



**School of Engineering**



Civil and Environmental Engineering

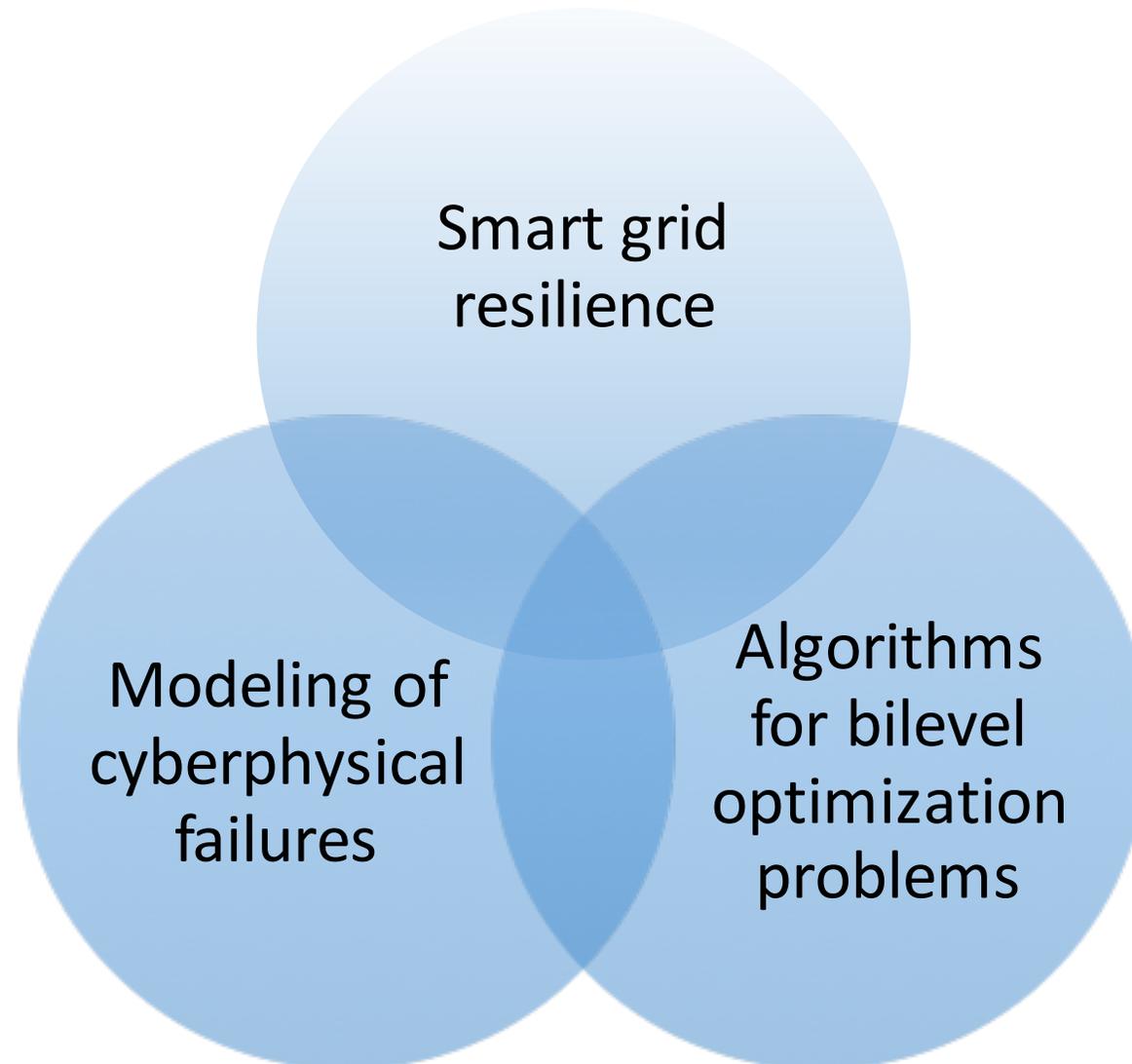
# Towards Improving the Resilience of Power Systems

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August 30, 2018

**Collaborators: Saurabh Amin, Ian Hiskens**

# Research Focus



# Outline

- Motivation: Resilience-Aware operations
- Attack models and Problem formulation
- Main results

# Cyber-Physical disruptions



- Hurricane Maria  
(September 2017)
- Customers facing blackouts for months



- Metcalf Substation (April 2013)
- Sniper attack on 17 transformers
  - Telecommunication cables cut
  - 15 million \$ worth of damage
  - 100 mn \$ for security upgrades



- Ukraine attack (Dec '15, '16)
- First ever blackouts caused by hackers
  - Controllers damaged for months

# Research challenge

Existing literature considers:

- Physical security of transmission networks
- DC powerflow models

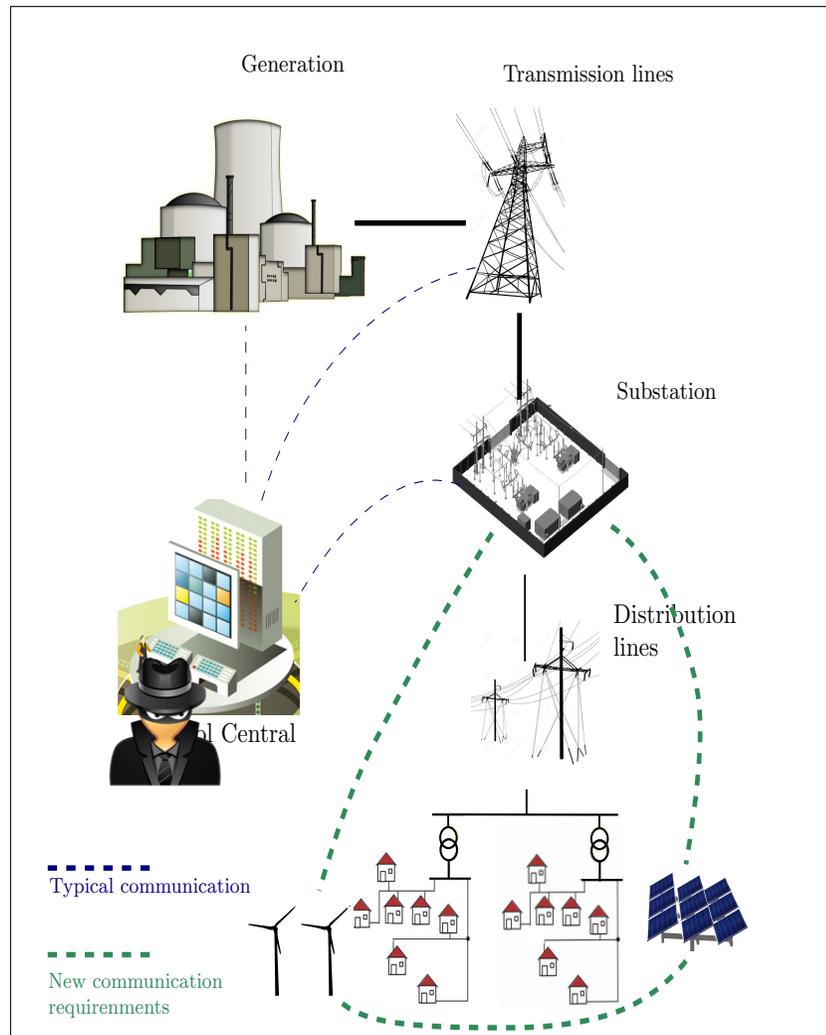
Limited focus on:

- Smart Distribution networks (DNs)
- Optimal attacker/defender strategies based on:
  - Network topology
  - Tradeoffs in resource allocation

My approach combines:

- Physics-based optimal attack
- Semantics-aware software memory attack

# Distribution network attack scenarios



- Agent
  - Disgruntled employee
  - External hacker
  - Buggy SCADA implementation
- NESCO Vulnerabilities (EPRI)
  - Mass remote disconnect of smart meters
  - Simultaneous disconnect of DERs
  - Rapid overcharging of electric vehicles
- Impact: **supply-demand disturbances** (sudden or prolonged)

# Background: Security-constrained OPF

- **Economic Dispatch** problem to ensure an operational power system **despite contingencies**
- Accounts for **appropriate corrective actions** for the said contingency

## Main issues

- Only captures N-k contingencies for small k. Typically  $k = 1$  or  $2$
- Assumes a priori fixed set of contingencies
- Does not model strategic attacker-induced failures

A. Monticelli, et al. - "Security-Constrained Optimal Power Flow with Post-Contingency Corrective Rescheduling"

J. A. Momoh, et al. - "A review of selected optimal power flow literature to 1993. II. Newton, linear programming and interior point methods"

# Our formulation: Resilience-Aware OPF

Stage I

Minimize

$C_{\text{allocation}}$  +

Stage II

Maximize

Stage III

Minimize

$C_{\text{post-contingency}}$

Over all  
allocations

Over all  
disruptions

Over all  
responses

Subject to

- Network constraints
- Component constraints
- Voltage constraints

# Resilience-Aware OPF (3-Stages)

Pre-contingency  
state

Worst-case post-  
contingency state

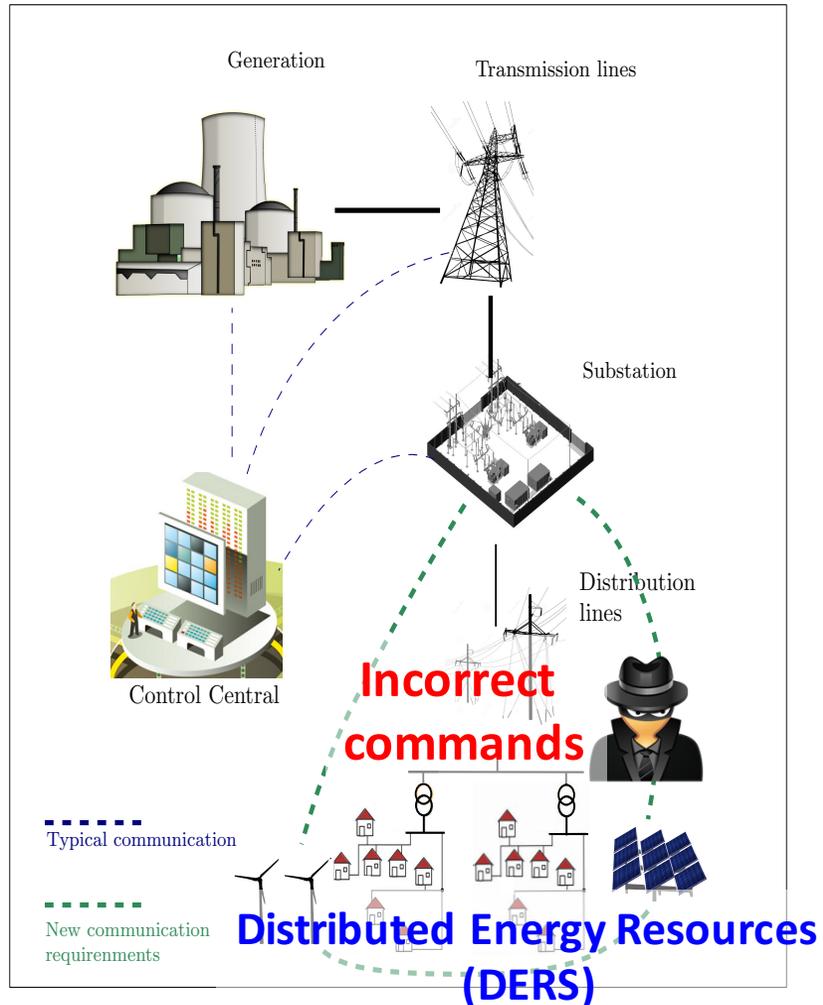
$$\min_{a \in \mathcal{A}} C_{alloc}(a) + \max_{d \in \mathcal{D}} \min_{u \in \mathcal{U}} L(a, d, u)$$

RAOPF  
(Stages II and III)

Subject to

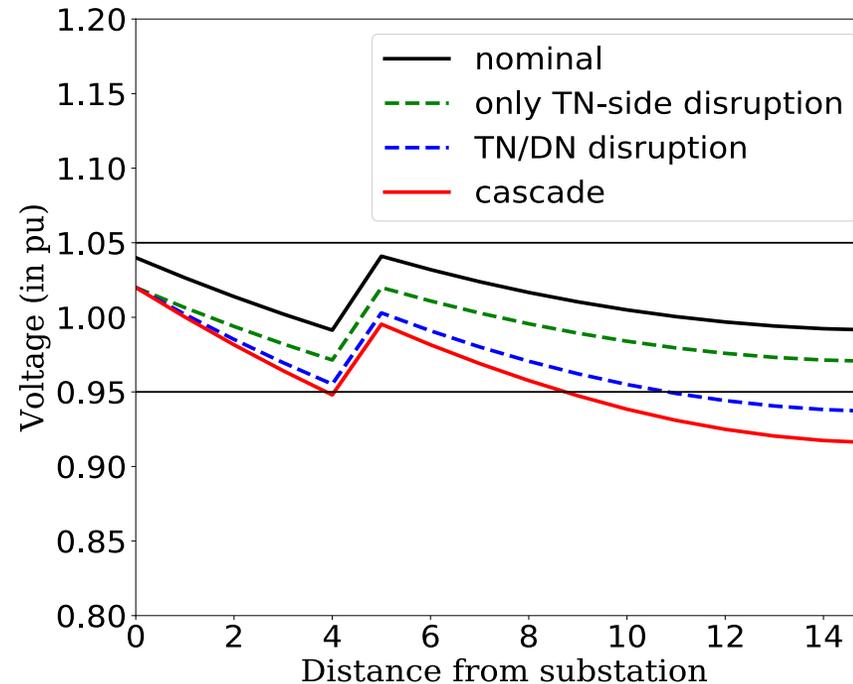
- Network constraints
- Component constraints
- Voltage constraints

# A specific attack scenario



## Adversary:

- Hack DER SCADA and disrupt DERs
- Create supply-demand disturbance
- Cause frequency and voltage violations
- Induce network failures (cascades)



