12.810 Dynamics of the Atmosphere

Hadley cells and zonally symmetric circulations
Mean meridional streamfunction ($10^{10}$ kg s$^{-1}$) (annual mean)

Hadley cells: key for determining precipitation in the tropics and subtropics

(ERA40 reanalysis 1980-2001)
Mean meridional streamfunction ($10^{10}$ kg s$^{-1}$)
(annual mean)
Mean meridional streamfunction ($10^{10}$ kg s$^{-1}$): different seasons

(Era40 reanalysis 1980-2001)
Mean meridional streamfunction \( (10^{10} \text{ kg s}^{-1}) \): different seasons

DJF

JJA

(ERA40 reanalysis 1980-2001)
Basic aspects we would like to understand about the tropical atmosphere
Mean meridional streamfunction ($10^{10}$ kg s$^{-1}$): different seasons

Strong cross-equatorial cell

(ERA40 reanalysis 1980-2001)
Zonal and time-mean zonal wind (m/s)

Subtropical jet
Zonal and time mean temperature (K) (annual average)

Weak horizontal temperature gradients in tropics

(ERA40 reanalysis data 1980-2001)
Observed angular momentum mainly varies with latitude, smaller at higher latitudes

Black contours: Mass flux streamfunction (contour interval $25 \times 10^9$ kg/s)
Gray contours: Angular momentum (contour interval $0.1 \Omega a^2$, with values decreasing monotonically away from the equator)
Superrotation may occur on Earth in a very warm climate.

**Fig 1.**

| Color shading: mean zonal wind (m/s) |
| Contours: standard deviation of zonal wind (m/s) |
| Arrows: Eliassen-Palm flux (see later in course) |

Aquaplanet simulations,
Caballero and Huber, GRL, 2010
List 1. Held-Hou model assumptions

- Zonally symmetric circulation on sphere
- No time dependence (steady)
- Boussinesq and hydrostatic approximations
- Rigid wall at upper \((z=H)\) and lower \((z=0)\) boundaries (upper is stress free, lower has drag)
- Radiation is represented by “Newtonian” relaxation of potential temperature to a specified radiative-equilibrium potential temperature \(\theta_E(z, \Phi)\)
To review the Boussinesq approximation:

- Holton section 2.8
- Vallis section 2.5 for the anelastic approximation (Boussinesq approximation is a special case of the anelastic approximation in which variations in the reference density are neglected in the mass continuity equation)
Angular momentum conserving (AMC) region is shaded
Radiative equilibrium (RE) region is unshaded

Fig 2. Angular momentum conserving (AMC) region is shaded
Radiative equilibrium (RE) region is unshaded
Application of matching conditions to find solution to Held-Hou model

![Graph](image)

**Fig 3.** The $\langle \theta_m \rangle$ curve is moved up or down until the two shaded areas have equal area

Adapted from Held and Hou 1980
Discuss with your neighbor:

How would the latitudinal extent of the Hadley cell on Titan (a moon of Saturn) compare to that of the Earth?

Titan day is $\sim 16$ Earth days
Titan radius is 0.4 Earth radii
Fig 4. Numerical solution of Held-Hou model for $v=1 \text{m}^2/\text{s}$

Streamfunction

Contour interval for zonal wind is 5 m/s
Streamfunction is in m$^2$/s rather than kg/s

Adapted from Held and Hou 1980
List 2. Properties of Held-Hou model solution that are similar to observations

• Subtropical jet

• Surface easterlies in deep tropics and westerlies at higher latitudes (Hadley cell transports zonal momentum poleward)

• $u=0$ line slopes equatorward with height

• Meridional temperature gradients are smaller than in radiative equilibrium ($\theta_E \propto \Phi^2$, $\theta_m \propto \Phi^4$)

• Hadley cell width 20-30 degrees (coincidence?)
List 3. Properties of Held-Hou model solution that are different to observations

- Missing Ferrel cell and surface westerlies at midlatitudes because these are related to eddies

