TOPICS IN WIND FARM LAYOUT OPTIMIZATION: ANALYTICAL WAKE MODELS, NOISE PROPAGATION, AND ENERGY PRODUCTION

by

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Graduate Department of Mechanical and Industrial Engineering
University of Toronto

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Abstract

Topics in Wind Farm Layout Optimization: Analytical Wake Models, Noise Propagation, and Energy Production

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Wind farm layout optimization (WFLO) is the design of wind turbine layout, subject to various financial and engineering objectives and constraints. The first topic of this thesis focuses on solving two variations of WFLO that have different analytical aerodynamic models, and illustrate deep integration of the wake models into mixed-integer programs and constraint programs. Formulating WFLO as MIP and CP enables more quantitative analysis than previous studies could do with heuristics, and allows the practitioners to use an off-the-shelf optimization solver to tackle the WFLO problem. The second topic focuses on another version of WFLO that has two competing objectives: minimization of noise and maximization of energy. A genetic algorithm (NSGA-II) is used. Under these two objectives, solutions are presented to illustrate the flexibility of this optimization framework in terms of supplying a spectrum of design choices with different numbers of turbines and different levels of noise and energy output.
Dedication

To my grandparents.
Acknowledgements

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Chapter 1

Introduction

The main topic of this thesis is to apply operations research tools to wind farm layout optimization (WFLO), or equivalently turbine micro-siting, and validating their usefulness in this context. Current results, compared with previous work, show that there are many more opportunities in applying sophisticated OR tools to WFLO.

Wind energy installation has experienced a tremendous increase in the past decade. The Canadian Wind Energy Association envisioned Canada to have 55 GW of wind energy installation by 2025, equivalent of 20% of the country’s energy needs [1]. The United States has seen annual growth between 5 and 10 GW since 2007, with a total installed capacity of 43GW through the third quarter of 2011 [2].

Wind farms produce power by harvesting the energy of air flow. Commercial wind farms usually span several kilometers (with 10 to 100 turbines), and wind turbine rotors range up to 40–50m in radius [3]. Under such conditions, turbines are close enough to influence each other’s performance, due to aerodynamic interactions (wakes). Therefore, the placement of wind turbines directly impacts the efficiency of the wind farm. Furthermore, the locations of wind turbines are also of environmental and social concern – line of sight, noise propagation, and disruption to animal migratory patterns.

The wind farm layout optimization problem is concerned with the optimal location of turbines within a fixed geographical area to maximize energy capture under stochastic wind conditions. Previously this problem appeared only in the engineering research literature [4–9], where much effort has been spent on developing metaheuristics [4–6,10] for variations of this problem.

One focus of this work is to apply mathematical programming tools to wind farm layout optimization. In Chapter 3, constraint programming and mixed-integer programming techniques are used to solve the problem, focusing on the comparison of CP and MIP models, while examining the suitability of two different analytical wake models in these math programs.

Apart from the novel application of different mathematical programming tools, this thesis also explores different application areas of WFLO. In Chapter 3, two analytical wake models are studied, one being more accurate than the other. We provide a novel mixed-integer program that can incorporate the more accurate wake model. Chapter 4 focuses on the bi-objective optimization considering noise propagation and energy production, a new topic that explores the entire spectrum of wind farm layout design with regard to energy and noise performance.
Chapter 2

Literature Review

Although most current wind farm layout optimization studies focus on the layout of turbines (also the focus of this thesis), we also want to explore additional optimization topics that are closely related: layout of infrastructure, and sizing of energy storage system. Our ability to model and solve these optimization problems influences the performance of wind farms on multiple dimensions: profitability, environmental footprint, and social responsibility. Each problem is explored individually with emphasis on modelling and optimization techniques. Directions of current research and suggestions for future research are discussed at the end of the review.

Overall, we found that the current scope of the turbine and infrastructure layout literature is limited to simple problem set-up: flat terrain, simple cost functions, and little integration of the turbine and infrastructure problems. The energy storage system sizing problem is closely related to the dynamic problem of wind energy dispatch. For all three problems, the main research contributions have been from the engineering literature. Based on these observations, we propose new research directions.
2.1 Introduction: Three Problems

Wind energy is a fast growing sector of renewable energy. At the same time, related research activities have flourished in the past decade. Figure 2.1 suggests a similar growth trend for the amount of publications in this area and the total installed capacity of wind energy.

![Figure 2.1: Wind energy: research activities [11] and global installed capacity [12].](image)

The main purpose of this review is to present the state-of-art optimization methods related to the design of commercial wind farms. It examines optimization models and techniques in the following three problems: turbine placement, infrastructure layout, and energy storage system sizing. In other words, our review is driven by the practical needs that a wind farm developer faces.

Turbine layout design is an important stage in wind energy capture for two reasons. First, good layout design increases the total energy production and reduces fatigue loads on turbines, because aerodynamic interactions between turbines can be mitigated by strategically spreading out turbines in a wind farm. Second, wind farm projects usually have multiple stakeholders, and financial and energy performances are only the baseline – the wind farm layout should comply with local environment rules, and incur minimum disturbance to the surrounding natural and human activities. Similarly, infrastructure layout is related to road routing, overall cost, and environmental/social footprint. Therefore, sometimes the infrastructure layout problem is simultaneously considered with the turbine layout problem.

On the other hand, the penetration of wind energy could be limited by its intermittency and fluctuation in the wind resource. An energy storage system is used as a buffer between the wind farm and the grid. With the appropriate power rating (charge/discharge rates) and energy capacity, an energy storage system can improve the wind energy quality. Current energy storage systems are still costly [13],...
the discussion on optimal sizing of energy storage systems is of concern.

2.1.1 Optimization Model Accuracy and Precision

An optimization task often begins with a model – a set of mathematical equations called objective functions and constraints – that describes the problem at hand. Ideally, this model should accurately reflect the physics of the problem, and also be easily solvable. In mechanical design, there is sometimes trade-off between the accuracy and the tractability of the model, because the physics of mechanical systems are often more complex than what the optimization algorithms can easily handle. For this review, we define these two aspects as model accuracy and model precision – the more accurate a model is, the more closely we are using math to describe the system behaviour; and the more precise a model is, the faster/easier we will get to a good/optimal solution.

By our definition, the accuracy of model depends on the types of mathematical constraints and objectives used to model the problem, and the precision depends on the types of optimization algorithms available for such model (simplex algorithm for linear programs [14], branching-type algorithms for combinatorial problems [15], metaheuristics for general nonlinear programs [16].)

The following problems are interdisciplinary, and they fall into the research scope of mechanical, aerospace and electrical engineering, operations research, and statistics. Through the presentation and comparison of different models and optimization methods, this review illustrates the landscape of current literature. The following three sections illustrate each problem individually, with summary and discussion at the end of each section. An overall discussion is presented at the end of the review, including an outlook to the future research directions.

2.2 Turbine Placement

2.2.1 Problem Description

For large commercial wind farms, multiple wind turbines are installed, and the efficiency of the wind farm is highly sensitive to the positioning of these turbines [4]. Wind farm layout optimization refers to the optimization task that chooses the best turbine positions [4–9, 17–55, 98]. Optimality can be defined as, for example, maximum energy capture during the lifetime of the project. Apart from turbine locations, sometimes the number and type of turbines are decision parameters [42].

This turbine layout optimization problem resembles a number of other problems. When translated into mathematical models, this optimization problem falls into one of two basic solution approaches – discrete and continuous problems. Discrete optimization, in this context, usually involves partitioning of wind farm land, and using binary variables to indicate the absence and existence of a turbine in a particular cell. In a continuous model, the decision variables are, for example, the Cartesian coordinates of turbine locations. The discrete location version of this approach is similar to a generalized vertex packing problem (without explicit considering of aerodynamic interactions, or wakes) [56] and the p-dispersion-sum and maximum diversity problems (with wakes) [17, 39]. The continuous version corresponds to a circle or eclipse packing problem (without wakes) [8, 18].
Objective Functions

Most models have a single objective function: maximum energy production and maximum profit. Some studies, such as [28], combine energy production and cost (profit) into a single objective function using a weighted sum approach.

Some studies keep different goals as separate objective functions. Sišbot et al. [48] used genetic algorithm to simultaneously maximize power production and minimize installation cost. Kusiak and Song [36] used bi-objective evolutionary algorithms to maximize energy production and minimize constraint violation: if all turbines are within the wind farm boundary and are all more than four rotor-diameters away from each other, the second objective function would have an optimal value of zero. This unconstrained algorithm produces a Pareto set of potential solutions. Kwong et al. [9] proposed a bi-objective optimization considering both the energy generation and noise propagation. Considering noise as a second objective instead of a constraint, results from this model offer the wind farm designers a spectrum of design choices. It also demonstrates that the bi-objective optimization usually yields very different solutions than those from single objective (energy maximization) optimization. In particular, bi-objective optimization with energy and noise considerations would push turbines further away from the receptors (e.g., residents), and changes propagate into transportation, logistics, infrastructure layout, and financing of the wind farm.

Land

There are two main approaches for modelling turbine locations – continuous and discrete. A continuous-location model allows turbines to be placed anywhere within the farm, subject to boundary and proximity constraints. A discrete-location model only allows turbines to be placed at a finite number of places.

Mosetti et al. [4] formulated the problem with a number of abstractions to the wind farm layout problem in their seminal paper. An idealized farm is discretized into a 10 by 10 square grid. Each cell has side length of five rotor diameters; and turbines could only be placed in the centre of a grid. Some later studies use finer grids or allow turbines to be placed continuously in the wind farm. Instead of dividing the land into 5D-cells, Mittal [41] proposed a model with grid spacing of 1/40D. Aytun Ozturk [18], Chowdhury et al. [24], Wan et al. [52] and Kusiak [36] allowed turbines to be placed anywhere on the farm, subject to the proximity constraints. Mittal compared results with those from Mosetti et al. and confirmed that a finer grid size could lead to superior solution: more energy production for the same problem setup.

Most publications discussed in this chapter consider the terrain to be regularly shaped and two dimensional. For the vertical direction (elevation), roughness length is always considered because it determines the wake expansion pattern [57]. Roughness length is a characteristic value for the roughness of the ground: the smoother the terrain, the smaller its roughness length. Although larger scale terrain features could change wind speed at rotor height and limit turbine positions [58], little work was found to include the effect of terrain orography. We think that a major hurdle is that in a complex terrain, the propagation of wakes is not well-understood, and therefore researchers do not have the analytical tools to describe the turbine interactions in an optimization model. A recent publication by Saavedra-Moreno et al. [46] considers the effect of orography on local wind speed. However, this work does not mention the propagation of wakes in a complex terrain.

To account for the impact of land configurations on wind farm performance, Chowdhury et al.
[24] examined the wind farm efficiency and cost per kWh of electricity as a function of different land orientations and aspect ratios. Chen and MacDonald [22] argued that different sections of a wind farm are of different importance. The authors claimed that this model can elicit insight regarding strategic land acquisition. Kwong et al. [9] introduced a similar idea by plotting a spatial histogram of turbine positions through a complete genetic algorithm run.

Wind Resource and Wake Modelling

The wind energy production of a turbine is a function of wind speed at rotor height. The turbulence intensity of wind impacts energy production [59], but it is not directly modelled in any study examined here. The propagation of aerodynamic wakes changes the amount of energy that can be extracted by turbines, increases fatigue cost, and reduces energy quality. It is generally modelled by the Jensen Wake Model [57]. The Jensen Wake Model is also known as the PARK model [60], and it is used in the commercial software package Wind Atlas Analysis and Application Program (WAsP) [61] as well as research papers. The following equation describes the wind speed deficit downstream from a turbine [62]:

\[
\text{Wind Speed Deficit} = \left(1 - \frac{U}{U_0}\right) = \frac{2b}{(1 + k R + k)} ^2
\]  

(2.1)

Where \(U_0\) is the free-stream wind speed, \(U\) is the downstream wind speed, \(b\) is the axial induction factor, \(x\) is the downstream distance, \(R\) is the turbine rotor radius, and \(k\) is the wake spreading constant:

\[
k = 0.5 \ln \frac{z}{z_0}
\]  

(2.2)

Here \(z\) is the turbine hub height, and \(z_0\) is the terrain roughness length.

There are several ways to describe the combination of wakes. One is to directly combine the wind speed deficits from multiple upstream turbines. Another is to combine the squares of overlapping wind speed deficits:

\[
\left(1 - \frac{U}{U_0}\right)^2 = \sum_i \left(1 - \frac{U_i}{U_0}\right)^2
\]  

(2.3)

For a more insightful comparison and validation of wind farm wake models, the readers are referred to the work of Renkema [98]. In Chapter 3, we focus on mathematical programming models that can incorporate two different analytical wake models, and compare the physical accuracy and computational performance.

Choice of Turbine Type and Number

Since turbines constitute a significant portion of the overall wind farm cost [42], the choices of turbine types and numbers are influential variables in wind farm optimization research. Mustakerov and Borissova [42] formulated a turbine layout optimization model that considers variable turbine types and numbers. In their analysis, turbines have different power ratings, diameters, heights, and nominal speeds. Their results support the conclusion that using larger turbines is more cost effective than using smaller ones. Chowdhury et al. did a comparative study of how three factors – rotor diameters, number of turbines and wind farm size – influence the power generation and cost of wind farms [23]. They
concluded that optimal wind farm layout is sensitive to turbine types, and using mixture of different types of turbines in a wind farm leads to better overall performance.

