Boesky Insiders, Informational Inefficiency and Allocative Efficiency*

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Abstract

A dominant, net buyer of a certain asset receives a private signal that is correlated with its mean value. We call this insider a Boesky Insider when the quality of the received signal is such that the future value of the asset can be predicted with certainty. We show that even an *infinitesimal probability* that the insider is a Boesky Insider results in informational inefficiency of prices. We identify reasonable conditions where, if one insists that the equilibrium be continuous in the signal, all equilibria are Strongly Informationally Inefficient in the sense that no information is conveyed.

The informational inefficiency not withstanding, a regime that allows insider trading can result in greater liquidity and is, in an ex-ante sense, Pareto superior when compared to a regime in which insider trading is banned. We conclude therefore, that notions of fairness must be assigned greater priority in judging the desirability of insider trading.

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1 Introduction

"The public's out there throwing darts at a board, sport. I don't throw darts at a board - I bet on sure things" - Gordon Gecko in *Wall Street.*

The notion that agents have heterogeneous beliefs about the value of assets is neither new, nor surprising. In some sense, all participants in securities markets believe that they have superior information to those with whom they trade. In this sense we all consider ourselves insiders to a certain degree. The fact that other traders have different views, based on different information does not lead to a fear of trading, nor a precipitous decline in the liquidity of markets. Yet what if the person with whom we were trading might know the value of that security for sure? What if they were the CEO of a company about which there was takeover speculation; of a company which was about to announce a large earnings downgrade, or upgrade; of a company which was about to file for bankruptcy? Market participants, regulators and commentators fear the possibility of just such an insider, and it is a major motivation for bans on insider trading.

We examine the above issues by modelling a securities market involving a dominant, price setting insider. The insider receives private information about a parameter that is correlated with the future value of a certain asset, and sets the unit price at which she is willing acquire it. If this estimate is perfect, so that the insider *knows* the future value, then we call her a Boesky Insider[1]. A significant innovation of this work is to allow for the presence of such insiders in an otherwise typical model used to study information transmission involving dominant traders.

Our findings are surprising. For instance, it turns out that even if the likelihood of a Boesky Insider is *infinitesimally small*, prices are necessarily informationally inefficient in that sense that in every equilibrium the price is a constant for a positive mass of signals. What is more, if one were to follow the literature and restrict attention to equilibria that are continuous (let alone linear) in the signal, then the equilibrium is informationally strongly inefficient, i.e. fully pooling.

The above is in sharp contrast with the traditional law and economics view, dating at least to Manne (1961), that insider trading promotes informational efficiency. We therefore go on to study the welfare effects of informational inefficiency by relating them to the outcomes under a regulatory regime that bans insider trading. We show that despite this informationally inefficiency, a regulatory regime which requires full disclosure of private information prior to trade, *and is even able to perfectly enforce this*, may not be welfare increasing. We provide a class of examples where regulation leads to a reduction in welfare.

In the remainder of this introduction we present a brief intuition for the informational inefficiency, discuss the regulatory aspects of insider trading and provide a brief review of the related literature. Section 2 contains the main results on informational inefficiency, namely Proposition 1 and Proposition 2. Section 3 introduces the legal restriction on insider trading and discusses welfare effects of the informational inefficiency and the relative merits of regulation, the main result here being Proposition 5. Section 3 can be read largely independently of the previous section. Section 4 concludes the paper with a discussion of various aspects of the model and results.

[1] The colorful term "Boesky" insider is suggested by the activities of the infamous insider trader Ivan Boesky.

1.1 Brief Intuition For Informational Inefficiency

In the model we study, the insider privately observes a parameter $\tilde{S}$ that is correlated with the mean of a risky asset. She then acts as a Stackelberg leader and sets a price $p(\tilde{S})$ at which she is willing to acquire the asset. The insider is risk neutral while the small traders are risk averse. When the signal $\tilde{S}$ is a noisy estimate of the mean value of the asset, for risk sharing reasons, there are gains from trade even if it becomes common-knowledge. On the other hand if a particular realization of the signal, say $\tilde{S} = s^*$, is fully informative of the future value of the asset so that the dominant trader is a Boesky insider, there is no further risk. Consequently if s^* were common-knowledge among all traders in the market, the otherwise risky asset and the risk-free bond are perfect substitutes.

Given the above observation, if there were to be a fully revealing equilibrium in which a positive quantity is traded, then the dominant trader is forced to set the price of the asset given s^* to be such that the return on the asset equals the return on the risk-free bond. Otherwise there would be an arbitrage opportunity. To illustrate the argument, assume that equilibrium price is continuous in the signal. (The main result does not involve this restriction.) For values of the signal close to s^*, the differential on the mean return between the bond and the risky asset is negligible. Therefore at a noisy signal sufficiently close s^*, the risk averse traders would strictly prefer to sell almost their entire endowment of the risky asset and invest the proceeds in the bond. Incentive compatibility conditions require the quantity traded to be non-decreasing and force the outsiders to trade their entire endowment for all signals higher than s^*, thus leading to a contradiction that prices is fully revealing.

In a nutshell, the fact that there is a Boesky insider constrains the return on the asset at s^* to equal that of the risk-free bond. Even though the probability of this event is infinitesimal, due to incentive compatibility, it will have repercussions for all other values of s^*

1.2 Regulation

There is an extensive literature, both in Economics and in Law, that addresses the merits of allowing insiders to trade on the basis of privileged information. A highly informative account of this debate can be found in Ausubel [1990b]. The argument made in favor of insider trading maintains that such trading is essential for prices and other public variables to aggregate information. The counter argument is that it leads to a loss in public confidence and ultimately lower liquidity and welfare. Given our results that the mere presence of Boesky Insiders is enough for prices to not reveal information, our purpose in Section 3 is to evaluate the above counter argume.

The regulated regime we consider is a stylization of common regulatory regimes and, in the context of the present model, is designed to be informationally efficient. This rule stipulates that, after receiving the private signal, the insider has two options. She can either abstain from trading in the market, or she can make her private information public and then trade. To bias the case in favor of regulation, public declarations of information assumed to be are perfectly verifiable. Given the structure of our model, it turns out that it is always advantageous for an insider to make her information public and trade on that basis. We refer to this regulated regime as the *RE* regime.

Proposition 5, obtains a comparison of the PE regime and the *RE* regime offers with several interesting insights. First, the pooling equilibrium we construct is such that the liquidity is identical under both regimes. Therefore, informational inefficiency does not imply lower liquidity. Second, depending on the parameter values, the PE regime can Pareto Dominate the *RE* regime while the converse can never be the case. The reason for this is as follows. Under the *RE* regime, it turns out that the quantity acquired by the *Insider* is a constant. The price, however, varies. By construction, the supply under the PE regime is the same as that under the *RE* regime but the price, being in a pooling equilibrium, is also constant. Consequently, the PE regime offers the outsider additional insurance. This makes it possible for an outsider to supply the same quantity as in the *RE* regime at a lower average price and still be as well off. As the *Insider* is risk-neutral, the lower average price at which she acquires the same quantity improves her payoff also. In fact, we show that when the signals are uniformly distributed over a large range, the payoffs under the unregulated PE regime Pareto dominate those under the regulated *RE* regime.

1.3 Related Literature

The early theoretical literature on the efficiency effects of insider trading argued that insiders would reveal their private information (see Allen [1981],Verrecchia [1982],Glosten and Milgrom [1985],Grossman [1976] and Grossman [1989], among others.). These papers assumed a competitive market in the sense that the large trader could not affect the price of the asset through their strategic choice. They find the equilibria that are separating and consequently conclude that prices are informationally efficient. (See Ausubel [1990a] and Ausubel [1990b] however.) The informational efficiency of prices and other actions has also been considered in imperfectly competitive environments, including the works of Gould and Verrecchia [1985], Grinblatt and Ross [1985], Kyle [1985], Kyle [1989], Laffont and Maskin [1990], Altug [1991], Bhattacharya and Spiegel [1991], Benabou and Laroque [1992] and Fishman and Hagerty [1992], among others. Most of these papers restriction attention to strategies that are linear in the signal received.