- Subtropical jet too strong (~90 m/s rather than 25 m/s in annual mean) because eddies cause deviation from angular momentum conservation
Fig 5. Seasonal Hadley cells: location of maximum radiative-equilibrium temperature off the equator

\( \Phi_0 \): latitude where \( \theta_E \) reaches a maximum
\( \Phi_1 \): latitude of dividing streamline

The winter (cross-equatorial cell) is stronger, and the peak upward motion occurs near \( \Phi_0 \) rather than \( \Phi_1 \)

Lindzen and Hou, JAS, 1988
Fig 6. Upper-level zonal wind is still symmetric about the equator (because assuming angular momentum conserved)

Max $\theta_E$ at equator ($\Phi_0=0$)

Max $\theta_E$ off equator ($\Phi_0=6$ degrees)

Ascent with $u=0$ occurs here

Lindzen and Hou, JAS, 1988
Fig 7a. Can solve for temperature using nonlinear thermal wind balance and weak winds at surface (as before)

\[ \text{Max } \theta_E \text{ at equator (} \Phi_0 = 0) \]

\[ \text{Max } \theta_E \text{ off equator (} \Phi_0 = 6 \text{ degrees)} \]

Temperatures have been normalized by reference temperature \( \theta_0 \)

*Lindzen and Hou, JAS, 1988*
Fig 7b. Big difference between $\theta_E$ and $\theta_m$ when $\Phi_0 \neq 0$  
$\Rightarrow$ need strong cross-equatorial Hadley cell to transport energy!

Max $\theta_E$ at equator ($\Phi_0=0$)

Max $\theta_E$ off equator ($\Phi_0=6$ degrees)

Temperatures have been normalized by reference temperature $\theta_0$

Lindzen and Hou, JAS, 1988
Fig 7c. Upper level easterlies and thermal wind relation ⇒ temperature reaches a minimum at the equator

Max $\theta_E$ at equator ($\Phi_0=0$)

Max $\theta_E$ off equator ($\Phi_0=6$ degrees)

Temperatures have been normalized by reference temperature $\theta_0$

Lindzen and Hou, JAS, 1988
Fig 8a. Small asymmetry in radiative forcing leads to large asymmetry in the circulation

Max $\theta_E$ at equator ($\Phi_0=0$)

Max $\theta_E$ off equator ($\Phi_0=6$ degrees)

Streamfunction units of $10^{10}$kg/s; contour interval changes between panels

Lindzen and Hou, JAS, 1988
Fig 8b. Winter cell (cross-equatorial cell) is much stronger than summer cell.

Max $\theta_E$ at equator ($\phi_0=0$)

Max $\theta_E$ off equator ($\phi_0=6$ degrees)

Streamfunction units of $10^{10}$kg/s; contour interval changes between panels.

Lindzen and Hou, JAS, 1988
Hadley circulation crosses angular momentum contours in summer cell (eddies important), but much less so in cross-equatorial winter cell (e.g. monsoons)

Fig. 9

Mass flux streamfunctions (black contours) and angular momentum (gray contours) Streamfunction contour interval is $25 \times 10^9$ kg/s. Contour interval for angular momentum is $0.1 \Omega a^2$, with values decreasing monotonically away from the equator.

Walker & Schneider, JAS, 2006
Fig. 10

Relevance to Monsoons:
Meridional overturning in Asian Monsoon sector
(streamfunction in black contours)

Before onset

After onset

Contours: streamfunction (CI $50 \times 10^9$ kg s$^{-1}$)
Gray lines: angular momentum
Color shading: eddy mom. flux div.
Sector defined as 70E-100E

Bordoni and Schneider, Nature Geoscience, 2008
Dramatic contrast in the seasonal cycle of precipitation in two cities
Dramatic contrast in the seasonal cycle of precipitation in two cities.
Zonal wind in Asian Monsoon sector (black contours)

Before onset

After onset

Contours: zonal wind (CI 6 m s\(^{-1}\)) with dashed for easterly
Color shading: eddy mom. flux div.
Sector defined as 70E-100E

Bordoni and Schneider, Nature Geoscience, 2008