# A 3-regime picture

## Grid-connected regime

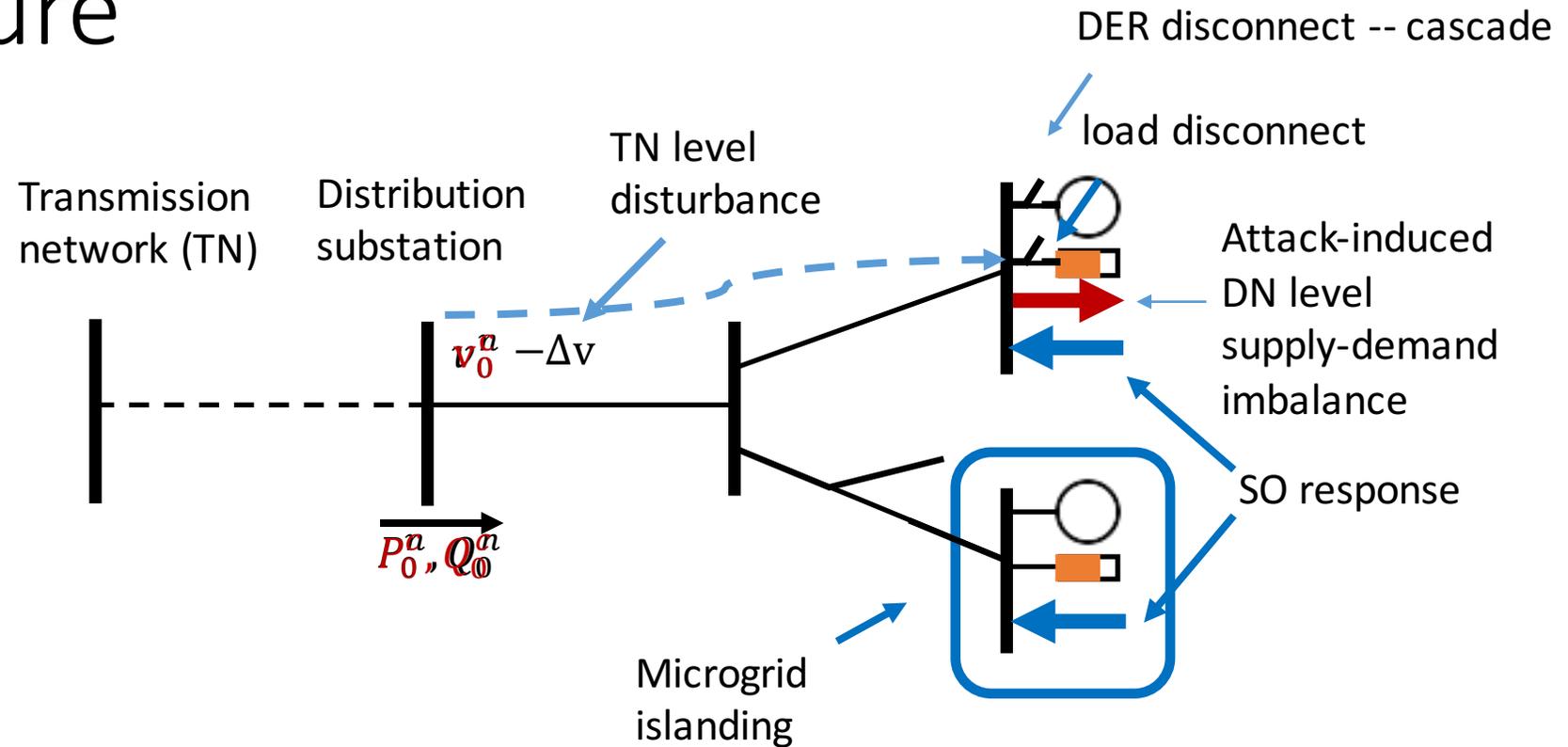
- Can absorb the impact of disturbances

## Islanding mode regime

- Larger disturbances may force microgrid islanding

## Cascade regime

- High severity voltage excursions, then more DER disconnects (cascades), more load shedding



When TN and DN level disturbances clear, the system can return to its nominal regime

# Our approach

Most attacker-defender interactions can be modeled as

- Supply-demand imbalance induced by attacker
  - Control (reactive and proactive) by the system operator
- 
- Abstraction: Bilevel (or multilevel) optimization problems
- 
- Supplements simulation based approaches
    - For example, co-simulation of cyber and power simulators

# Resilience-aware OPF (Stages II and III)

Stage II - Adversarial node disruptions

a. Which nodes to compromise ( $\delta$ )?

... can include other attack models

Stage III - Optimal dispatch / response ( $x^c$ )

a. Exercise load control or not

b. Disconnects loads/DGs?

c. Maintain voltage regulation

... possible to consider frequency regulation

Goals:

1. Identify critical nodes

2. Determine optimal response

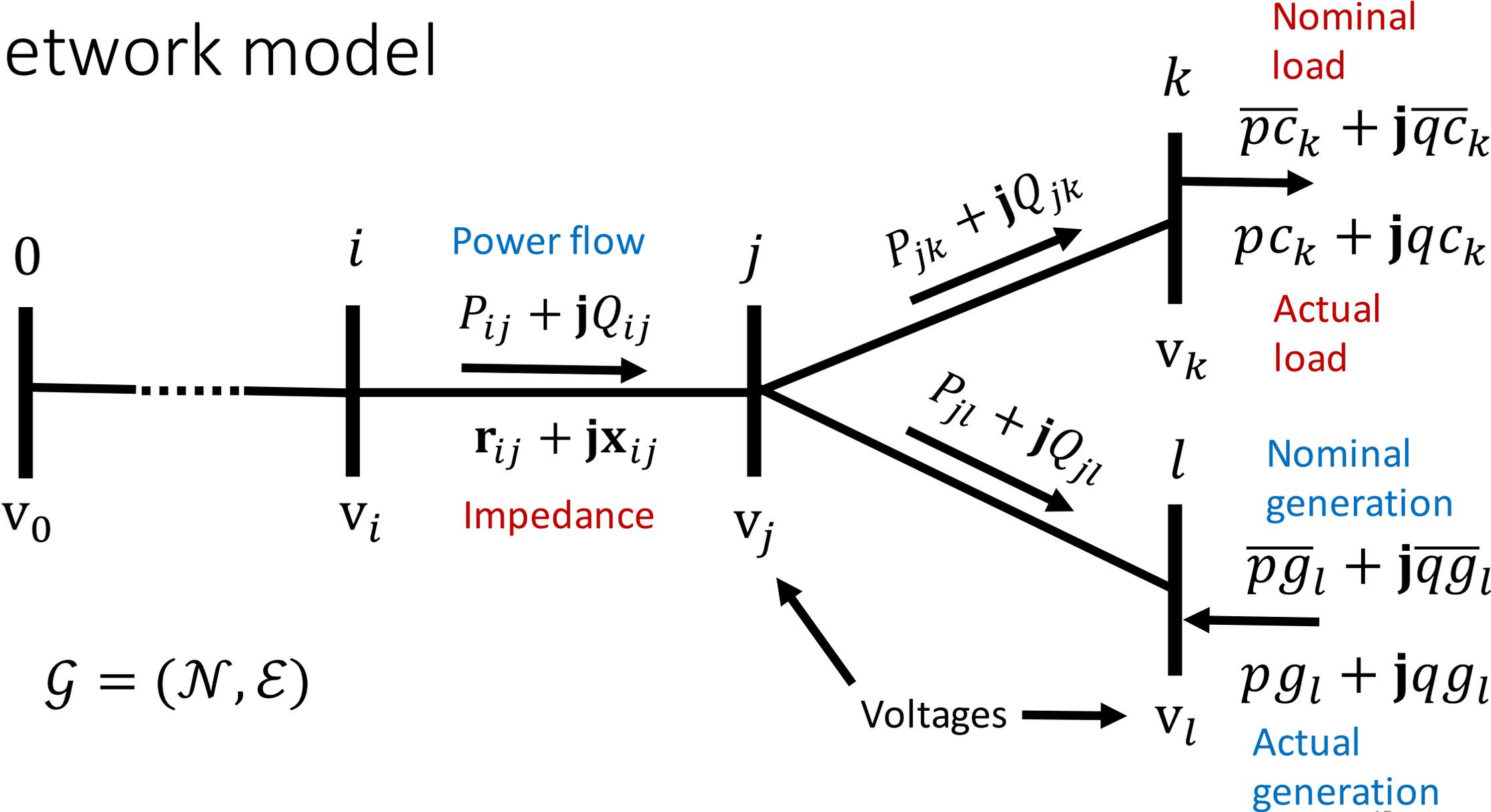
# Modeling of Grid-connected/Cascade regimes

$$\max_{d \in \mathcal{D}} \min_{u \in \mathcal{U}} L(d, u)$$

Subject to

- Network constraints
- Component constraints
- Voltage constraints

# Network model



# Defender model in Grid-connected regime

- Defender response: only load control
- $u = \beta$
- $\beta_i \in [\underline{\beta}_i, 1]$ : load control parameter at node  $i$

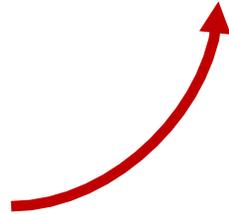
$$pc_i = \beta_i \overline{pc}_i, \quad qc_i = \beta_i \overline{qc}_i$$

Defender response:

How much load control should be exercised?

# Losses in Grid-connected regime

$$L^{\text{GC regime}} = \text{Cost of active power supply} + \text{Cost of loss of voltage regulation} + \text{Cost of load control}$$

$W_{AC}P_0$  

$W_{VR}t$  

$\sum_{i \in \mathcal{N}} W_{LC,i}(1 - \beta_i)$  

Where

$$t \geq \max_{i \in \mathcal{N}} |v_0^{\text{nom}} - v_i|$$

# Defender model in Cascade regime

Defender response: load control, connectivity control

$$u = (\beta, kg, kc)$$

$$kg_i = \begin{cases} 1, & \text{if DG } i \text{ is disconnected} \\ 0, & \text{otherwise.} \end{cases}$$
$$kc_i = \begin{cases} 1, & \text{if load } i \text{ is disconnected} \\ 0, & \text{otherwise.} \end{cases}$$

Connectivity constraints are mixed-integer linear:

- Connected implies no violations
- Violation implies not connected

$$kg_i = 0 \quad \Rightarrow \quad v_i \in [\underline{vg}_i, \overline{vg}_i]$$
$$v_i \notin [\underline{vg}_i, \overline{vg}_i] \quad \Rightarrow \quad kg_i = 1$$

Voltage  
bounds for DG



Defender response:

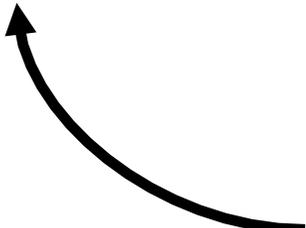
Which loads and DGs to disconnect?

Similarly  
for loads!

# Losses in Cascade regime

$$L^{\text{CS regime}} \equiv L^{\text{GC regime}} + \text{Cost of load disconnection}$$

$\sum_{i \in N} W_{\text{SD},i} k c_i$



# Attacker model

Attacker strategy:  $d = (\delta, \Delta v_0)$

$$\delta_i = \begin{cases} 1, & \text{if node } i \text{ is attacked} \\ 0, & \text{otherwise.} \end{cases}$$

Attacker's resource budget  $\sum_i \delta_i \leq k$

- $\Delta v_0$ : amount by which substation voltage drops
  - Due to physical disturbance or temporary fault in the TN

Attacker strategy:

- Which nodes to compromise?

# Effect of attacker actions

- DER disruption makes its output zero.

$$\begin{aligned}k g_i &\geq \delta_i \\p g_i &= (1 - k g_i) \overline{p g_i} \\q g_i &= (1 - k g_i) \overline{q g_i}\end{aligned}$$

- TN-side disturbance impacts substation voltage

$$V_0 = V_0^{\text{nom}} - \Delta V_0$$

# Linear power flows

Power conservation

$$P_{ij} = \sum_{k:j \rightarrow k} P_{jk} + pc_j - pg_j$$

$$Q_{ij} = \sum_{k:j \rightarrow k} Q_{jk} + qc_j - qg_j$$

Voltage drop

$$v_j = v_i - 2(\mathbf{r}_{ij}P_{ij} + \mathbf{x}_{ij}Q_{ij})$$

$$\mathbf{v}_0 = \mathbf{v}_0^{\text{nom}} - \Delta \mathbf{v}$$

System state

$$\mathbf{x} = (pc, qc, pg, qg, \mathbf{v})$$

# Cascade regime

$$\mathcal{L} := \max_{d \in \mathcal{D}} \min_{u \in \mathcal{U}} L^{\text{CS regime}}(d, u)$$

Subject to

- Network constraints
- Component constraints
- Voltage constraints

This is a mixed-integer bilevel linear program: **NP-hard!**

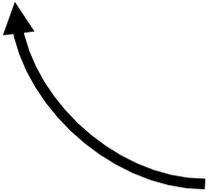
# Islanding regime

$$\max_{d \in \mathcal{D}} \min_{u \in \mathcal{U}} L^{\text{MI regime}}(d, u)$$

Subject to

- Network constraints
- Component constraints
- Voltage constraints

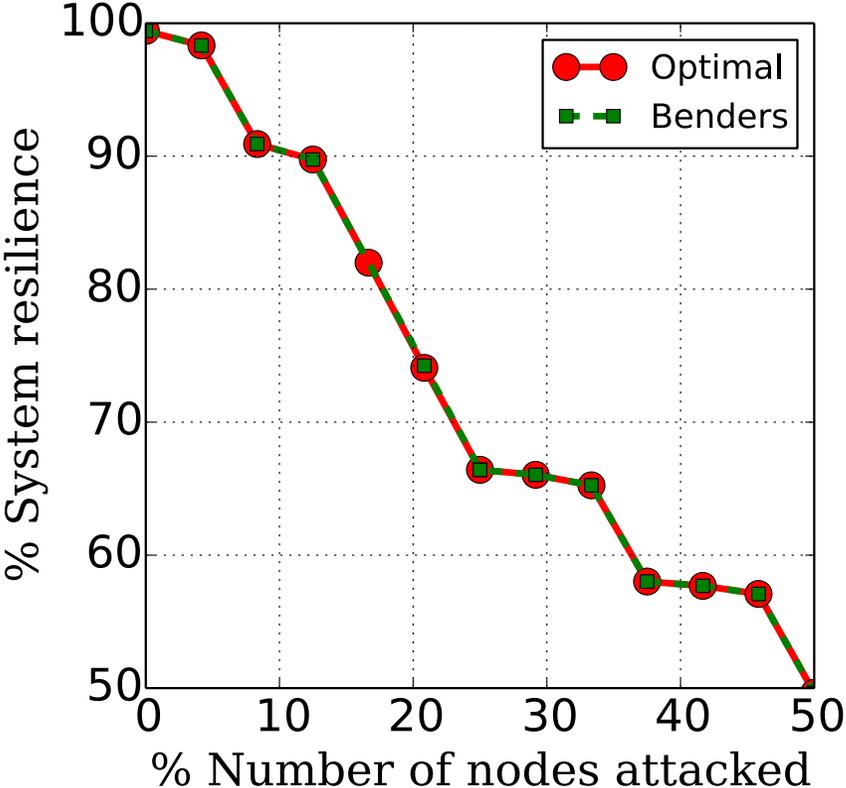
$$L^{\text{MI regime}} \equiv L^{\text{CS regime}} + \text{Cost of islanding}$$

$$\sum_{(i,j) \in \mathcal{X}} W_{\text{MG},ij} km_{ij}$$


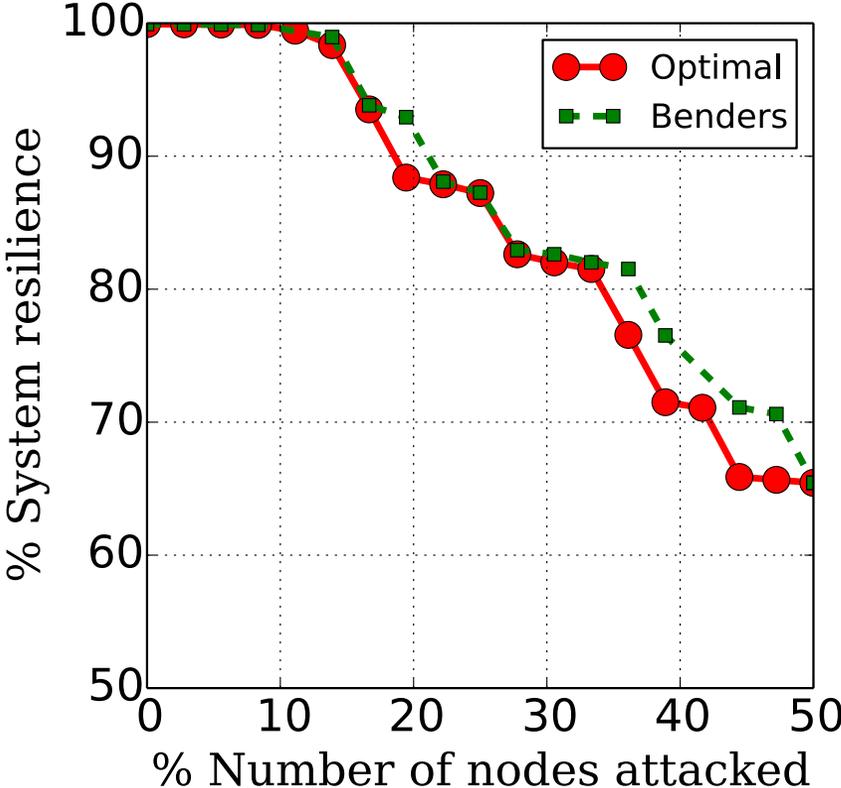
# System resilience

- $\mathcal{L}_{max} = \sum_{i \in N} W_{SD,i}$  : maximum loss
  - Cost of disconnection of all loads
- System resilience
  - Percentage decrease in system performance relative to maximum loss
  - $= 100 \left( 1 - \frac{\mathcal{L}}{\mathcal{L}_{max}} \right)$