Ausubel [1990b] and Laffont and Maskin [1990] are among the comparatively few contributions that, to our knowledge, compare the informational and allocative efficiency of insider trading. The latter is an important reconsideration of most of the earlier literature. Laffont and Maskin criticize the preoccupation with linear equilibria in most other works and focus instead on the equilibrium that is most favorable to the insider. Indeed, in the binary signal model they consider, any strategy is trivially linear. They show that the most favorable equilibrium for the insider (whenever such an equilibrium exists) is the one that is fully pooling. The reason that a pooling equilibrium improves the payoff relative to the separating equilibrium is that, in their model, the expected equilibrium price is more or less the same in a separating and a pooling equilibrium. To ensure incentive compatibility however, the quantity traded must be constrained and lower in a separating equilibrium which in turn implies that the insider prefers the pooling equilibrium. Depending on the parameter values, this may even benefit the outsiders. Adopting the Pareto principle for ranking the two equilibria, they then argue that an insider may be able to influence beliefs at an ex-ante stage so that the pooling equilibrium emerges and thus prices are informationally inefficient.

Our model is related to Laffont and Maskin [1990] but is different in two key respects. First, we allow the type of the insider to be a continuous variable. Second, we introduce the notion that there may be Boesky insiders, albeit with an infinitesimal probability. If one insists on the equilibrium being continuous, which as a requirement is clearly weaker than linearity, and is a condition that is not relevant in Laffont and Maskin [1990], our finding is that every equilibrium is fully pooling. Therefore the issue of ranking equilibria that differ in informational content does not arise.

The rationale for why the ban on insider trading does not benefit the insider is also very different to the role played by quantity constraints in the Laffont and Maskin's explanation of a pooling equilibrium being favorable for the insider. In fact, in our model the ban on insider trading acts as a pre-commitment device enabling the insider, being a Stackelberg leader, to fully exploit her dominant position by allowing her to credibly reveal her private information. In particular, she will not face the quantity constraints that are implied by a separating equilibrium. Therefore one might expect that the insider will necessarily prefer the *RE* regime to the PE regime. This, however, is not the case, the reason being that in the PE regime the constant pooling price faced by the outsiders offers them additional insurance. This allows them to supply the same expected quantity as they would under the *RE* regime but for a lower (expected) price.

Ausubel [1990b] offers a public confidence argument[2] for regulating insider trading by introducing an ex-ante investment stage involving potential insiders and outsiders. Potential insiders may prefer regulation at the later trading stage so that it allows for greater investment at the ex-ante stage. Regulation can Pareto-dominate the unregulated regime. Our work omits the ex-ante investment stage although in our model too there is a break down of public confidence resulting from the presence of perfectly informed insiders. The loss in public confidence notwithstanding, regulation may be Pareto-dominated, in contrast to the findings of Ausubel [1990b]. The difference can be explained in part by the fact that there is imperfect competition in our model whereas the markets are perfectly competitive in Ausubel's model. We discuss this further in Section 4.1.

Finally, others have noted that a more informative trading process may have ambiguous welfare effects depending on how the additional information impinges on the risk sharing opportunities. These are often related to the so called Hirshleifer Effect. (See Hirshleifer [1971], Dow and Rahi [2000] and also Marin and Rahi [2000].) Unlike here, these conclusions typically rely on the removal of some extraneous risk factors.

2 The Model

There are two periods, Date 1 and Date 2. Two assets are traded at Date 1, one being a risky asset whose random value at Date 2 is $\tilde{V}$ and the other is a bond that offers a risk free gross return of $R \geq 1$. Prior to trading, another random variable $\tilde{S}$ is realized which is informative of

[2] Benabou and Laroque [1992] also offers a related raionale for regulating Insider trading.

the future value of the risky asset. Assume that the tuple $(\tilde{S}, \tilde{V})$ is jointly distributed according to a probability distribution function $F(\cdot,\cdot)$ with $[a, b] \times \mathbb{R}$ being the support. Let $F(\cdot|s)$ denote the conditional distribution of $\tilde{V}$ given that $\tilde{S} = s$. We assume that the unconditional density of $\tilde{S}$ is continuous and positive on $[a, b]$. Thus it is implicit that there is a continuum of signals. Also assume that $\mu(s) = E[\tilde{V}|\tilde{S} = s]$ is positive, continuous and strictly increasing.

There is a single *Insider*, whom we shall refer to as Player D, who observes the realization of $\tilde{S}$ prior to trade. We shall say that Player D is of type s when she receives the signal that $\tilde{S} = s$. Player D does not have an initial endowment of the of the risky asset but has m units of money. Assuming risk-neutrality, the payoff of type s if she acquires q units of the risky asset at the unit price of ρ is

$$V(\rho, q, s) = mR + (\mu(s) - \rho R)q \tag{1}$$

The market also consists of a continuum of identical *outsiders*. Each outsider begins with an initial endowment of w units of money and one unit of the risky asset. If an outsider sells an amount q of her holdings of the risky asset at a unit price of ρ, then her future wealth is $\tilde{w} = wR + \rho qR + (1 - q)\tilde{V}$.

Outsiders do not observe the signal directly but may learn from the prices. Suppose that after they learn, their revised beliefs regarding the distribution of $\tilde{S}$ is given by a probability distribution function $G(\cdot)$. Note that $G(\cdot)$ is endogenous in equilibrium. Nevertheless, for a given $G(\cdot)$, we can write the expected utility from selling q units at the unit price of ρ as

$$U(q, \rho, G) = \int_a^b E[u(\tilde{w})|\tilde{S} = s]dG(s) \tag{2}$$

where $u(\cdot)$ is the vNM utility of wealth and the expectation that appears in the integrand is with regard to the distribution of $\tilde{V}$ conditional on various signals. Throughout we assume that outsiders are strictly risk averse and that $u(\cdot)$ is twice continuously differentiable.

We model price formation as a simple non-cooperative game of incomplete information. After observing the signal, the *Insider* acts as a Stackelberg leader and sets the price at which she is willing to acquire the risky asset. Each outsider observes the price and forms beliefs about the signal observed by the *Insider*. The *Insider*'s strategy is therefore a function $p : [a, b] \longrightarrow \mathbb{R}_+$. An *outsider's strategy* is a function $Q : \Re_+ \longrightarrow [0, 1]$ where $Q(\rho)$ is the quantity of risky asset that she will supply if offered a unit price of ρ.

Let $\{G_\rho\}_{\rho \geq 0}$ be a family of probability distribution functions, with some measurable subset of $[a, b]$ being the support of each member of this family. We shall refer to $\{G_\rho\}_{\rho \geq 0}$ as as a *system of beliefs* and use it to represent the information of an outsider: given a price ρ, an outsider believes that the *Insider*'s type is distributed according to G_ρ.

Definition 1. *A Perfect Bayesian Equilibrium (PBE) consists of a strategy profile $\{p, Q\}$ and a system of beliefs $\{G_\rho\}_{\rho \geq 0}$ such that:*

i) G_ρ is consistent with Bayes Rule for every $\rho \geq 0$.

ii) $p(s) \in argmax_\rho V(\rho, Q(\rho), s)$ for all s.

iii) $Q(\rho) \in argmax_q U(q, \rho, G_\rho)$ *for all* $\rho \geq 0$.

Condition (i) forces the small traders to have rational expectations while Conditions (ii) - (iii) require that traders' choices are optimal given their beliefs.