In most studies where wind farm cost appears in the objective function, the cost is approximated by a function of the number of turbines. In contrast, Elkinton [27] included electrical grid cost and foundation cost in their optimization model. A recent study by Réthoré et al. [45] for a Denmark offshore wind farm also models electrical grid cost, foundation cost, and turbine fatigue cost in the cost function. It is expected that more complex (realistic) cost models will continue to be developed.

### 2.2.2 Optimization Methods

#### Evolutionary Algorithms

Evolutionary algorithms [63] (EA) are nature-inspired metaheuristic optimization algorithms: the algorithm searches for global optimality through iterative sampling, evaluation, and enhancement of solutions. An evolutionary algorithm usually does not assume any underlying landscape of the search space like some other algorithms do (such as simplex algorithm requiring a problem to be linear [14]). Therefore, it is widely applicable in many areas of engineering, including the wind farm layout optimization problem. In this context, the candidate wind farm layouts are repeatedly filtered, combined, and mutated based on the principle of survival of the fittest. Mutation is basically a random perturbation of the current layouts, and it helps maintain diversity in the solution pool. Fitness of a layout is based on its objective function evaluation and potentially some other factors, such as how diverse the solutions are. Typically, the most expensive computation of an EA is the evaluation of objective functions. Most studies in the wind farm layout optimization literature, especially those using evolutionary algorithms, did not fully record computational details. For a good discussion on the computational performance of wind farm layout optimization algorithms, readers are referred to [50], in which the authors compared their local search algorithm with a previously reported algorithm [51] and the optimization feature in the industrial tool OpenWind [60].

Mosetti et al. [4] proposed a genetic algorithm approach to automate the wind farm layout optimization task. Their model divides a square farmland into a 10 by 10 grid. Each chromosome of the genetic algorithm is an array of 100 elements representing whether or not each cell contains a turbine. Grady [5] built on Mosetti’s idea and further examined the performance of genetic algorithms in this problem. In a simple case of linearly placing a few turbines, this work validates the solution of a genetic algorithm with the analytical solution. The GA is run on a 10 by 10 grid with longer time and larger population sizes than Mosetti et al. did. Grady demonstrated the accuracy of genetic algorithm in these test cases. Papers by Mosetti et al. and Grady employed test cases with non-uniform wind speeds and directions, i.e., the wind resource is described with a number of probability terms, each representing the portion of time that wind comes from a particular direction at a certain speed.

Huang [34] posed a distributed genetic algorithm approach to reduce the computational time. Distributed genetic algorithm divides the search space into several smaller regions (demes), so that each part requires fewer individuals and generations to reach a good solution. With the correct parameters to control migration between demes, Huang claimed that better solutions could be found in less time. Two opportunities for extension are discussed: the relative independence between the demes naturally leads to computational parallelization; and local search in a deme can be utilized to improve search speed. The latter has been realized in subsequent studies. For example in the work by Serrano et al. [29], a local
search method is combined with genetic algorithm to improve solution quality. Other hybrid algorithm models have been proposed by Elkinton et al. [27] and Réthoré et al. [45].

Elkinton et al. [27] use a genetic algorithm to generate a set of initial solutions, and feed these solutions into a greedy heuristic for further improvement. Conceptually, the genetic algorithm finds solutions that maintain good diversity, and the greedy heuristic improve on them by finding better neighbour solutions. Neighbours refer to the wind farm layouts that are close in proximity in the design space, such as those that differ only by the positions of a small subset of turbines. In all test cases examined, the hybrid algorithm outperforms the two individual algorithms alone, i.e., with better and more consistent solutions.

Another hybrid algorithm involving genetic algorithms is the TopFarm project. Réthoré et al. [45] developed a multi-stage model to progressively find better solutions. The first stage utilizes a genetic algorithm, and the second stage relies on a local search algorithm to improve the first stage solutions. In addition to searching on a finer grid, the second stage includes a more comprehensive cost and revenue model, as well as a more refined wind resource probability distribution. The authors noted that convergence during global search was slow. Therefore, large problems demand fine-tuning of this approach, such as parallelization.

As Réthoré et al. [45] and Fagerfjäll [8] implied, computational time can be a roadblock against the application of genetic algorithms in problems with practical constraints and large number of turbines. However, little effort has been spent on recoding and comparing the computational times of applying different genetic algorithms to different models. As an exception, Wagner et al. [51] applied the Covariance Matrix Adaptation based Evolutionary Strategy (CMA-ES) to the optimization problem and recorded the detailed computational performance. This variation of evolutionary algorithm focuses the search in more promising regions of the design space [64]. The authors applied CMA-ES to two- to six-turbine cases as those used in Kusiak [36], and showed better quality results. Cases of up to 1000 turbines are also presented. Computational times are clearly documented. For the 1000-turbine case, 20 processors were used for parallel computing, and one wake loss calculation for 1000 turbines takes approximately 30 seconds on an Intel Xeon E7530 (1.87GHz) processor. In total, an offspring pool of 20 was run for 20,000 generations. The total run time was 12 days. The authors also noted that the speedup of parallelization was sub-linear: adding more cores does not lead to a proportional speed-up. This could potentially be improved, as they did not study the parallelization of the problem formally.

More recently, multi-objective genetic algorithms [65, 66] are applied to this problem. Kusiak and Song [36] used Strength Pareto Evolutionary Algorithm 2, or SPEA2 [65], to maximize energy and minimize turbine location violation: turbines cannot be placed too close to each other or outside of the circular wind farm boundary. In the work by Kwong et al. [9], Non-dominated Sorting Genetic Algorithm II (NSGA-II [66]), solves an energy-maximization and noise-minimization model. The value of the latter is calculated according to the ISO standard on noise propagation [67].

Particle Swarm Optimization

Particle swarm optimization (PSO) [68] has roots in evolutionary strategies. Similar to genetic algorithms, PSO is initialized with a set of random solutions, which can be interpreted as positions in the search space. In addition to these positions, PSO also assigns velocities to each solution to guide the search, thus the analogy about swarm (i.e., bird flock) behaviour.

Application of PSO in wind farm layout problem first appeared in 2009 [19]. Since then some
researchers have studied this application. Most recently, Chowdhury et al. published several studies on this topic [6,23,24], focusing on non-traditional factors that influence the maximum power production – such as turbine types and land configuration. In the work by Bilbao and Alba [19], a genetic algorithm and a particle swarm optimization algorithm are compared in several test cases. The authors found the general performance of these two algorithms to be similar. In one of the test cases, convergence is achieved after 10,000 to 15,000 function evaluations for six to eighteen turbines. This is comparable with the evolutionary algorithm model used in Kusiak and Song’s work [36], in which optimization for six turbines converges after 100 generations. Assuming each generation contains 100 individuals, their algorithm would require about 10,000 function evaluations. In this scenario, both PSO and evolutionary algorithms need roughly the same number of function evaluations to achieve convergence. However, we do not intend to make a definite judgement since convergence in two different algorithms is not directly comparable.

Branch and Bound (Mixed Integer Programming)

A mixed integer (linear) programming (MIP) model consists of linear objective functions, linear constraints, and continuous/discrete variables. Commonly used methods for solving MIP include branch and bound, which divides up the search space by partitioning the domains of a subset of variables (branching), and relaxing the sub-problems so that the remaining variables become continuous and can be solved with efficient algorithms [14]. Solutions from relaxed sub-problems are used to guide the overall search.

In the context of wind farm layout optimization, when wind farm land is discretized, the turbine placement problem can be modelled as a MIP problem. In such a formulation, the decision variables are the existence of turbines at a finite set of locations. At every location, a turbine either exists (value of 1) or does not (value of 0). In several studies, Fagerfjäll [8] and Donovan et al. [7, 25, 69] explored the application of branch and bound methods in wind farm optimization.

Results from these studies show that using a MIP model and branch and bound solver is a promising direction. Apart from using a strong commercial/academic MIP solver (such as Gurobi [70] or CPLEX [71]), more experienced researchers can improve the solver efficiency by providing user-defined branching strategies. One of Donovan’s papers [25] explores the design of such strategies. In general, branch and bound provides a distinctive advantage over other heuristics since it provides an upper bound on how far away the current solutions are from optimality. This characteristic has not been fully exploited for the current problem.

Simulated Annealing

Simulated Annealing (SA) is another metaheuristic search method, and it draws analogy from statistical physics. Similar to the annealing process in which a substance stabilizes to its minimum-energy state, SA iteratively improves the solution until convergence criteria are met. SA for the current optimization problem starts with an initial layout of turbines, and this layout is perturbed randomly. If the new layout is better, perturbation is accepted. Otherwise the modification could be rejected – the probability of rejection increase as the search continues. Several publications [20, 33, 72] have demonstrated the feasibility of SA in wind farm optimization problems. In the study done by Herbert-Acero et al. [33], SA showed better or equal performance than genetic algorithm in three one-dimensional cases.

Tables 2.1, 2.2, and 2.3 summarize several characteristics of the papers we discussed.
Table 2.1: Annotation for Tables 2.2 and 2.3

<table>
<thead>
<tr>
<th>Turbine type/number as variables</th>
<th>Land geometry</th>
<th>Wind regime</th>
<th>Wake and Power modelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>All fixed</td>
<td>Rectangular</td>
<td>Unidirectional, uniform speed</td>
<td>Turbine interaction and power production are polynomial</td>
</tr>
<tr>
<td>One of type or number of turbines is a variable</td>
<td>Convex</td>
<td>Many directions and speeds</td>
<td>Wake interaction are analytical</td>
</tr>
<tr>
<td>Both are variables</td>
<td>Nonconvex</td>
<td>Contains continuous distribution and/or location-dependent wind speeds</td>
<td>Wake interactions are more complex than Jensen model; or power calculation includes curtailment and realistic power curves</td>
</tr>
</tbody>
</table>

2.2.3 OpenWind

OpenWind is an open-source, industry-standard software package for WFLO. According to Wagner et al. [50], the OpenWind [60] layout optimizer produces layouts that are about 4% less efficient than the results from their algorithms. Since they used an older version of OpenWind, we have included a new comparison between the most recent version of OpenWind (version 01.04.00.1026) and their results.

![Figure 2.2: Comparing OpenWind version 1.4 with Other optimizers.](image)

In Wagner et al., each optimization run starts with a given initial layout. For each number of turbines, 30 instances were run. In our case, since we do not have access to their starting layouts, we initialized the problem with a random feasible layout every time for 10 runs, where feasibility is defined as the
Table 2.2: Comparison of Some Current Turbine Layout Studies (a)

<table>
<thead>
<tr>
<th>Algorithm Type</th>
<th>Max number of turbines</th>
<th>Turbine type and number as variables</th>
<th>Land geometry</th>
<th>Wind regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archer et al. (2011)</td>
<td>MIP BnB</td>
<td>30</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Aytun Ozturk and Norman (2004)</td>
<td>Greedy</td>
<td>n/a</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Bilbao and Alba (2009)</td>
<td>GA; PSO</td>
<td>50</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Bilbao and Alba (2009)</td>
<td>SA</td>
<td>50</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Chen and Macdonald (2011)</td>
<td>GA</td>
<td>30</td>
<td>+</td>
<td>+++</td>
</tr>
<tr>
<td>Chowdhury et al. (2010)</td>
<td>PSO</td>
<td>10</td>
<td>++</td>
<td></td>
</tr>
<tr>
<td>Chowdhury et al. (2011)</td>
<td>PSO</td>
<td>20</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Chowdhury et al. (2012)</td>
<td>PSO</td>
<td>30</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>Donovan (2006)</td>
<td>MIP BnB</td>
<td>20</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Du Pont and Cagan (2010)</td>
<td>EPS</td>
<td>60</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Elkinton et al. (2008)</td>
<td>GA + Greedy</td>
<td>10</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Emami and Noghreh (2010)</td>
<td>GA</td>
<td>40</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Fagerjäll (2010)</td>
<td>MIP BnB</td>
<td>30</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>González et al. (2010)</td>
<td>GA</td>
<td>10</td>
<td>+</td>
<td>+++</td>
</tr>
<tr>
<td>Grady (2005)</td>
<td>GA</td>
<td>40</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Herbert-Acero et al. (2009)</td>
<td>GA; SA</td>
<td>10</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>Huang (2009)</td>
<td>GA + HC</td>
<td>50</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Kwong et al. (2012)</td>
<td>GA</td>
<td>50</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Kusiak and Song (2010)</td>
<td>EA</td>
<td>10</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Mittal (2010)</td>
<td>GA</td>
<td>50</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Mora et al. (2007)</td>
<td>EA</td>
<td>50</td>
<td>+++</td>
<td>+</td>
</tr>
<tr>
<td>Mosetti et al. (1994)</td>
<td>GA</td>
<td>30</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Mustakerov and Borissova (2010)</td>
<td>n/a</td>
<td>130</td>
<td>+ + +</td>
<td>+</td>
</tr>
<tr>
<td>Réthoré et al. (2011)</td>
<td>GA + SLP</td>
<td>20</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Wagner et al. (2012)</td>
<td>LS</td>
<td>1000</td>
<td>+</td>
<td>+++</td>
</tr>
<tr>
<td>Wan et al. (2010)</td>
<td>PSO</td>
<td>40</td>
<td>+</td>
<td>++</td>
</tr>
</tbody>
</table>

Branch and bound (BnB), genetic algorithm (GA), particle swarm optimization (PSO), evolutionary algorithm (EA), sequential linear programming (SLP), and local search (LS).

