A Distributed Strategy for Electricity Distribution Network Control in the face of DER Disruptions

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Outline

1. Threat Model of DER Disruptions

2. Maximin formulation
   - Centralized control strategy
   - Distributed control strategy

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Increasing DER penetration, controllable loads

Bidirectional communication infrastructure.

Communication between the control central (C.C) and DERs or controllable loads are susceptible to threats.
“Renewable electricity companies in Europe reportedly were targeted by cyberattackers at a clean power website from which malware was passed to visitors, thus giving the attackers access to the power grid.”

- Richard J. Campbell, Cybersecurity Issues for the Bulk Power System.
DER operation under nominal conditions

- Set-points $\tilde{sg}^{\text{nom}}$ are communicated from C.C. to the DERs
- DER controllers enforce these using set-point tracking
An attacker or a control center can introduce incorrect set-points $\tilde{g}^a$ that lead voltage and frequency below (or above) the permitted thresholds $V, f$.

This could cause disconnection of DERs\(^1\) resulting in a cascading failure\(^2\).

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\(^2\)Abraham Ellis, “IEEE 1547 and High Penetration PV”
Model the attacker-defender interaction as a Stackelberg game and specify the worst case attack that maximizes frequency and voltage deviation from its nominal operation points.

Design a defender strategy that minimizes the attacker impact in frequency deviation from its nominal value and loss of voltage regulation.
Contributions

- **Centralized strategy**
  - Formulate the Stackelberg game as a bilevel optimization problem that can be solved in a centralized manner.
  - Provide new set-points for the non-compromised nodes

- **Distributed strategy**
  Find the new set-points for the non-compromised nodes using:
  - Local voltage and frequency information.
  - Location of the worst-affected node (i.e., the node with the lowest voltage).
Power flow model

Power generated at node $i$:

$$s_g^i = p_g^i + jq_g^i$$

Power consumed at node $i$:

$$s_c^i = p_c^i + jq_c^i$$

We have that

$$P_{ij} = \sum_{k:j \rightarrow k} P_{jk} + r_{ij} \ell_{ij} + p_{cj} - p_{gj}$$

$$Q_{ij} = \sum_{k:j \rightarrow k} Q_{jk} + x_{ij} \ell_{ij} + q_{cj} - q_{gj}$$

$$\nu_j = \nu_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) \ell_{ij}$$

$$\ell_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{\nu_i}$$

where $S_{ij} = P_{ij} + jQ_{ij}$ denote the complex power flowing on line $(i,j) \in \mathcal{E}$. $\nu_j = |V_j|^2$, $\ell_{ij} = |I_{ij}|^2$, and $z_{ij} = r_{ij} + jx_{ij}$ the impedance.

DER model

$$-\sqrt{s_g^2 - (p_g)^2} \leq q_g^i \leq \sqrt{s_g^2 - (p_g)^2}$$

$$p_g^i \geq 0$$

Voltage constraints

$$\nu_i \leq \nu_j \leq \bar{\nu}_i$$
Frequency model

The DERs synchronize their frequencies with the system frequency rapidly due to low inertia.

The maximum drop in frequency after supply loss ($f_{dev}$) is given by:

$$f_{dev} = -H^{BG} P_{e,dev} = -H^{BG} (P_0 - P_0^{nom}),$$

where $H^{BG}$ is a constant that depends on the synchronous generator, and $P_0^{nom}$ is the real power flowing into substation under nominal conditions.

Frequency constraint

$$f_{\underline{dev}} \leq f_{dev} \leq f_{\overline{dev}}$$
Attacker Model

We denote $\psi = (\delta, \tilde{p}_g^a, \tilde{q}_g^a)$ the attacker strategy

- $\delta$ is a vector whose elements $\delta_i = 1$ if DER $i$ is compromised and zero otherwise.
- $\tilde{p}_g^a$: Active power set-points induced by the attacker.
- $\tilde{q}_g^a$: Reactive power set-points induced by the attacker.