2.1 The Boesky Insiders

The model described above is close to the standard framework of much of the finance and economics literature involving rational expectations[3]. The main difference is in how we treat the conditional variance of the asset value, i.e. $\sigma_s^2 = Var(\tilde{V}|\tilde{S} = s)$. If one assumes that σ_s^2 is a constant, the model is the same as the one studied by Laffont and Maskin [1990], except for the fact that we allow for a continuum of signals while they deal with binary signals. The assumption of a constant variance does not allow one to speak of one type of insider as being better informed than another. One extreme form in which this distinction can be made is to think of the signal as either "noisy" or "accurate", in which case σ_s^2 takes one of two values $\{\sigma^2, 0\}$ where $\sigma^2 > 0$. When $\sigma_s^2 = 0$, the signal s is accurate and noisy otherwise. The type of *Insider* who observes an accurate signal will be called a Boesky Insider. We *do not* always assume that the quality of the signal takes binary values as above. However, an innovation of this work is allow for the presence of "Boesky Insider" and study its implication for informational efficiency.

How reasonable is the hypothesis that a person can have perfect knowledge of the future price of a security? The case involving Ivan Boesky[4] is indicative of the plausibility of the hypothesis that a signal can be fully informative. Indeed, one might imagine numerous other situations where an insider could be perfectly informed about the future value of a security. The situation where a tender offer is being considered is a classic case. In many instances the recommendation of the board will be sufficient to ensure that the offer will be accepted. An insider who knew that the board had decided to recommend the offer would know that the share price would become equal to the offer price. A company which is about to sign a merger agreement will have a definitive value known to those who know that the agreement will be signed. A company which is going to perform a share buyback will have a share price equal to the buyback price. If a company is about to file for bankruptcy (in the sense that its liabilities exceed its assets) the value of its equity is zero. These situations are widespread. They also arise where the information does not imply a specific price of an underlying asset but derivative securities, side contracts or other financial instruments are involved. For instance: when the holder of barrier options, so called "knock-ins" or "knock-outs", receives information that the price of the underlying asset will more above or below a certain price, or where a "collar" is in place[5] which means that the holder is certain of their payoff outside a defined band of prices of the underlying asset.

[3]See Grossman [1989] for example.

[4]An infamous example of such a case involved Ivan Boesky himself. As the *Wall Street Journal* reported "Mr. Boesky testified that Mr. Mulheren responded 'I understand' after the arbitrager told him 'it would be great if Gulf & Western [Industries Inc. stock] traded at 45.' Mr. Mulheren then bought 75,000 shares of the stock just before the market closed, causing the price to climb to $ 45 – which triggered an agreement by G&W to buy back Mr. Boesky's huge stake" - 7/25/91

[5]This consists of buying a "cap" (say through the purchase of a call option) and selling a "floor" (say through the sale of a put option). A "zero cost collar" is a special case where the cost of the cap is equal to the proceeds generated by the sale of the floor.

The above scenarios depict several instances where certainty of information translates directly into perfect knowledge of the value of a security. It is important to distinguish here between certain knowledge of an event taking place which will affect the price of a security, and certain knowledge of the price of a security. It is the latter which we mean by a Boesky Insider. Clearly there are many more instances of the former than the latter. Yet, as we shall see, that for the purposes of our analysis there need only be an infinitesimally small probability that a Boesky Insider exists.

2.2 Boesky Insiders and Informational Inefficiency

We shall say a type $\hat{s}$ is fully revealed in an equilibrium if the equilibrium pricing strategy $p(\cdot)$ is such that $p(s) = p(\hat{s})$ implies $s = \hat{s}$. Now note that it is a dominant strategy for an *Insider* of type s to offer a price higher than $\mu(s)/R$. The prices that are actually offered then may be so low in relation to the risk attitudes of an outsider such that she is not willing to supply any of her endowment of the risky asset. Consequently, the price may be strictly increasing in the signal and fully reveals the price but if no trade occurs at these prices, the fact that information if fully transmitted is a moot point. The following definition takes this into account. Let $q(s) = Q(p(s), s)$ denote the quantity that is traded in state s.

Definition 2 (Informationally Inefficient Equilibrium). *An equilibrium is said to be informationally inefficient if there exists $\alpha < \beta$ such that on the interval (α, β), the function $p(\cdot)$ is a constant and $q(\cdot)$ is positive.*

Our main result requires certain regularity assumptions on the optimal supply of the risky asset by an outsider in the case where a signal is common-knowledge. First let

$$\mathcal{B} = \{s \in [a, b] : \sigma_s^2 = 0\}$$

be the set of all Boesky Insiders. The supply behavior is fairly straightforward when a signal $s \in \mathcal{B}$ becomes common-knowledge. If offered a price ρ, the otherwise risky asset now offers a sure return of $\mu(s)/\rho$. Simple arbitrage considerations reveal that if $\mu(s)/\rho < R$, she should sell her entire endowment of the asset and invest the proceeds in the bond and should retain her endowment if $\mu(s)/\rho > R$. If $\mu(s)/\rho = R$, both the asset and the bond offer an identical return with no risk and the outsider remains indifferent to supplying an arbitrary fraction of the asset.

Now consider the case when $s \notin \mathcal{B}$ becomes common-knowledge. Let $x(\rho, s)$ denote the utility maximizing supply of the asset when the outsider is offered a price of ρ in this case. Now note that the asset offers an return of $\mu(s)/\rho$ with a variance $\sigma_s^2 > 0$. If this return is not higher than R, the asset is dominated in its risk-return profile by the bond and hence[6] $x(\rho, s) = 1$. Therefore, whenever the price is at least $\mu(s)/R$ the outsider sells her entire endowment. Let $\rho_h(s)$ denote the smallest price such that $x(\rho, s) = 1$ whenever $\rho \geq \rho_h(s)$. Clearly $\rho_h(s) \leq \mu(s)/R$. Also let $\rho_\ell(s)$ be the largest price such that $\rho \leq \rho_\ell(s)$ implies $x(\rho, s) = 0$. The first assumption asserts that the optimal supply is unique and well behaved when prices are in the intermediate range $[\rho_\ell(s), \rho_h(s)]$.

Assumption 1. *Suppose $s \notin \mathcal{B}$. Then $x(\cdot, s)$ is strictly increasing in the interval $(\rho_\ell(s), \rho_h(s))$*

[6] Formally, this can be seen through a routine application of Jensen's inequality.

The next assumption is essentially a continuity restriction on $x(\rho, \cdot)$ with respect to the second argument.

Assumption 2. *Let $\{s_n\}$ be an infinite sequence such that $s_n \notin \mathcal{B}$ for all n and $\lim_{n\to\infty} = s^*$. Then $\lim_{n\to\infty} x(\mu(s^*)/R, s_n)) = 1$.*

Proposition 1. *Suppose Assumption 1 and Assumption 2 hold. If the set of Boesky Insiders is not empty and $\mathcal{B} \neq \{a\}$, every equilibrium in which trade occurs with a positive probability is informationally inefficient.*

Proof. See Appendix. □

A sketch of this proof is presented toward the end of this section. We precede this with three important remarks regarding the above result. First, note that the result does not depend on the probability of the event $\mathcal{B}$. This set could have a zero probability measure and yet the equilibrium is informationally inefficient.

Second, a particular environment where Assumption 1 and Assumption 2 are satisfied occurs when the outsiders display constant absolute risk aversion and a signal is either fully informative or equally noisy. The following makes this precise.

Assumption 3. *Suppose $\tilde{V}$ can be decomposed as*

$$\tilde{V} = \mu(\tilde{S}) + \tilde{\epsilon}_s \tag{3}$$

where $\tilde{\epsilon}_s$ is normally distributed with mean 0 and variance σ^2 provided $s \in \mathcal{B}$ and is degenerate at 0 when $s \notin \mathcal{B}$.

If an outsider exhibits constant absolute risk aversion of say $\gamma > 0$, then Assumption 3 gives us[7]

$$x(\rho, s) = \begin{array}{cc} 1 & \text{if } \mu(s) \leq \rho R, \\ 1 - \frac{\mu(s)-\rho R}{2\nu} & \text{if } \mu(s) - 2\nu \leq \rho R \leq s, \\ 0 & \rho R \leq \mu(s) - 2\nu \end{array} \tag{4}$$

where $\nu = \gamma\sigma^2/2$. It is now easy to verify that $\rho_\ell(s) = \mu(s) - 2\nu$, $\rho_h(s) = \mu(s)/R$ and that Assumption 1 and Assumption 2 hold.