# Benders Decomposition vs. Optimal



Grid-connected and Cascade regime



Grid-connected, cascade, and Islanding regime

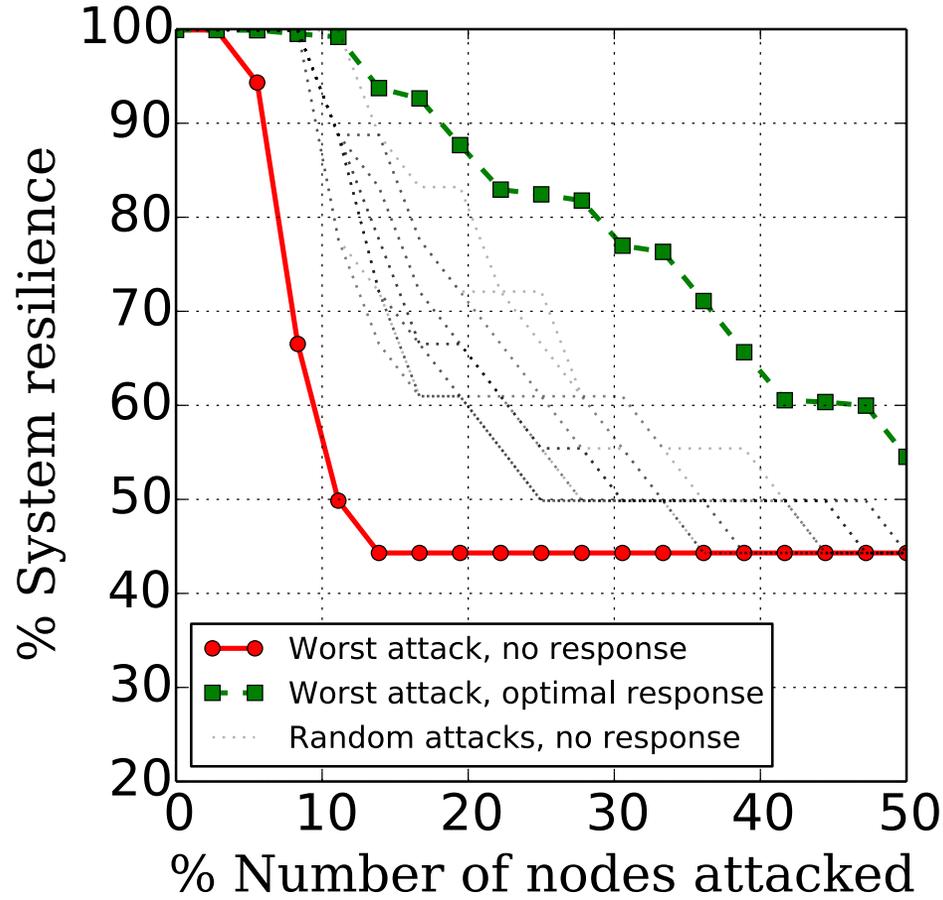
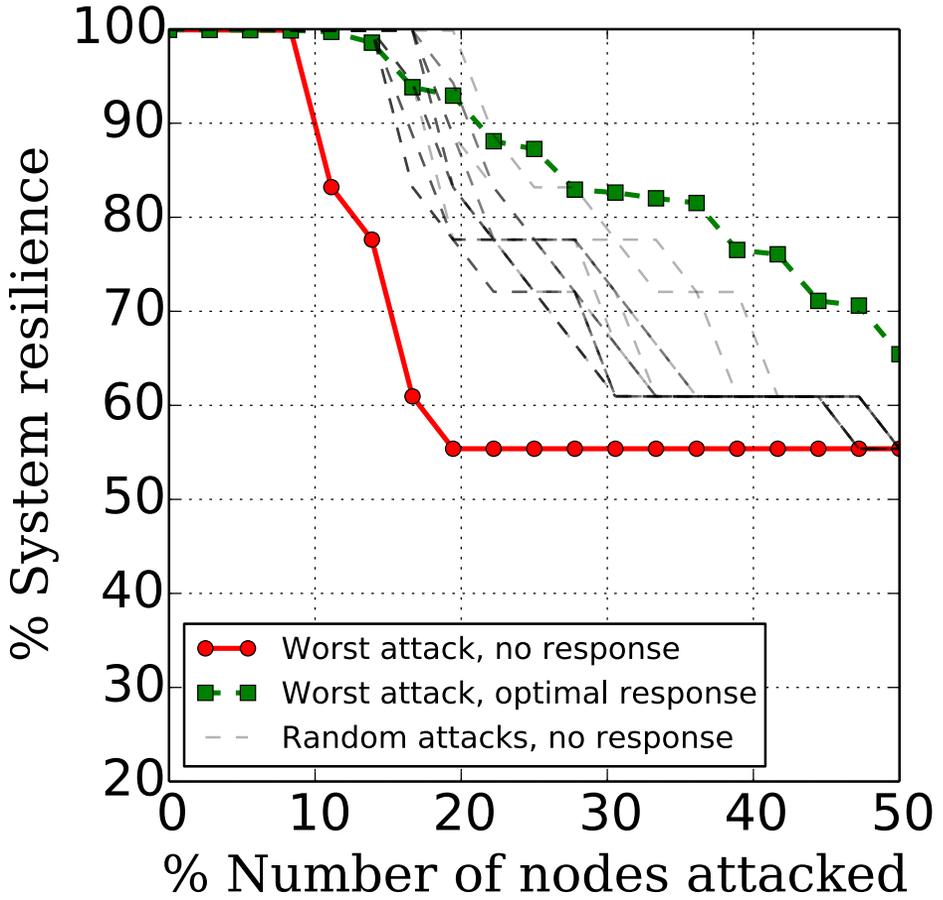
# Uncontrolled (multi-round) cascade

In reality, defender may not be able to instantaneously detect and identify attack, and optimally respond to it

## No response cascade algorithm

- Initial contingency
- For  $r = 1, 2, \dots$ 
  - Compute power flows
  - Determine the nodes that violate the voltage bounds
  - Disconnect the loads or non-controllable DGs accordingly

# Uncontrolled vs Cascade regime



N = 36

# Performance of Benders Decomposition

Entries are resilience metric of DN (in percentage), number of iterations (written in brackets), time (in seconds), attack cardinality.			
$\mathcal{R}_{\text{target}}$	N = 24	N = 36	N = 118
99	98.75, (3), 0.04, 1	98.96, (11), 0.22, 5	98.52, (27), 1.86, 14
95	91.15, (6), 0.08, 2	93.82, (13), 0.27, 6	94.66, (39), 3.34, 17
90	89.75, (10), 0.16, 3	88.08, (15), 0.34, 8	89.94, (50), 5.44, 26
85	82.41, (11), 0.18, 4	82.93, (17), 0.4, 10	84.96, (69), 9.23, 44
80	74.38, (14), 0.26, 5	76.99, (21), 0.52, 14	79.71, (86), 613.42, 52
75	74.38, (14), 0.26, 5	71.1, (23), 0.59, 16	Failure
65	58.01, (20), 0.41, 9	Failure	
55	49.65, (23), 0.47, 12		
45	Failure		

$$\text{Res}^{\text{Worst-case}} = \left(1 - \frac{L}{L^{\text{max}}}\right) 100 \%$$

# Summary (so far)

- Resource allocation and dispatch in electricity DNs
  - under strategic cyber-physical failures
  - Multi-regime defender response
- Benders decomposition approach for solving bilevel MILPs
- Structural results on worst-case attacks and defender response



# Learning of Power Transmission Dynamics from partial PMU observations

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August 30, 2018

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Sidhant Misra, Marc Vuffray**

# Motivation

- State estimation
  - Optimal resource allocation for improved resiliency
  - Secure and efficient operations
- Dynamic model estimation
  - Detection of faults/attacks
  - Prompt and accurate response
- Data-driven approach

# Preliminaries

- Dynamical equation:  $x_{t+1} = Ax_t + Fv_t$
- $A \in R^{N \times N}$  : dynamic matrix;
- $x_t \in R^N$  : state vector
- $v_t \in R^N$  : Noise vector
- $F$  : Noise-scaling matrix

# Assumptions

- Temporal independence of noise vectors
  - $v_i$  and  $v_j$  are independent for all  $i \neq j$
- Spatial independence of noise vectors
  - $F$  is a diagonal matrix (there is no spatial mixing of noise)

# Learning under full observability

Given: observations  $x_t$  for  $t = 1, 2, \dots, n + 1$

Result:

- Maximum likelihood estimator of A [1]

$$\hat{A} = \Sigma_1^{-1} \Sigma_0$$

Where

$$\Sigma_0 = \frac{1}{n} \sum_{t=1}^n x_t x_t' \quad \text{and} \quad \Sigma_1 = \frac{1}{n} \sum_{t=1}^n x_{t+1} x_t'$$

- Also the solution of least squares regression

# Linear Swing Dynamics model

- Network  $(\mathcal{V}, \mathcal{E})$
- $\mathcal{V}$  set of nodes,  $N = |\mathcal{V}|$  number of nodes
- $\mathcal{E}$  set of edges

Swing equation

$$M_i \ddot{\theta}_i + D_i (\dot{\theta}_i - \omega^0) = P_i^{(m)} - P_i^{(e)}$$

- $P_i^{(m)}$  : mechanical power input
- $-P_i^{(e)}$  : electrical power output

# Power system model

Using change of variables

- $\delta_i$  : phase deviations from steady state values
- $\omega_i$  : relative generator rotor speed relative nominal frequency

$$M_i \dot{\omega}_i + D_i \omega_i = - \sum_{(i,j) \in \mathcal{E}} \beta_{ij} (\delta_i - \delta_j) + \delta P_i$$

$$\begin{bmatrix} \dot{\delta} \\ \dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} 0_{N \times N} & I_{N \times N} \\ -M^{-1}L & -M^{-1}D \end{bmatrix}}_{A_d} \begin{bmatrix} \delta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & M^{-1} \end{bmatrix} \begin{bmatrix} 0_N \\ \delta P \end{bmatrix}$$

# Discrete dynamical model

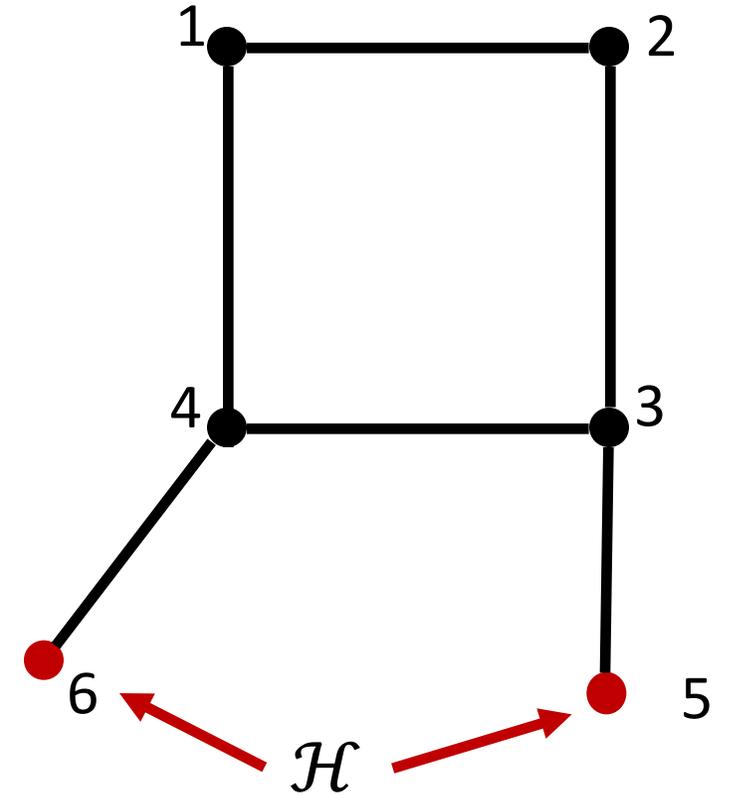
- Using discretization with timestep  $T$ 
  - $A = (I + A_d T)$

$$\underbrace{\begin{bmatrix} \delta_{t+1} \\ \omega_{t+1} \end{bmatrix}}_{x_{t+1}} = \underbrace{\begin{bmatrix} I_{N \times N} & T I_{N \times N} \\ -T M^{-1} L & I_{N \times N} - T M^{-1} D \end{bmatrix}}_A \underbrace{\begin{bmatrix} \delta \\ \omega \end{bmatrix}}_{x_t} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & T M^{-1} \end{bmatrix}}_F \underbrace{\begin{bmatrix} 0_N \\ \delta P \end{bmatrix}}_{v_t}$$

$$x_{t+1} = A x_t + F v_t$$

# Learning under partial observability

- $\mathcal{H} \subseteq \mathcal{V}$  set of hidden nodes (without PMUs)
- $\mathcal{O} = \mathcal{V} \setminus \mathcal{H}$  set of observable nodes (with PMUs)



# Rearrangement of dynamic matrix

$$\begin{bmatrix} \delta_{t+1}^{\mathcal{O}} \\ \omega_{t+1}^{\mathcal{O}} \\ \delta_{t+1}^{\mathcal{H}} \\ \omega_{t+1}^{\mathcal{H}} \end{bmatrix} = \begin{bmatrix} A_{\mathcal{O}\mathcal{O}} & A_{\mathcal{O}\mathcal{H}} \\ A_{\mathcal{H}\mathcal{O}} & A_{\mathcal{H}\mathcal{H}} \end{bmatrix} \begin{bmatrix} \delta_t^{\mathcal{O}} \\ \omega_t^{\mathcal{O}} \\ \delta_t^{\mathcal{H}} \\ \omega_t^{\mathcal{H}} \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} 0 \\ v_t^{\mathcal{O}} \\ 0 \\ v_t^{\mathcal{H}} \end{bmatrix}$$

By change of notation,

$$\begin{bmatrix} y_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} B & C \\ D & E \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} u_t \\ w_t \end{bmatrix}$$

## Problem statement

- Given measurements from observable nodes  $y_t$  for  $t = 1, 2, \dots, n$
- Goal: To recover dynamic matrix  $A$ 
  - Or equivalently, recover sub-matrices  $B, C, D, E$

