Eliminating the Transverse Instabilities of Kerr Solitons

Charalambos Anastassiou,[1,2] Marin Soljačić,[1,3] Mordechai Segev,[1,2] Eugenia D. Eugenieva,[4]
Demetrios N. Christodoulides,[4] Detlef Kip,[1,5] Ziad H. Musslimani,[6] and Juan P. Torres[1,7]
[1]*Physics Department and the Solid State Institute, Technion, Haifa 32000, Israel*
[2]*Electrical Engineering Department, Princeton University, Princeton, New Jersey 08544*
[3]*Physics Department, Princeton University, Princeton, New Jersey 08544*
[4]*Electrical Engineering and Computer Science Department, Lehigh University, Bethlehem, Pennsylvania 18015*
[5]*Physics Department, Universität Osnabrück, 49069 Osnabrück, Germany*
[6]*Mathematics Department, Technion, Haifa 32000, Israel*
[7]*Department of Signal Theory and Communications, Polytechnic University of Catalonia, Jordi Girona 1-3, Campus Nord D3, 08034 Barcelona, Spain*
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We show analytically, numerically, and experimentally that a transversely stable one-dimensional $[(1 + 1)D]$ bright Kerr soliton can exist in a 3D bulk medium. The transverse instability of the soliton is completely eliminated if it is made sufficiently incoherent along the transverse dimension. We derive a criterion for the threshold of transverse instability that links the nonlinearity to the largest transverse correlation distance for which the 1D soliton is stable.

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Research on optical spatial solitons has made much progress during the past decade: new systems that support solitons have been identified, solitons of more than one transverse dimension have been demonstrated, and a whole range of soliton interactions was explored [1]. Despite the diversity of the physical systems that support them, solitons are a universal phenomenon and share many common features [1], one of which is transverse instability (TI) [2–10]. TI is a symmetry breaking instability: almost all solitons [11] of a particular dimension that propagate in a higher dimension system [by having a uniform wave function in the additional dimension(s)] are unstable to perturbations in the dimension(s) in which they are uniform. TI occurs because perturbations in the dimension of uniformity have nothing to restrain them from growing (driven by the nonlinearity) and breaking the soliton up.

In the particular case of a spatial optical $(1 + 1)D$ soliton that is self-trapped in one dimension x, is uniform in the transverse dimension y, and is propagating along z, TI causes the soliton to break up along y into an array of 2D filaments [2–10]. The transverse wavelength of these perturbations is usually much larger than the soliton width [2–4]. Transverse instability is especially severe for Kerr nonlinearities and prohibits spatial 1D Kerr solitons in a bulk medium. This is why spatial $(1 + 1)D$ Kerr solitons have to be launched in a planar waveguide configuration, in which the y confinement is much narrower than the self-trapped (soliton) width in x [5,6]. TI actually occurs for solitons in any nonlinearity, including, for example, quadratic solitons [7] and photorefractive solitons [8,9]. Interestingly, saturation arrests transverse instability [10] but never completely eliminates it. In fact, it is the suppression of TI due to saturation that facilitates the observation of stable 1D solitons in a bulk photorefractive crystal for more than ten diffraction lengths [8].

Thus far, in order to avoid TI, experiments with 1D solitons were conducted in either planar waveguides [5,6] or nonlinearities in which TI was greatly suppressed [8]. Here we demonstrate how to produce a truly stable stripe Kerr soliton propagating in a 3D bulk medium without suffering from transverse instability. We show that if the soliton is made "sufficiently" incoherent in its transverse dimension y, then TI is completely eliminated.

First, recall incoherent solitons made of partially incoherent light [12]. They are multimode (speckled) beams of which the instantaneous amplitude varies randomly with time. If such beams are launched into a noninstantaneous self-focusing medium, so that the response time of the nonlinearity greatly exceeds the fluctuation time, then self-focusing is driven solely by the average intensity. Then, the incoherent beam induces a multimode waveguide and guides itself in it by properly populating the guided modes, thus forming an incoherent soliton [12–18].

