherent [17]. In fact, this limiting case corresponds to the results found in Ref. [14]. Figure 3(a) shows the evolution of such a second order antidark soliton resting on a coherent background ($Q_0 = 0$) when again $\varepsilon^2 = 2$, $I_0 = 1$, $x_0 \simeq 5.6$ μm, and $n_2 I_0 = 6.1552 \times 10^{-4}$. This figure was obtained by solving the associated vector evolution equations in a Kerr system with the input modes given by Eqs. (4) in the limit $(Q_0, Q) \rightarrow 0$. Clearly after a certain distance ($\sim$0.5 cm) this solution disintegrates as a result of modulation instability, since the background is fully coherent. This instability was observed experimentally in the case of a first-order antidark vector soliton in a self-focusing environment [15].

On the other hand, by making the background partially incoherent this instability can be totally eliminated. For example, Fig. 3(b) shows the stable propagation of an antidark incoherent soliton when the initial background coherence length is $l_c \simeq 5.5$ μm. The evolution of this antidark beam launched from a stationary sounce is simulated using the coherent density approach [5,7] in a Kerr environment. The parameters used in this simulation are $x_0 \simeq 5.6$ μm, $I_0 = 1$ a.u., $\varepsilon^2 = 2$, $n_2 I_0 = 6.1552 \times 10^{-4}$, $n_0 = 2.3$, and $\lambda_0 = 0.5$ μm. The width of the Gaussian power spectrum used in this simulation was $\theta_0 = 15.7$ mrad. As one can see, the beam eventually settles into a partially coherent antidark soliton. Even more importantly, this antidark soliton is stable, i.e., the modulation instability has been totally eliminated for this degree of incoherence. As previously found [16], the modulation instability of the background can be avoided if $\theta_0^2 \gtrsim n_2 I_0/[2\ln(2) n_0^2]$ which is the case for the parameters used in the above example. Similar results were also found with saturable nonlinearities as in the case of biased photorefractives [18]. The fundamental difference between coherent MI and incoherent MI can be understood intuitively in the following manner. A small periodic perturbation on a coherent beam remains periodic and maintains its modulation depth during linear diffraction, thus any nonlinearity, small as it may be, increases the modulation depth and leads to instability. This is why coherent MI has no threshold. On the other hand, a perturbation upon an incoherent beam diminishes its modulation depth during linear diffraction. The nonlinearity has to overcome this "washout" effect in order to lead to instability. This is why incoherent MI requires a threshold for its existence. In our simulations, we have also found that the correlation length of the antidark soliton of Fig. 3(b) becomes asymptotically similar to that of Figs. 1(b) and 1(c).

In parallel with the above theoretical and numerical studies, we performed a set of experiments in order to observe incoherent antidark solitons. The nonlinear material used is a photorefractive strontium barium niobate crystal (with an effective electro-optical coefficient of 260 pm/V), which provides a saturable noninstantaneous nonlinearity necessary to observe incoherent solitons [4,8]. The incoherent light source was obtained by passing a laser beam through a diffuser rotating much faster than the response time of the nonlinearity. This provides a partially incoherent background beam with controllable degree of coherence [4,8]. The experimental setup is similar to that employed for bright incoherent solitons [4], with the exception that now both the soliton and the background beam are extraordinarily polarized, and the background beam is broad and partially spatially incoherent. No additional ordinarily polarized background beam was used for dark illumination [4,8]. Typical experimental results are presented in Fig. 4 for two different degrees of spatial coherence. In both cases, the central bright beam at the input had a FWHM of 18 μm, and the ratio between the