Turbines obeying proximity rules in the OpenWind environment. For each of the ten cases (ten different numbers of turbines), we recorded the average percentage difference between the initial and final layouts in terms of energy production. Turbine Distribution Algorithm (TDA) is the algorithm developed by Wagner et al., and OpenWind 2008 refers to the OpenWind optimizer performance recorded in the same study. We use the same problem set up (turbine characteristics, land configuration, and wind regime) as Wagner et al. did. Although we cannot directly compare the results since the initial layouts are different, we do not see any statistically significant difference between OpenWind 2008 and 2012 in the cases given (Fig. 2.2).
Table 2.3: Comparison of Some Current Turbine Layout Studies (b)

<table>
<thead>
<tr>
<th>Study</th>
<th>Wake and Position</th>
<th>Stopping Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ + + D(discrete)</td>
<td></td>
<td>n/a</td>
</tr>
<tr>
<td>Aytun Ozturk and Norman (2004)</td>
<td>+</td>
<td>C(ontinuous) n/a</td>
</tr>
<tr>
<td>Bilbao and Alba (2009)</td>
<td>+ + + D</td>
<td>max function evaluations: 5m</td>
</tr>
<tr>
<td>Bilbao and Alba (2009)</td>
<td>+ + + D</td>
<td>max generations: 1.5m</td>
</tr>
<tr>
<td>Chen and Macdonald (2011)</td>
<td>+ +</td>
<td>D max function evaluations: 5m</td>
</tr>
<tr>
<td>Chowdhury et al. (2010)</td>
<td>+ +</td>
<td>C max eval.: 15k</td>
</tr>
<tr>
<td>Chowdhury et al. (2011)</td>
<td>+ +</td>
<td>C max eval.: 60k</td>
</tr>
<tr>
<td>Chowdhury et al. (2012)</td>
<td>+ +</td>
<td>C max eval.: 15-25k</td>
</tr>
<tr>
<td>Donovan (2006)</td>
<td>+ +</td>
<td>D max gen.: 3k</td>
</tr>
<tr>
<td>Elkinton et al. (2008)</td>
<td>+ +</td>
<td>D stop if stall for 50 gen</td>
</tr>
<tr>
<td>Emami and Noghreh (2010)</td>
<td>+ +</td>
<td>D max gen.: 100</td>
</tr>
<tr>
<td>Fagerfjäll (2010)</td>
<td>+ +</td>
<td>D n/a (convergence within 100 gen.)</td>
</tr>
<tr>
<td>González et al. (2010)</td>
<td>+ + + D</td>
<td>n/a</td>
</tr>
<tr>
<td>Grady (2005)</td>
<td>+ +</td>
<td>D max gen.: 3k</td>
</tr>
<tr>
<td>Huang (2009)</td>
<td>+ +</td>
<td>D max gen (GA): 2.5k</td>
</tr>
<tr>
<td>Kwong et al. (2012)</td>
<td>+ +</td>
<td>C dynamic max gen and stall length</td>
</tr>
<tr>
<td>Kusiak and Song (2010)</td>
<td>+</td>
<td>C max gen.: 100</td>
</tr>
<tr>
<td>Mittal (2010)</td>
<td>+ +</td>
<td>D n/a</td>
</tr>
<tr>
<td>Mora et al. (2007)</td>
<td>n/a</td>
<td>D max gen.: 100</td>
</tr>
<tr>
<td>Mosetti et al. (1994)</td>
<td>+ +</td>
<td>D max gen.: 400</td>
</tr>
<tr>
<td>Mustakerov and Borissova (2010)</td>
<td>n/a</td>
<td>D n/a</td>
</tr>
<tr>
<td>Wagner et al. (2012)</td>
<td>+ +</td>
<td>C max eval.: 200k</td>
</tr>
</tbody>
</table>

2.3 Infrastructure Layout

2.3.1 Problem Description

Infrastructure layout optimization refers to the design of supporting structure and networks in a wind farm, including turbine foundations, electrical collection system, road network, and control, monitoring and data collection system [73]. Usually the objective of an infrastructure layout optimization task is to maximize system reliability and minimize cost. Junginger et al. [74] provide a breakdown of wind farm investment cost. For modern onshore wind farms, foundations represent 5-10% and internal grid connection represents 10-15% of the total investment cost. For offshore wind farms, these portions are 15 – 25% and 15 – 30% respectively.

The main purpose of a foundation is to reliably support the turbine during its lifetime. Therefore foundation design should consider different load profiles and local ground conditions [73] – constructing a foundation at a lower load-bearing region of a wind farm could lead to higher cost. Load-bearing capacity is the soil’s capacity to support the loads applied to the ground, in this case, the turbine weight and the dynamic loads such as the momentum due to wind flow.
Electrical system consists of transformers, cables, energy storage systems, and switchgear for grid-connection. The cable network is a costly and crucial part of the collection system and is subject to optimization [73]. For offshore wind farms, cable planting and maintenance is costly. Various works have introduced cable redundancy in their optimization model [75–77]. The electrical collection network redundancy increases power generation reliability and reduces contingency costs associated with on-site maintenance at offshore wind farms. Choices of the AC/DC transmission systems are also analyzed to showcase their cost behaviours [78] offshore.

For onshore wind farms, access roads are constructed between turbines, and from the wind farm to existing roads. It is sometimes subject to optimization. In one of the studies reviewed [8], the authors assume that the road and cable networks overlap each other. The same study also includes existing roads in the wind farm road network to reduce overall construction cost.

In the long run, better efficiency and reliability of electrical collection system play an important role in cost reduction. Some studies have chosen to include electrical loss into operating cost in their objective functions [8,62].

### 2.3.2 Models and Optimization Techniques

#### Onshore Wind Farms

González et al. [32] formulated a bottom-up model to optimize the Net Present Value (NPV) of onshore wind farm, considering initial investment and long term costs and revenues induced by the turbines and the supporting infrastructure. In their model, cost of foundations depends on the specific ground conditions (high and low bearing capacities of soil). Costs of roads are proportional to the length of roads. Electrical infrastructure cost model includes three components: the internal medium voltage (MV) distribution network, substations, and high voltage (HV) evacuation lines. Each component carries two types of cost: initial investment cost for construction, and resistive electrical loss.

One contribution of [32] is the comparison between sequential optimization and simultaneous optimization of both wind turbine positions and infrastructure layout. In sequential optimization, the wind turbine layout problem is solved first, and turbine positions are used as parameters for the second problem – infrastructure layout optimization. In simultaneous optimization, these two problems are solved concurrently with a hybrid algorithm including GA and Prim’s algorithm [79]. The sequential optimization yielded a NPV within 0.5% of result from the simultaneous optimization.

In Fagerfjäll’s model [8], foundation cost varies with location. This model assumes that road and cable layouts are identical within the wind farm. Existing roads and highways are included to shorten road networks. Other variables include the types of cables and transformers.

The overall problem is formulated as a MIP, and solved with a branch and bound method. Fagerfjäll attempted to simultaneously solve the turbine layout and infrastructure layout problem, but no details are included in the paper since the author could not solve any nontrivial case within reasonable time. For the attempted small cases, the author reported that simultaneous optimization did not give significant reduction in total wind farm cost compared with sequential optimization of turbine layout and infrastructure layout.
Offshore Wind Farms

Offshore wind farms have received more attention for their electrical system design in the past due to higher cost. In Zhao et al.’s work [76], the authors focused on one type of electrical system (direct current), and used genetic algorithms to minimize cost by varying the connection paths of electrical system and types of transformers. Turbine positions are predetermined prior to the infrastructure optimization. One important constraint in this model is the minimum reliability. An overall reliability function is based on the probability of each component’s failure rate. Among other things, transmission line redundancy is included for better reliability. In a subsequent paper [77], the authors further investigated the suitability of genetic algorithms for this problem. Four different variations of GA are tested. Ability to avoid premature convergence is the benchmark for testing, and they concluded that, unsurprisingly, the best GA variation is the one that keeps the solutions diverse. Convergence criteria are defined as any one of the three conditions: reaching predetermined number of generations, stabilization of best fitness value in population, and convergence of population fitness values.

Elkinton et al. [62] presented a more comprehensive model that considers the investment costs and long term costs, such as operations and maintenance (O&M) cost and electrical loss. In another study by Elkinton et al., greedy heuristic and genetic algorithm are compared for offshore wind farm layout problems [27]. Sensitivity analysis shows that wake effect has the most impact, while power curve, electrical loss, and availability of system are the next most impactful factors.

Lumbreras and Ramos [75] proposed the use of Benders decomposition method [14, 80] to solve the MIP they proposed. Benders decomposition breaks large/complex problems into smaller sub-problems. The MIP model emphasizes different wind scenarios and reliability of cables and transformers. Separation of wind scenarios is one of their ways to decompose the problem. One advantage of Benders decomposition is the ability to separate the overall mixed integer program into smaller master problem in MIP, and a linear program as sub-problem. In practice, linear programs are much easier to solve than mixed integer programs.

In a more recent study done by Réthoré et al. [45], a multi-stage algorithm is applied to the offshore test case of Middelgrunden, Denmark. This algorithm solves the problem in two steps. In the first step, genetic algorithm finds relatively good solutions over a relatively large coverage of search space (coarse search), and the second stage incorporates a local search algorithm to find more refined solutions. In this model, foundation cost has linear dependency on water depth. This relatively simple infrastructure model – not considering electrical components – is solved simultaneously with the turbine layout model.

Banzo and Ramos [81] created a stochastic programming model considering investment costs and long term reliability of electric power system, focusing on the types and connection pattern of cables. Stochastic programming models include uncertainty in parameters by using a distribution, instead of a single value, to describe a parameter. In this case, wind speed (Rayleigh distribution) and system reliability are stochastic variables. A case study with different levels of model simplifications was used to validate the stochastic programming approach.

In summary, offshore wind farm has much higher costs associated with civil and electrical infrastructure components compared with onshore wind farms. Hence, many works have been published in the literature studying the different types and layouts of such components. Due to the harshness of operating environment, offshore maintenance is more costly and complex than onshore operations. Therefore, many studies look at reliability as a separate objective/constraint.

The layout of electrical system at an offshore wind farm is already hard considering the increased
complexity. Many studies focused on optimizing the electrical system layout at a given wind farm with established turbine layout. Most works did not include the civil infrastructure costs as a design factor. One exception is the OWFLO project [62]. At the same time, several different optimization techniques have been explored in the recent years, showing the increase of research interest in this area and the potential for more efficient solution methods.

Tables 2.4 and 2.6 summarize several studies in this area. We first note that the number of publications we found is significantly less than the number of publications on turbine layout optimization, and all of them were published within the past ten years. Among the existing publications, the scope of infrastructure varies greatly from one paper to another. We note that the simultaneous optimization of turbine and infrastructure layouts is the holy-grail of wind farm design for some engineering companies [82], but currently no algorithms have efficiently solved large (>20 turbines) problems instances.

Given such observations, at least two directions can be exploited for future work. First, multi-objective algorithms can be explored to solve for optimal cost and reliability at the same time. Second, the trade-off between minimizing energy loss due to wakes and minimizing electricity loss due to cable network could be explored in detail, since reducing turbine interference requires a sparse placement of turbines, while reducing the resistive loss of electricity calls for a more regular/tight clustering of turbines. A potential way to solve this combined turbine-infrastructure problem is to use a decomposition model similar in spirit to SOM3 (Chapter 3).

Table 2.4: Wind farm infrastructure layout optimization (a).

<table>
<thead>
<tr>
<th></th>
<th>Optimization Scope</th>
<th>Algorithm</th>
<th>Max # of turbines</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Onshore</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>González et al. (2011)</td>
<td>+++</td>
<td>EA</td>
<td>≤ 10</td>
</tr>
<tr>
<td>Fagerfjäll (2010)</td>
<td>+++</td>
<td>MIP BnB</td>
<td>≤ 10</td>
</tr>
<tr>
<td><strong>Offshore</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zhao et al. (2004)</td>
<td>++</td>
<td>GA</td>
<td>100</td>
</tr>
<tr>
<td>Elkinton et al. (2006)</td>
<td>+ + +</td>
<td>GA + Greedy</td>
<td>20</td>
</tr>
<tr>
<td>Lumbrares and Ramos (2011)</td>
<td>++</td>
<td>Benders decompo</td>
<td>30</td>
</tr>
<tr>
<td>Réthéoré et al. (2011)</td>
<td>++</td>
<td>GA + SLP</td>
<td>20</td>
</tr>
<tr>
<td>Li et al. (2008)</td>
<td>++</td>
<td>GA</td>
<td>50</td>
</tr>
<tr>
<td>Banzo and Ramos (2011)</td>
<td>++</td>
<td>Stochastic Programming</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 2.5: Annotation for Table 2.4.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>Partial system</td>
</tr>
<tr>
<td>++</td>
<td>Including electrical or civil system</td>
</tr>
<tr>
<td>+++</td>
<td>Simultaneous optimization (turbine locations) attempted</td>
</tr>
</tbody>
</table>
Table 2.6: Wind farm infrastructure layout optimization (b).