A clue that TI could be completely eliminated for solitons was given by two recent discoveries: modulation instability (MI) of incoherent light [19] and elliptical incoherent solitons [14,20]. MI belongs to the same family of symmetry breaking instabilities as TI does, and it occurs when a plane wave (or a very broad beam or pulse) is launched into a self-focusing medium. If this plane wave is fully coherent, it breaks up into a train of filaments due to MI. Recently, it has been shown theoretically and experimentally [20] that MI does exist also for incoherent light, but it occurs only if the nonlinearity exceeds a well-defined threshold. The MI threshold is determined by the coherence of the light. If the nonlinearity is below threshold, then MI is eliminated and the wave is stable. This generic idea has enabled the observation of antidark solitons [21], which were thought to be unstable in conservative nonlinear systems [22]. The new finding of

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incoherent elliptical solitons [14,20] is based on having different coherence function for the two transverse dimensions of self-trapping. Combing these ideas, one can generate a 1D soliton that is fully coherent in x (direction of trapping), partially incoherent but uniform in y, and propagating along z. The intimate relation between TI and MI suggests that TI of incoherent beams should also exhibit a threshold for its existence. Therefore, if the degree of coherence in y is such that TI is below the threshold, all transverse perturbations are suppressed and TI is eliminated. This is the core idea of our Letter. The idea of using the threshold to eliminate TI applies to any type of nonlinearity, yet we will concentrate on the Kerr nonlinearity for two reasons. First, wave propagation in Kerr media is described by the cubic nonlinear Schrödinger equation (NLSE) which is one of the most general soliton equations [23]. Generally speaking, the NLSE describes envelope solitons in dispersive wave systems with weak symmetric anharmonicity. Second, the effect of TI for Kerr solitons is very strong and we can demonstrate a convincing difference between having TI and eliminating TI by making the soliton incoherent along y.

An incoherent beam can be represented as a series of coherent speckles that change, on average, every coherence time τ_{coh}. We define $B(x_1, y_1 x_2, y_2, z) = \langle E^*(x_2, y_2, z, t) E(x_1, y_1, z, t)\rangle$, the spatial correlation function, $E(x, y, z, t)$ being the slowly varying amplitude. The $\langle\,\rangle$ denote averaging over the response time of the medium τ, which is much larger than τ_{coh}. From the paraxial wave equation we get [17]

$$\frac{\partial B}{\partial z} - \frac{i}{k}\left\{\frac{\partial^2 B}{\partial x \partial \rho_x} + \frac{\partial^2 B}{\partial y \partial \rho_y}\right\} = \frac{ik}{n_0}\{\delta n(x_1, y_1, z) - \delta n(x_2, y_2, z)\}B\,, \quad (1)$$

where z is the propagation direction, k is the carrier wave number, n_0 is the bias refractive index, δn is the nonlinear contribution to the refractive index, $x = (x_1 + x_2)/2$, $y = (y_1 + y_2)/2$ are the middle point coordinates, and $\rho_x = x_1 - x_2$, $\rho_y = y_1 - y_2$ are the difference coordinates. When $\rho_x = \rho_y = 0$, $B(x, y, \rho_x, \rho_y, z)$ is the time-averaged intensity $I(x, y, z)$. Let $B_S(x, y, \rho_x, \rho_y) = u(x + \rho_x/2)\, u^*(x - \rho_x/2) A_n(\rho_y)$ be a z-independent solution of Eq. (1). It represents a 1D soliton stripe, which is self-trapped and fully coherent in x, while being uniform and incoherent along y with an angular spectrum of $A_n(\rho_y)$. $u(x)$ is determined by the nonlinearity and can be taken to be real without loss of generality. To study TI, we add a small perturbation B_1 to B_S where $B_1 \ll B_S$. The nonlinear index change in Kerr media is $\delta n(I) = \gamma I$, where γ is the nonlinear coefficient (n_2). Linearizing Eq. (1) yields

$$\frac{\partial B_1}{\partial z} - \frac{i}{k}\left\{\frac{\partial^2 B_1}{\partial x \partial \rho_x} + \frac{\partial^2 B_1}{\partial y \partial \rho_y}\right\} = \frac{ik}{n_0}\gamma\left\{\begin{array}{l} [B_1(x + \frac{\rho_x}{2}, y + \frac{\rho_y}{2}, \rho_x = 0, \rho_y = 0, z) - B_1(x - \frac{\rho_x}{2}, y - \frac{\rho_y}{2}, \rho_x = 0, \rho_y = 0, z)]B_S(x, y, \rho_x, \rho_y) \\ + [B_S(x + \frac{\rho_x}{2}, y + \frac{\rho_y}{2}, \rho_x = 0, \rho_y = 0) - B_S(x - \frac{\rho_x}{2}, y - \frac{\rho_y}{2}, \rho_x = 0, \rho_y = 0)]B_1(x, y, \rho_x, \rho_y, z) \end{array}\right\}. \quad (2)$$