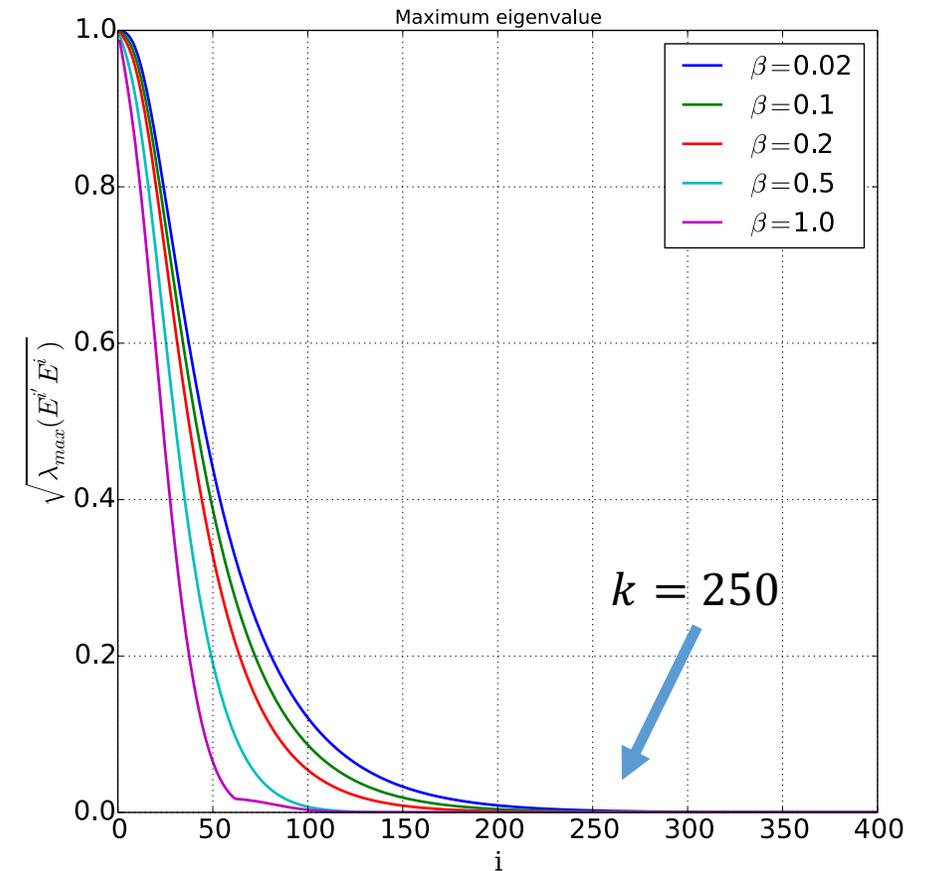
# Some simple observations

- Stable system implies

$$|\lambda_{max}(E)| \leq |\lambda_{max}(A)| < 1$$

- Thus,  $E^k \approx 0$  for sufficiently large  $k$

- Large susceptance values imply more unstable system



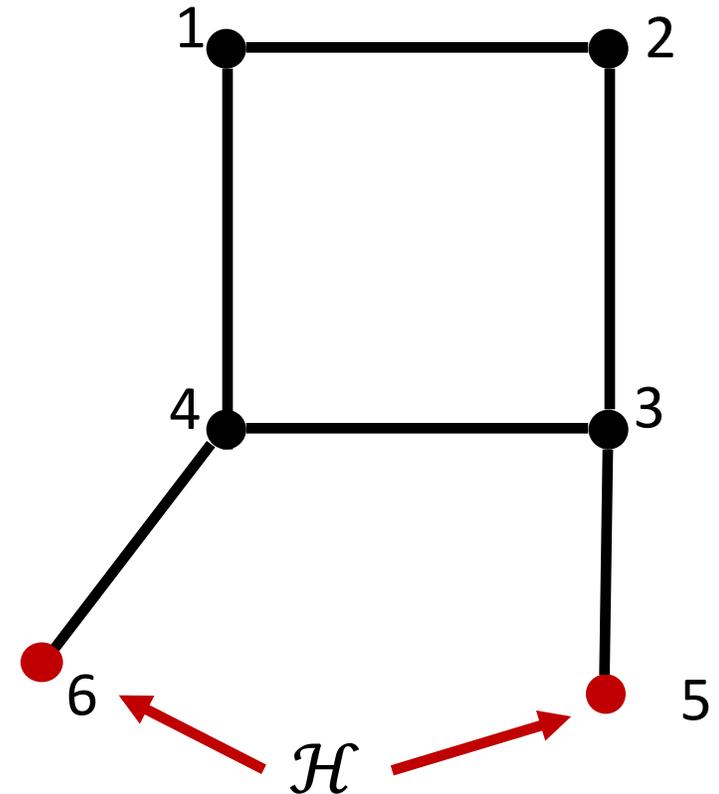
# Eliminating hidden node measurements

$$\therefore y'_{t+k+1} = [y'_{t+k} \ y'_{t+k-1} \ \cdots \ y'_t] \begin{bmatrix} B' \\ (CD)' \\ \vdots \\ (CE^{k-1}D)' \end{bmatrix} + \left( [G \ CH \ \cdots \ CE^{k-1}H] \begin{bmatrix} u_{t+k} \\ w_{t+k-1} \\ \vdots \\ w_t \end{bmatrix} \right)'$$

$$y'_{t+k} = Y'_t X + \eta_t$$

# Connectivity restrictions

- Each observable node is connected to at most one hidden node
  - $|\{(o, h) \in \mathcal{E} : h \in \mathcal{H}\}| \leq 1 \forall o \in \mathcal{O}$
- Each hidden node is connected to exactly one observable node
  - $|\{(o, h) \in \mathcal{E} : o \in \mathcal{O}\}| \leq 1 \forall h \in \mathcal{H}$



# Some simple properties

$$y'_{t+k+1} = [y'_{t+k} \ y'_{t+k-1} \ \cdots \ y'_t] \begin{bmatrix} B' \\ (CD)' \\ \vdots \\ (CE^{k-1}D)' \end{bmatrix} + \left( [G \ CH \ \cdots \ CE^{k-1}H] \begin{bmatrix} u_{t+k} \\ w_{t+k-1} \\ \vdots \\ w_t \end{bmatrix} \right)'$$

$$y'_{t+k} = Y'_t X + \eta_t$$

## Properties

- $G$  is diagonal by assumption
- Under connectivity restriction, for all  $m = 0, 1, \dots, k - 1$ ,  $CE^m H$  is of the form  $\begin{bmatrix} 0 & 0 \\ x & 0 \end{bmatrix}$ , where
  - $x \in R^{O \times \mathcal{H}}$  with
    - exactly 1 non-zero entry per column, and
    - at most 1 non-zero entry per row.

# Implications

For timesteps  $t = i, i + k, i + 2k, \dots$

- The noise vectors  $\eta_t$  satisfy both temporal and spatial independence
- Thus, we can use least squares estimator

$$\begin{bmatrix} y'_{t+k+1} \\ y'_{t+2k+1} \\ \vdots \\ y'_{t+ck+1} \end{bmatrix} = \begin{bmatrix} Y'_t \\ Y'_{t+k} \\ \vdots \\ Y'_{t+ck} \end{bmatrix} X + \eta_t$$

$$r \approx SX$$

# Least squares estimator

- $\hat{X} = (S'S)^{-1}(S'r)$ , or equivalently,

$$\begin{bmatrix} B' \\ (CD)' \\ \vdots \\ (CE^{k-1}D)' \end{bmatrix} = \begin{pmatrix} \begin{bmatrix} \Sigma_0 & \Sigma_1 & \cdots & \Sigma_k \\ \Sigma_{-1} & \Sigma_0 & \cdots & \Sigma_{k-1} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{-k} & \Sigma_{-k+1} & \cdots & \Sigma_0 \end{bmatrix} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_k \\ \Sigma_{k-1} \\ \vdots \\ \Sigma_0 \end{pmatrix}$$

Where

$$\Sigma_i = \frac{1}{l-j+1} \sum_{j=1}^l y_{jk+i} y'_{jk}$$

- Allows, recovery of B matrix in a straightforward manner.

# Recovering submatrices C, E, and D

- Under the connectivity restrictions,  $C$  and  $D$  are sparse matrices such that  $C = \begin{bmatrix} 0 & 0 \\ \mathbf{x} & 0 \end{bmatrix}$ ,  $D = \begin{bmatrix} 0 & 0 \\ \mathbf{z} & 0 \end{bmatrix}$  and  $E^i = \begin{bmatrix} R_{1i} & R_{2i} \\ R_{3i} & R_{4i} \end{bmatrix}$ 
  - $\mathbf{x} \in R^{0 \times \mathcal{H}}$  with exactly 1 non-zero entry per column and at most 1 non-zero entry per row.
  - $\mathbf{z} \in R^{\mathcal{H} \times 0}$  with exactly 1 non-zero entry per row and at most 1 non-zero entry per column.
  - $R_{ji} \in R^{\mathcal{H} \times \mathcal{H}}$  is a diagonal matrix for  $j = 1, 2, 3, 4$  and  $i = 1, 2, \dots$
- Hence, given values of  $CE^1D$ ,  $CE^2D$  and  $CE^3D$ , are relatively simpler non-linear expressions of entries in C, E and D.

# Concluding remarks

## Summary

- Connectivity restriction can be leveraged to learn the dynamical model with partial observability.
- These properties may be applicable to other domains
- Identifying properties of non-linear optimization model

## Future work

- Relaxing assumptions such as connectivity restriction and using smaller values of  $k$ .

Questions?

Thank you

# Benders Decomposition approach

- Reformulate **budget-k-max-loss** problem as **target-loss-min-cardinality** problem. Let  $L_{target}$  be minimum target loss.

Attacker Master problem

- Initialize with no cuts

$$\min \sum_i \delta_i$$

s. t. Bender cuts

$$\delta_i \in \{0,1\}$$

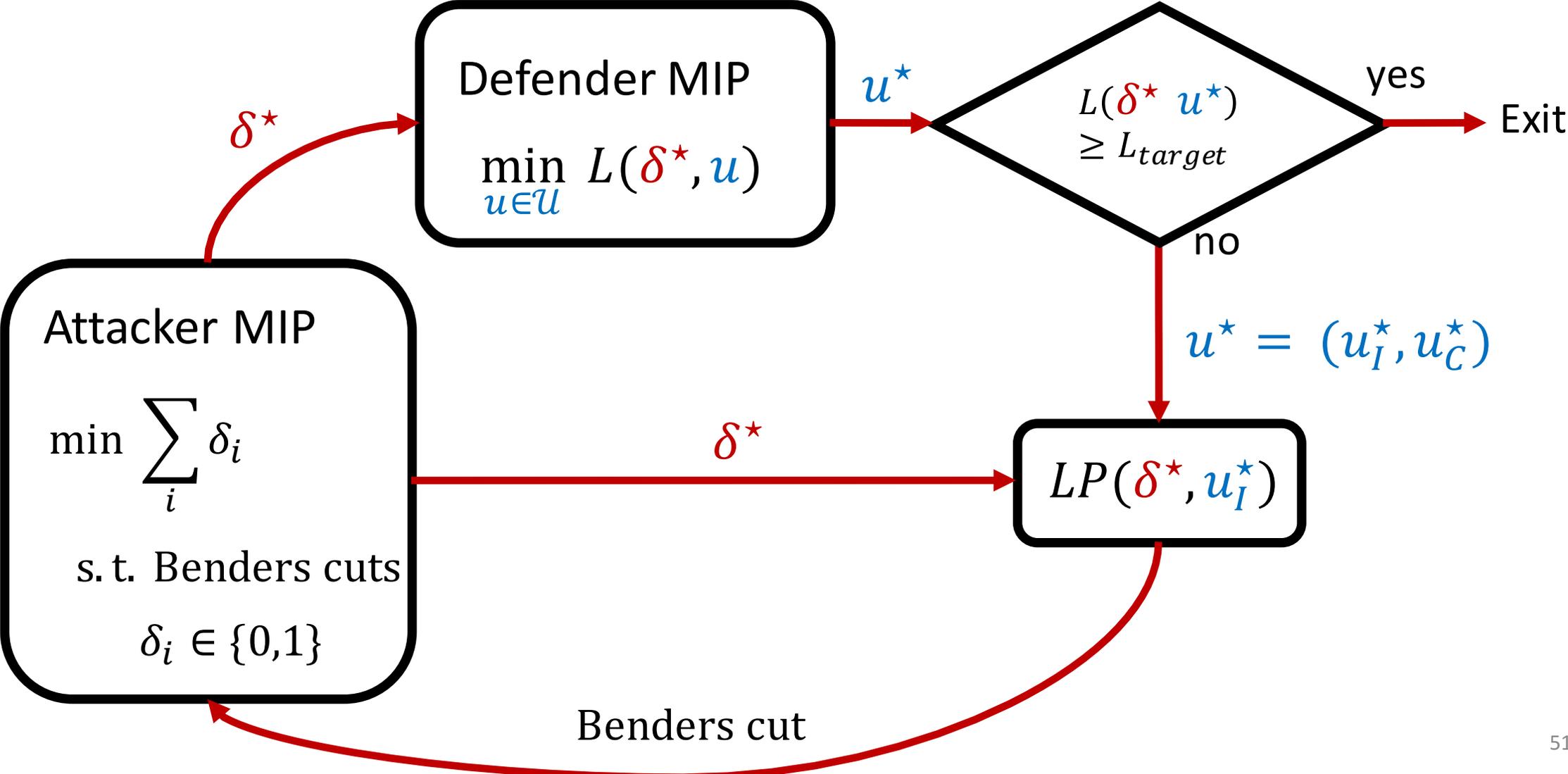
Defender problem (Same as Stage III)

$$\min_{u \in \mathcal{U}} L(\delta, u)$$

s.t.