The maximum injected power by each DER forms a semicircle due to

$$- \sqrt{sg^2_i - (\tilde{p}_g^a)^2} \leq \tilde{q}_g^a \leq \sqrt{sg^2_i - (\tilde{p}_g^a)^2}$$

Attacker’s resource constraint:

$$\sum_{i \in \mathcal{N}} \delta_i \leq M$$
**Defender model**

The defender action is given by \( \phi = (\gamma, \tilde{p}_g^d, \tilde{q}_g^d) \),

- \( \gamma_i \in [\gamma_i, 1] \) the portion of controlled loads.
- \( \tilde{p}_g^d \) is the defender new active power set-points.
- \( \tilde{q}_g^d \) is the defender new reactive power set-points.

The maximum injected power by each DER forms a semicircle due to

\[
-\sqrt{sg_i^2 - (\tilde{p}_g_i^d)^2} \leq \tilde{q}_g^d \leq \sqrt{sg_i^2 - (\tilde{p}_g_i^d)^2}
\]
Loss functions

We define the following cost functions

**Loss of voltage regulation**

\[
L_{VR} := \max_{i \in \mathcal{N}} W_i (\nu_i - \nu_i)_+, \]

**Loss of frequency regulation**

\[
L_{FR} := C (f_{dev} - f_{dev})_+, \]

where \(W_i\) and \(C\) are the costs or importance given to voltage and frequency regulation respectively, and \(a_+ = \max(a, 0)\).
The composite loss function is \( L(\psi, \phi) = L_{VR} + L_{FR} \).

The objective of the attacker (defender) is to maximize (minimize) the loss function as follows:

\[
\begin{align*}
\max_{\psi} \quad \min_{\phi} \quad L(\psi, \phi) \\
\text{s.t.} \quad & \delta_i \tilde{s}_{g_i}^a + (1 - \delta_i) \tilde{s}_{g_i}^d \quad \forall \quad i \in \mathcal{N} \\
& \sum_{i \in \mathcal{N}} \delta_i \leq M
\end{align*}
\]

subject to the power flow and operational constraints.

This is a non-linear, non-convex, mixed-integer, bilevel optimization problem and is NP-hard. \(^3\)

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Optimal attacker strategy under fixed defender response

Assume linear Power Flows.
Let $\Delta_j(\nu_i)$, $\Delta_j(f)$ denote the change of $\nu_i$, $f$, respectively due to compromise of node $j$.
$\Delta_j(\nu_i, f) := W_i \Delta_j(\nu_i) + C \Delta_j(f)$.
For $J \subseteq \{1, \ldots, M\}$, $\Delta_J(\nu_i, f) = \sum_{j \in J} \Delta_j(\nu_i, f)$.
The following greedy algorithm can be used to find $\delta$ that generate the worst impact.

**Algorithm 1: Optimal Attack Algorithm**

1. Calculate $\nu_i \ \forall \ i$ when no attack.

2. for $i \in \mathcal{N}$ do
   3. for $j \in \mathcal{N}$ do
      4. compute $\Delta_j(\nu_i, f)$
   5. Sort $j$ s in decreasing order of $\Delta_j(\nu_i, f) \rightarrow (\pi_1, \ldots, \pi_N)$.
   6. Set $J_i^* = (\pi_1, \ldots, \pi_M)$.
   7. Calculate $\Delta_{J_i^*}(\nu_i, f)$.

8. Find $\hat{i} = \arg \max_{k \in \mathcal{N}} \Delta_{J_k^*}(\nu_i, f) - W_k \nu_k$

9. Return $J_{\hat{i}}^*$.
Centralized Control Strategy

Limitations

- Centralized response - heavy communication requirements
- Assumes perfect knowledge of the attack plan.