The third remark concerns the extent of informational inefficiency. Proposition 1 only rules out the possibility of fully revealing equilibria. In principle it may be that the extent of this inefficiency is minimal. But are there reasonable assumptions under which the extent of inefficiency is significant? The answer turns out to be affirmative for an interesting class of environments. For example, it will be evident from the sketch of the proof of Proposition 1 that a Boesky Insider can never be revealed. Therefore if $\mathcal{B}$ is a countable dense subset of $[a, b]$, then $\mathcal{B}$ would still be of measure zero and yet no state will be fully revealed. In fact, one can prove a stronger result which we present as Proposition 2 below.

[7]See Chapter 4, Huang and Litzenberger [1990] for the algebra leading to Formula (4).

Definition 3 (Informationally Strongly Inefficient Equilibrium). *An equilibrium is said to be informationally strongly inefficient if there exist constants ρ^* and $q^* > 0$ such that $q(s) > 0$ implies $q(s) = q^*$ and $p(s) = \rho^*$.*

In other words, when an equilibrium is informationally strongly inefficient, whenever there is trade, a constant amount is traded at a constant price.

An equilibrium is said to be continuous if $p(\cdot)$ is continuous on the domain where $q(\cdot)$ is positive.

Proposition 2. *Suppose $\mathcal{B} \neq \emptyset$ but $\text{Prob}(\mathcal{B}) = 0$, that Assumption 1-Assumption 3 hold and that $u(\cdot)$ displays non-decreasing absolute risk aversion. Then every continuous equilibrium is informationally strongly inefficient.*

Proof. See Appendix. □

Sketch of Proof of Proposition 1. Let $v(s) = (\mu(s) - p(s))q(s)$ so that the equilibrium profit of a type s *Insider* is $mR + v(s)$. In what follows, we shall ignore the constant term mR and refer to $v(s)$ as the equilibrium profit of the *Insider* of type s. Also, to keep the presentation simple, assume that $\mathcal{B} = \{\hat{s}\}$ for some $\hat{s} > a$.

Recall that in any equilibrium, $p(s) \leq \mu(s)/R$ for all s. Using routine arguments from the theory of incentives it can be shown that $p(\cdot)$ and $q(\cdot)$ are both non-decreasing. (See Lemma 1 in the Appendix.) This monotonicity must hold in *every* equilibrium. We use this monotonicity property to show that $v(s) = 0$ if and only if $q(s) = 0$. (See Lemma 2 in the Appendix.). Define

$$s^* = \sup\{s \,:\, q(s) = 0\} \tag{5}$$

First suppose $\hat{s} > s^*$. Let, if possible, $\hat{s}$ be fully revealed. As $\hat{s}$ is a Boesky Insider, it then becomes common-knowledge that the otherwise risky asset in fact offers a sure return of $\mu(\hat{s})/p(\hat{s})$. No-arbitrage conditions require that this return equal the risk-free rate R, which means $v(\hat{s}) = 0$, contradicting the definition of s^*. Therefore $\hat{s}$ cannot be fully revealing, which is to say there exists $s' \neq \hat{s}$ such that $p(s') = p(\hat{s})$. Choose $\alpha = \min\{s', \hat{s}\}$ and $\beta = \max\{s', \hat{s}\}$ and appeal to the monotonicity of $p(\cdot)$ and $q(\cdot)$ to conclude that these functions are positive and constant in the interval (α, β).

Next, suppose that $\hat{s} \leq s^*$. We claim that the equilibrium must be pooling in a small right neighborhood of s^*. Indeed, by way of contradiction, suppose there is an $\epsilon > 0$ such that $p(\cdot)$ and $q(\cdot)$ are both strictly increasing in the interval $(s^*, s^* + \epsilon)$. There are no Boesky Insiders to the right of s^*. Lemma 4 in the Appendix shows that $p(\cdot)$ is right continuous at s^* and moreover $p(s^*) = \mu(s^*)/R$. As every type in this interval is fully revealed, we have $q(s) = x(p(s), s)$. To illustrate the remainder of the arguments, suppose that $x(\cdot,\cdot)$ is given by (4) to obtain

$$\begin{aligned} q(s^*) &= \lim_{s \downarrow s^*} q(s) \\ &= \lim_{s \downarrow s^*} 1 - \frac{\mu(s) - p(s)R}{2\nu} \\ &= 1 \end{aligned}$$

which in turn implies $q(s) = 1$ for all $s \geq s^*$, contradicting the fact that $q(\cdot)$ is strictly increasing. The argument for the general case is similar and does not rely on the specific functional form for $x(\cdot,\cdot)$. The details are in the Appendix. □

2.3 Existence of Equilibrium

In case of binary signals, the setup considered by Laffont and Maskin [1990], a pooling equilibrium may not exist[8] unless the signals are sufficiently close, although a separating equilibrium always exists. Here the presence of Boesky Insiders rule out the possibility of a separating equilibrium. Therefore the question of existence of an equilibrium is an important one. We do not have an general proof for existence of a pooling equilibrium. (See Proposition 4 however.) We are however able to offer a simple set of conditions to check if a candidate price quantity pair can be supported as a fully pooling equilibrium outcome. First define

$$\pi(s) = \max_{\rho}(\mu(s) - \rho R)x(\rho, b) \tag{6}$$

so that $mR + \pi(s)$ is the maximum profit that a trader who is actually of type t can obtain when the outsiders believe that she is of the highest type b.

Proposition 3. *Suppose that $\mu(\cdot)$ is concave. A tuple (ρ^*, q^*) can be supported as the outcome of a fully pooling equilibrium if:*

$$0 \leq R\rho^* \leq a \tag{7a}$$

$$q^* = \arg\max_{q} U(q, F, \rho^*) \tag{7b}$$

$$v(a) \geq \pi(a) \text{ and } v(b) \geq \pi(b) \tag{7c}$$

$$\text{where } v(s) = mR + (s - \rho^* R)q^*.$$

Proof. Let $\rho^m(s) = \arg\max_\rho (s - \rho)(x(\rho, b))$ and $q^m(s) = x(\rho^m(s), b)$. By definition, $\pi(s) = (s - \rho^m(s))q^m(s)$. By the Envelope Theorem, $\pi'(s) = q^m(s)$. Arguments similar to those used to prove that the equilibrium quantity is non-decreasing in signals (See Lemma 1) can also be used to show that $q^m(\cdot)$ is (weakly) increasing in s and hence $\pi(\cdot)$ is convex.

We construct an equilibrium by assigning the following beliefs for a small trader regarding $\tilde{S}$: If she observes a price different from ρ^*, she believes that that dominant trader is the highest type, namely b. If she observes ρ^*, her belief continues to be the prior distribution $F(\cdot)$.

[8]See Proposition 9 of their paper.

Given $\rho^* < a/R$, it is clear that Player D is better off trading the equilibrium $q^* > 0$ than not trading, perhaps by setting a very low price. Given the strategy of the outsiders, the profit of the *Insider* if she asks for ρ^* is $mR + v(s)$ whereas her profit upon deviation cannot exceed $mR + \pi(s)$. To ensure that choosing ρ^* is a best response, we must have $\pi(s) \leq v(s)$ for all s. By (7c) we are assured that $v(\cdot)$ lies above $\pi(\cdot)$ at the extreme points a and b. However since $\mu(\cdot)$ is concave, $v(\cdot)$ is also concave while $\pi(\cdot)$ is convex which means $v(\cdot)$ lies above $\pi(\cdot)$ everywhere on the interval $[a, b]$.

That q^* is a best response to the strategy of Player D is clear by construction. □

The importance of the above result will be apparent when we apply it to demonstrate a particular allocation as an equilibrium outcome in Proposition 4.