- Network constraints
- Component constraints
- Voltage bounds

# Benders Decomposition approach



# Benders Decomposition approach

$$LP(\delta^{iter}, u_I) \equiv \begin{aligned} &\min c^T y \\ &s. t. Ay \geq b + Q\delta^{iter} \end{aligned}$$

Fixed attacker strategy for current iteration

Response with fixed integer values

Benders cut

$$\lambda^{*T} (b + Q\delta) \geq L_{target} + \epsilon$$

Optimal dual vector solution to LP

Right hand side of LP

Small number  $\approx 10^{-6}$

# Technical Detail

- Bad Benders cuts may arise
  - If no Stage III constraints have non-zero coefficients for both attack variables and continuous inner variables
  - Which indeed is the case in our problem!
  - May perform as badly as brute force!
- Suggestion! Approximate reformulation?
  - Ensure positive coefficients of attack variables in constraints having continuous inner variables
  - Significant computational speed-up
    - Solutions for 118 node network obtained in less than 2 minutes
  - Approximation error produces sub-optimal min-cardinality attacks

# Resilience-Aware OPF - Trilevel formulation

pre-contingency  
state  $x^o$

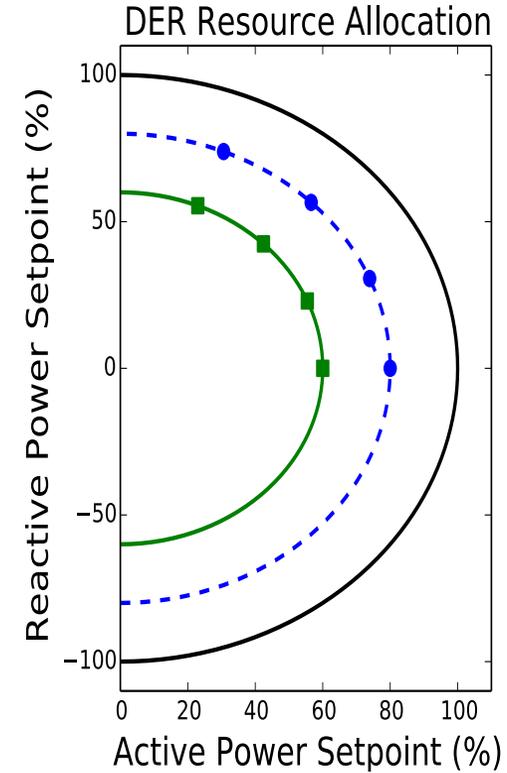
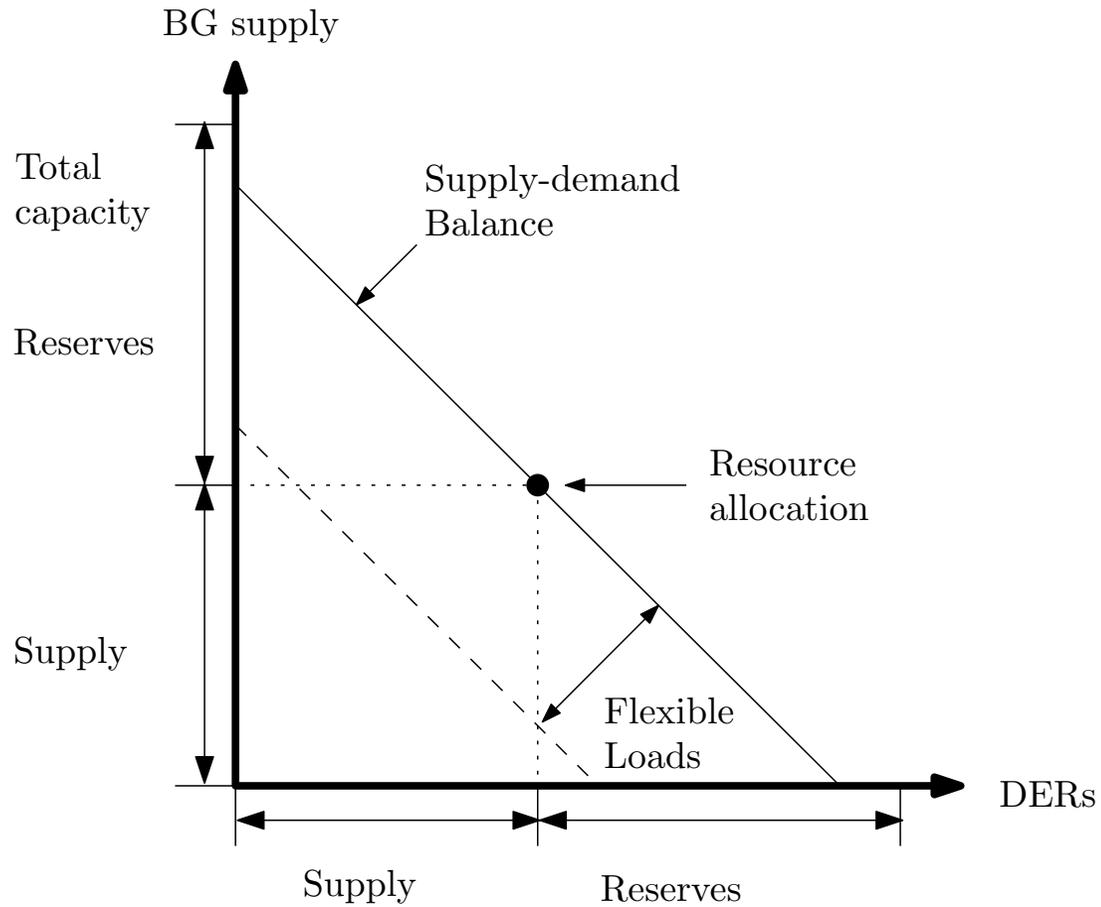
post-contingency  
state  $x^c$

$$\min_{a \in \mathcal{A}} C_{alloc}(a) + \max_{d \in \mathcal{D}} \min_{u \in \mathcal{U}} L(a, d, u)$$

Subject to

- Network constraints
- Component constraints
- Voltage constraints

# Resiliency-aware Resource Allocation (Stage I)



Stage I - Allocation of DERs over radial networks

a. Size and location

b. Active and reactive power setpoints ( $x^n$ )?

Suppose, some controllable DERs are not vulnerable to attack.

# Resiliency-Aware OPF - Trilevel formulation

Frequency deviation model

$$f^{\text{nom}} - f^c = -f^{\text{reg}}(P_0^o - P_0^c)$$

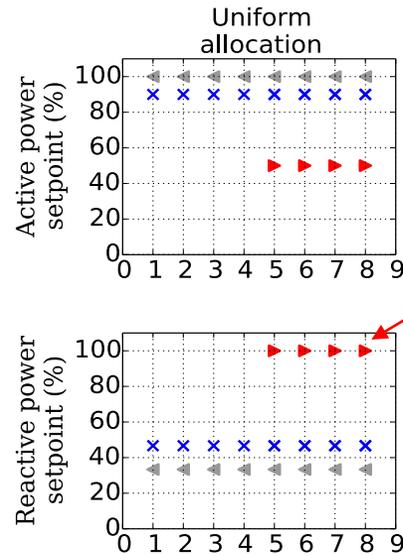
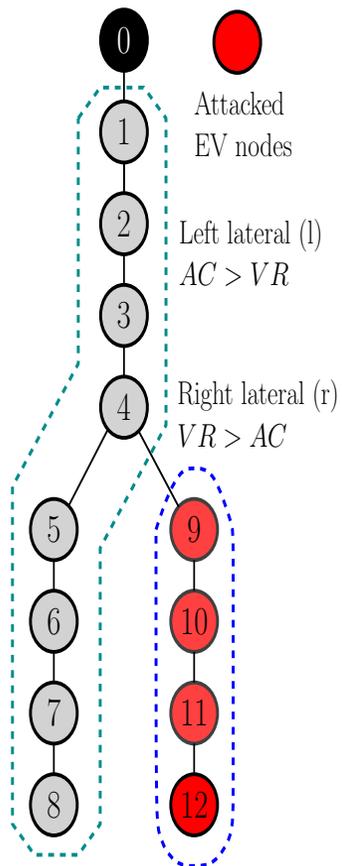
Voltage deviation model

$$v^{\text{nom}} - v_0^c = -v^{\text{reg}}(Q_0^o - Q_0^c)$$

Pre-contingency resource allocation

$$a = (pg^o, qg^o)$$

# Defender Response and Allocation: Diversification



- Some DERs contribute to  $L_{VR}$  more than  $L_{AC}$ , and vice versa
- Diversification holds for “heterogeneous allocation” with downstream DERs with more reactive power
- Post-contingency losses are the same for uniform vs. heterogeneous resource allocations
- Pre-contingency voltage profile is better for heterogeneous resource allocation

# Going from LPF to NPF

Lower and upper bound the optimal loss for non-linear power flows with optimal losses computed using linear power flows.

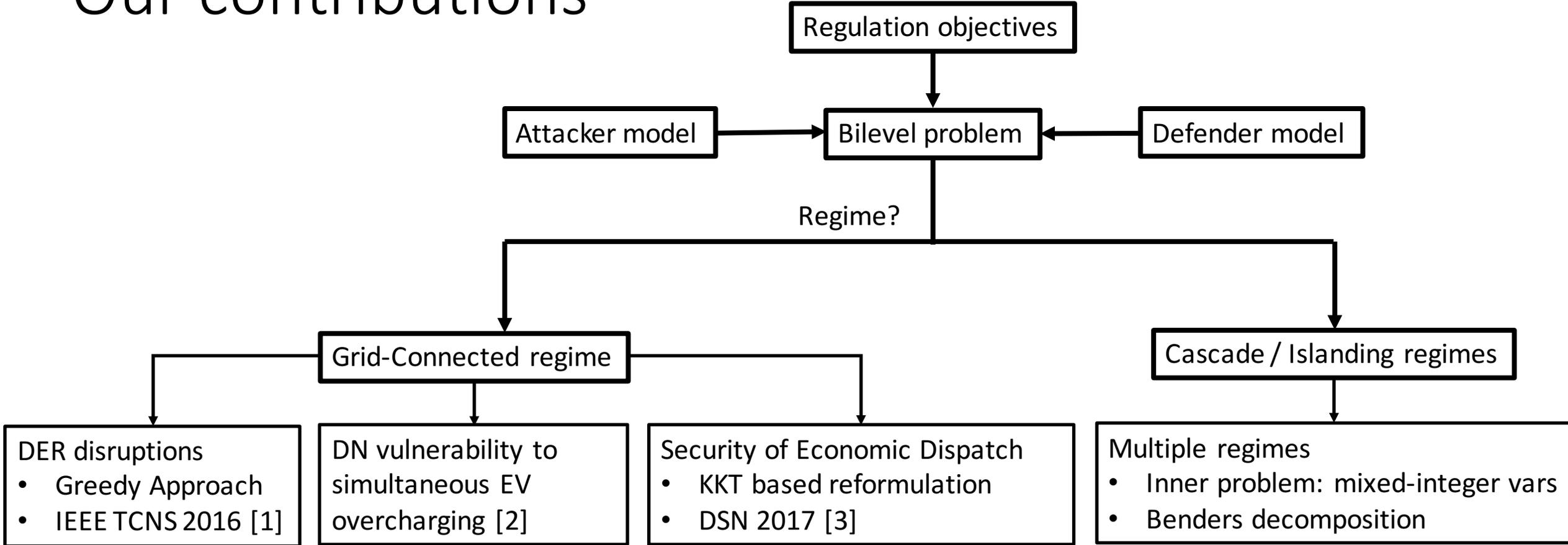
**Theorem:** Let  $\mathcal{L}$ ,  $\hat{\mathcal{L}}$ , and  $\check{\mathcal{L}}$  denote the optimal losses using NPF, LPF, and  $\epsilon$ -LPF respectively. Then,

$$\hat{\mathcal{L}} \leq \mathcal{L} \leq \check{\mathcal{L}} + \frac{\underline{\mu}N}{2\underline{\mu} + 4}.$$

## Remarks

- For  $\underline{\mu} = 0.5$ ,  $N = 37$ ,  $\frac{\underline{\mu}N}{2\underline{\mu} + 4} = 3.7$ . With typical  $\epsilon$  (max. ratio of line loss to power flows), the gap between the bounds is small (3-5%).

# Our contributions

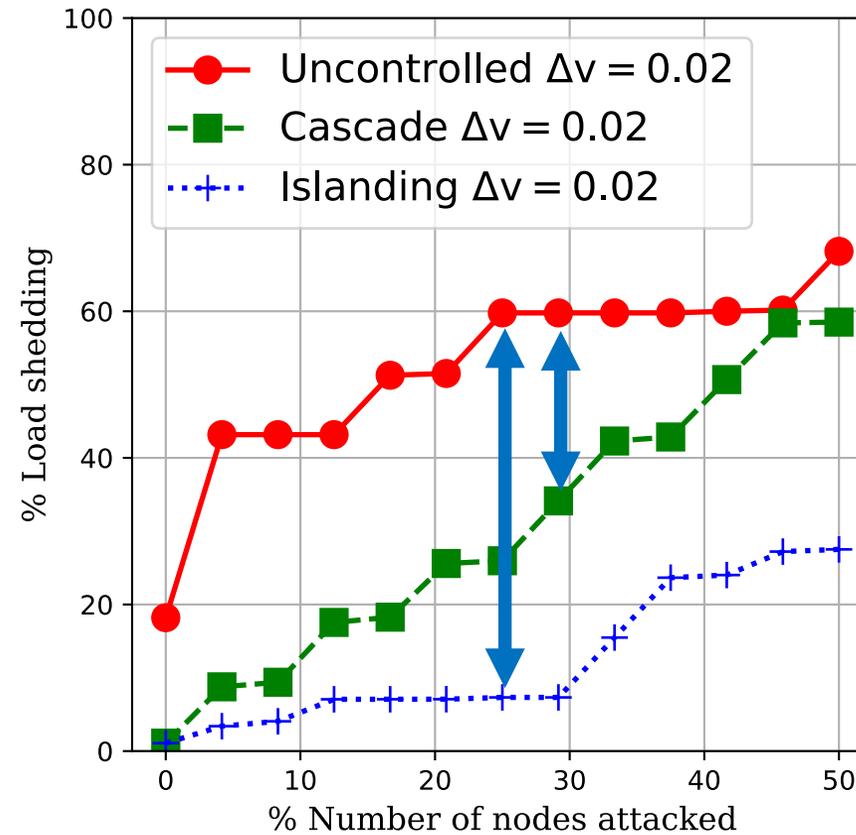
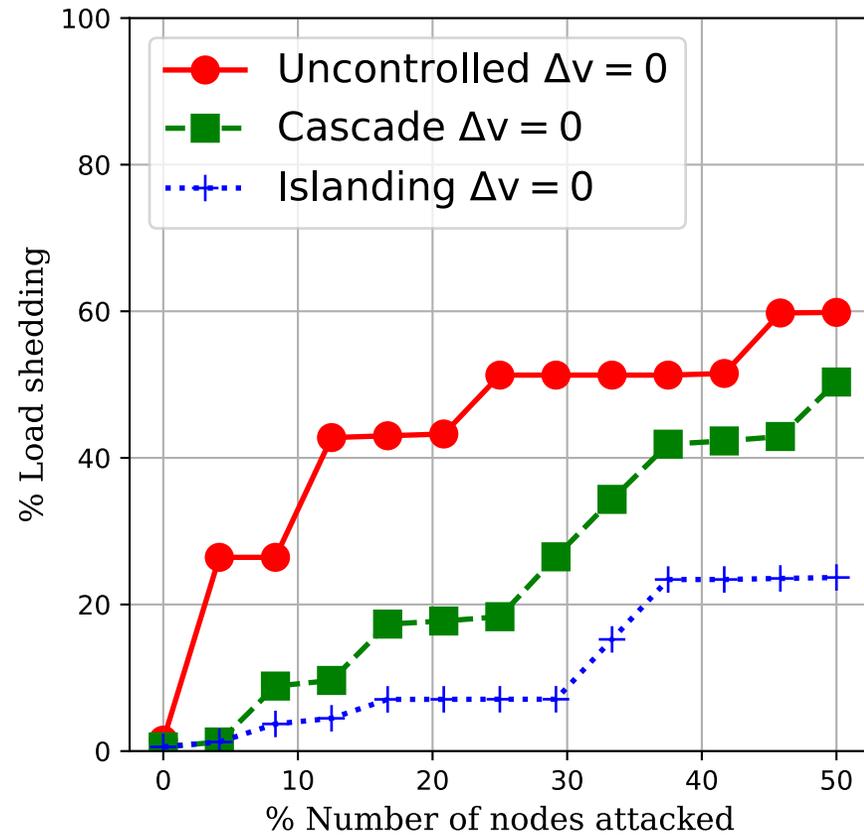


[1] Shelar D. and Amin. S - "Security assessment of electricity distribution networks under DER node compromises"

[2] Shelar D., Amin. S and Hiskens I. – "Towards Resilience-Aware Resource Allocation and Dispatch in Electricity Distribution Networks"

[3] Shelar D., Sun P., Amin. S and Zonouz S. - "Compromising Security of Economic Dispatch software"