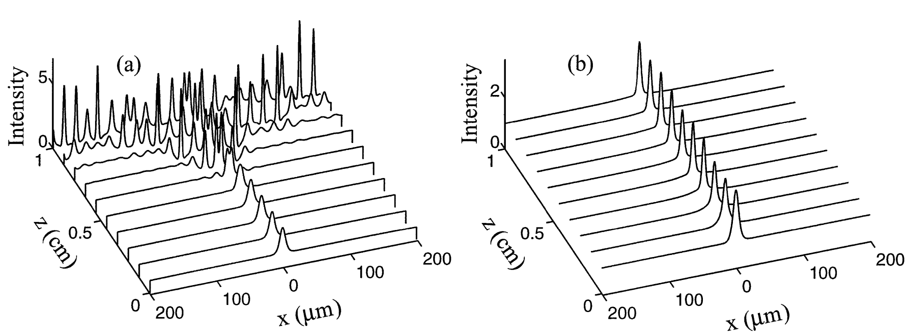

FIG. 3. (a) Propagation of an unstable multicomponent antidark soliton when the background is coherent. (b) Intensity evolution of a partially incoherent antidark soliton when initially $l_c \simeq 5.5$ μm. For both cases, the initial intensity FWHM of the beam is 10 μm, $\varepsilon^2 = 2$, $n_2 I_0 = 6.1552 \times 10^{-4}$, $n_0 = 2.3$, and $\lambda_0 = 0.5$ μm.

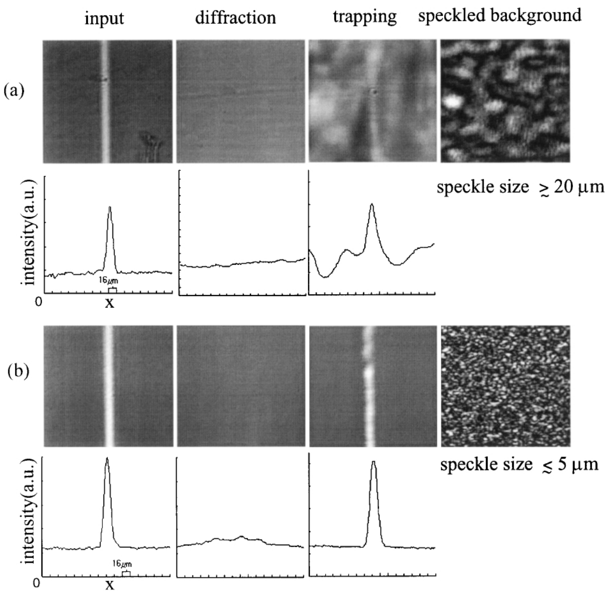

FIG. 4. Experimental observation of (a) an unstable and (b) a stable partially coherent antidark soliton when their intensity FWHM is 18 μm and the applied bias field is 1320 V. The first column depicts the input intensity, the second column shows the output diffracted beam, and the third shows the output self-trapped beam. The fourth column shows the speckled structure of the beam.

peak intensity and the background intensity was about 4. Since the crystal is 20 mm long, the diffracted beam at the exit face of the crystal is almost invisible without changing the attenuation for the camera. Figure 4(a) shows the unstable trapping of an antidark beam, when the speckle size of the background (associated with the coherence length) is at least 20 μm. For such a partially incoherent wave front, the nonlinearity is well above the threshold for incoherent modulation instability, and therefore stable trapping cannot be realized. But, as we reduce the coherence length of the background beam down to about 5 μm, stable trapping of antidark solitons is achieved [Fig. 4(b)]. In both cases, the trapping voltage we used is 1320 V over the 5.6 mm wide crystal. Applying a higher voltage while keeping the coherence of the beam fixed leads to modulation instability that prevents stable trapping. These experimental results are in good agreement with the numerical simulations of Fig. 3. This is the first demonstration of stable antidark spatial solitons in nonlinear media. Moreover, it is a proof of concept that incoherent modulation instability occurs only when the nonlinearity exceeds a certain threshold, as predicted very recently [16].

In conclusion, we have demonstrated both analytically and experimentally that *stable* partially incoherent antidark solitons are possible in noninstantaneous nonlinear media provided that the degree of spatial incoherence of the soliton background is such that modulation instability is suppressed. This new class of solitons always involves bound states as well as radiation modes. Our experimental results are in agreement with the predicted theoretical behavior.

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