How to choose the defender set-points to minimize in a distributed manner the impact of the attacker action?
Distributed Control Strategy

1. Attack detected
2. Exchange voltage information
3. Find the worst affected node
4. Find critical node
5. Establish new set-points
Distributed Control Strategy

- Attack detected
- Exchange voltage information
- Find the worst affected node
- Find critical node
- Establish new set-points

Each node compares the received (and its own) voltage values and transmits the smallest to its neighbors.
Distributed Control Strategy

Attack detected

Exchange voltage information

Find the worst affected node

Find critical node

Establish new set-points

The worst affected node is the node with the lowest voltage
Distributed Control Strategy

Attack detected

Exchange voltage information

Find the worst affected node

Find critical node

Establish new set-points

The worst affected node is the node with the lowest voltage

Key assumption: The location of the worst-node $t$ does not change before and after the defender response.
Distributed Control Strategy

Attack detected

Exchange voltage information

Find the worst affected node

Find critical node

Establish new set-points

Critical node:
It is the node that partitions the graph into two disjoints subsets $\mathcal{N}_f, \mathcal{N}_v$ of $\mathcal{N}_0$. $j \in \mathcal{N}_f$ contribute to frequency regulation and $j \in \mathcal{N}_v$ to voltage regulation.
Distributed Control Strategy
Finding the critical node

$\mathcal{P}_i$ is the set of edges on the path between root node and node $i$.

$\mathcal{P}_i = \{(0,a), (a,g), (g,j)\}$

$\mathcal{P}_i \cap \mathcal{P}_j = \{(0,a)\}$

Critical node:

For $t$ the worst affected node, let $n_{jt} = |\mathcal{P}_j \cap \mathcal{P}_t|$ denote the number of edges on the intersection of the paths $\mathcal{P}_j, \mathcal{P}_t$. Let $\kappa = \frac{CH^{BG}}{2W \sqrt{r^2 + x^2}}$. Then, the critical node

$\tau = \arg \min_{n_{jt} \geq \kappa} |\mathcal{P}_j|$ and it is unique due to the tree topology of the DN.
Distributed Control Strategy

- Attack detected
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Distributed Control Strategy

- Attack detected
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**Frequency regulation**
\[
\tilde{p}_{g_i}^d = s_{g_i}, \quad \tilde{q}_{g_i}^d = 0.
\]

**Voltage regulation**
\[
\begin{align*}
\tilde{p}_{g_i}^d &= \frac{r s_{g_i}}{\sqrt{r^2 + x^2}}, \\
\tilde{q}_{g_i}^d &= \frac{x s_{g_i}}{\sqrt{r^2 + x^2}}.
\end{align*}
\]
Result

DER-disconnect strategy: \( \tilde{g}^d_i = 0 + j0 \).

Theorem

Under constant \( R/X \) ratio, if \( \hat{L}_d \), \( \hat{L}_c \) and \( \hat{L}_0 \) are the maximin losses with distributed, centralized, and DER-disconnect strategies, then

\[
\frac{\hat{L}_d - \hat{L}_0}{\hat{L}_c - \hat{L}_0} \geq \cos\left(\frac{\angle z_u}{2}\right),
\]

where \( z_u \) is the impedance per unit length.

For standard IEEE DNs, this competitive ratio is \( \approx 0.92 \).
Optimal Power Injection

Using the distributed strategy for the aforementioned example, we find the set of nodes that contribute to frequency and voltage regulation. The critical node is 3 and the worst affected node is 6.
The OPF set points do not contribute to frequency regulation, but they maintain better voltage levels.

The set-points obtained with the distributed method are suboptimal solutions.

For $C = 1000$ and $W = 700$, the required power from the substation is lower for the centralized case, which implies fastest frequency regulation.
Conclusions and Future Work

- We proposed a novel formulation that allow us to minimize the impact of attacks that affect set-point information.
- With the distributed strategy it is possible to react to attacks using only local information and predefined set-points, but it is necessary to ensure secure communication between DERs.
- In the future, we will analyze the case in which the worst-affected node change after applying a contingency strategy.