12.812: Problem set 4

1. Mountain torque

We showed in class that the zonal-mean torque on the atmosphere due to mountains results from the difference in surface pressure from the east to the west side, and that it may be written as

\[
\frac{1}{2\pi} \int_{z_{\text{min}}}^{\infty} \sum_i \left( p_{Ei}^s - p_{Wi}^s \right) dz,
\]

where \( z_{\text{min}} \) is the minimum in surface geopotential height at the latitude in question, \( p_{Ei}^s \) and \( p_{Wi}^s \) are the time-mean surface pressures at the intersection of mountain \( i \) with level \( z \) on the eastern and western sides, respectively, and \( \sum_i \) is a sum over all the mountains that reach \( z \) at that latitude. While this formula makes intuitive sense, it is difficult to evaluate in practice. Show that it may be rewritten in terms of surface geopotential height gradient and surface pressure as

\[
- \left[ p_s \frac{\partial z_s}{\partial \lambda} \right],
\]

where \( \left[ \cdot \right] \) denotes the zonal mean, \( p_s \) is time-mean surface pressure, \( z_s \) is surface geopotential height, and \( \lambda \) is longitude. (Hint: one approach is to rewrite \( p_{Ei}^s - p_{Wi}^s \) as \( \int_{\lambda_W}^{\lambda_E} \frac{\partial p_s}{\partial \lambda} d\lambda \) and use a Heaviside function in \( z_s - z \) to extend the \( \lambda \) integral to all longitudes.)

2. Hadley cell strength and width under climate change

In class, we discussed how extratropical eddies influence both the strength and poleward extent of the Hadley cell. Suppose the Hadley cell is in a regime in which extratropical eddies control both its strength (measured by \( v \) in the upper branch) and poleward extent (measured by the latitude of the poleward edge). How would the Hadley cell strength and Hadley cell extent respond to each of the following changes?

a) An increase in extratropical eddy kinetic energy by 10%.

b) An increase in the pressure depth of the troposphere by 10%. The pressure depth is defined here as \( p_s - p_t \) where \( p_s \) is the surface pressure and \( p_t \) is the pressure at the tropopause.

c) An decrease in the magnitude of the pole-to-equator temperature gradient by 10%.

You should assume that all other factors (such as lapse rates) remain unchanged in each case. Note that the answer may be “no effect” in some cases.