<table>
<thead>
<tr>
<th></th>
<th>Investment</th>
<th>Operations &amp; Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Civil</td>
<td>Electrical</td>
</tr>
<tr>
<td><strong>Onshore</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>González et al.  (2011)</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>Fagerfjäll (2010)</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td><strong>Offshore</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zhao et al.      (2004)</td>
<td></td>
<td>++</td>
</tr>
<tr>
<td>Elkinton et al.  (2006)</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>Lumbreras and Ramos (2011)</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Réthoré et al.   (2011)</td>
<td>+ *</td>
<td></td>
</tr>
<tr>
<td>Li et al.        (2008)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Banzo and Ramos  (2011)</td>
<td>++</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2.7: Annotation for Table 2.6.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Annotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>Some indication of including it</td>
</tr>
<tr>
<td>++</td>
<td>Explicitly stated that they are included as parameters</td>
</tr>
<tr>
<td>+ + +</td>
<td>Explicitly stated that they are a function of other parameters</td>
</tr>
<tr>
<td>*</td>
<td>Included reliability of lifetime fatigue of foundations</td>
</tr>
</tbody>
</table>

2.4 Wind Energy Storage Size

2.4.1 Problem Definition

The last topic in this review is also in the scope of optimal wind farm design – optimal sizing of energy storage system (ESS). The main purpose of such system is to dampen the energy production fluctuations since wind is intermittent and cannot be perfectly predicted. The two important components of size are the rated power and rated energy of a storage device. Rated power of storage refers to its ability to supply power (kW or MW). The rated energy is the total capacity of the storage (kWh or MWh). Some modelling and optimization techniques [83] for wind farm ESS can be broadly applied to other renewable energies that have intermittent energy supply.

We want to note that the notion of optimal sizing is a relatively loose terminology in this context, since the optimal size of an ESS depends on the way it is used. Therefore, the sizing problem often appears in conjunction with the unit commitment and economic dispatch of wind energy. But to stay within the scope of this review regarding wind farm development, we focus on the papers that have emphasis on sizing instead of unit comment or economic dispatch.

2.4.2 Models and Solution Approaches

Korpaas et al. [83] presented a model for the scheduling and operation of energy storage at wind power plants that are integrated to energy market. A dynamic programming algorithm is used. Based on wind
speed forecast (24-hour horizon), the amount of available wind energy for the planning horizon can be calculated, and a bid (constant wind energy supply for 24 hours) can be placed. In terms of sizing, they used nine options (three power and three energy choices). Hence the authors did not use an optimization algorithm, but rather a trial-and-error approach to demonstrate the application of their bid scheduling model.

Brunetto and Tina [84] proposed linear programming and dynamic programming models for the optimal operation of wind power plant storage system. They are concerned with the hydrogen storage method (electrolyser and fuel cell). Similar to the work by Korpaas et al. [83], the ESS size is a parameter instead of a decision variable in the core optimization (scheduling) model. The authors estimated the effect of ESS size on profit by using a few scenarios (with different ESS sizes).

Bludszuweit et al. [85] and Pinson et al. [86] both proposed models for the probabilistic sizing of ESS. Bludszuweit et al. proposed a model to find optimal probabilistic size by penalizing the “unserved wind energy”. Unserved energy represents the lost profit due to one of two reasons: ESS having a slower charging speed than that of wind energy generation; and ESS being saturated. Therefore, their model’s decision variables include both the optimal rated power and optimal capacity of ESS. The model by Pinson et al. focuses on the optimal capacity of ESS, and includes more aspects of financial loss, such as the loss due to insufficient energy supply. These two models should be used for designing wind farms in different markets. The model of [85] is applicable for wind farms operating in an isolated system, where wind energy is a price-setter. The inability to meet customer demand does not incur a financial penalty and the model is accurate. The model of [86] can be applied to grid-integrated wind farms, because the model accounts for both over-delivery and under-delivery, which is accurate for such environment. Pinson et al. also introduced the idea of dynamic sizing – the wind farm owner can rent instead of purchase ESS, because the optimal ESS size might change depending on the level of uncertainty in wind forecast.

Dutta and Sharma [13] presented a stochastic linear program model to optimize the size of energy storage system to increase reliability (ability to follow plans). Both energy capacity and power discharge capacity are considered. Two different optimization goals are used: minimal under-generation (loss-of-load); and minimal energy deviation – both over- and under-generation. Uncertainties in wind and load forecasts are modelled by Gaussian distributions. In their model, a large penalty is given for over- or under-delivery. As a result, this model gives the minimum storage size necessary to guarantee a zero loss or spillover.

Round-trip efficiency of ESS is significant practically. Korpaas et al. [83], Brunetto and Tina [84], and Pinson et al. [86] incorporated less-than-100% efficiency parameters into the models, while Bludszuweit et al. [85] and Dutta and Sharma [13] assumed no energy loss during ESS charging and discharging.

Although the financial penalty due to energy loss and spillover is included in cost/revenue models, none of these five studies explored the significance of ESS depreciation cost. Knowing that ESS depreciation rate depends on the frequency of charging/discharging, and charging/discharging efficiencies could be modelled, the depreciation cost could also be modelled in a similar way. To complete the description of ESS in an energy market with renewable sources, the following section introduces market conditions as well as the technological and financial aspects of different energy storage systems.
2.4.3 Related Topics on Energy Storage Systems

This review does not cover the operational strategy of ESS. It is worth mentioning that understanding of energy markets [87] can facilitate a good grasp of the choice, operation, and value of ESS. Li et al. [88] present a review of some of the earlier works in ESS sizing and operation. A more recent work by Kim and Powell [89] introduces an optimal commitment policy for wind farm with ESS. Pinson et al. [86] gave a concise description about deregulated energy markets, such as the Scandinavian Nord Pool market.

Studies have been done to introduce different types of ESS. Schainker [90] briefly reviewed commercially viable ESS. Chen et al. [91] and Ibrahim et al. [92] gave more comprehensive description and analyses about the technical characteristics of different storage technologies.

When penetration of wind and other intermittent energy sources is low, the uncertainty in energy supply does not create extra pressure on the grid, since this fluctuation from supply can be simply represented by a (inverted) fluctuation in demand. However, as wind energy penetration increases, there is pressure to reduce the volatility of supply level at the source.

From another perspective, the goal of installing ESS at a wind farm is to increase the quality of wind energy, thus bring benefits to wind farm owners, energy markets, and the society. For example, ESS enables the wind farm owners to take advantage of energy arbitrage opportunities in a deregulated energy market [93]. Some types of ESS generate significantly less greenhouse gas emissions than others [94]. In addition to its energy arbitrage and emissions attributes, ESS can impact the society on a larger scale (e.g. consumer surplus) [95].

2.5 Discussion

Three optimization problems are presented in this review. Overall, we see a variety of optimization methods, for examples the genetic algorithm, as well as hybrid algorithm that takes advantage of global search method and the accuracy of local search method. No single algorithm or modelling approach has dominated any of the three problems. On one hand, the efficiency of algorithms needs to be recorded rigorously. On the other hand, a wider spectrum of methods can potentially facilitate a deeper understanding of these optimization problems. Genetic algorithms and other metaheuristic methods can form a good basis for research in this area. However, these metaheuristics alone can hardly drive the solution quality to a higher level. Methods that require more tailoring can potentially lead to the discovery of deeper mathematical structure of these problems.

The current solutions techniques for these problems do not represent the variety of operations research techniques that are applied to other problems. For example, a problem that is closely related to wind farm layout optimization is the maximum diversity problem (MDP) [96]. While MDP has been studied more extensively in the facility planning literature, there has been no application of MDP solution techniques to solving the wind farm layout problem. In general, little attention has been drawn from the operations research, artificial intelligence, and related fields on these three problems (on the other hand, unit commitment of wind energy has attracted a lot of attention from the dynamic programming community).

The turbine and infrastructure layout problems are still constrained within simple problem set-ups. For example, most articles assume the terrain to be smooth (such as farm lands). We think that a major reason is that little work is known on approximating for turbine interactions with closed-form equations in complex terrains. If such equations indeed exist, there is a good chance that researchers can
develop some realistic optimization models for wind farm projects that are located in rough terrains. In addition, simulation optimization [97] can be called to tackle the task of understanding the design space and optimization when the wind farm performance can only be accurately evaluated with simulations.
Chapter 3

Analytical Wake Models

Previously, the wind farm layout optimization problem has been modelled as a maximum diversity (or \( p \)-dispersion-sum) problem, but such a formulation cannot capture the nonlinearity of aerodynamic interactions among multiple wind turbines. We present the first constraint programming (CP) and mixed-integer linear programming (MIP) models that incorporate such nonlinearity. Our empirical results indicate that the relative performance between these two models reverses when the wind scenario changes from a simple to a more complex one. We also propose an improvement to the previous maximum diversity model and demonstrate that the improved model solves more problem instances.
3.1 Introduction

An interesting feature of the wind farm layout optimization problem that sets it apart from standard location problems is the aerodynamic interaction among multiple turbines. In a basic scenario where only two turbines are present, the turbine downstream is said to be in the wake region of the upstream turbine, and it experiences a loss in energy production due to the reduction in wind speed and increase in turbulence intensity [57]. In practice, a turbine that is downstream of multiple turbines is affected by all upstream turbines simultaneously, and the overall effect is a nonlinear function of individual wakes. There are different analytical equations to describe the superposition of multiple wakes, some being closer to the physical reality than the others [98].

It is difficult to incorporate the more accurate wake equations into a mathematical programming model due to their nonlinearity: currently only heuristics [4–6, 9] include the most accurate wake models. Our goals are to computationally improve existing mixed-integer programming (MIP) models and incorporate more accurate wake models into constraint programming (CP) and MIP models.

The contributions of this chapter are: the proposal of two novel mathematical programming models (CP and MIP) that can describe the physics of the problem more accurately than the previous MIP models; the extension of a previous MIP model so that the solution quality and time are improved; the comparison of four models on twelve problem instances, with varying wind scenario complexity, turbine numbers, and wind farm grid resolution; and the elicitation of insights from the experiments to suggest future research directions.

3.2 Problem Definition and the Physics of Wake Modelling

3.2.1 Description of the Problem

Wind farm site selection, or wind farm siting, is based on, among other factors, meteorological conditions, topological features of the site, and accessibility for construction and grid transmission [73]. After siting, wind farm developers optimize the layout of the turbines according to prescribed objectives and constraints in a process called micro-siting. In a typical case, design engineers try to maximize the expected profit and minimize hazardous side-effects during wind farm construction and operation [73] (Section 2.2.1). This is a challenging task because there are many objectives and constraints, and every site is different. To limit our scope, we consider the maximization of energy capture of a wind farm as our only objective, as it is closely related to the long term profit of the wind farms and it is well accepted in the wind farm optimization literature [4–6]. We further assume that the wind farm land is flat, and all turbines are of the same type.

We use the same problem setup that Mosetti et al. [4] proposed. The objective is to maximize the wind farm’s overall power generation capability. There are three types of constraints:

1. Proximity: turbines must be placed five diameters apart to avoid structural damage induced by strong aerodynamic interactions;
2. Boundary: Turbines must be placed within the wind farm boundaries;
3. Turbine number: The number of turbines is fixed.

The reason that the total number of turbines is fixed – instead of bounded by a maximum number of turbines – is due to practical considerations. During wind farm development, the total number of turbines is determined prior to the design process, by government regulations and the local electricity
grid interconnection capacity among other factors. However, to explore the design space more fully, a given model can always be solved multiple times with different numbers of turbines.

As mentioned in the previous section, we use a discrete representation of wind farm: land is decomposed into a set of cells, where each cell can only accommodate one turbine. This approach is common in the literature [4,5,7] (Section 2.2.1).

3.2.2 The Physics: Wake and Energy Models

While some constraints of this problem are similar to vertex packing [99], undesirable facility location [100], and circle packing problems [101], the objective function is unique to wind farm layout optimization. In particular, the energy capture at each turbine is proportional to the cube of wind speed at that location. In turn the wind speed at a turbine is a nonlinear function of the distances to its upstream turbines. Note that “upstream” is relative to the wind direction, which varies over time.

Although wind changes speed and direction frequently, we assume that the turbine can re-orient its rotor towards the upcoming wind direction. We further assume that there is no power loss during the transient states. Overall, the yearly wind frequency data at each direction fits well into a Weibull distribution [58]. In the literature, it is a common practice to discretize the yearly wind frequency data into multiple directions and multiple speeds [4–6], so that the total energy production is the weighted sum of energy produced at each wind state (speed and direction). Then the expected power is only different from the expected energy by a scalar (the number of seconds per year). Therefore we only deal with the expected power in this work to simplify calculations.

**Single Wake**

The downstream region of a wind turbine, with increased level of turbulence and decreased energy, is called the wake region (Fig. 3.1). Equation (3.1), first proposed by Jensen [57], describes the propagation of a single wake. Parallel arrows in Fig. 3.1 represent wind direction and speed. The region with lower wind speed (shorter arrows) is the wake region. The two ellipses represent a turbine. The key idea of Jensen’s wake model is momentum conservation within the wake region. In addition, wind speed is assumed to be uniform and non-turbulent across the circular wake cross-section.

Here $R$ is the wake radius immediately after rotor; $r$ is the downstream wake radius; $r_0$ is the rotor radius; $u_\infty$ is the free stream wind speed; $u_r$ is the wind speed immediately behind the rotor; $u$ is the

![Figure 3.1: Jensen [57] wake model.](image)
speed of wind at downstream distance \( x \); \( \alpha \) is the wake decay constant; \( z_0 \) is the roughness of terrain; \( z \) is the turbine height; and \( a \) is the axial induction factor (the percentage reduction in wind speed from the free stream to the region close to turbine) [3]:

\[
\pi R^2 u_r + \pi \left( r^2 - R^2 \right) u_\infty = \pi r^2 u
\]

(3.1)

where \( u_r = (1 - a)u_\infty \), \( r = R + \alpha x \), \( R = r_0 \sqrt{\frac{1-a}{1-2a}} \) and \( \alpha = 0.5/\ln\left( \frac{z}{z_0} \right) \).