We seek solutions in the form $B_1(x, y, \rho_x, \rho_y, z) = \exp(gz)\exp(i\alpha y)L(x, \rho_x)A_f(\rho_y) + \exp(g^* z)\exp(-i\alpha y) \times L^*(x, -\rho_x)A_f^*(-\rho_y)$, where α is the transverse wave number, g is the TI growth rate (gain), and $A_f(\rho_y)$ is the angular spectrum. The necessary condition $B_1(x, y, \rho_x, \rho_y, z) = B_1^*(x, y, -\rho_x, -\rho_y, z)$ is satisfied [17,19]. Substituting B_1 into Eq. (2) gives

$$gL(x, \rho_x)A_f(\rho_y) - \frac{i}{k}\left\{\frac{\partial^2 L(x, \rho_x)}{\partial x \partial \rho_x} A_f(\rho_y) + i\alpha \frac{dA_f(\rho_y)}{d\rho_y} L(x, \rho_x)\right\} = \frac{ik}{n_0}\gamma\left\{\begin{array}{l} [L(x + \frac{\rho_x}{2}, 0)A_f(0)e^{i\alpha\rho_y/2} - L(x - \frac{\rho_x}{2}, 0)A_f(0)e^{-i\alpha\rho_y/2}]u(x + \frac{\rho_x}{2})u(x - \frac{\rho_x}{2})A_n(\rho_y) \\ + [u(x + \rho_x/2)^2 - u(x - \rho_x/2)^2]L(x, \rho_x)A_f(\rho_y)A_n(0) \end{array}\right\}. \quad (3)$$

We are interested in determining the threshold condition: to find the conditions under which the growth rate $g(\alpha)$ goes from a positive value to a negative value for all α. The procedure of determining the threshold applies to any form of spatial coherence (angular power spectrum), but for simplicity, we consider an initial Gaussian angular power spectrum $A_n(\rho_y) = \exp[-(\rho_y \theta_0 k/2)^2]$, where θ_0 defines the degree of coherence (correlation distance). The higher θ_0 the more incoherent the soliton is. For a fully coherent soliton, if we were to calculate the growth rate g as a function of transverse wave number α, then g starts from 0 (at $\alpha = 0$), increases and reaches a maximum positive value (at the wave number with the largest growth rate), and then drops back to 0 at α associated with the "cutoff wavelength" [2–4]. This means that for a coherent soliton, the growth rate is positive (and TI exists) for a band of wave numbers α between zero and the cutoff wave number. For a soliton that is partially coherent in y, we expect that for θ_0 small enough (a beam that is coherent enough), $g(\alpha)$ will be positive in a band of wave numbers, just as the coherent case. But, as θ_0 increases, this band becomes narrower until it completely disappears at some value θ_{0T}. If θ_0 is larger than this value, then TI is eliminated. We therefore expect that, at the threshold $\theta_0 = \theta_{0T}$, the two boundary points at which $g(\alpha) = 0$ (one at $\alpha = 0$ and the other at the cutoff wave number) coincide. Thus, we seek the value of θ_0 at which (i) $g(\alpha = 0) = 0$ and (ii) $g'(\alpha = 0) = 0$. We solve Eq. (3) by expansion while retaining up to O(α). This eliminates the first term on the left-hand side (LHS) because $g = O(\alpha^2)$. The growth rate of the transverse instabilities, g, is independent of x [2–4] even though the actual shape of the perturbations depends on x. We can therefore seek solutions of Eq. (3) under the conditions (i) and (ii) at the center of the soliton, i.e., at $x = \rho_x = 0$, and assume that the threshold we find is the same everywhere on the soliton. It can be easily shown (by expanding into derivatives with respect to x_1 and x_2) that the second term of the LHS is zero for $x_1 = x_2$, i.e., for $\rho_x = 0$. Thus, from Eq. (3) we get