3 Regulated vs. Unregulated Insider Trading

The analysis thus far has suggested that the presence of Boesky Insiders leads to some informational inefficiency. In this section, we discuss some of the welfare effects and implications for insider trading laws. In particular we compare the unregulated trading regime of the previous section with a regulated voluntary disclosure regime that is most commonly observed in many financial markets[9]. Under such regulation, once the *Insider* receives the signal, she has one of two options. She can publicly declare the signal she has received and then trade. If she chooses not to disclose, then she must abstain from trading.

It is a widely held belief that insider trading laws are among the hardest to implement. Yet, here we shall assume that there are no such difficulties and any non-compliance is immediately detected. Given that our main result here shows that regulation can lead to Pareto inferior outcomes relative to informationally inefficient unregulated equilibrium, this assumption provides the most hospitable environment for making a case for regulation. Altering this assumption will only strengthen our results.

For reasons relating to mathematical tractability, we find it necessary to conduct the rest of the analysis under additional assumptions. While the analysis is less general than in the previous sections, as we shall see, our assumptions cover the most often chosen frameworks in the literature.

Assumption 4. *$\mu(s) = s$ and $u(w) = -\mathrm{e}^{-\gamma w}$ so that outsiders display constant absolute risk aversion.*

Most of the analysis in this section, will impose Assumption 3 and Assumption 4. It is useful to bear in mind the discussion following Proposition 1 where it was argued that in this case, $x(\cdot,\cdot)$

[9] See Ausubel [1990b] for instance for a discussion of different kinds of regulation.

is given by (4). The following expressions that play a role in the discussions to follow.

$$\nabla = \frac{E[e^{-\gamma \tilde{S}}]}{E[e^{-\gamma \tilde{S}/2}]} - e^{-\gamma\delta/2}, \qquad \delta = \frac{E[\tilde{S}e^{-\gamma\tilde{S}/2}]}{E[e^{-\gamma\tilde{S}/2}]},$$

$$\text{and} \quad (\rho^{pe}, q^{pe}) = (\frac{(\delta - \nu)}{R}, \frac{1}{2})$$

In the above, the expectation is taken over $\tilde{S}$ using its unconditional distribution $F(\cdot)$ while $\nu = \gamma\sigma^2/2$. Note that the parameter δ depends only the conditional prior distribution of $\tilde{S}$, whereas ν depends on the degree of risk-aversion and the variance of the idiosyncratic risk. Also consider the following inequalities:

$$\nu \leq \delta \leq \nu + a \tag{8a}$$

$$\nu \geq \frac{(b-a)^2}{4(b-\delta)}. \tag{8b}$$

The region where the above inequalities hold is given a visual representation as the shaded region in Figure 1. The next proposition shows that it is in this region that (ρ^{pe}, q^{pe}) can be supported as the outcome of a strongly informationally inefficient equilibrium.

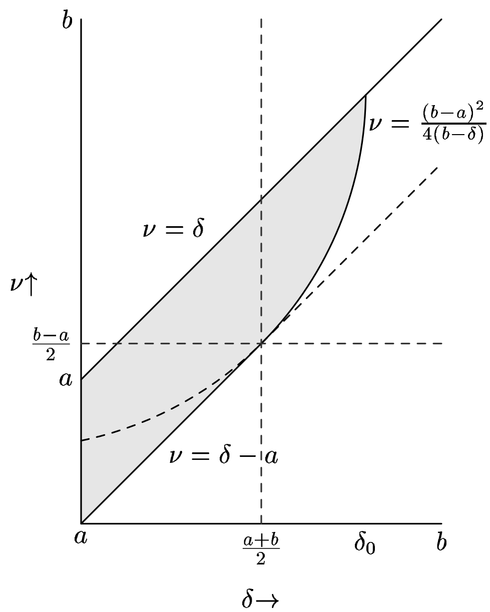

FIGURE 1. Equilibrium exists if (δ, ν) lies in the shaded region.

The picture is constructed as follows. Clearly $a < \delta < b$ while $\nu \geq 0$. Inequality (8a) requires that (δ, ν) lie between the two parallel lines $\nu = \delta$ and $\nu = \delta - a$. The RHS of Inequality (8b), regarded as a function of δ, is increasing, convex and is supported by the line $\nu = \delta - a$, with $\delta = (a + b)/2$ being the point of tangency. It also has a fixed point at $\delta_0 = b + \sqrt{a(2b - a)}$ which, it may be verified lies to the left of b. The picture is drawn schematically with these points in mind. The shaded region is the set described by the Inequalities (8b-8b).

Proposition 4. *Suppose Assumption 3 and Assumption 4 hold and the parameters (ν, δ) satisfy (8a) if $\nu \leq (b-a)/2$ and also (8b) if $\nu \geq (b-a)/2$. Then (ρ^{pe}, q^{pe}) can be supported as a fully pooling equilibrium. The corresponding ex-ante equilibrium payoffs are*

$$\begin{aligned} V_{PE} &= mR + (\bar{S} - \delta + \nu)/2 \\ U_{PE} &= -e^{\gamma w_0 + 3\gamma\nu/4} e^{-\gamma\delta/2} E[e^{-\tilde{S}/2}]. \end{aligned}$$

Proof. See Appendix. □

Proof of Proposition 4 merely involves verifying that the outcome satisfies the various conditions listed in Proposition 3. Hereafter, take (ρ^{pe}, q^{pe}) to be the outcome when there is no regulation.

We shall refer to the above pooling equilibrium as the PE regime and the regulated outcome as *RE* regime. The following result shows how the traders welfare compares between these regimes depending on simple relationship between ∇ and δ.

Proposition 5. *Suppose that the hypotheses of Proposition 4 hold and in addition $\nu \leq a$.*

1. *If $\delta > \bar{S}$, regulation makes the* Insider *better off and the outsiders worse off.*
2. *If $\delta < \bar{S}$, regulation makes the* Insider *worse off. The outsiders are also worse of if (and only if) $\nabla > 0$.*

Proof. For almost all s, by revealing her private information, a type s *Insider* faces the supply curve $x(\cdot, s)$, which is given by Eq. (4). By hypothesis $\nu \leq a$. Therefore, taking the reaction of the outsiders as a given, the *Insider* type s maximizes profits by setting a price

$$p^{re}(s) = (s - \nu)/R. \tag{9}$$

and acquiring positive maximum profit by quantity $q^{re} = 1/2$. The expected price is therefore $\rho^{re} = \bar{S} - \nu$ and her wealth under RE in state s is $\tilde{w} = w_0 R + s - \nu + \tilde{\epsilon}/2$ for all[10] s. By substituting appropriately in (1- 3), we the respective ex-ante utilities of the *Insider* and a typical outsider are

$$\begin{aligned} V_{RE} &= mR + \nu/2 \\ U_{RE} &= -e^{-\gamma w_0 R + 3\gamma\nu/4} E[e^{-\gamma\tilde{S}}] \end{aligned}$$

Direct algebraic manipulation shows that

$$V_{PE} > V_{RE} \quad \Leftrightarrow \quad \delta < \bar{S} \tag{10}$$

$$U_{PE} > U_{RE} \quad \Leftrightarrow \quad \nabla > 0. \tag{11}$$

To complete the proof, it suffices to show that ∇ is necessarily positive whenever $\delta > \bar{S}$. This can

[10] This is true for all s as the assumption $\nu \leq a$ ensures that acquiring $q^{re} = 1/2$ at the price $p^{re}(s)$ is individually rational.

be seen as follows:

$$
\begin{aligned}
E[\mathrm{e}^{-\gamma\tilde{S}}] &- e^{-\gamma\delta/2}E[\mathrm{e}^{-\gamma\tilde{S}/2}] \\
&> E[\mathrm{e}^{-\gamma\tilde{S}}] - e^{-\gamma\bar{S}/2}E[\mathrm{e}^{-\gamma\tilde{S}/2}] \\
&> E[\mathrm{e}^{-\gamma\tilde{S}}] - E[e^{-\gamma\tilde{S}/2}]E[\mathrm{e}^{-\gamma\tilde{S}/2}] \qquad \text{(Since } e^{-x} \text{ is convex)} \\
&\geq 0
\end{aligned}
$$

where the last inequality follows from the fact that $E[X^2] \geq E[X]^2$ for any random variable X. $\square$

Though the above proof is somewhat mechanical, fortunately it is possible to offer some intuition for the welfare comparisons presented in Proposition 5. Note that under both the regimes, the *Insider* acquires an amount $q^{pe} = q^{re} = 1/2$, independent of the signal she receives. The important difference between the regimes is that under the *RE* regime, the price is a random variable $p^{re}(\tilde{S})$ whereas under the PE regime, the price is the constant p^{pe}. It is this variation in prices that causes the various welfare effects.