Figure 3.2 describes the wind speed recovery after an upstream turbine, based on the previous equations.

**Multiple Wakes: Sum-of-Squares**

Following Renkema [98], we write the effective wind speed in the wakes of multiple turbines as:

\[
\mathbb{E}[u_{id}] = u_{id,\infty} \left[ 1 - \left( \sum_{j \in U_{id}} \left( 1 - \frac{u_{ijd}}{u_{id,\infty}} \right) \right)^2 \right]
\]

(3.2)

\( u_{id} \) and \( u_{id,\infty} \) are the wind speeds at turbine \( i \) at wind state (speed and direction) \( d \) with and without wake interactions respectively, where \( d \in \mathcal{D} \), the set of all possible wind states; \( U_{id} \) is the set of upstream turbines for turbine \( i \) at wind state \( d \); \( u_{ijd} \) is the wind speed at turbine \( i \) due to a single wake from upstream turbine at \( j \), which can be obtained by (3.1). Compared with other wake combination models, the sum-of-squares model is the most accurate analytical model and is used in current wind farm layout design software packages [98,102].

Based on this model, the expected power production of the wind farm can be calculated as:

\[
\text{expected power} = \sum_{i=1}^{m} \sum_{d \in \mathcal{D}} \frac{1}{3} u_{id}^3 p_d
\]

(3.3)

where \( m \) is the total number of turbines and \( p_d \) is the probability of wind state \( d \) occurring, subject to \( \sum_{d \in \mathcal{D}} p_d = 1 \).

Note that we will be using average power (watts) instead of total energy (kilo-watt hours) in the
maximize \[ \sum_{i=1}^{n} \sum_{d \in D} \frac{1}{3} x_i \left( u_{id,\infty} \left[ 1 - \sqrt[3]{\sum_{j \in U_{id}} x_j \left( 1 - \frac{u_{ijd}}{u_{id,\infty}} \right)^2} \right] \right)^3 p_d \]

subject to \[ \sum_{i=1}^{n} x_i = k \]
\[ x_i + x_j \leq 1 \quad \forall (i,j) \in E \]
\[ x_i \in \{0, 1\} \quad \forall i = 1, \ldots, n. \]

Figure 3.3: SOM1: a constraint programming model.

objective function because they are equivalent for our purpose, and the former is easier to represent.

Multiple Wakes: Linear Superposition

Another way to account for multiple wakes in the energy production calculation is to use a direct linear superposition of power deficits. It is more easily representable in the mathematical programming models [7,8], because we can pre-calculate the pairwise interactions between two locations, then “activate” the interactions with binary variables indicating the existence of turbines at those locations, and sum up the interactions linearly:

\[
\text{expected power} = \sum_{i=1}^{m} \sum_{d \in D} \left( \frac{1}{3} u_{id,\infty}^3 - \sum_{j \in U_{id}} \frac{1}{3} \left( u_{id,\infty}^3 - u_{ijd}^3 \right) \right) p_d \tag{3.4}
\]

and again \( u_{ijd} \) can be obtained from (3.1).

With the physics introduced, we want to make a note on the representation of these equations in our optimization models. Although the power calculation equations (3.2) and (3.4) appear to be nonlinear in wind speeds, we can actually remove some of the nonlinearity due to the choice of discrete optimization models. As illustrated by Donovan [7], the linear superposition model (3.4) is completely linear, because all the wind speed terms can be calculated prior to the optimization, since the candidate turbine locations \((i, j)\) and wind states \(D\) are known in the discrete representation.

However, linearizing the wake model given by (3.2) is a non-trivial task, even though the wind speed terms can be pre-calculated. The next section will first introduce a CP model that directly represents the nonlinearity in its objective function, and then describe our novel approach that can incorporate the physics of (3.2) into a mixed-integer linear program.

3.3 Optimization Models

3.3.1 Sum-of-Squares Optimization Models (SOM)

The following three optimization models are based on the more accurate way of accounting for multiple wakes (3.2).
maximize \( \sum_{i=1}^{n} z_i \)

subject to

\( \sum_{i=1}^{n} x_i = k \)
\( x_i + x_j \leq 1 \quad \forall (i, j) \in \mathcal{E} \)
\( z_i \leq \sum_{d \in D} \frac{1}{3} u_{d, \infty}^3 p_d x_i \quad \forall i = 1, \ldots, n \)
\( z_i \leq M \left( |S_i| - \sum_{j \in S_i} x_j \right) + w_{i, S_i} \quad \forall S_i \subset I \setminus i \quad (\ast) \)
\( x_i \in \{0, 1\} \quad \forall i = 1, \ldots, n. \)

Figure 3.4: SOM2: a mixed-integer programming model.

**CP and MIP Models.**

Figure 3.3 presents the SOM1 CP model. The binary decision variable \( x_i \) represents whether there is a turbine at location \( i \); \( n \) is the total number of grid points; \( k \) is the total number of turbines; and \( \mathcal{E} = \{(i, j)\mid \text{grid } i \text{ and grid } j \text{ cannot both have turbines due to proximity constraint}\} \). This set is determined by the proximity constraint and the grid resolution. We choose equality for the constraint \( \sum_{i=1}^{n} x_i = k \) for practical reasons – the total number of turbines is usually determined prior to the optimization based on project financing and government regulations. For the problem instances used in this work, the optimal energy production is an increasing function of \( k \) [26].

In Figure 3.4, we present SOM2, a MIP sum-of-squares model where the nonlinearity is dealt with via a potentially exponential number of constraints. The auxiliary variable \( z_i \) represents the average power production at each location \( i \). The key of this model are the constraints indicated by \((\ast)\). \( M \) is a sufficiently large constant. In general, \( w_{i, S_i} \) is the maximum amount of power convertible when all cells with indices in \( S_i \) have turbines and all cells with indices in \( I \setminus S_i \) do not; \( I \) is the set of all turbine location indices \( \{1, \ldots, n\} \); and \( S_i \) is a set of turbine locations not including \( i \). \( w_{i, S_i} \) is calculated according to (3.2) and (3.3).

**A Decomposition Model.**

It is not hard to see that the number of constraints \((\ast)\) is exponential in \( n \) due to the requirement \( (\forall S_i \subset I \setminus i) \). Therefore, rather than experimenting with SOM2, we propose a third model, SOM3, which can be understood as a decomposition of SOM2.

In Fig. 3.5, a MIP master problem is formulated that includes all constraints of SOM2 except for those indicated with \((\ast)\). After solving the master problem, a sub-problem calculates the actual power according to (3.2) and (3.3) as follows:

1. Evaluate the turbine layout power considering full wake effects based on \( x^t \) (the solution from the master problem at iteration \( t \)) by substituting it into...
maximize \[ \sum_{i=1}^{n} z_i \]
subject to
\[ \sum_{i=1}^{n} x_i = k \]
\[ x_i + x_j \leq 1 \quad \forall (i, j) \in \mathcal{E} \]
\[ z_i \leq \sum_{i=1}^{n} \sum_{d \in \mathcal{D}} \frac{1}{3} u_{id, \infty} p_d x_i \quad \forall i = 1, \ldots, n. \] (cuts)
\[ x_i \in \{0, 1\} \quad \forall i = 1, \ldots, n. \]

Figure 3.5: SOM3: A mixed-integer programming model of the master problem.

\[ \sum_{i=1}^{n} \sum_{d \in \mathcal{D}} \frac{1}{3} x_i^t \left( u_{id, \infty} \left[ 1 - \sqrt{\sum_{j \not\in \mathcal{U}_d} x_j^t \left( 1 - \frac{u_{id}}{u_{id, \infty}} \right)^2 } \right] \right)^3 p_d \]

2. If it is evaluated to the same as the objective value from the master problem or the maximum solution time is reached, terminate; otherwise:
3. Generate cuts in the form of \[ z_i \leq g_i(x^t) \], where \[ g_i(x^t) \] is defined by (3.5) and (3.6); return to the master problem.

The master problem is then re-solved with the new cuts. In the first iteration, the master problem assumes that there is no wake interaction at all. In each subsequent iteration, the cuts refine the modeling of turbine interactions. The master problem does not represent the interaction of a specific group of turbines unless the related cuts are added. Therefore, the master problem always over-estimates the true objective value.

Instead of solving the master problem to optimality, we run it with a time limit of \( T \) seconds. In our experiment, we choose \( T_0 = 30 \) seconds for the first iteration. \( T \) is increased by 5 seconds each time the current best master solution is the same as the previous iteration. In other words, if the master problem produces the same solution as the previous iteration and it does not converge to the subproblem value, the algorithm will keep running with no new cuts generated, therefore getting stuck in a loop. This will happen if the master problem is unable to make any new progress in a new iteration (compared with the previous iteration) within the prescribed time limit.

**Cuts** We propose two types of cut: a no-good cut and a 3-cut. The former is presented in Equation (3.5). \( M \) is a large constant; \( x_j^t \) is the \( j \)th component of \( x^t \); \( w_{i, A} \) is the maximum amount of power convertible when all cells with indices in \( A \) have turbines; \( w_i, w_{i,j} \) and \( w_{i,jk} \) are short forms for \( w_{i,\emptyset}, w_{i,j} \) and \( w_{i,jk} \), following the definition of \( w_{i, A} \).

\[ g_i(x^t) = M \left( |S_i| - \sum_{j \in S_i} x_j \right) + w_{i,S_i} \quad \forall i = 1, \ldots, n . \] (3.5)

In practice, the no-good cuts alone are inefficient in large problem instances, because an exponential number of them are required to correctly shape the feasible region and the information of each cut is
maximize \[ \sum_{i=1}^{n} \sum_{d \in D} \left( \frac{1}{3} u_{id,\infty} x_i - \sum_{j=1}^{n} \frac{1}{3} (u_{ijd,\infty} - u_{ijd}) y_{ij} \right) p_d \]

subject to
\[ \sum_{i=1}^{n} x_i = k \]
\[ x_i + x_j \leq 1 \quad \forall (i, j) \in \mathcal{E} \]
\[ x_i + x_j - 1 \leq y_{ij} \quad \forall i, j = 1, \ldots, n. \]
\[ y_{ij} \geq 0 \quad \forall i, j = 1, \ldots, n. \]
\[ x_i \in \{0, 1\} \quad \forall i = 1, \ldots, n. \]

Figure 3.6: LSOM1 [7]

minimal when there are many wind states and location cells. Therefore, we propose another type of cut to increase the speed of refinement of the representation of turbine interactions. Equation (3.6) presents the 3-cuts.

\[ h_{ijk}(x^i) = w_i + (w_{i,j} - w_i)x_j + (w_{i,jk} - w_{i,j})x_k \quad \forall j, k \in \mathcal{S}_i, i = 1, \ldots, n . \quad (3.6) \]

For each downstream turbine \( i \), there are \( \binom{\mathcal{S}_i}{3} \) cuts generated. The power of 3-cuts lie in their accurate description of the interaction between a group of three turbines (thus the name 3-cut). In practice, the closest few upstream turbines have the most significant influence on a downstream turbine (see Fig. 3.2).

The following proposition states that the three-turbine interaction accurately describes the feasible region:

**Proposition 3.1.** The cut \( z_i \leq w_i + (w_{i,j} - w_i)x_j + (w_{i,jk} - w_{i,j})x_k \) is tight (cutting off all infeasible values for \( z_i \) assuming no turbines are “on” except for \( j, k \)) at \( (x_i, x_j, x_k) = (1, 0, 0), (1, 1, 0), \) and \((1, 1, 1)\).

**Proof.** When \( (x_i, x_j, x_k) = (1, 0, 0), (1, 1, 0), \) and \((1, 1, 1)\), the cut reduces to \( z_i \leq w_i, z_i \leq w_{i,j}, \) and \( z_i \leq w_{i,jk} \) respectively. These values are tight by definition of \( w_i, w_{i,j}, \) and \( w_{i,jk} \). When \( (x_i, x_j, x_k) = (1, 0, 1) \), the cut reduces to \( z_i \leq w_i + w_{i,jk} - w_{i,j} \). Since \( z_i \leq w_{i,k} \) by the definition of \( w_{i,k} \), and the combination of power deficits is sub-linear (3.2 and 3.3), \( w_i + w_{i,jk} - w_{i,j} \geq w_{i,k} \). Therefore \( z_i \leq w_i + w_{i,jk} - w_{i,j} \) is not tight and does not cut off any feasible region.

Overall, the no-good cuts ensure that the problem can eventually reach the true optimality while 3-cuts increase the communication between subproblem and master to speed up convergence.