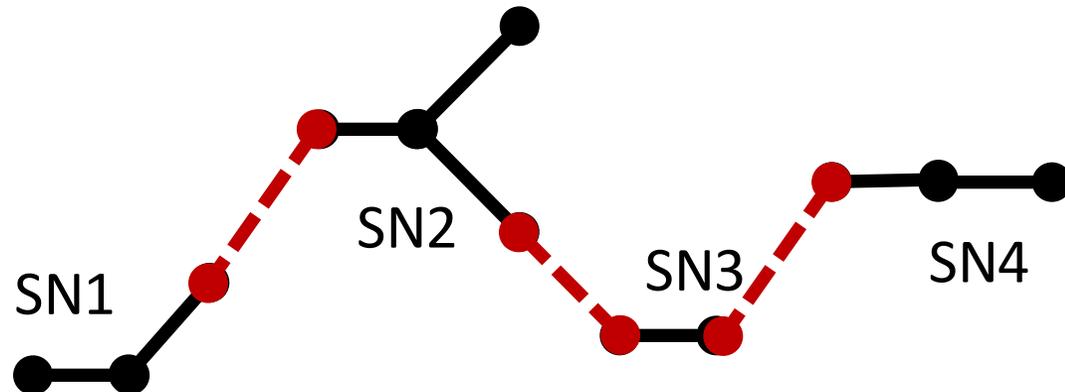
# Uncontrolled vs Cascade vs Islanding



Value of timely  
islandings

N = 24

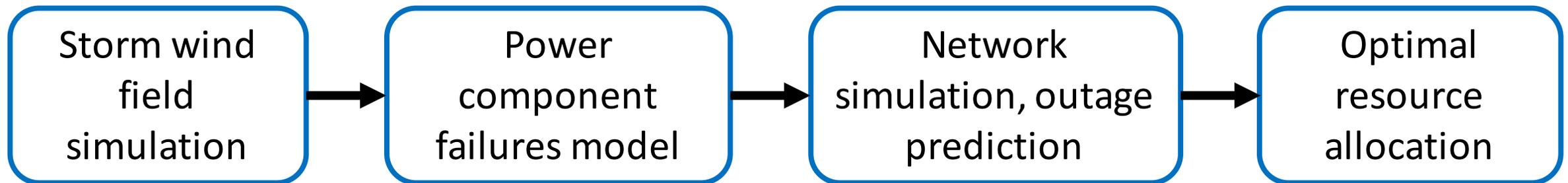
# Strategic deployment of portable DERs for post-hurricane power restoration efforts



- A simpler problem
  - Given
    - set of subnetworks
    - repair times of lines
    - inventory of portable DERs with varying capabilities
  - Question
    - What is optimal deployment of portable DERs such that lost demand is minimized?

# Portable DERs for power restoration

- More challenging problem
  - What is the optimal deployment of portable DERs before the hurricane to minimize expected lost demand?



# Technical detail

Original constraints

$$\begin{aligned}kg_i &\geq \delta_i \\pg_i &= (1 - kg_i) \overline{pg}_i \\qg_i &= (1 - kg_i) \overline{qg}_i\end{aligned}$$

LP constraints

$$\begin{aligned}0 &\geq 0 \\1 &\geq 0 \\1 &\geq 1\end{aligned}$$

Reformulated constraints: Choose  $\eta = 10\epsilon$

$$\begin{aligned}kg_i &\geq \delta_i \\pg_i &= (1 - (1 - \eta)kg_i - \eta\delta_i) \overline{pg}_i \\qg_i &= (1 - (1 - \eta)kg_i - \eta\delta_i) \overline{qg}_i\end{aligned}$$

Cases

$$\delta_i = 1, kg_i = 1 \checkmark$$

$$kg_i = 0, \delta_i = 0 \checkmark$$

$$kg_i = 1, \delta_i = 0 ?$$

# Going from LPF to NPF

**Theorem:** Let  $\mathcal{L}$ ,  $\hat{\mathcal{L}}$ , and  $\check{\mathcal{L}}$  be optimal solutions to attacker-defender game under NPF, LPF, and  $\epsilon$ -LPF respectively; and denote the optimal losses by, respectively. Then,

$$\hat{\mathcal{L}} \leq \mathcal{L} \leq \check{\mathcal{L}} + \frac{\underline{\mu}N}{2\underline{\mu} + 4}.$$

## Remarks

- Voltages for  $\hat{\mathcal{L}}$  (*resp.*  $\check{\mathcal{L}}$ ) upper (*resp.* lower) bound voltages for  $\mathcal{L}$
- Power flows for  $\hat{\mathcal{L}}$  (*resp.*  $\check{\mathcal{L}}$ ) lower (*resp.* upper) bound power flows for  $\mathcal{L}$
- For  $\underline{\mu} = 0.5, N = 37, \frac{\underline{\mu}N}{2\underline{\mu}+4} = 3.7$ . With typical  $\epsilon$  (max. ratio of line loss to power flows), the gap between the bounds is small (3-5%).
- Better bounds can be derived

# Two simpler problems

$$\begin{array}{l} \widehat{\mathcal{L}} \text{ (LPF model)} \\ \check{\mathcal{L}} \text{ (\epsilon-LPF model)} \end{array} \equiv \left\{ \begin{array}{l} \max_{\delta} \min_{\phi} L(x(\delta, \phi)) \\ \text{s. t. constraints,} \\ \text{linear power flow (LPF) or (\epsilon - LPF)} \end{array} \right.$$

$$\text{LPF state: } \hat{x} = [\hat{v}, \hat{\ell}, sc, sg, \hat{S}] \in \hat{\mathcal{X}}$$

$$\hat{S}_{ij} = \sum_k \hat{S}_{jk} + s_j + \cancel{z_{ij} \ell_{ij}}$$

$$\hat{v}_j = \hat{v}_i - 2\mathbf{Re}(\bar{z}_{ij} \hat{S}_{ij}) + \cancel{|z_{ij}|^2 \ell_{ij}}$$

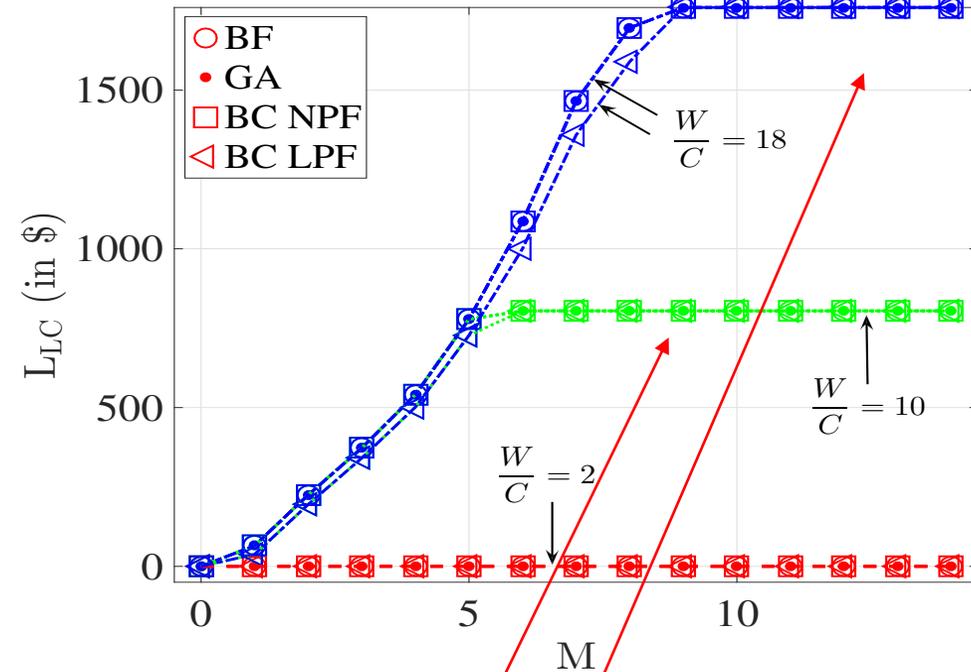
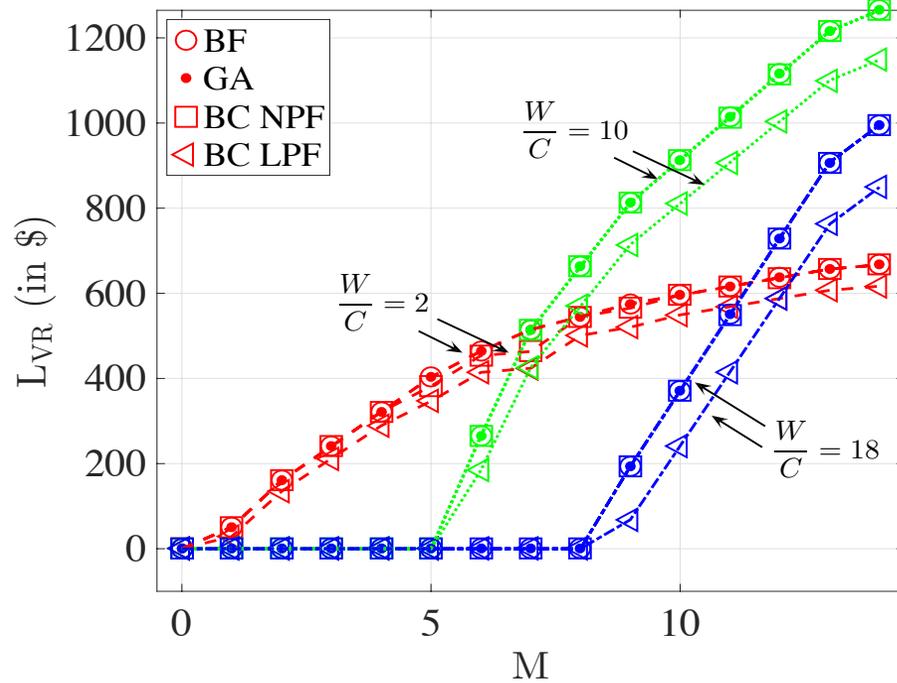
$$\epsilon\text{-LPF state: } \check{x} = [\check{v}, \check{\ell}, sc, sg, \check{S}] \in \check{\mathcal{X}}$$

$$\check{S}_{ij} = \sum_k \check{S}_{jk} + (1 + \epsilon)s_j$$

$$\check{v}_j = \check{v}_i - 2\mathbf{Re}(\bar{z}_{ij} \check{S}_{ij})$$

$\epsilon$  chosen based on the size of the tree network and the max ratio of line losses to power flows

# Structure of attacks



- Downstream nodes are more critical for voltage regulation
- Greedy approach computes “near-optimal” solutions
- Load control is not effective for higher intensity attacks
- Load control reaches higher saturation levels for higher weightage for  $L_{VR}$

# Defender model (Cascade regime)

Defender response:  $u = (\beta, kg, kc)$

$$kg_i = \begin{cases} 1, & \text{if DG } i \text{ is disconnected} \\ 0, & \text{otherwise.} \end{cases}$$
$$kc_i = \begin{cases} 1, & \text{if load } i \text{ is disconnected} \\ 0, & \text{otherwise.} \end{cases}$$

Connectivity condition:

$$kg_i = 0 \quad \Rightarrow \quad v_i \in [\underline{vg}_i, \overline{vg}_i]$$
$$v_i \notin [\underline{vg}_i, \overline{vg}_i] \quad \Rightarrow \quad kg_i = 1$$



Voltage  
bounds for DG  
Similarly  
for loads!

Defender response:

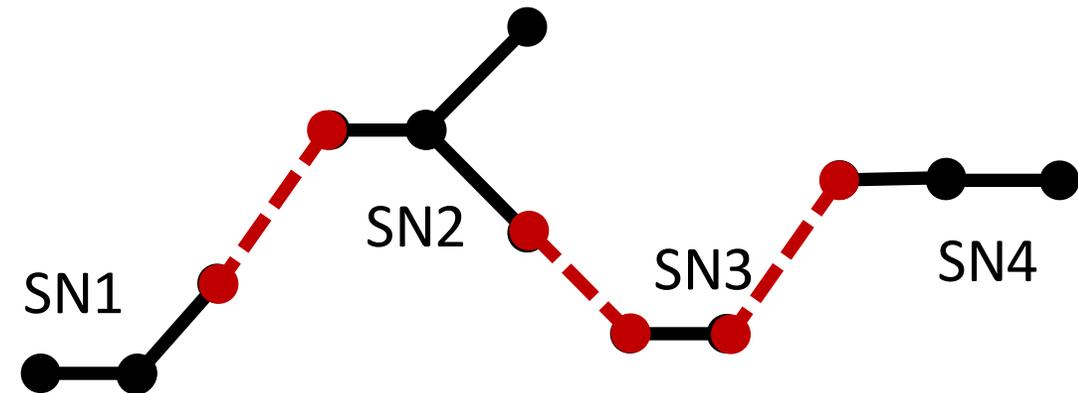
Which loads and DGs to disconnect?

# Strategic deployment of portable DERs for post-hurricane power restoration efforts

- Damage to lines result in subnetworks (SNs)
- Usual restoration steps are:
  - Repair the damaged lines
  - Connect to main grid
  - Restore the power supply



- How can portable DERs help?



# Literature survey

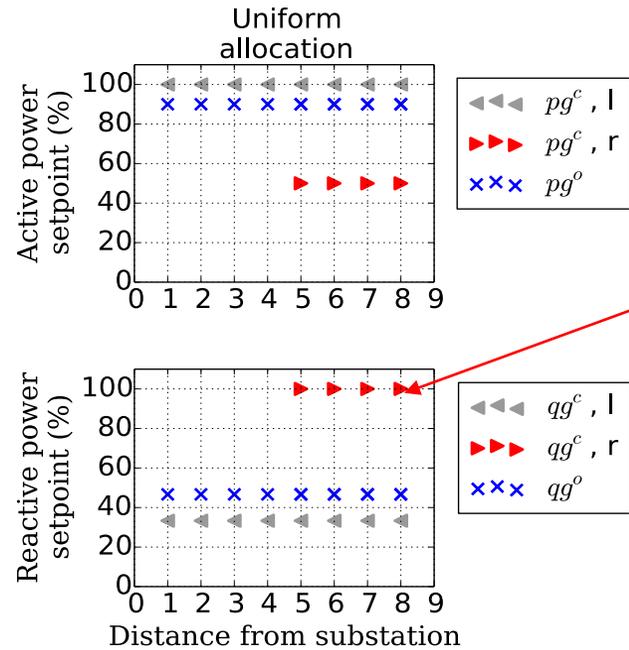
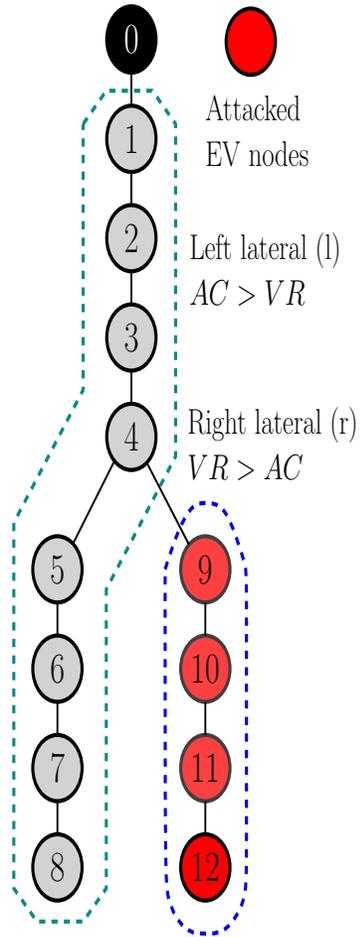
## **(T1) Interdiction and cascading failure analysis of power grids**

