1. In class we showed that the QGPV is conserved for adiabatic inviscid flow: $D_q q / Dt = 0$. Suppose now that we include a potential temperature tendency $J$ from latent heating such that the thermodynamic equation in the QG approximation is

$$\frac{D \theta}{Dt} + \omega \frac{\partial \theta_0}{\partial z} = J.$$ (a) Derive an equation for $D_q q / Dt$ that includes the effects of latent heating.

(b) Next you will investigate the response of QGPV to an isolated region of (positive) latent heating in the mid-troposphere. Fig. 1 shows a vertical profile of latent heating versus height. Make a sketch of the resulting tendency of QGPV ($D_q q / Dt$) versus height. Explain how the changes in $q$ are consistent with the relationship between static stability anomalies and PV anomalies that we discussed in class.

![Figure 1: Idealized vertical profile of potential temperature tendency $J$ due to latent heating.](image)

2. In this problem you will derive some important properties of vertically propagating Rossby waves. Consider a QG Rossby wave that is propagating upwards (in the group velocity sense), with a uniform background zonal velocity $u_0$ and constant $N^2$. For simplicity, you may assume that the perturbation streamfunction is of the form

$$\psi' = \psi_0 e^{z/(2H)} \sin(kx + ly + mz - \omega t).$$ (1)

(a) Derive an expression for the vertical component of the group velocity and use it to show that the phase tilt is westward with height.

(b) Derive an expression for the meridional potential temperature flux, $\overline{v \theta'}$, where the overbar denotes a zonal average in $x$, and show that the flux is poleward in both hemispheres.
3. Write a short plan for your 12.810 project. Include a description of why the topic is interesting and/or important as a project in atmospheric dynamics. If you are doing a literature review, include a list of some of the important papers you are planning to cover. If you doing a research project, include an outline of the activity and the goal(s). Do not exceed one page of text (excluding references).