### 3.3.2 Linear Superposition Optimization Models (LSOM)

Previous MIP models use a simpler (and less accurate) [98] calculation of energy: the power deficits from individual wakes are combined linearly to account for the total power loss. The following two MIP models are based on such linear superposition technique. The first model (LSOM1) was originally proposed by Donovan [7], while the second one (LSOM2) is our extension of LSOM1.
maximize \[ \sum_{i=1}^{n} z_i \]
subject to \[ \sum_{i=1}^{n} x_i = k \]
\[ x_i + x_j \leq 1 \quad \forall (i, j) \in \mathcal{E} \]
\[ z_i \leq \sum_{i=1}^{n} \sum_{d \in D} \frac{1}{3} u_{id,\infty} p_d x_i \quad \forall i = 1, \ldots, n. \quad (\dagger) \]
\[ z_i \leq \sum_{i=1}^{n} \sum_{d \in D} \left( \frac{1}{3} u_{id,\infty} - \sum_{j=1}^{n} \frac{1}{3} (u_{id,\infty} - u_{ijd}) x_j \right) p_d \quad \forall i = 1, \ldots, n. \quad (\ddagger) \]
\[ x_i \in \{0, 1\} \quad \forall i = 1, \ldots, n. \]

Figure 3.7: LSOM2

Figure 3.6 presents the LSOM1 model where \( \frac{1}{3} (u_{id,\infty} - u_{ijd}) \) is the power reduction at wind state \( d \) at turbine \( i \) due to the presence of upstream turbine \( j \). These values can be calculated prior to running the optimization (3.4). Variable \( y_{ij} \) indicates whether there are turbines at both positions \( i \) and \( j \), and so \( y_{ij} \) is 1 if both \( x_i \) and \( x_j \) are 1, and 0 otherwise. \( \mathcal{E} \) is the set of cell pairs \((i, j)\) that are too close to both host turbines. Due to the use of the simpler linear superposition model of upstream turbines, the model over-estimates the energy deficit [98].

Other location problems in the literature such as the maximum diversity problem (MDP) [96] and the \( p \)-dispersion-sum (pDS) problem [103] are similar to Donovan’s model. However, we have not seen any application of the state-of-art MDP/pDS solution algorithms to this wind farm layout optimization model.

In LSOM1, the \( y_{ij} \) variables are always equal to the product of \( x_i \) and \( x_j \), indicating if there are turbines at both places. Since \( i, j \in \mathcal{I} \), there are in total \( |\mathcal{I}|^2 \) \( y_{ij} \) variables. As our experiments below demonstrate, a high resolution of the wind farm grid with a complicated wind regime results in too many \( y_{ij} \) variables for reasonable performance. To address this weakness we propose LSOM2.

Figure 3.7 presents the LSOM2 model. It does not have \( y_{ij} \) variables. Instead, we use \( z_i \) to represent the power production at location \( i \). If there is no turbine at location \( i \), then the right hand side of constraint (\( \ddagger \)) is zero, and in most cases it is tighter than (\( \ddagger \)). So, if no turbine appears at \( i \), then there will be no power production from location \( i \). If there is a turbine at \( i \), then in general constraint (\( \ddagger \)) is tighter due to the extra negative terms (deduction of power due to upstream turbines). The value that \( z_i \) is, therefore, calculated by the total available power subtracting the linear combination of power loss due to wakes.

However, LSOM2 is not equivalent to LSOM1. When \( x_i = 0 \) and constraint (\( \dagger \)) is \( z_i \leq 0 \), constraint (\( \ddagger \)) may become \( z_i \leq -c \), where \(-c\) is a negative value. This is because the linear superposition model over-estimates power loss, making it possible for the right hand side of (\( \ddagger \)) to be negative. With the current problem setup (2km by 2km land) and turbine proximity constraint (turbine diameter of 40m, at least 5 diameters apart), this situation does not arise during our WR36 experiments (described in the next section). For one-directional wind regimes (WR1), this situation is also avoided with less than 50
3.4 Experiment Setup

All models were implemented with Microsoft Visual C++ Express 2010 and IBM ILOG CPLEX 12.3. Twelve benchmark instances [4–6, 9, 26] (referred to as WR$q$-$n$-$k$) were used to test the performance of models. WR1 refers to the wind regime of 1 directional wind (from west to east) and WR36 refers to the wind regime with wind coming from 36 directions at different speeds (Fig. 3.8).

Experiments were run on a Dell Vostro 460 with Core i5-2500 CPU (3.30GHz) and 64-bit Windows 7 OS. Since CPLEX solvers are deterministic by default, only one run of each instance was performed. Common parameters are: $z = 60m$, $z_0 = 0.3m$, $R = 20m$, and the wind farm is 2km by 2km.

Since the SOM models evaluate the power production by sum-of-squares (3.2) and the LSOM evaluation is based on linear superposition (3.4), the four models are not directly comparable. We therefore compare the solution quality in Table 3.1 by a posteriori re-evaluating the LSOM solutions based on the (more accurate) sum-of-squares method. The power production values in brackets indicate the objective function value of the LSOM solutions.

3.5 Results

Table 3.1 summarizes the performance of the four models by comparing the expected power and solution times. The MIP optimality gaps are included where applicable. Columns $n$ and $k$ represent the total numbers of cells and turbines. For LSOM1 and LSOM2, there are two power values: the power calculated with the sum-of-squares wake model and, in parentheses, the objective function value based on linear superposition model. Overall, LSOM2 outperforms the other models in most cases in terms of solution quality. SOM3 and LSOM2 can solve problem instances with high grid resolution and high wind data resolution (WR36-400), while the other two models cannot even initialize these instances within an hour due to the size of the model.
SOM1 vs. SOM3

Table 3.1 shows that SOM3 solves more instances than SOM1. In the higher resolution case (WR36-400), the SOM1 objective function expression must account for many wind directions and turbine pairs leading to memory saturation during the model creation phase and the inability to start search within an hour. SOM3 also performs better in WR1 cases. We believe that the simpler turbine interactions for WR1 instances are accurately captured by no-good and 3-cuts.

For a more detailed examination of these results, Figs. 3.9–3.11 present the evolution of solution quality over time for SOM1 and SOM3 for selected problem instances. In the single-direction scenario (WR1), SOM3 consistently outperforms SOM1. For the WR36 instances where SOM1 was able to run, SOM3 performs much worse than SOM1 (Fig. 3.11). To understand these results, recall that SOM3 is a decomposed model where the improvement from iteration to iteration is based on cuts representing information of turbine interactions. In the WR1 cases, every cell has about 10 (20) upstream cells, because the wind farm resolution is 10 by 10 (20 by 20). During the optimization, the better layouts often have turbines spaced out in the wind direction, thus the 3-cut, although only describing the interactions between a few turbines, already contains enough information for the master problem to make good decisions. However, in the WR36 cases, every turbine has $k-1$ upstream turbines and the 3-cut only expresses the impact of the most significant two upstream turbines. Therefore, SOM3’s search for better objective value in WR36 instances is often stalled due to the lack of effective cuts.

SOM3 also improves more slowly in the WR1-400 instances (Fig. 3.10) than in the WR1-100 instances (Fig. 3.9). There are more combinations of three turbines in the former instances and therefore more 3-cuts must be generated for the master problem to improve its objective value. Eventually SOM3 catches up with SOM1 because the 3-cuts describe the interactions of all turbines reasonably accurately.

LSOM1 vs. LSOM2

Table 3.1 shows that the power production calculated by linear superposition method (in brackets) is always lower than the sum-of-squares calculation. For some values of $n$ and $k$, the problem cannot be solved to optimality within an hour, while some other instances are solved in less than 10 seconds. This observation confirms with other work on MDP and $p$-dispersion-sum problems [39]: increase in $n$ with fixed $k$ will often lead to longer solution time, and increase in $k$ while $n$ is fixed often leads to longer solution time too (except for when $k$ is close to 0 or $n$). Current state-of-art MDP algorithms are benchmarked on problem instances with similar dimensions (in $n$ and $k$) as our instances [39].
Table 3.1: Comparison of solutions based on sum-of-squares power calculation (\(^\circ\): experiments took more than one hour to setup; boldface: better objective value). Numbers in brackets are the original objective values (based on linear superposition method) of LSOM solutions.
Table 3.1 clearly shows that LSOM2 outperforms LSOM1 in solution quality in all but one instance (WR1-400-40). In the WR1-400-40 case, LSOM1 outperforms LSOM2 in terms of the revised power calculation, however, LSOM2 is “misled” by the objective function (in brackets). Thus if we compare the LSOM1 and LSOM2 power productions based on the sum-of-squares evaluation, LSOM2 strictly dominates LSOM1 in solution quality. In terms of computation time, LSOM2 performs similarly to LSOM1 except the WR1-100-30 and WR1-100-40 instances. A closer look at the CPLEX solution log
Chapter 3. Analytical Wake Models

3.6 Discussion

LSOM1 represents the state-of-art solution model for this wind farm layout optimization problem. Our LSOM2, an extension of LSOM1, outperforms it and the other two models we proposed on most instances. Since the SOMs represent the first time that the most accurate analytical wake equations [98] are modelled with constraint programming and mixed-integer programming, there is much to learn about the performance and potential opportunities for the SOMs. We describe several promising research directions.

For SOM1, nonlinearity appears only in the objective function, thus we could apply nonlinear solvers that are based on linear solvers (e.g., SCIP [104]). The SOM3 cuts capture information in two ways: no-good cuts capture interaction between all $k$ turbines in very specific layouts, while the 3-cuts capture information from a wider range of layouts, but limited to the interaction among three turbines. We can potentially apply the same idea of 3-cut and generate constraints that inform the master problem more effectively, without having to generate too many of them (e.g., 4,5-cut).

A straightforward hybridization would be the sequential application of SOM1 and SOM3, where we start the problem with low resolution (coarse grid) and progressively increase it. In this case, we could solve SOM1 in the initial stages, utilizing the constraint propagation of CP solvers for the proximity constraints, and then solve the problem with SOM3 in the later stages while fine-tuning the turbine positions, utilizing the fact that this fine-tuning focuses on clusters of closely located turbines and that such information can be effectively captured by 3-cut (or 4,5-cut).

Finally, it is interesting to observe that although the LSOMs employ less accurate power models (i.e., their optimal solutions are not the same as the optimal solutions of SOM), LSOMs can still produce good solutions even when benchmarked by the more accurate power calculations. We plan to explore this in more detail once the SOMs are improved.

3.7 Conclusion

We have presented the wind farm design layout problem and proposed two models to incorporate the nonlinearity in the problem. The first model (SOM1) is a direct formulation of the problem in constraint programming. While having promising performance under complex wind scenarios, the major drawback of this approach is the “curse of dimensionality” – the growth of the numbers of variables and terms quickly exceeds reasonable computational capacity. A second decomposed MIP model (SOM3) performs well in the simple wind regimes, because no-good and 3-cuts can accurately describe the turbine interactions. However with more complicated wind regimes, SOM3 is unable to improve its early feasible solutions due to the weakness of the current cuts. We also presented a novel extension of an existing LSOM model. The LSOM models are based on a less accurate model of power productions, thus having different objective functions than the SOMs. However, the models can be solved more quickly and
achieve high quality solutions when a posteriori evaluated with the more accurate sum-of-squares power calculation.

In summary, we have presented two new models for the wind farm layout optimization problem. These CP and MIP models are the first mathematical programming models that capture the wind turbine interactions by modelling the sum-of-squares equations – most accurate analytical multi-wake modelling in the literature [98]. We also presented an extension (LSOM2) to a previous MIP model (LSOM1) and demonstrated improved solution quality and time. Based on the experimental study, we think that the most promising directions for future work include the strengthening of cuts for SOM3 and hybridization of SOM1 and SOM3.
Chapter 4

Noise Propagation and Energy Production

Past studies on wind farm layout optimization have focused on energy-maximization, cost-minimization or revenue-maximization objectives [4, 5, 7]. As land is more extensively exploited for onshore wind farms, wind farms are more likely to be close to residential locations. Therefore governments, developers, and landowners are increasingly aware of wind farms’ environmental impacts. After considering land constraints due to environmental features, noise generation remains the main environmental/health concern for wind farm design. Therefore, noise generation is sometimes included in optimization models as a constraint. Here we present continuous-location models for layout optimization that take noise and energy as objective functions, in order to fully characterize the design and performance spaces of the optimal wind farm layout problem. Based on Jensen’s wake model and ISO-9613-2 noise calculations, we used single- and multi-objective genetic algorithms to solve the optimization problem. Results from the bi-objective optimization model illustrate the trade-off between energy generation and noise production by identifying several key parts of Pareto frontiers. In particular, the different regions of a Pareto frontier represent very different layout types that the wind farm developers can choose from, depending on the emphasis on either objective. We show that the multi-objective genetic algorithm produces a spectrum of good layouts, many of which comply with the noise regulation policy.
4.1 Introduction

Wind energy is still facing resistance in North America due to health and environmental concerns. The government of Canada has published a series of reports regarding noise generation of wind farms [105-107]. Regardless of whether wind farm noise has negative health impact, it concerns both the developers and the residents near wind farms. Therefore noise is an important factor in wind farm design.

Due to the aerodynamic nature of both energy capture and noise generation, these are usually competing factors, meaning that the more energy we capture with a given set of wind turbines, the more noise it might generate for surrounding residential communities. Many aspects affect the noise generation of wind turbines. On one hand, the operation of turbine’s mechanical components produces noise. On the other hand, wind flowing through turbine generates another type of noise. The wake interactions between turbines can also change the noise level and propagation pattern. Therefore, the faster turbines operate (higher revolutions per minute, or rpm), the more noise they generate. An indirect effect appears when the number of turbines increases in a given area with the goal of increasing the energy capture. In such situations, the average expected distance between the turbines and a noise receiver at the wind farm boundary decreases, thus increasing the sound level measured at the receiver’s location.