- R. Baldick, K. Wood, D. Bienstock: Network Interdiction, Cascades
- A. Verma, D. Bienstock: N-k vulnerability problem
- D. Papageorgiou, R. Alvarez, et al.: Power network defense
- X. Wu, A. Conejo: Grid Defense Planning

## **(T2) Data-integrity attacks**

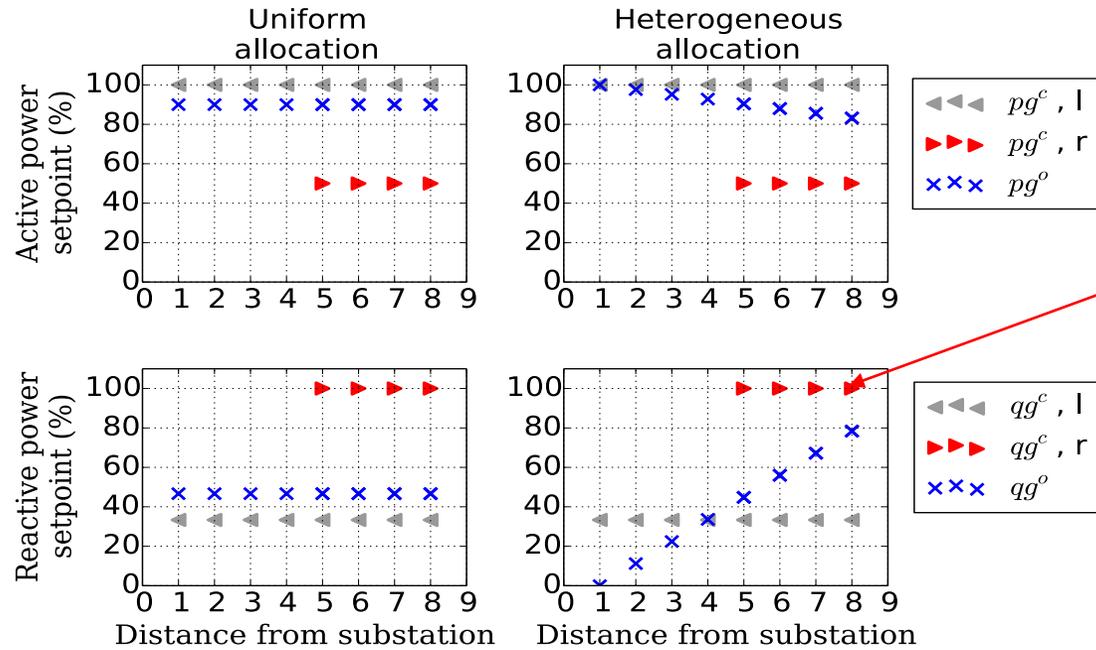
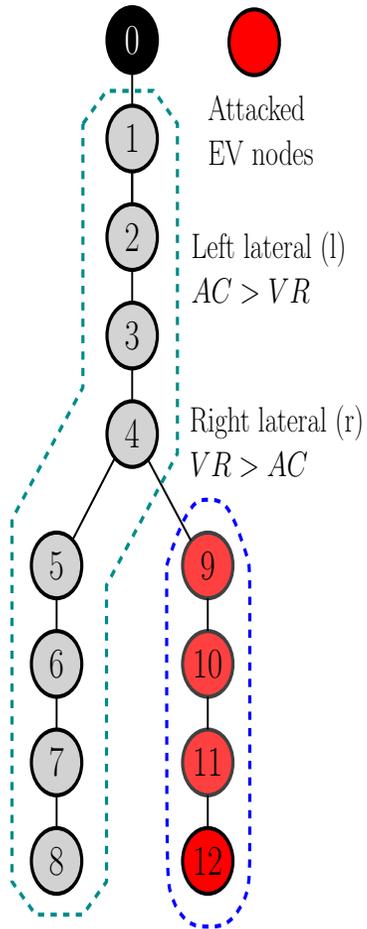
- E. Bitar, K. Poolla, A. Giani: Data integrity, Observability
- H. Sandberg, K. Johansson: Secure control, networked control
- B. Sinopoli, J. Hespanha: Secure estimation and diagnosis

# Defender Response and Allocation: Diversification



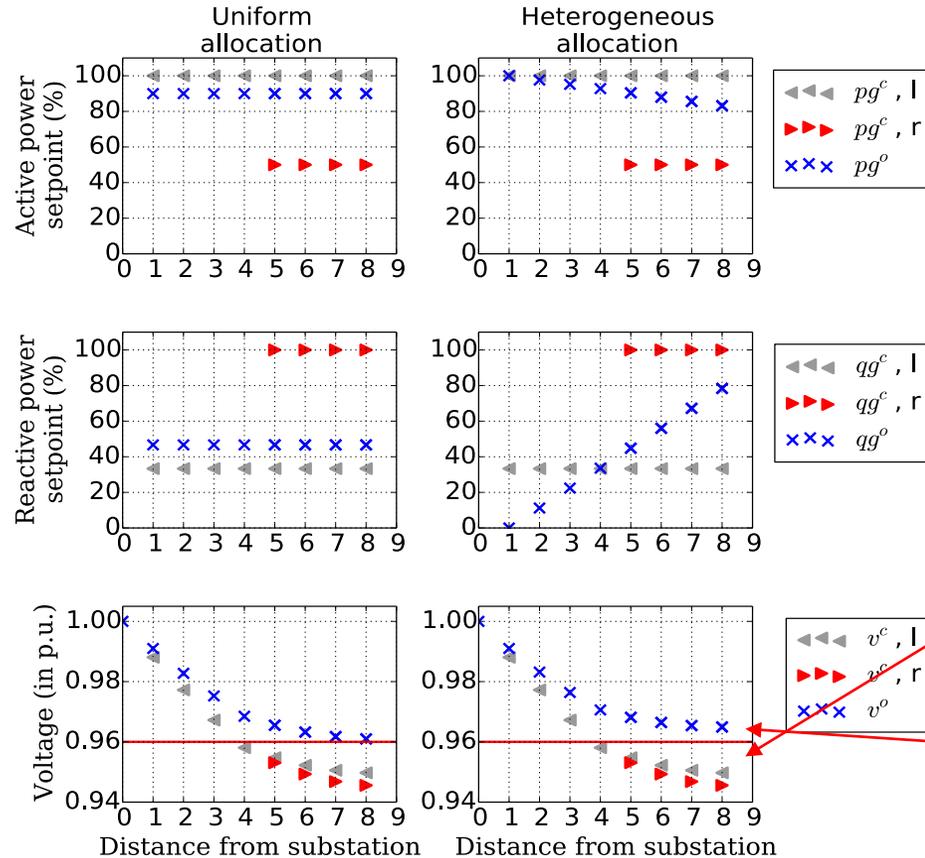
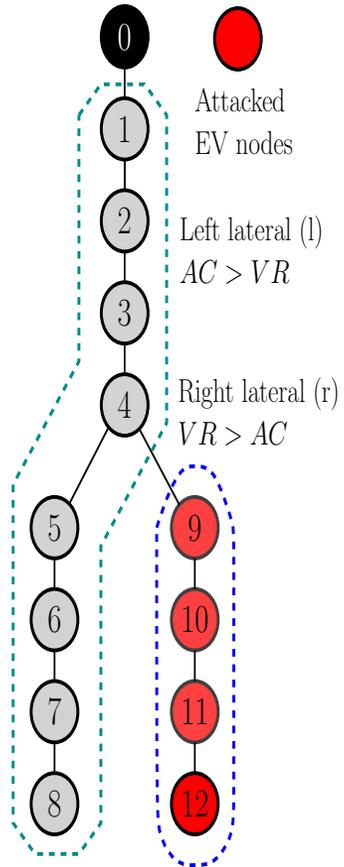
- Some DERs contribute to  $L_{VR}$  more than  $L_{AC}$ , and vice versa

# Defender Response and Allocation: Diversification



- Diversification holds for “heterogeneous allocation” with downstream DERs with more reactive power

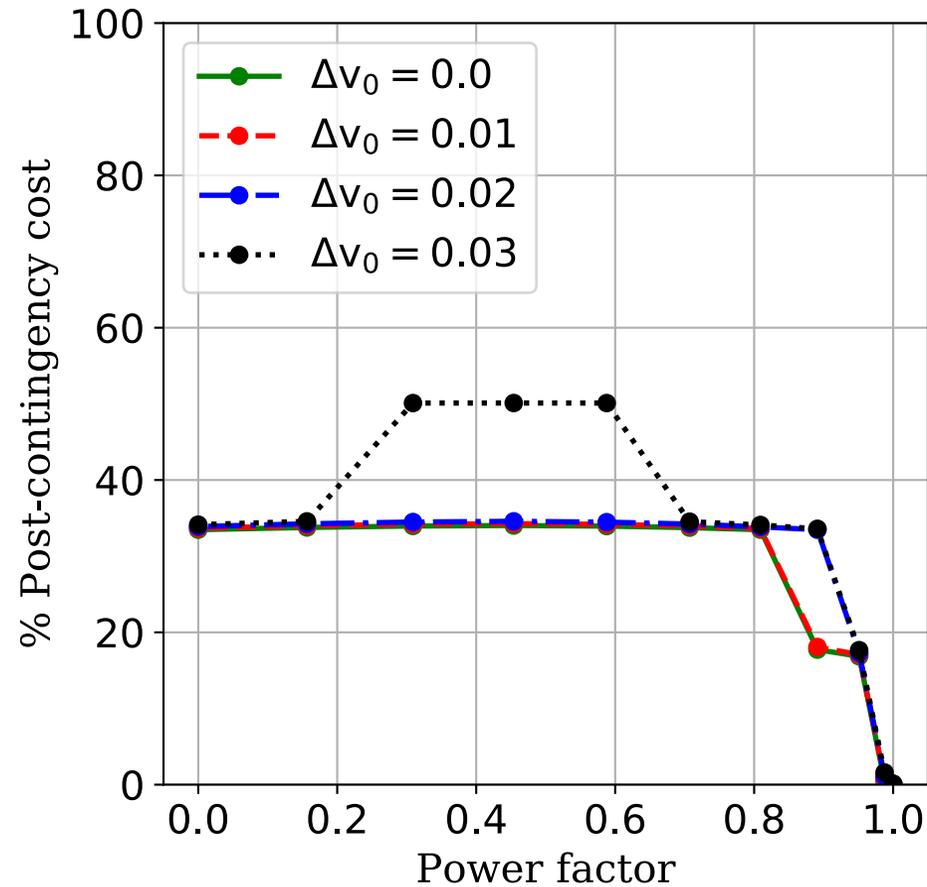
# Defender Response and Allocation: Diversification



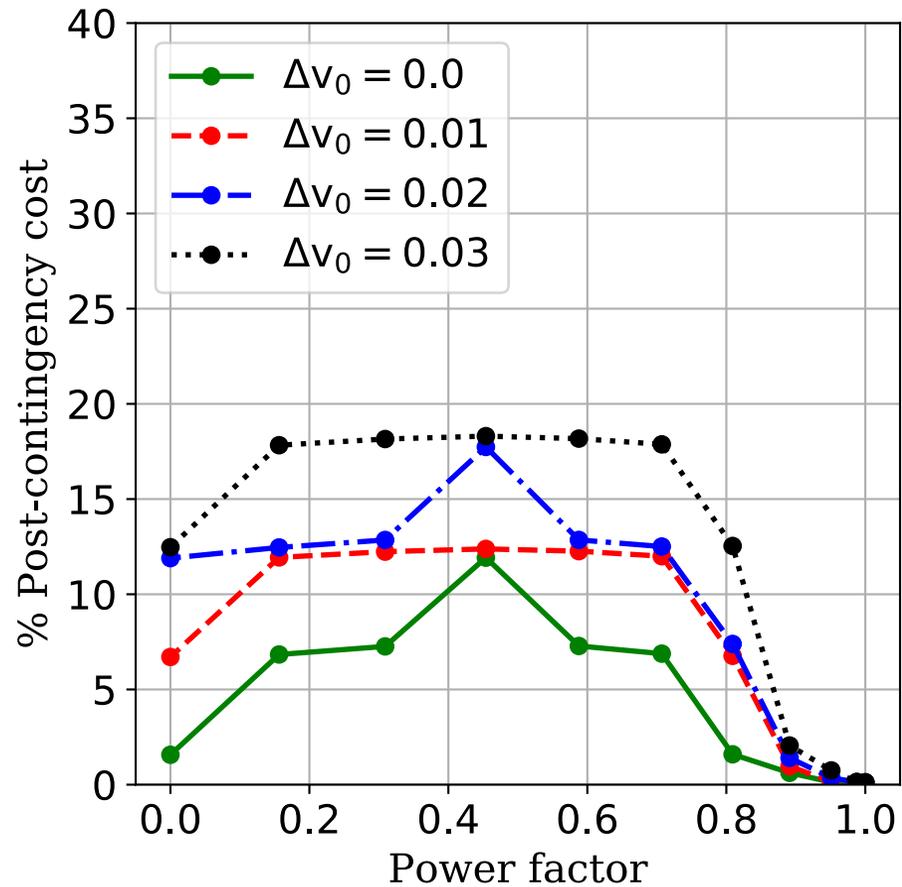
- Post-contingency losses are the same for uniform vs. heterogeneous resource allocations
- Pre-contingency voltage profile is better for heterogeneous resource allocation

Heterogeneous resource allocation can support more loads than uniform one.