$$\frac{\alpha}{k}\frac{dA_f(\rho_y)}{d\rho_y}L(0,0) = -\frac{kI_0}{n_0}\gamma L(0,0)\alpha\rho_y A_f(0) \times \exp[-(k\theta_{0T}\rho_y/2)^2], \quad (4)$$

where $I_0 = u(0)^2$ is the peak intensity of the soliton. It is unlikely that a small perturbation will alter the coherence statistics of the soliton (especially here that propagation effects, given by g, are of the order of α^2 and are ignored). Thus, we assume that $A_f(\rho_y) = A_n(\rho_y)$. Equation (4) gives the threshold condition $\theta_{0T}^2 = 2\Delta n_0/n_0$, where $\Delta n_0 = \gamma I_0$ is the maximum change in the refractive index. One can actually calculate, using numerical methods similar to [10], the function $g(\alpha)$ and from it obtain the threshold for any angular distribution function [24].

To verify the analytic predictions, we perform simulations using the coherent density approach [12]. We launch a 1D Kerr soliton with a Gaussian angular power spectrum, $A(\rho_y) = \exp[-(\rho_y\theta_0 k/2)^2]$, for various values of θ_0. In this example, $n_0 = 2.3$, $\lambda = 0.5$ μm in vacuum, FWHM = 9 μm, which yields a $\Delta n_0 = 0.000\,105\,6$ and an analytic prediction of the threshold of $\theta_{0T} = 0.55°$. Our results are displayed in Fig. 1, where we show images of the intensity distribution of the soliton and cross sections of the intensity along y for $x = 0$. Figure 1(a) shows the input soliton at $z = 0$, and Fig. 1(b) shows a fully coherent soliton ($\theta_0 = 0°$) after 0.8 cm of propagation. As clearly depicted there, TI breaks the soliton up into a train of 2D filaments. As we approach the threshold, the TI gain is getting smaller: As we set θ_0 to 0.5°, it takes a 4.5 cm propagation to exhibit signs of TI [Fig. 1(c)]. To show that TI is completely eliminated when the nonlinearity is below threshold, we increase θ_0 to 0.56°. As shown in Fig. 1(d), after 4.5 cm of propagation there are absolutely no signs of TI.

Our experiments are conducted in a photorefractive SBN:75 crystal in a setup similar to that of [12]. The beam is made spatially incoherent by passing it through a rotating diffuser. The rotating diffuser provides a new phase and amplitude distribution every $\tau_{\text{coh}} \sim 1$ μs, which is much shorter than the response time of the medium $\tau \sim 1$ s. Unlike all previous experiments with incoherent solitons, here we need to generate a beam which is very narrow and fully coherent in x, yet uniform and partially incoherent y. To do that, we use a cylindrical lens which focuses the beam only in the y direction onto the rotating diffuser. Then, by moving the focal point of this lens closer (farther away) from the diffuser, we increase (decrease) the coherence in y. The x coherence is not affected by the translation of this lens. After the diffuser, the beam is collimated (to ~2 cm) and passed through a narrow (along x) slit. The slit is made narrower than the speckle size in x, and it effectively creates a 1D beam that is narrow and coherent in x and "infinitely" long (uniform) and incoherent in y. The slit is then imaged to the input face of the crystal. We get a reasonable estimate of the correlation distance by stopping the diffuser and measuring the average speckle size at the crystal input plane. Finally, we use an orthogonally polarized background beam that covers the crystal uniformly as necessary for photorefractive screening solitons [8]. The input and output faces of the crystal are imaged onto a CCD camera.

