

12.810 Problem set 2

Due on April 7th, 2020

Need help?: Office hours Friday 1-2pm; Also email or drop by as needed.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

1. In this problem you will use a Matlab script available on the class website (in the problem set section: lee_wave.810.m) to solve for a mountain wave. The script uses the Boussinesq approximation and assumes constant buoyancy frequency ($N = 0.01\text{s}^{-1}$) and basic-state horizontal wind u_0 . The domain is 2-dimensional (x-z) of width 70km and height 15km. The ridge is a single Gaussian, centered at $x = 35\text{km}$, of amplitude 500m and of standard deviation 3km. Note that a Gaussian ridge includes contributions from a wide range of horizontal wavelengths. Running the script plots the vertical velocity and streamlines.
 - (a) Run the script for basic-state wind $u_0 = 5, 20, \text{ and } 120 \text{ m s}^{-1}$ and discuss the resulting wave in each case. Include vertical and horizontal propagation in your discussion. Indicate on the graphs of streamlines the positions where clouds might be seen if the atmosphere is close to saturation (e.g. upstream, downstream, over the ridge).
 - (b) Using the analytic results from class, calculate the vertical wavelength in the *hydrostatic* limit for $u_0=5 \text{ m s}^{-1}$. Compare this vertical wavelength with what you find from running the script for this u_0 . Do you think the wave is close to the hydrostatic limit?
 - (c) The width of the Gaussian ridge as measured by its standard deviation is 3km. For sinusoidal topography of wavelength 6km, use the analytic results from class to calculate the critical u_0 at which the wave doesn't propagate in the vertical (becomes evanescent). Then run the script for the Gaussian ridge for a range of values of u_0 and estimate the critical u_0 for an evanescent wave. Is the critical value for the Gaussian ridge well predicted by the result for the sinusoidal topography of wavelength 6km? Can you explain what sets the critical speed for the Gaussian ridge in this domain?
 - (d) The script assumes that the waves are linear. Assess the validity of the linear approximation for each of $u_0 = 5, 20, \text{ and } 120 \text{ m s}^{-1}$. (Hint: the disturbance wind u' is given by the variable `uxz` in the script)
2. This problem gives you practice thinking about log-pressure coordinates which are often used in atmospheric dynamics. You will show how the buoyancy frequency in log-pressure coordinates is related to buoyancy frequency in height coordinates. Let z be the log-pressure coordinate defined in class and N the corresponding buoyancy frequency where

$$N^2 = \frac{R\Pi}{H} \frac{d\theta}{dz}.$$

Here R is the gas constant, $\Pi = (p/p_0)^\kappa$ is the Exner function, and $H = RT_*/g$ is the scale height based on a reference temperature T_* . The buoyancy frequency in regular height coordinates (denoted \tilde{z}) is

$$\tilde{N}^2 = \frac{g}{\theta} \frac{d\theta}{d\tilde{z}}.$$

Show that (assuming hydrostatic balance) these two frequencies are related by

$$N^2 = \left(\frac{T^2}{T_*^2} \right) \tilde{N}^2.$$

If you use the regular density in your derivation, be sure to denote it $\tilde{\rho}$ to avoid confusion.

3. We showed in class that the amplitude of an internal gravity wave typically increases as the wave propagate upwards through the atmosphere. Eventually, the wave may become so large in amplitude that the atmosphere becomes locally statically unstable ($\partial\theta/\partial z < 0$). This is one type of wave breaking, and convection and mixing ensues.

We will work in log pressure coordinates. Consider a resting basic state with constant buoyancy frequency N defined by

$$N^2 = \frac{R\Pi}{H} \frac{d\theta_0}{dz}, \quad (1)$$

where $\theta_0(z)$ is the potential temperature in the basic state. Suppose that the perturbation geopotential in the wave is given by

$$\phi' = \Phi_0 e^{z/2H} \cos(kx + mz - \omega t), \quad (2)$$

where Φ_0 is real but the notation is otherwise as in class.

Assuming that the vertical wavelength is much smaller than H , show that the level z_b at which the wavebreaking begins to occur ($\partial\theta/\partial z = 0$) is approximately given by

$$z_b \simeq 2H \log \left(\frac{N^2}{m^2 \Phi_0} \right), \quad (3)$$

where log is natural logarithm. Thus, as expected, the breaking occurs higher in the atmosphere for a smaller amplitude wave and a larger buoyancy frequency in the basic state.