Traditionally, wind farm design engineers and researchers have included noise as a constraint in their optimization model [8]. This means that when they considered noise as a design factor, they usually tried to find the wind farm design with maximum energy (or minimum cost per energy), while keeping the noise levels below a certain threshold. There are a few potential limitations to this approach. If the optimization is done manually, as it is typically done in the wind energy industry, wind farm design is an iterative and lengthy process, involving stages of layout design for maximum energy, checks for compliance with environmental restrictions (for example, noise), and refinement of the layout based on infrastructure considerations. Also, feasible solutions might be scarce, if they exist at all, so that it takes a long time to find an acceptable layout, or decide that the project is not profitable. Feasible solutions, in the optimization sense, refer to the layouts that satisfy all environmental, infrastructure and financial constraints, including the noise regulations. On the other hand, if optimization relies on computer, designers will locate feasible solutions faster, but might focus only on one final solution without getting any insights on the design trade-offs, sensitivity, robustness and other acceptable design alternatives. In summary, there is a need for a computational approach for optimization of noise-constrained wind farm layouts that is capable of (a) finding the optimal solution, as well as a set of feasible solutions with acceptable performance, and (b) elucidating the design trade-offs and sensitivity of the solution to changes in the position of individual turbines.

We propose a different approach to this problem. Our method considers both noise minimization and energy maximization as objectives, using a probabilistic optimization algorithm, namely Genetic Algorithms (GAs). This way, we hope to identify whether noise and energy generation are truly competing factors; and if so, what the relationship between noise and energy is. In addition, by analysing the populations of solutions generated by the Genetic Algorithm, we can gain insights into characteristics of layouts that are associated with good performance. In other words, our goal in using this approach is to understand the trade-off between energy and noise in wind farm layout design and to assist engineers in formulating design guidelines.

Common threads across existing WFLO studies are: (a) the objective functions used: maximum energy capture, minimum cost of energy, or a weighted sum of energy capture and cost, (b) a pre-
determined number and type of turbines, and (c) minimum turbine proximity and convex-polygonal wind farm boundaries are the only constraints enforced during the optimization.

On a broader scope, in our future work we intend to solve the full-scale, comprehensive wind farm layout optimization problem, including major aspects of the problem: Energy capture, environmental impact and cost. As a first step towards that vision, in this chapter we will focus on understanding the energy-noise trade-off in wind farm layout design.

The remainder of this chapter is organized as follows. In the following section, we describe the models we used to predict energy capture and noise generation/propagation in the wind farm. Then, we present a description of the optimization method used in this work, namely the multi-objective genetic algorithm with fitness assignment by non-dominated sorting. Then, we present our test case, followed by our results in two aspects: (a) validation of our models against industrial-grade software, and (b) single- and multi-objective optimization. We close with our concluding remarks and a discussion of future work.

4.2 Wind Farm Modelling

4.2.1 Wake Modelling

An analytical, closed-form wake model is used to quantify the aerodynamic interaction between turbines. This model was first proposed by Jensen [8], who developed it by considering that momentum is conserved within the wake, and that the wake region expands linearly in the direction of the flow, as shown in Fig. 4.1.

To determine the effective wind speed experienced by a turbine located within another turbines wake, the momentum balance equation can be written as

$$\pi r_r^2 u_r + \pi \left(r_1^2 - r_r^2\right) u_o = \pi r_1^2 u$$

where $r_r$ is the wake radius immediately after the turbine, $r_1$ is the radius of the wake at any position $x$ measured downstream, $u_o$ is the free stream wind speed, $u_r$ is the wind speed immediately behind the rotor, and $u$ is the speed of wake a downstream distance $x$. According to Betz’s theory [3], the wind speed immediately behind the rotor is approximately $1/3$ of the free stream speed, and with the assumption of a linearly expanding wake, the downstream speed can be calculated as
Chapter 4. Noise Propagation and Energy Production

Figure 4.2: Wind speed along a single wake’s centerline, as a function of distance normalized with the turbine diameter.

\[
\begin{align*}
\frac{u}{u_0} &= \left(1 - \frac{2}{3} \left(\frac{r_1}{r_r}\right)^2\right) \quad (4.2)
\end{align*}
\]

where

\[
\begin{align*}
r_1 &= r_r + \alpha x \quad (4.3)
\end{align*}
\]

\(\alpha\) is the entrainment constant, also known as the wake decay constant, and is calculated (empirically) as

\[
\alpha = \frac{0.5}{\ln \left(\frac{z}{z_o}\right)} \quad (4.4)
\]

where \(z\) is the hub height and \(z_o\) is the surface roughness of the terrain, both in metres. Fig. 4.2 shows the variation of wind speed as a function of position along the wake’s centerline.

For turbines under the influence of multiple wakes, an effective wind speed can be calculated from the sum of kinetic energy deficits from upstream turbines, same as the sum-of-squares model (3.2) in Chapter 3. Note that this is a superposition approach that assumes that kinetic energy deficits can be aggregated. Although this is a simplification of the complex fluid dynamics involved in wake merging, this approach has been used extensively in previous work, especially for optimization purposes, and it is used in commercial software for wind farm design. For more complex models of wake dynamics, the reader can refer to [108]. The effective speed of the turbine inside \(n\) wake regions can therefore be expressed as

\[
\begin{align*}
\frac{u}{u_0} &= \left(1 - \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(1 - \frac{u_i}{u_0}\right)^2}\right) \quad (4.5)
\end{align*}
\]

where \(u_i\) is the reduced wind speed in a single wake due to upstream turbine \(i\).
Based on the effective wind speed at the turbine rotor, the power produced by the turbine can be calculated through the manufacturer-supplied power curve. Without loss of generality, in this work we follow previous work [4,5,26] and use a simplified expression for a turbine’s power production: power is a simple continuous function of the local effective speed at hub height. Hence, when the farm is subjected to a uniform wind speed, the total power extracted from $K$ wind turbines is expressed in the following equation:

$$P_{tot} = \sum_{i=1}^{K} \frac{1}{3} u_i^3$$  \hspace{1cm} (4.6)$$

Finally, we note that the annual energy production (AEP) of wind farm is defined as the integration of power production (kW) over time (h). This is an expected value of a random variable, as it is based on the probability distribution of wind speeds and directions. Hence, it is calculated as

$$\text{AEP} = 8766 \sum_{i} \sum_{j} \sum_{k} F_{ijk} P_{ijk}$$  \hspace{1cm} (4.7)$$

where $i$, $j$ and $k$ are indices over the number of wind directions, speeds and the number of turbines, respectively, $F_{ijk}$ is the probability of wind coming at (free stream) speed $v_i$ from direction $\theta_j$ at turbine location $k$, and $P_{ijk}$ is the corresponding power generated by that turbine.

Overall, this set of equations for single wake, multiple wakes, and power curve are the same as (3.1, 3.2, 3.3) in Chapter 3.

4.2.2 Noise Modelling

In the context of the ISO-9613-2 standard [67], receptors are the locations where the sound level is to be measured or predicted. In wind farm layout design, all residence located within the wind farm terrain, or within a certain neighbourhood, are considered receptors for noise calculation purposes.

In a practical setting, the equivalent continuous downwind octave-band sound pressure level (SPL) at each receptor location is calculated for each point source, at each of the eight octave bands with nominal mid-band frequencies from 63 Hz to 8 kHz [67],

$$L_f = L_w + D_c - A$$  \hspace{1cm} (4.8)$$

where $L_w$ is the octave-band sound power emitted by the source, $D_c$ is the directivity correction for sources that are not omni-directional, $A$ is the octave-band attenuation, and $f$ is a subscript indicating that this quantity is calculated for each octave band.

Several octave-band weightings are available to convert the sound pressure levels in (4.8) to an effective SPL. For wind farm layout applications, it is customary to use A-weighted sound pressure levels [106]. The equivalent continuous A-weighted downwind sound pressure level at specific location can be calculated from summation of contributions of each point sound source at each octave band, as follows

$$L_{avg} = 10 \log \left( \sum_{i=1}^{n_s} \left( \sum_{j=1}^{8} 10^{0.1(L_f(i,j)+A_f(j))} \right) \right)$$  \hspace{1cm} (4.9)
Figure 4.3: Sound Pressure Level (A-weighted) as a function of distance from the source.

$n_s$ is the number of point sound sources, $j$ is the index representing one of the eight standard octave-band mid-band frequencies, and the $A_f(j)$ are the standard A-weighting coefficients.

The attenuation term ($A$) in (4.8) is the sum of different attenuation effects

$$A = A_{\text{div}} + A_{\text{atm}} + A_{\text{gr}} + A_{\text{bar}} + A_{\text{misc}}$$

due to geometrical divergence ($A_{\text{div}}$), atmospheric absorption ($A_{\text{atm}}$), ground effects ($A_{\text{gr}}$), sound barriers ($A_{\text{bar}}$) and miscellaneous effects ($A_{\text{misc}}$). In this model, it is assumed that the attenuation due to sound barriers and miscellaneous effects are insignificant. Further detail of the calculation procedure can be found in the ISO 9613-2 document [67]. An illustration of the behaviour of the SPL as a function of distance with respect to the source is shown in Fig. 4.3. This model does not consider the effect of wind speed and direction on the propagation of sound.

4.3 Optimization with Genetic Algorithms

Genetic algorithms (GAs) [109] are probabilistic search algorithms inspired by the concept of natural selection and survival of the fittest. GAs search through the solution space by keeping a population (set) of solutions, which are ranked according to their fitness to solve the optimization problem (e.g. objective function values), and evolved through many generations. An important advantage of GAs is that they do not require information about the gradient of the solutions, therefore avoiding problems with the non-convexity and non-continuity of the solution space. This characteristic of GAs makes them well suited for the wind farm layout problem. On the other hand, GAs typically exhibit slow rates of convergence, thus increasing the computation cost and runtime of the optimization. In this work, we will not focus on improving the runtime behaviour and/or convergence rate of the algorithm. Rather, we will exploit its advantages to characterize the design space of the wind farm layout.

There are several variants GAs that are suited for Multi-Objective Optimization (MOO) problems, such as the Strength-Pareto Evolutionary Algorithm (SPEA, SPEA-2) [65,110], and the Non-Domination Sorting Genetic Algorithm (NSGA, NSGA-II) [66,111], among others [112]. Both SPEA-2 and NSGA-II have been shown to have similar performance over an array of test functions. This is expected since the
algorithms are very similar, the main difference being the method used to convert multiple objective function values to a unique metric of fitness. In this work, we use the NSGA-II algorithm; its main steps are shown in Fig. 4.4.

In the wind farm layout problem, \( n_{\text{pop}} \) initial layout patterns are generated randomly, and the corresponding objective values (energy generation, noise level) are evaluated. For each individual in the population, a rank is assigned according to their non-domination status (Non-Dominated Sort) and the distance between the solution and its neighbours in the objective space (Crowding Distance). One layout dominates the other if the first is no worse than the latter in all objectives, and strictly better in at least one objective. The \( n \)-dimensional objective space is defined as the cartesian space where each axis represents an objective, for a total of \( n \) objectives. In the Parent Selection stage, parents for the next generation are chosen based on the rank and the crowding distance via binary tournament. Solutions with lower rank values are preferred, as the ranks are assigned so that the current Pareto front has rank 1. Crowding distance is used as a secondary fitness value to break ties when comparing solutions based on rank. After parents are selected, an offspring generation of size \( n_{\text{off}} \) is created by combination (crossover) and mutation of the layout patterns of the parent generation.

Previously, many studies focused on using binary variables to represent the wind farm layout (a sequence of 1’s and 0’s indicating the existence of turbines at potential sites). In this work, we consider the turbines to be continuously located across the wind farm. Under this setup, the combination of parents is done by individually combining each pair of turbines to produce a new turbine location. In other words, we rank the turbine coordinates in both parents based on their appearance in the decision
variable. Then we take turbine 1’s coordinates \((x_1, y_1)\) from parent 1, and turbine 1’s coordinates \((a_1, b_1)\) from parent 2, and produce the turbine 1 coordinates of child layout as \((\lambda x_1 + (1 - \lambda)a_1, \lambda y_1 + (1 - \lambda)b_1)\), where \(\lambda\) is a random number in \([0, 1]\). We repeat this process until we have new coordinates generated for all turbines.

After evaluating the objective function values of the offspring population, the offsprings are merged with the parent population, and new rank and crowding distance values are assigned. Elitism is implemented by keeping only the best (i.e. rank 1) or the first \(n_{\text{pop}}\)-best solutions for the next generation (iteration) of the algorithm. The readers are referred to [66] for more details on the algorithm and its implementation.

4.4 Test Cases

Fig. 4.5 shows the problem scenario with WR1 – a wind regime with only one direction of wind with uniform speed, following Mosetti et al. [4], Grady [5] and others. WR36 is the second set of cases with a more complex wind regime, as described by the probability distribution in Fig 4.6. In previous work such as [4,5], a discretized version of the optimization problem was solved by defining a square grid over the wind farm terrain. In this work, we allow turbine positions to vary continuously, to more closely reflect the setting found in layout design practice. Note that we do not enforce proximity constraints in our optimization, as we would like to see them arise naturally from the optimization objectives. In previous work, proximity constraints were enforced, either directly as actual constraints, or indirectly using the discrete version of the problem.

The next section presents our results. First, we comment on the validation of our implementation of the energy and noise models, by comparing our results with previous work and/or an industry-grade, open-source software for wind farm design and analysis, OpenWind [113]. Then, we present the trade-off surface for the multi-objective, energy-noise optimization. Finally, we compare our results for the single-
objective optimizations, and discuss the potential implications for wind farm layout design practice.

