

Modulation Instability of Spatially-Incoherent Light Beams and Pattern Formation in Incoherent Wave Systems

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Modulation Instability (MI) is a universal process that appears in most nonlinear wave systems in nature. Because of MI, small amplitude and phase perturbations (from noise) grow rapidly under the combined effects of nonlinearity and diffraction (or dispersion, in the temporal domain). As a result, a broad optical beam (or a quasi-CW pulse) disintegrates during propagation, leading to filamentation or to break-up into pulse trains. Modulation instability is largely considered as a precursor of solitons, because the filaments (or pulse trains) that emerge from the MI process are actually trains of almost ideal solitons. Over the years, MI has been systematically investigated in connection with numerous nonlinear processes. Yet, it was always believed that MI is inherently a coherent process and thus it can only appear in nonlinear systems with a perfect degree of spatial/temporal coherence. Earlier this year however, we have theoretically demonstrated¹ that MI can also exist in relation with partially-incoherent wave-packets or beams.

Modulation instability in nonlinear incoherent environments reveals several new features that have no counterpart in coherent wave systems. The most important new features are as follows. (I) The existence of a sharp threshold for the nonlinear index change, below which perturbations (noise) on top of a uniform input beam decay and above which a quasi-periodic pattern forms. (II) The threshold depends upon the coherence properties of the input beam: the threshold increases with decreasing correlation distance (decreasing spatial coherence). The intuition behind these features and the fundamental difference between MI in coherent and in incoherent wave systems can be understood in the following manner. A small periodic perturbation on a coherent beam remains periodic and maintains its modulation depth during linear diffraction. Thus, any self-focusing nonlinearity, no matter how small, increases the modulation depth and leads to instability. This is why coherent MI has no threshold. On the other hand, a perturbation upon an incoherent beam diminishes its modulation depth during linear diffraction. The nonlinearity has to overcome this "washout" effect in order to lead to instability. This is why incoherent MI has a threshold: it occurs only if the nonlinearity is strong enough to overcome the diffusive washout due to diffraction. Furthermore, the more incoherent the beam is, the higher the MI threshold.

Recently, we have made the first experimental observation of incoherent MI.² We have shown that in a nonlinear partially coherent system (a nonlinear system of weakly-correlated particles) patterns can form spontaneously (from noise) when the nonlinearity exceeds the threshold, and a periodic train of one-dimensional filaments emerges. At a higher value of nonlinearity, the incoherent 1D filaments display a two-dimensional instability and break up into self-ordered arrays of light spots. A typical experimental result is shown in Fig.1. We emphasize that in all the pictures displayed in this figure, the

correlation distance is much shorter than the distance between two adjacent stripes or filaments.