# Effect of power factor on losses



$N = 12$



$N = 36$

# Optimal attacker set-points

Typically,

- **Small line losses:** in comparison to power flows
- **Small impedances:** sufficiently small line resistances

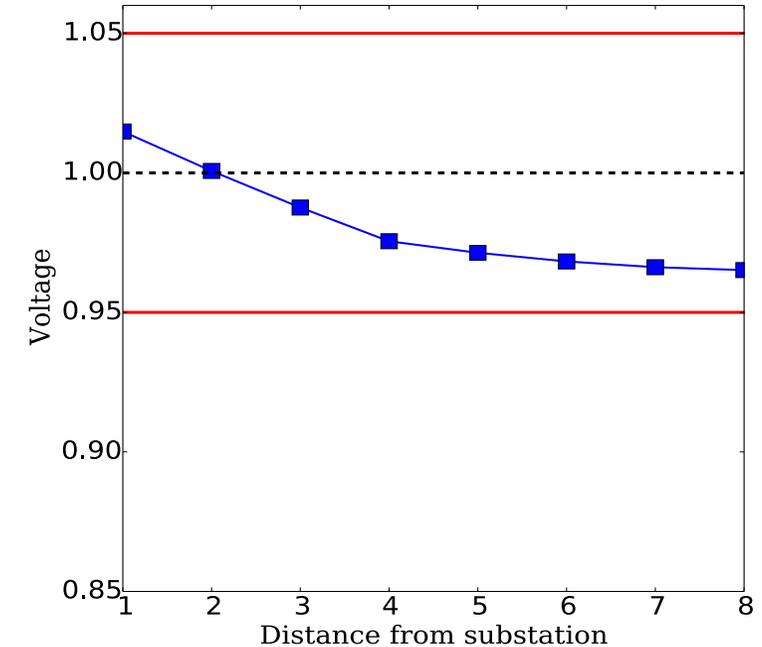
Assume for simplicity:

- **No reverse power flows:** power flows from substation to downstream

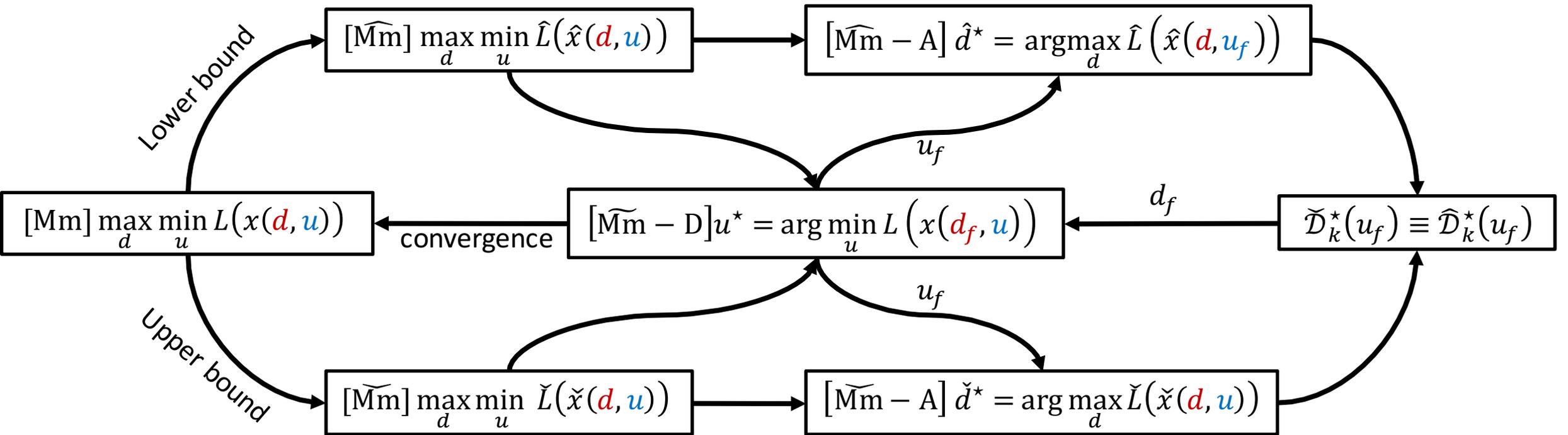
**What are optimal attacker set-points?**

**Proposition:** For a defender action  $\phi$ , and given attacker choice of  $\delta$ , the optimal attacker set-point is given by:

$$pd^{a^*} = 0, \quad qd^{a^*} = -j \overline{sg}_i$$



# Greedy Approach



For fixed defender action:

- For a fixed attacker action, the ordering of nodes with respect to their voltages remain the same between  $\hat{L}$  and  $\check{L}$
- For any fixed node, the ordering of optimal attacker actions with respect to their impact on this node remains the same between  $\hat{L}$  and  $\check{L}$

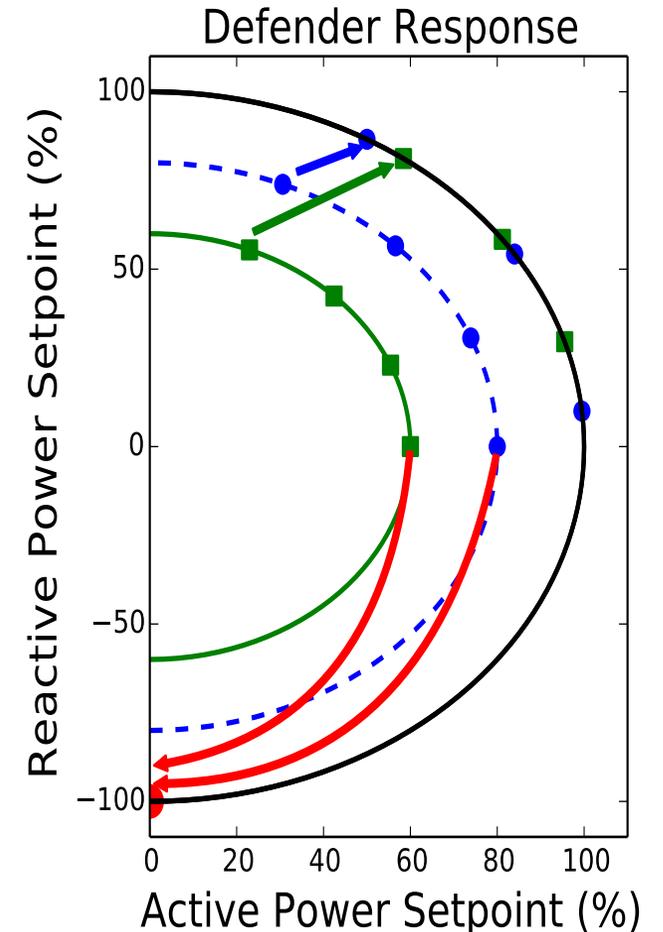
# Defender model

- Defender response:  $u = (pr, qr, \beta)$
- $pr_i, qr_i$  : active and reactive power output of reserves (controllable DGs) at node  $i$ 
  - $0 \leq pr_i \leq \overline{pr}_i, pr_i^2 + qr_i^2 \leq \overline{sr}_i^2$
- $\beta_i \in [\underline{\beta}_i, 1]$ : load control parameter at node  $i$ 
  - $pc_i = \beta_i \overline{pc}_i, qc_i = \beta_i \overline{qc}_i$

Defender response:

How to optimally dispatch reserves?

How much load control should be exercised?



# Optimal interdiction plan: fixed defender choices

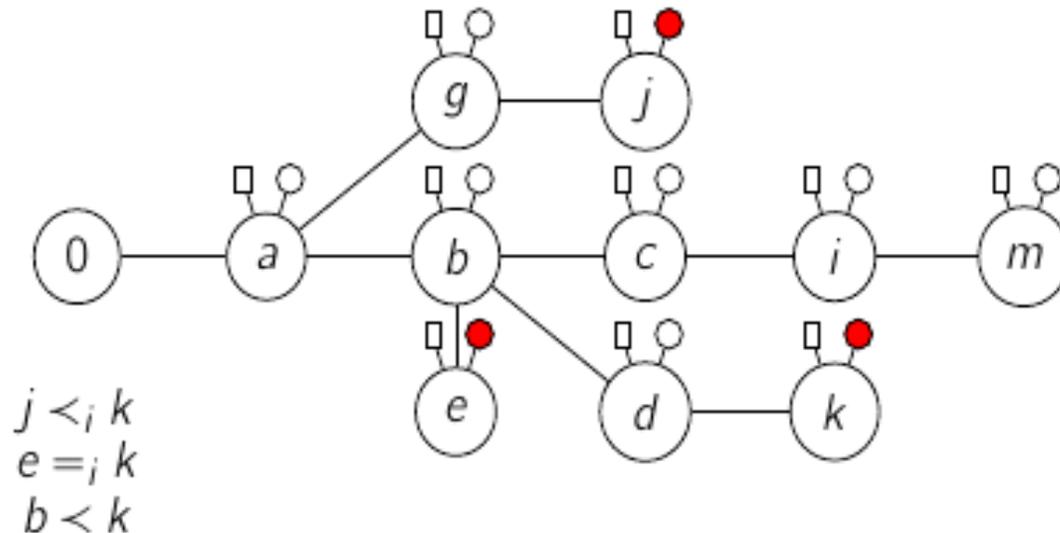
## Proposition

For a tree network, given nodes  $i$  (pivot),  $j, k \in \mathcal{N}$  :

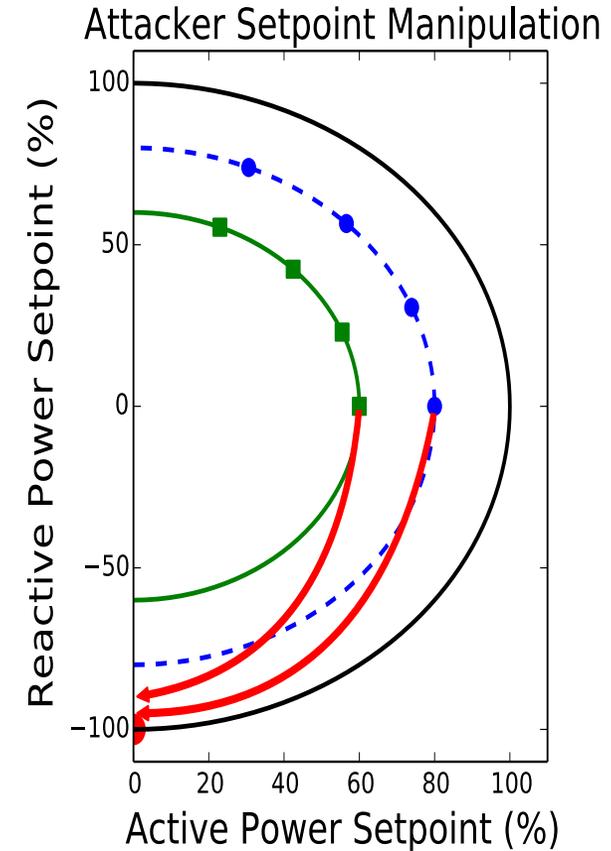
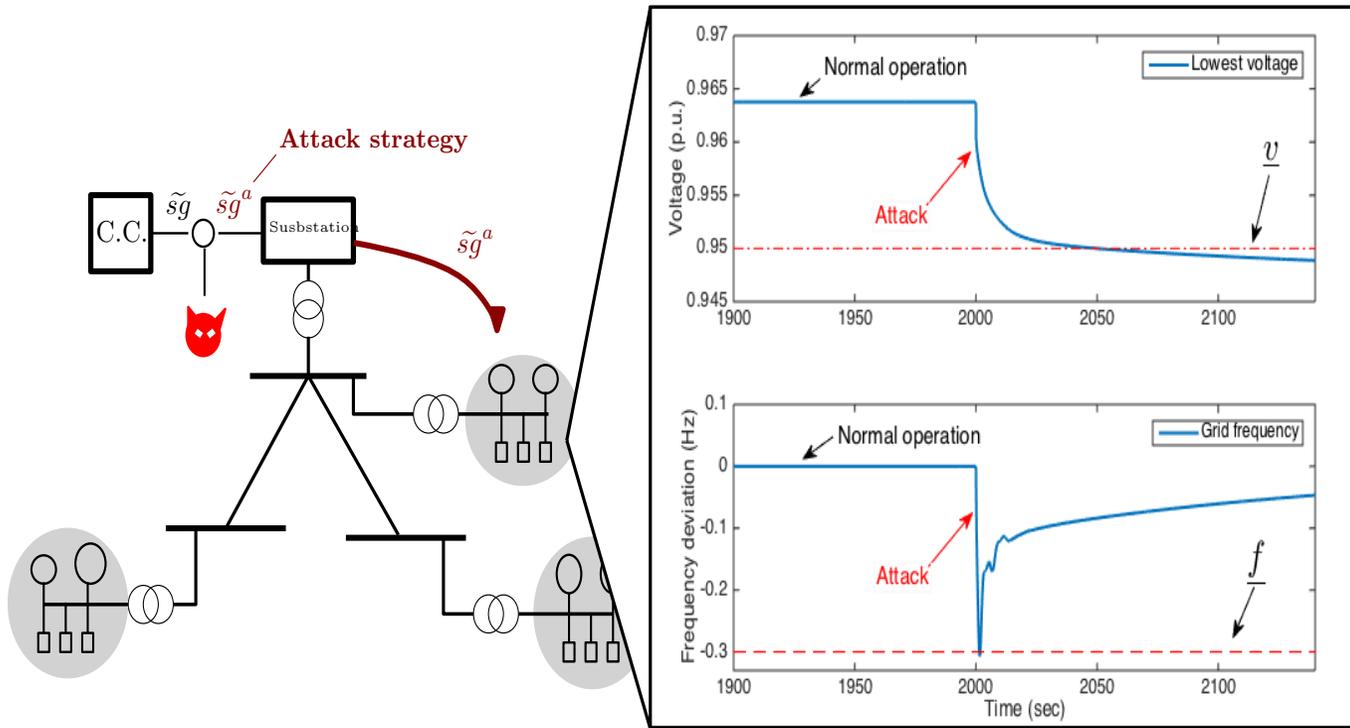
- If DGs at  $j, k$  are homogeneous and  $j$  is before  $k$  w.r.t.  $i$ , then DG disruption at  $k$  will have smaller effect on  $v_i$  (relative to disruption at  $j$ )
- If DGs at  $j, k$  are homogeneous and  $j$  is at the same level as  $k$  w.r.t.  $i$ , then DG disruptions at  $j$  and  $k$  will have the same effect on  $v_i$

$$\Delta_j(v_i) < \Delta_k(v_i)$$

$$\Delta_e(v_i) \approx \Delta_k(v_i)$$



# Resiliency-aware Resource Allocation (Stage II)



Stage II - Adversarial node disruptions

- Which nodes to compromise ( $\delta$ )?
- Set-point manipulation ( $sp^a$ )?

... can include other attack models

# Resiliency-aware Resource Allocation

Stage I - Allocation of DERs over radial networks

- Size and location
- Active and reactive power setpoints ( $x^n$ )?

Stage II - Adversarial node disruptions

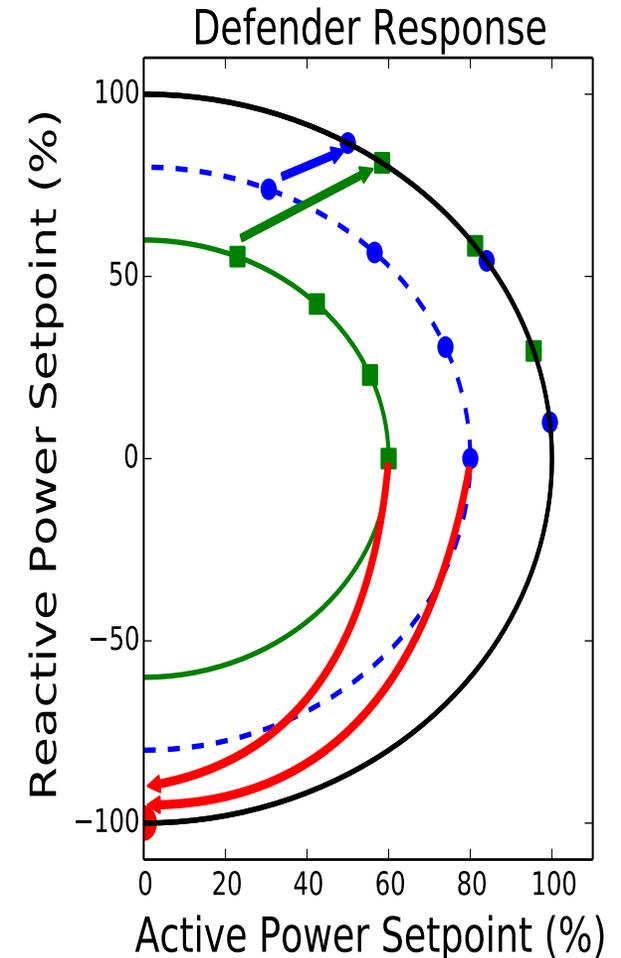
- Which nodes to compromise ( $\delta$ )?
- Set-point manipulation ( $sp^a$ )?

Stage III - Optimal dispatch / response ( $x^c$ )

- Maintain voltage
- Exercise load control or not

## Goals:

- Determine the best resource allocation
- Identify vulnerable / critical nodes
- Determine optimal dispatch post-contingency



# Resiliency-aware Resource Allocation

Stage II - Adversarial node disruptions

- a. Which nodes to compromise ( $\delta$ )?
- b. Set-point manipulation ( $sp^a$ )?

Stage III - Optimal dispatch / response ( $x^c$ )

- a. Maintain voltage
- b. Exercise load control or not

Goals:

1. Identify vulnerable / critical nodes
2. Determine optimal dispatch post-contingency