Table 4.1: Wind turbine parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbine hub height ($z$)</td>
<td>60 m</td>
</tr>
<tr>
<td>Terrain Roughness Length ($z_0$)</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Rotor Radius ($r_r$)</td>
<td>40 m</td>
</tr>
<tr>
<td>Thrust Coefficient ($C_T$)</td>
<td>0.88</td>
</tr>
<tr>
<td>Power Curve</td>
<td>$0.3u^3$ kW</td>
</tr>
<tr>
<td>Noise Generation ($L_w$)</td>
<td>100 dB</td>
</tr>
</tbody>
</table>

4.5 Results and Discussion

4.5.1 Validating the Models

The first task in our optimization effort was the implementation of the wake and noise models for a wind farm. We chose C++ for its computational efficiency.

To validate our implementation, we evaluated the annual energy production (AEP) and maximum noise level of two layouts with different numbers of turbines using (a) our implementation of the models, and (b) OpenWind [113], an open source, industry-grade software. Table 4.2 shows the predicted energy performance according to these models, and their difference expressed as a percentage of the OpenWind prediction, which is assumed to be correct [60]. Figures 4.7a and 4.7b show a comparison of the predicted sound pressure level inside the wind farm terrain.

4.5.2 Energy Maximization

We present the results from our energy optimization runs in comparison with several results from previous work in Table 4.3 and 4.4. The previous studies have used the same problem setup. It is notable
Table 4.2: Comparison of Annual Energy Production predictions of 30 turbines between current implementation and OpenWind [113].

<table>
<thead>
<tr>
<th>Turbine</th>
<th>Current Study</th>
<th>OpenWind</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR1</td>
<td>132.38 GWh</td>
<td>132.17 GWh</td>
<td>+ 0.2</td>
</tr>
<tr>
<td>WR36</td>
<td>225.88 GWh</td>
<td>230.48 GWh</td>
<td>- 2.0</td>
</tr>
</tbody>
</table>

(a) Our implementation of ISO-9613-2 standard [67].
(b) OpenWind [113].

Figure 4.7: Noise Contours

that although NSGA-II is not specifically designed for single objective optimization, its performance is comparable to the algorithms used in the previous studies in terms of final solution quality. This is an advantage since, in some practical situations, an optimization algorithm that can perform single- and multi-objective optimization tasks can quickly adapt to different design criteria.

We also want to note that that each study implements its own code for wake expansion, but there is no standard and explicit way of dealing with the wakes. For example, when a turbine rotor is partially inside the influence of an upstream turbine, most previous studies did not mention how to calculate the impact. The only exception is a recent work done by Archer et al. [17]. This has probably caused the inconsistency between our evaluation of the energy production from previous studies’ optimal layouts (e.g. Du Pont and Cagan [26] reported an efficiency of 100% for WR1, 30 turbines optimal layout, while we found the same layout to be 88.35% efficient with our model). The discrepancy is possibly due to the difference in their energy evaluation method and ours in (4.1) - (4.7).

4.5.3 Multi-objective Optimization

Once the models were validated, we focused our efforts on the optimization. Figures 4.10 and 4.11 show the spatial histogram of turbine locations for multiple layouts from the multi-objective NSGA-II algorithm. In other words, the figure shows the relative frequency with which one or more turbines were placed on a given cell during the optimization process.

A wind farm designer can extract valuable information from Fig. 4.10 and 4.11. For example, it is
Table 4.3: Comparison of Maximum Energy Production Efficiency (%): Current study (NSGA-II and models in Chapter 3), Grady [5], and Du Pont and Cagan [26], WR1. Full efficiency (100%) refers to the hypothetical case of no wake effects.

<table>
<thead>
<tr>
<th>No. of Turbines</th>
<th>Wind Farm Efficiency (%)</th>
<th>NSGA-II</th>
<th>Grady</th>
<th>Du Pont</th>
<th>SOM1</th>
<th>SOM2</th>
<th>LSOM1</th>
<th>LSOM2</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
<td>99.79</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>98.98</td>
<td>-</td>
<td>-</td>
<td>99.74</td>
<td>99.77</td>
<td>99.77</td>
<td>99.77</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>98.14</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>30</td>
<td></td>
<td>91.77</td>
<td>95.13</td>
<td>88.35</td>
<td>95.54</td>
<td>95.98</td>
<td>95.95</td>
<td>95.97</td>
</tr>
<tr>
<td>35</td>
<td></td>
<td>91.16</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>39</td>
<td></td>
<td>92.29</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>93.05</td>
<td>-</td>
<td>-</td>
<td>89.78</td>
<td>90.83</td>
<td>90.83</td>
<td>90.83</td>
</tr>
<tr>
<td>45</td>
<td></td>
<td>88.51</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.4: Comparison of Maximum Energy Production Efficiency (%): Current study (NSGA-II and models in Chapter 3), Mosetti et al. [4], Grady [5], and Du Pont and Cagan [26], WR36. Full efficiency (100%) refers to the hypothetical case of no wake effects.

<table>
<thead>
<tr>
<th>No. of Turbines</th>
<th>Wind Farm Efficiency (%)</th>
<th>NSGA-II</th>
<th>Mosetti</th>
<th>Grady</th>
<th>Du Pont</th>
<th>SOM1</th>
<th>SOM2</th>
<th>LSOM1</th>
<th>LSOM2</th>
</tr>
</thead>
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<tr>
<td>15</td>
<td></td>
<td>96.58</td>
<td>96.69</td>
<td>-</td>
<td>94.51</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>95.37</td>
<td>-</td>
<td>-</td>
<td>95.67</td>
<td>95.42</td>
<td>95.84</td>
<td>95.85</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>93.78</td>
<td>-</td>
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clear that (as we expect from our intuition), if the goal of the optimization is to minimize sound levels at the boundary of the farm, optimal layout configurations will tend to have only a few turbines near the boundaries, with the tendency to concentrate towards the centre of the farm. On the other hand, if the focus is on maximizing energy production, turbines tend to stay further from each other, spreading into the boundary regions. In addition, by correlating both figures we can see which land cells are critical to achieve a design that both increases AEP and decreases SPL. In particular, there are several areas with larger sampling frequencies, meaning that layouts with turbines in these locations tend to perform better according to our optimization objectives.

Note that the spatial histograms used in this work are a novel way of presenting the optimization results. Their usefulness is based on the assumption that, due to the selection procedure implemented in the optimization algorithm, its long-term sampling distribution provides information about the probability of a given location being part of an optimal layout. This is valuable information for the wind farm developers, as it quantifies the importance of a given piece of land for an optimal wind farm design. In other words, looking at the spatial histograms enables the designer to leverage data generated during...
Chapter 4. Noise Propagation and Energy Production

Fig. 4.8 and 4.9 show the estimated Pareto frontiers for WR1 and WR36 cases respectively. They are not the exact Pareto frontiers because we have no proof of their global optimality. Each point in the graphs corresponds to a layout, and the coordinates indicate its energy and noise performance. In this problem setup, maximum energy production and minimum noise propagation are indeed competing objectives – as seen by the general trend of increase in noise level with more energy production (for a certain number of turbines).

A point in the top region of a Pareto front corresponds to layouts that have superior energy performance (the x-axis is in reversed). Not surprisingly, the maximum amount of energy achievable increases monotonically as a function of total number of turbines. In general, the layouts indicated by the top of the Pareto fronts produce relatively more energy (Fig. 4.10). Similarly, the layouts that correspond to the lower part of a Pareto front produce less energy (Fig. 4.11). To look at these properties from another perspective – the more spread out the turbines are, the less wake effects they experience in general, and vice versa (Fig. 4.10 and 4.11).

In most Pareto fronts, we can see discontinuities, and also premature stops at both ends of the fronts. This is due to those regions of the Pareto fronts being more difficult to reach from a randomized initial set of layouts, i.e., there are fewer layouts (percentage-wise) that correspond to the extrema of the Pareto for WR1, 15- and 20-turbine cases. Because our problem is continuous location with square wind farm, there is no inherent discontinuity in the design space.

In both cases, there are almost crossing of Pareto fronts – more obvious in WR1, but certainly likely in WR36 cases too.

Adding one more turbine can significantly introduce more energy, considering that the total energy
efficiency is probably close to 90% (Tables 4.3 and 4.4). Therefore, adding an extra turbine allows the layout to be much more “compact” while having a similar amount of energy production.

Suppose that the noise regulation permits wind farms that produce less than a certain amount of noise at receptors, then the exact amount of the threshold is very critical. If the noise regulation level falls towards the top of the Pareto fronts, then there is a lot of freedom in terms of choosing the layouts, and more importantly, there is a unique answer to the total number of turbines that we should purchase (subject to the financial analysis of the marginal benefit of one additional turbine). In other words, a typical energy-noise point in the top region pretty much corresponds to only one number of turbines.

However, if the noise level is at the “crossing” area, then there would be multiple numbers of turbines that satisfy the same energy-noise performance criteria – this triggers another “degree of freedom” in the wind farm design process. And as shown in Fig. 4.12 and 4.13, two layouts (with different numbers of turbines) that generate similar amounts of energy/noise can look very differently. This is of practical interest because there are several economic decisions that are related: the total number of turbines to be purchased, the cost of purchasing one extra/less turbine and its affiliated infrastructure, and the price and licensing difficulty in getting land permits (the 40-turbine case obviously has a less environmental/social footprint). In addition, it is possible that with another type of turbine, the positions of the curves would shift entirely, making the “crossing” area much different than the ones exhibited. Therefore, that is also an decision making tool that we can use – incorporating different types of turbines and see the exact performance of the turbines.
Figure 4.10: Spatial histogram indicating the frequency of turbines positions for layouts in the top 10% of the energy performance (2000m by 2000m wind farm). Darker colour indicates higher frequency of turbine placement.

Figure 4.11: Spatial histogram indicating the frequency of turbines positions for layouts in the bottom 10% of the energy performance (2000m by 2000m wind farm). Darker colour indicates higher frequency of turbine placement.

4.6 Conclusion

Although wind farm layout optimization, like many other engineering design problems, is inherently multi-objective, previous studies rarely touched on the multi-objective optimization of this problem. In this work, we presented the application of Non-dominated Sorting Genetic Algorithm to this problem, with the goal of validating this algorithm for this problem, and more importantly, extract insights from the single and multi-objective optimization tasks. We used the same experimental setup as several previous studies to facilitate comparison. In addition to demonstrating that NSGA-II is suitable for both single and multi-objective optimization, we spent the most effort in analyzing the Pareto frontiers of the energy-noise performance of various numbers of turbines. We discussed the importance of several regions of the Pareto fronts, and we provided a qualitative sensitivity analysis of how the regulation noise limit would impact the design of the wind farm significantly, due to the close connection with parameters...
such as total number of turbines, land utilization, energy production, noise propagation, and potentially turbine types. This insight, along with the application of NSGA-II for the energy-noise optimization of wind farm design, is a novel contribution. There are several natural directions for future studies. For example, the computational efficiency of the NSGA-II for this problem and related multi-objective optimization tasks; and the inclusion of other objectives. On the problem instance side, we can also extend the setup to include real cases where terrain is discontinuous, and/or the noise receptors are not exclusively at the boundary. We also want to note that we chose a continuous-optimization task, different than the majority of other wind farm layout optimization problems. Therefore incorporating discontinuity could be an interesting topic, and it can potentially yield more accurate (higher quality) layout designs for this problem.
Chapter 5

Future Work

This thesis presents two different approaches in solving the wind farm layout optimization problem. The mathematical programming approach is applied to the single-objective, discrete-location optimization of wind farm layouts. While the mathematical programming models require more tailoring (the deep integration of different analytical wake models in the MIP and CP models in Chapter 3), the advantage is the measuring of the quality of the solution (with the natural inclusion of optimality gaps). Practically, the CP and MIP models can be solved directly with off-the-shelf optimization solvers. On the other hand, genetic algorithms (such as NSGA-II) perform well under more complex constraint and objective functions, since GAs adopt the blackbox approach. The flexibility of this optimization algorithm leads us to focus more on the comparison of different objectives, and the resulting layouts under such objectives.

Complex Terrain and Wake Modelling  In terms of the modelling scope, we want to note that an important extension of the WFLO is missing from the literature – the modelling of complex terrains. Given that the metaheuristics can generally incorporate complex objectives and constraints, we think that the major roadblock for such model is the lack of understanding of wake propagation in complex terrain. Therefore, new research in this direction can definitely help push forward the wind farm layout optimization problem.

Holistic System Design  Since infrastructure cost accounts for about 20% to 30% of the total costs for onshore wind farms (and even more for offshore), the simultaneous optimization including turbine layout and infrastructure layout is an interesting and practical research topic. A natural approach is to design the problem so that it is separable into sub-problems, and can be solved with decomposition methods (such as the Benders decomposition method in [75]).

Robustness  Uncertainty appears naturally in almost all engineering systems, including the wind energy systems. There is surprisingly little work on this topic, given that long-term wind resource cannot be accurately measured or predicted. In addition, uncertainty also comes from financial situations, turbine performance, weather conditions, terrain conditions, and system reliability. We have successfully shown that CP and MIP are reasonable tools for solving the WFLO problem, and the mathematical programming community is already very familiar with optimization under uncertainty (stochastic programming, robust optimization [115]). A natural next step is to apply robust optimization and stochastic programming techniques to the WFLO problem.
Bibliography