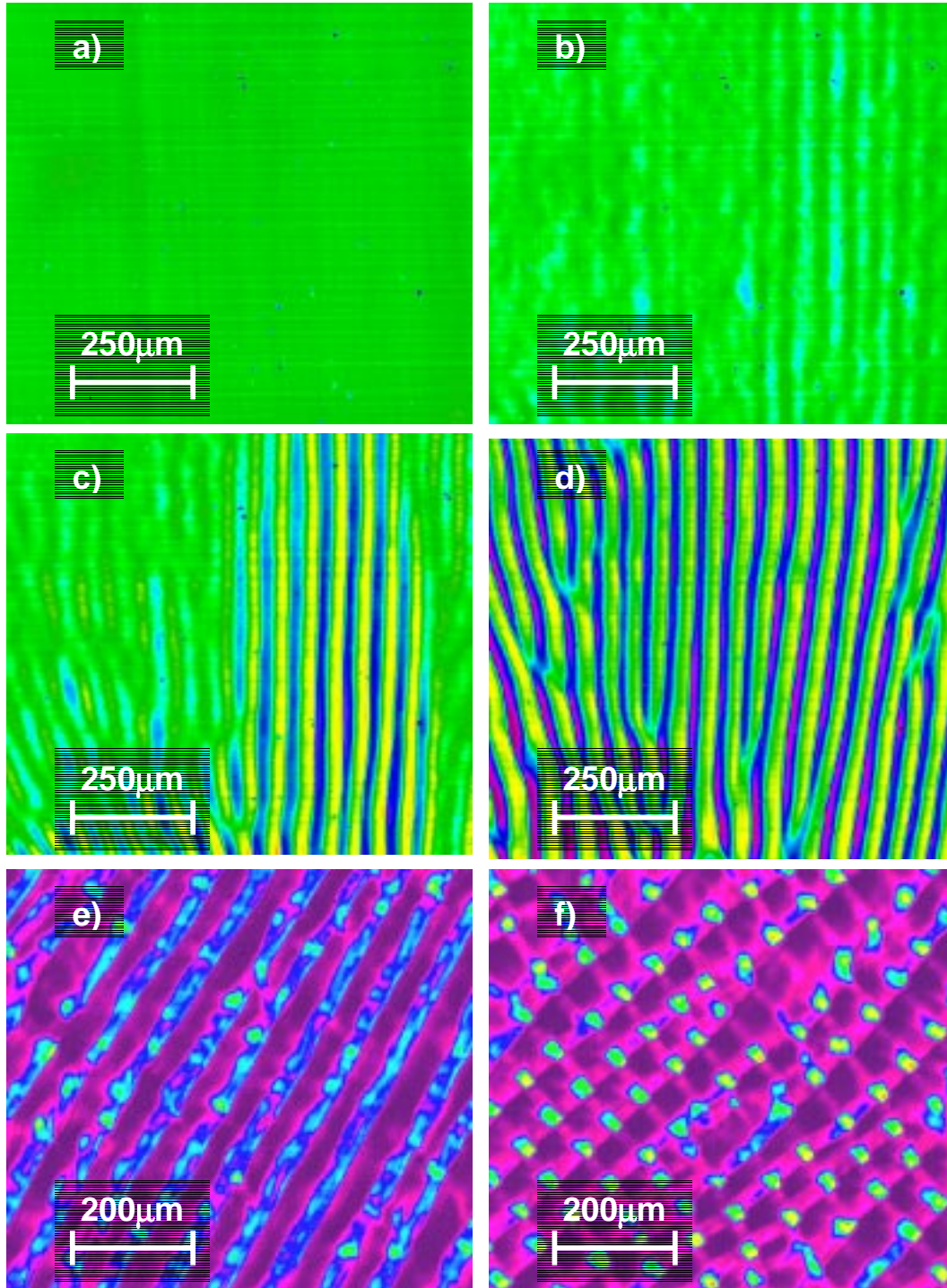
The ability to suppress MI has led to new families of solitons that have no counterpart whatsoever in the coherent regime. For example, it facilitated the observation of anti-dark solitons³: bright-like solitons superimposed on a uniform background. Such an entity was largely considered highly unstable and has never been observed before.⁴ Another example is the generation of stable (in the absolute sense) of (1+1) dimensional bright solitons in bulk Kerr media,⁵ once again – an entity that has been considered highly unstable since 1974.

The discovery of incoherent MI actually reflects on many other nonlinear systems beyond optics: it implies that patterns can form spontaneously (from noise) in nonlinear many-body systems involving weakly-correlated particles, such as, for example, atomic gases at temperatures near Bose-Einstein-Condensation (BEC) temperatures,⁶ electrons in semiconductors at the vicinity of the quantum Hall regime, high- T_c superconductors. We believe and hope that, as happened so many times in science, the best is yet to come.

This research is part of the MURI program on optical spatial solitons and is supported by the U.S. Army Research Office, the National Science Foundation, and the Air Force Office of Scientific Research, the Israel Science Foundation and by the Israeli Ministry of Science.

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Modulation Instability Figure: The intensity structure of a partially-spatially-incoherent beam at the output plane of the nonlinear photorefractive crystal. The sample is illuminated homogeneously with partially spatially incoherent light with a coherence length $l_c=17.5 \mu\text{m}$. The displayed area is $1.0 \times 1.0 \text{mm}^2$ (a-d) and $0.8 \times 0.8 \text{mm}^2$ (e,f), respectively. The size of the nonlinear refractive index change of the crystal is successively increased from (a) $\Delta n_0=0$ (the linear case), to (b) 3.5×10^{-4} , (c) 4.0×10^{-4} ,

(d) $4.5 \cdot 10^{-4}$, (e) $9 \cdot 10^{-4}$ (e), and (f) $1 \cdot 10^{-3}$. The plots (b-d) show the cases just below threshold (no features), at threshold (partial features), and just above threshold (features everywhere) for 1D incoherent MI that leads to 1D filaments. Far above this threshold, at a much higher value of the nonlinearity, the 1D filaments become unstable (e), and finally become ordered in a regular two-dimensional pattern (f).