After all the *Insider*, being risk-neutral, does not care about the idiosyncratic risk. The question of whether she prefers the PE or the *RE* regime is simply a question of comparing the (expected) prices, which are $\rho^{re} = \bar{S} - \nu$ and $\rho^{pe} = \delta - \nu$. The equivalence in Eq. (10) is then immediate. An outsider on the other hand cares about the idiosyncratic risk. When $\delta > \bar{S}$ the expected price she receives for her asset under *RE* regime, namely ρ^{re} is lower than the expected price under the PE regime, namely ρ^{pe}. What is more the former has a greater variance. Naturally, regulation makes the risk averse outsider worse off in this case. This leads to Part 1 of the Proposition.

In contrast, when $\delta < \bar{S}$, it does not necessarily follow that an outsider prefers regulation. Even though the expected price she receives for her endowment is now higher under regulation, the price is random. Therefore if the difference between δ and $\bar{S}$ is not too large, then she too, like the *Insider*, may well prefer the pooling equilibrium to regulated trade – the higher expected price being the risk premium. If this were true, then the pooling equilibrium is Pareto superior to the regulated outcome as stated by Part 2 of the Proposition.

The fact that the *Insider* may prefer the informationally inefficient PE regime must seem somewhat unintuitive. For, as one may recall from Laffont and Maskin [1990], the reason that a pooling equilibrium leads to higher utility for the *Insider* is due to the fact that the *Insider* has to constrain the quantity that she sells in order to credibly communicate her type. However, the regulation here acts as a pre-commitment device allowing her to credibly reveal her information. One might therefore have thought this possibility for the *Insider* to fully exploit her monopoly position without being quantity constrained to credibly reveal her information cannot be improved upon. Yet, the Proposition shows this need not be the case.

The resolution to the above puzzle comes from supply side. With full information, the quantity that is traded is a constant. The *Insider* is forced to charge a price that depends on her signal to induce the outsiders to supply the quantity q^{re} in each state. Consequently the price varies. To escape the variability in the price fluctuations, the outsiders may prefer to offer the same quantity for a lower price. It is evident then the likelihood of this being the case depends on the particulars

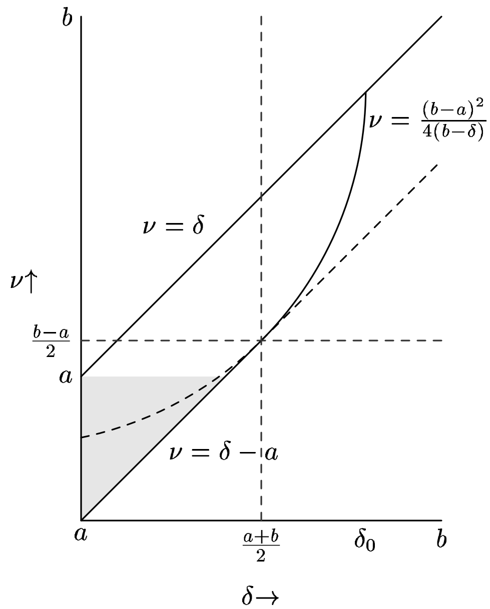

FIGURE 2. Proposition 5 applies only in the shaded region.

of the prior distribution $F(\cdot)$.

To the extent that the legal restriction works as a pre-commitment device it is similar to that Ausubel [1990b]. It is worthwile to remark that the restriction in his framework leads to the opposite conclusion to ours.

Some indication of when the parameters are such that PE Pareto dominates the *RE* regime can be had from considering the case where $(b-a)/2 > a$. In this case, the region in the parameter space to which Proposition 5 applies to is shown as the shaded triangle in Figure 2, as it is only in this region that $\nu \leq a$. Observe that the triangle does not change as b increases. However, δ is a function of b, and is likely to be increasing in b. Therefore for b sufficiently large, Part 1 of the Proposition may be inapplicable. This for instance is the case when F is uniformly distributed on $[a, b]$. Direct computation reveals that

$$\delta = 2 + \frac{ae^{-a/2} - be^{-b/2}}{e^{-a/2} - e^{-b/2}}$$

in which case $\lim_{b\to\infty} \delta = (a+2)$ and $\bar{S} = (a+b)/2 > \delta$. In this case, if the outsiders are better off, Part 2 of Proposition 5 must be true, that is the PE regime is Pareto Superior to *RE*.

Corollary 1. *Suppose that $u(w) = -e^{-w}$. Suppose F is uniformly distributed on $[a, b]$ such that $(b-a) > 4$. For all ν such that $\delta - a \leq \nu \leq a$, the informationally inefficient PE regime Pareto dominates the regulated and informationally efficient* RE *regime.*

Proof. Direct computation shows that

$$A = \frac{E(e^{-\tilde{S}})}{E(e^{-\tilde{S}/2})} = \frac{e^{-a/2} + e^{-b/2}}{2}.$$

The log derivative A is $-b/2$, while the log derivative of $\exp\{-\delta/2\}$ is $-\partial\delta/\partial b$ which is exponential in b . Therefore $e^{-\delta}$ decreases at a much faster rate than does A with respect to b. Moreover $\lim_{b\to a} Ae^{-a/2}$ and $\lim_{b\to\infty} e^{-a/2}/2$. Therefore $\lim_{b\to a}\nabla = 0$ (while $\lim_{b\to\infty}\nabla = e^{-a/2}(0.5 - e^{-1}) > 0$). Therefore, $\nabla > 0$ for all b. Since $\delta < a + 2$ and $\bar{S} = (a + b)/2$, we have $\delta < \bar{S}$. Apply Part 2 of Proposition 5 to complete the proof. □

4 Discussion and Conclusion

4.1 Public Confidence, Liquidity and Insider Trading

Insider trading was outlawed in most countries long ago[11]. These bans do not stop so-called corporate "insiders" (such as managers, officers and directors) from trading *per se*. The bans outlaw trading, by a person , on the basis of material, price-sensitive, non-public information. There are well documented fairness as well as efficiency arguments in favor of such legislation. Though the academic literature has focused almost exclusively on direct efficiency arguments, (Ausubel [1990b] is a notable exception), the public rationale for regulation of insider trading is heavily focused on notions of public confidence. It is frequently claimed by policymakers, commentators and market participants that insider trading "undermines public confidence" in securities markets. The following quotations are illustrative:

> "If the investor thinks he's not getting a fair shake, he's not going to invest, and that is going to hurt capital investment in the long run." - Arthur Levitt (former Chairman SEC and American Stock Exchange)[12]

> "Powerful Wall Street tycoons convinced former President Bill Clinton not to pardon junk-bond king Michael Milken in a furious last-minute battle, The Post has learned. Personal phone calls to Clinton and White House aides - arguing that the widely expected Milken pardon would undermine public confidence in financial markets - tipped the balance...The opponents of a pardon included Citigroup's influential Robert Rubin, a former treasury secretary in the Clinton administration."[13]

> "It should give impetus to those dealing in securities and thereby bring back public confidence." - Franklin D. Roosevelt proposing to Congress the legislation which became the Securities Act of 1933 (quoted in Ausubel [1990b]) [14].