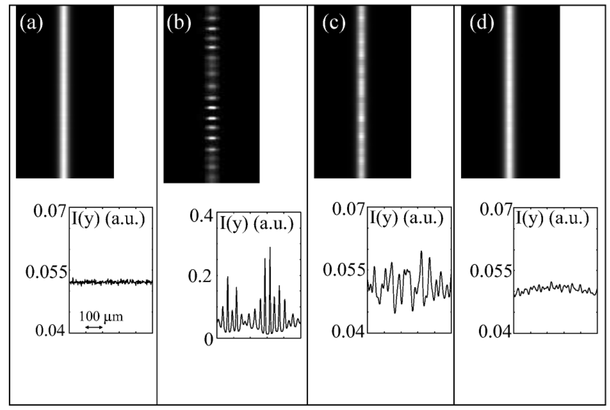

FIG. 1. Simulations of a 1D Kerr soliton with a Gaussian spectrum, for various degrees of coherence. The predicted threshold is $\theta_0 = 0.549°$. (a) Input intensity. (b) Output beam after 0.8 cm of propagation for a fully coherent beam $\theta_0 = 0°$; the soliton is destroyed by TI. (c) Output beam close to the threshold (for $\theta_0 = 0.5°$), after 4.5 cm of propagation: As the threshold is approached, TI gain is reduced and it takes a longer propagation before TI is evident. (d) Output beam for $\theta_0 = 0.56°$, which is below the threshold, after 4.5 cm of propagation. TI is completely eliminated.

The photorefractive nonlinearity is in general saturable but resembles the Kerr nonlinearity when the peak intensity of the soliton normalized to the background intensity is much smaller than unity [8,25]. In our case, this ratio is ~0.1. At this normalized intensity, a soliton that is fully coherent in both x and y exhibits strong transverse instability [8]. We then gradually increase the incoherence in y (decrease the speckle size) until the soliton becomes transversely stable, while keeping all other parameters (applied field, intensity) constant. Our results are shown in Fig. 2. The 12 μm FWHM input beam [Fig. 2(a)] linearly diffracts to a 60 μm output after 6 mm propagation in the crystal [Fig. 2(b)]. The nonlinearity is turned on with the application of 2.7 kV/cm and the beam self-traps forming a soliton in x. When the beam is fully coherent, the soliton suffers from TI and breaks up into filaments [Fig. 2(c)]. When the beam is made incoherent in y, but with a large speckle size (~100 μm), the nonlinearity is still above

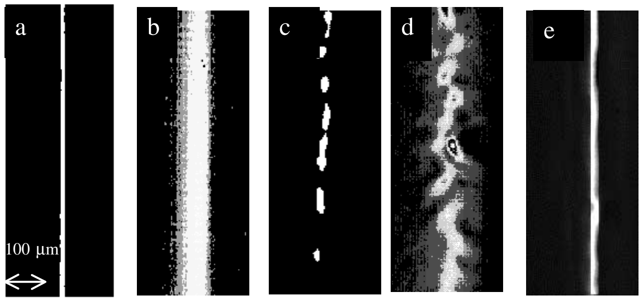

FIG. 2. Experiments in photorefractive SBN:75 in the Kerr regime (intensity ratio ~ 0.1). (a) Input 12 μm FWHM beam. (b) Output beam after 6 mm of linear diffraction when nonlinearity is off. (c), (d), and (e) show the output beams with an applied field of 2.7 kV/cm, for various degrees of coherence along y and all other parameters kept constant. (c) A fully coherent soliton breaks up into filaments because of TI. (d) The soliton is made incoherent along y but with large speckle sizes ~ 100 μm (small θ_0) and still displays a strong TI. (e) The speckle sizes are reduced to ~ 5 μm: TI is completely eliminated.

TI threshold and the beam suffers from TI [Fig. 2(d)]. Finally, by decreasing the speckle sizes to ~ 5 μm, TI is eliminated and we get a stable $(1 + 1)D$ soliton [Fig. 2(e)]. Thus, we have shown that a stable $(1 + 1)D$ soliton can propagate in a 3D bulk medium if it is made partially incoherent in the "uniform" transverse dimension. For the soliton to be stable, the degree of coherence in the "dimension of uniformity" must be such that the nonlinearity is below threshold for transverse instability.

In conclusion, we have derived the threshold for TI of $(1 + 1)D$ solitons that are fully coherent in their direction of trapping yet are partially incoherent in their direction of uniformity. We predicted that if the nonlinearity is below a well-defined threshold, then transverse instability of such 1D solitons is completely eliminated. We proved our results analytically, numerically, and experimentally, and showed that it is possible to generate stable 1D Kerr-like solitons in a 3D bulk material. This is the only method we know of for propagating truly stable 1D solitons in a bulk material. Our method applies to all types of saturable nonlinearities and could be used to eliminate TI in them as well. We believe that this work opens up a range of possibilities of eliminating transverse instabilities in many soliton systems, for example, instabilities of 1D dark solitons in bulk media, instabilities of ring beams (with and without topological charge) in self-focusing media, and more.

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