

12.810 Problem set 4

Due on May 5th, 2020

Submit your solution to: <https://www.dropbox.com/request/KvqJFJH8xYxpiW1vQ6Cf>

Need help?: Office hours Friday 1-2pm on zoom; Also email questions or to set up zoom meeting as needed.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

1. In class we showed that the QGPV is conserved for adiabatic inviscid flow: $D_g q/Dt = 0$. Suppose now that we include a potential temperature tendency J from latent heating such that the thermodynamic equation in the QG approximation is

$$\frac{D_g \theta}{Dt} + w \frac{\partial \theta_0}{\partial z} = J.$$

- (a) Derive an equation for $D_g q/Dt$ that includes the effects of latent heating.
- (b) Next you will investigate the response of QGPV to an isolated region of (positive) latent heating in the mid-troposphere. Fig. 1 shows a vertical profile of latent heating versus height. Make a sketch of the resulting tendency of QGPV ($D_g q/Dt$) versus height. Explain how the changes in q are consistent with the relationship between static stability anomalies and PV anomalies that we discussed in class.

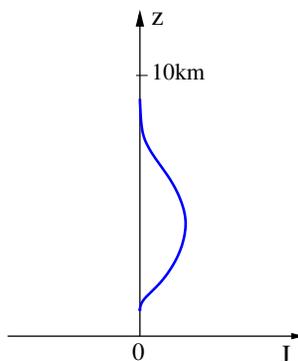


Figure 1: Idealized vertical profile of potential temperature tendency J due to latent heating.

2. In this problem you will derive some important properties of vertically propagating Rossby waves. Consider a QG Rossby wave that is propagating upwards (in the group velocity sense), with a uniform background zonal velocity u_0 and constant N^2 . For simplicity, you may assume that the perturbation streamfunction is of the form

$$\psi' = \psi_0 e^{z/(2H)} \sin(kx + ly + mz - \omega t). \quad (1)$$

- (a) Derive an expression for the vertical component of the group velocity and use it to show that the phase tilt is westward with height.

- (b) Derive an expression for the meridional potential temperature flux, $\overline{v'\theta'}$, where the overbar denotes a zonal average in x , and show that the flux is poleward in *both* hemispheres. Thus upward propagating Rossby waves (such as those generated by mountains and land/ocean contrasts) contribute to the poleward heat flux that helps to maintain the climate of higher latitudes.
3. Consider small-amplitude plane QG Rossby waves with wavevector (k, l, m) and frequency ω on a beta plane. Show that the Eliassen Palm flux \mathbf{F} is related to the group velocity \mathbf{c}_g and wave activity density A by $\mathbf{F}_y = \mathbf{c}_{gy}A$ and $\mathbf{F}_z = \mathbf{c}_{gz}A$. In your answer, use the log-pressure vertical coordinate and assume that the vertical wavelength is small such that $m \gg H^{-1}$, that the wave streamfunction has the form

$$\psi' = \Psi_0 \exp\left(\frac{z}{2H}\right) \sin(kx + ly + mz - \omega t), \quad (2)$$

where Ψ_0 is real, and that the background zonal flow U_0 and buoyancy frequency N are constant.