[11] For instance the Securities and Exchange Commission (SEC) has prohibited insider trading since the Securities and Exchanges Act of 1934 (Section 10(b) and SEC Rule 10b-5 generally and Section 14(e) and Rule 14e-3 with respect to takeovers) and strengthened such position with the Insider Trading Sanctions Act of 1984 and the Insider Trading and Securities Fraud Enforcement Act of 1988. Many other countries' bans are also long held.

[12] "The Epidemic of Insider Trading - The SEC is Fighting a Loosing Battle to Halt Stock-Market Abuses" *Business Week* 4/29/85

[13] *New York Post* 2/1/01

[14] As Ausubel [1990b] notes the Securities Act of 1933 was a forerunner to the Securities and Exchange Act of 1934, and was concerned with information disclosure in Initial Public Offerings ("IPOs").

The Public Confidence argument is directly concerned with the quality of information which insiders posses. Indeed, it is largely concerned with extreme cases. The rationale for prohibition of trading on "material, price sensitive, non-public information" is not to prevent market professionals doing their job. In the words of the *Wall Street Journal,* "Market professionals earn their keep by analyzing companies, crunching numbers, interviewing company executives. Their job, in other words, is to come up with material information not yet discovered by the market."[15]

The rationale for insider trading laws is not to prevent people trading when they *sense* something, rather when they *know* something. Such laws are designed to stop Boesky Insiders from trading. There is significant evidence that corporate insiders earn above market returns. Seyhun [1992] documents that in the United States "corporate insiders earned an average of 5.1 percent abnormal profits over a one-year holding period between 1980 and 1984, increasing further to 7.0 percent after 1984". This kind of insider trading, though it may be technically illegal, has no apparent negative effect on liquidity and public confidence. Indeed, the huge rise in the liquidity of US equities markets over the 1980s and '90s has been at least correlated with a more than fourfold rise in trading of corporate insiders (See Seyhun [1992]). Such trading, illegal or not, is rarely, if ever, pursued by the SEC. It is the Boeskies and Milkens who raise the spectre of diminished public confidence in securities markets. Not because they are insiders, but because they are Boesky Insiders.

The quantity traded in the equilibrium identified in Proposition 3 is as high as the quantity traded when insider trade is regulated. Therefore, trading on the basis of private information does not necessarily lead to lower liquidity. This observation is somewhat different from the findings of Glosten [1989] and Bajeux and Rochet [1989], who argue that insider trading decreases liquidity and Leland [1992] who as part of a general exploration of the efficiency of insider trading, finds similarly. A somewhat different argument is presented in Ausubel [1990b] which articulates an efficiency argument for public confidence in securities markets. Utilizing a two stage model in which there is first investment in a productive asset, and then merely exchange, he argues that insider trading leads to reduced confidence by outsiders and that this may in fact damage insiders though a reduction in investment. Our work does not directly address the important issue of public confidence. A reconsideration of Ausubel's work when the insiders differ in the quality of their information would be a fruitful exercise.

4.2 Robustness of Equilibria

One way to overcome the sensitivity of continuous PBE to zero probability events to assume that incentive compatibility conditions are satisfied except perhaps on a set of measure zero. This would rule out the impact of a zero probability event. Even if one assumes this position, Proposition 1 and Proposition 2 could be reinterpreted as a failure of equilibria to be robust to small perturbations. For one could just as well assume that $B = (-\epsilon, \epsilon)$ so that the set of Boesky Insiders is a set of measure zero. Any continuous equilibrium is necessarily fully pooling for all $\epsilon > 0$, even though the case $\epsilon = 0$ may allow for a fully separating equilibrium.

[15] *Wall Street Journal* 11/18/86.

4.3 Conclusion

The existing literature on insider trading has tended to over emphasize the role of informational efficiency. To achieve a more complete understanding of the allocative inefficiency, or otherwise, of insider trading, and possible regulatory remedies, one must consider a broader range of public confidence issues in addition to those considered here. This may involve combining features of the type of model considered in Ausubel [1990b] with those contained in this paper. Depending on the results of such work, it may be that the fundamental rationale for bans on insider trading is fairness, rather than efficiency.

5 Appendix

We remind the reader that we refer to $v(s) = (\mu(s) - p(s)R)q(s)$ as as the equilibrium profit of an *Insider* even though it is in fact $mR + v(s)$. The following inequality is the familiar incentive compatibility compatibility condition and must hold in any equilibrium:

$$v(s) \geq (\mu(s) - p(t)R)q(t) \quad \forall s, t. \tag{12}$$

The following Lemma involves standard arguments involving the the incentive compatibility conditions (12). Recall the definition of s^* from (5).

Lemma 1. *The following hold in any equilibrium*

1. *$q(\cdot)$ are non-decreasing. On the domain (s^*, β), $p(\cdot)$ is also non-decreasing. Moreover if $q(\cdot)$ is strictly increasing in an interval (α, β), then $p(\cdot)$ is also strictly increasing in this interval.*
2. *$v(\cdot)$ is non-decreasing and continuous.*
3. *If $p(\cdot)$ is continuous, then $q(\cdot)$ is also continuous.*

Proof of Lemma 1. In an equilibrium, a type s trader must have no incentive to choose the price $p(t)$, where $s \neq t$. From the above incentive compatibility condition, $(\mu(s) - \mu(t))q(t) \leq v(s) - v(t) \leq (\mu(s) - \mu(t))q(s)$ for all s, t. Recalling that $\mu(\cdot)$ is strictly increasing, it follows from this that $q(s) \geq q(t)$ whenever $s > t$.

To see that $p(\cdot)$ must also be non-decreasing on assume by way of contradiction that there exist $\alpha < s < t$ such that $p(s) > p(t)$. Since $q(s) < q(t)$, we have

$$\begin{aligned} v(s) &= (\mu(s) - Rp(s))q(s) \\ &< ((\mu(s)) - Rp(t))q(t) \end{aligned}$$

contradicting that $p(\cdot)$ is an equilibrium. We leave it to the reader to follow the above inequalities to conclude that $p(\cdot)$ is increasing if and only if $q(\cdot)$ is increasing. This completes Part 2.

Part 3 is immediate from Equation (12) while Part 4 is immediate from the definition of $v(s)$. $\square$

Note that an insider may make a zero profit for one of two reasons – either because she charges a price of $\mu(s)/R$ but trades a positive amount or she does not trade a positive amount, i.e. $q(s) = 0$. The following Lemma rules some of the possibilities.

Lemma 2. *The equilibrium profit of an* Insider *of type s is positive if and only if she trades a positive quantity.*

Proof of Lemma 2. Let s^* be as defined in (5) Due to the monotonicity $q(\cdot)$ and $v(\cdot)$ obtained in Lemma 1, it suffices to show that $v(s) > 0$ if and only if $s > s^*$. It is clear from the definition of $v(\cdot)$ that $s < s^*$ implies $v(s) = 0$. Continuity of $v(\cdot)$ also implies that $v(s^*) = 0$.

To show the converse that $v(s) = 0$ implies $s > s^*$, suppose, by way of contradiction that there exists a $s_1 > s^*$ such that $v(s_1) = 0$. by the monotonicity of $v(\cdot)$, $v(s) = 0$ for all $s \leq s_1$. Since $q(s) > 0$ in for all such s, it must be that that $p(s) = \mu(s)/R$ for all such s. Consider $s^* < t < s < s_2$ and use the fact that $\mu(\cdot)$ is strictly increasing to note that this contradicts (12). □

Lemma 3. *If a Boesky Insider is fully revealed, then her equilibrium profit is zero.*

Proof of Lemma 3. Let $s \in \mathcal{B}$. If $q(s) = 0$, then clearly $v(s) = 0$ and the proof is complete. Assume then $q(s) > 0$. When the s is common-knowledge, the otherwise risky asset is risk-free and offers a sure return of $\mu(s)/p(s)$. The fact that $q(s) > 0$ implies that this return must be the same as the risk free return or $p(s) = \mu(s)/R$, which too yields $v(s) = 0$. □

A final ingredient of our proof of Proposition 1 is the following technical lemma.

Lemma 4. *Let s^* be as defined in (5). If $s^* > a$, then $p(\cdot)$ is right continuous at s^* and moreover $p(s^*) = \mu(s^*)/R$.*

Proof of Lemma 4. Since $p(\cdot)$ is non-decreasing, $\rho^+ = \lim_{s \downarrow s^*} p(s)$ is well defined. Suppose $\rho^+ > \mu(s^*)/R$. Since $\mu(\cdot)$ is continuous, there exists a type $\hat{s} > s^*$ such that $p(s) > \mu(s)/R$. Choose $t \leq s^*$ and $s = \hat{s}$ to obtain a contradiction to (12). Now suppose $\rho^+ < \mu(s^*)/R$. Then there exists a type $\hat{s} > s^*$ such that $p(\hat{s}) < \mu(s^*)/R$. Now take $t = \hat{s}$ and $s = s^*$ to contradict (12) again. □

Proof of Proposition 1. Let s^* be as defined in (5). Suppose there exists a $s_1 \in \mathcal{B}$ such that $s^* < s_1$. If s_1 is fully revealed, then by Lemma 3, we have $v(s_1) = 0$ and by Lemma 2 $q(s_1) = 0$, thus contradicting (5). Therefore s_1 cannot be fully revealed and there must exist $s_2 \neq s_1$ such that $p(s_1) = p(s_2) = \rho^*$. By the monotonicity of $p(\cdot)$, we must have $p(s) = \rho^*$ for all s such that $\min\{s_1, s_2\} < s < \max\{s_1, s_2\}$. By Lemma 1, $q(s) = q(s_1) > 0$ in this interval, thus establishing the informational inefficiency of the equilibrium.

Now consider the case where $s^* \geq s$ for all $s \in \mathcal{B}$. In particular, this implies that $s^* > 0$. We shall argue that there is small right neighborhood of s^* in which both $p(\cdot)$ and $q(\cdot)$ are a constant, thus establishing the informational inefficiency of the equilibrium.

Indeed, suppose to the contrary and assume that there exists an $\epsilon > 0$ such that for all $s \in (s^*, s^* + \epsilon)$, $q(\cdot)$ and $p(\cdot)$ are both strictly increasing. Pick an infinite decreasing sequence $\{s_n\}$ from this interval such that $\lim_{n\to\infty} s_n = s^*$. By Lemma 4, we have $p(s^*) = \mu(s^*)/R$

and $q(s^*) = \lim_{n\to\infty} q(s_n) = \lim_{n\to\infty} x(p(s_n), s_n)$. Using these relations and Assumption 2, we conclude that $q(s^*) = 1$. By Lemma 1 then, $q(s) = 1$ for all $s \geq s^*$ contradicting the assumption that $q(\cdot)$ is strictly increasing in the region $(s^*, s^* + \epsilon)$. □

Proof of Proposition 2. Assume that $w = 0$ so that the outsider has no initial wealth. Also set $R = 1$. It will be clear to the reader that these are simplifications for notational convenience and do not affect the arguments used in this proof.

Given $s \in \mathcal{B}$, let $\tilde{w} = qp + (1-q)[\mu(s) + \tilde{\epsilon}]$. The expected utility of an outsider who knows that s is the signal is given by $E[u(\tilde{w})\big|s]$ where the expectation is taken with respect $\tilde{\epsilon}$. Let

$$\eta(\rho, q, s) = \frac{\partial E[u(\tilde{w})\big|s]}{\partial q}.$$

Now consider an arbitrary equilibrium. From Proposition 1, there exists an interval $[\alpha, \beta]$ such that the *Insider* chooses a constant price ρ^* and trades a constant quantity $q^* > 0$ whenever her type is drawn from this interval. If $\alpha = a$ and $\beta = b$, then we are done.

If possible, suppose $\beta < b$. Since the equilibrium is right continuous, there must be a small neighborhood of β, say $(\beta, \beta + \epsilon)$ for some $\epsilon > 0$ such that the price is fully revealing. Let $s \in (\alpha, \beta)$. As such a type trades a positive quantity $v(s) > 0$. Use Lemma 1-Lemma 4 to conclude that $(\beta, \beta + \epsilon) \cap \mathcal{B} = \emptyset$. Consequently for each $s \in (\beta, \beta + \epsilon)$, we have $q(s) = x(p(s), s)$ and hence $\eta(p(s), q(s)) = 0$ is the first order condition. Using continuity and taking the limit as $s \to \beta$ gives

$$\eta(\rho^*, q^*, \beta) = 0 \tag{13}$$

Now when an outsider observes the price ρ^*, her updated belief is that the *Insider*'s type is drawn from the interval (α, β). Her payoff from choosing a supply of q is $U(q, \rho^*, G)$ described in (2) where $G(\cdot)$ is the appropriate Bayesian posterior. Since q^* is the optimal supply, we have the following first order condition for an optimum:

$$\int_{\alpha}^{\beta} \eta(\rho^*, q^*, \tilde{S}) \mathrm{d}G(x) = 0 \tag{14}$$

Since $\mu(\cdot)$ is increasing and outsiders display non-decreasing absolute risk aversion, it follows from the work of Arrow [1971] that $\eta(\rho^*, q^*, \cdot)$ is decreasing, which means (14) and (13) are incompatible. Therefore $\beta = b$.

A similar contradiction can be obtained if $\alpha > a$. □

Proof of Proposition 4. We will verify that (ρ^{pe}, q^{pe}) satisfies Eq. (7b- 7c) and apply Proposition 3. Equality (8a) is a simple rearrangement of Eq. (7a).

To see that (ρ^{pe}, q^{pe}) satisfies Eq. (7b), first write the payoff of an outsider from choosing q

when facing a constant price ρ, given that the posterior belief continues to be the prior $F(\cdot)$:

$$\begin{aligned} U_I = U(q, F, \rho) = &- e^{-\gamma w_0 R - \gamma q \rho R + \gamma \nu (1-q)^2} \\ &\times E\big[e^{-\gamma(1-q)\tilde{S}}\big] \end{aligned}$$

U_I is concave in q. By differentiating $\log(-U_I)$ with respect to q yields the following first order condition for an optimum:

$$\rho = \frac{E[\tilde{S} e^{-\gamma(1-q)\tilde{S}}]}{E[e^{-\gamma(1-q)\tilde{S}}]} - \gamma(1-q)\sigma^2.$$

which clearly holds at $(\rho, q) = (\rho^{pe}, q^{pe})$.

It remains to verify the two inequalities in Eq. (7c). Checking that $v(b) \geq \pi(b)$ is relatively straightforward. For, $\pi(b)$ is the monopoly profit of the type b *Insider* under full information. This is achieved if she acquires half the endowment of the risky asset at a unit price $\rho = (b - \nu)/R$. On the other hand, in the proposed equilibrium, she acquires an identical quantity at the price ρ^{pe}. Since $\delta < b$, it follows that $\pi(b) \leq v(b)$.

Seeing $\pi(a) \leq v(a)$ is little more involved. When the supply is being determined as per $x(\rho, b)$, it is clear from Eq. (4) that unless a price of at least $b - 2\nu$ is offered, an outsider will refuse to supply a positive quantity. When ν is sufficiently low, so that there is not much idiosyncratic risk in the asset, the desired price $(b - 2\nu)$ will be greater than a. In this case, $\pi(a) = 0$ as no positive quantity will be acquired by the type a *Insider*. For such values of ν, $\pi(a) \leq v(a)$ holds trivially as long as Inequality (8a) holds. In contrast, when $\nu \geq (b - a)/2$, there interior solution to type a *Insider*'s maximization problem which yields

$$\pi(a) = \frac{\nu}{2}\Big(1 - \frac{b-a}{2\nu}\Big)^2 \tag{15}$$

Comparing the above with $v(a) = (a - \delta + v)/2$ shows that $\pi(a) \leq v(a)$ is equivalent to Inequality (8b). $\square$

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