

Correcting Finite Sample Biases in Conventional Estimates of Power Variation and Jumps

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Abstract

Commonly used estimators for power variation, such as bi-power variation (BV), tri-power quarticity (TP), and quad-power quarticity (QP) are significantly downwardly biased due to finite sampling and the daily volatility pattern, with the bias increasing as sample interval increases. This biases many results based on these estimators, including the Barndorff-Nielsen and Shephard (BN-S) jump test and the Jiang-Oomen jump test. This work is a first step in bias-correcting these results. Using a simple and intuitive simple scaling model, we derive analogous versions of BV, TP and QP that are scale-invariance, collapse to old definitions, are robust to finite sampling bias, and are asymptotic correct on average. We use these and similar estimators to bias-correct the BN-S and Jiang-Oomen jump tests. Simulation results confirm our theory, and tests on real stock returns suggest that most of the jumps commonly flagged are simply due to the daily pattern in volatility.

Key words: Intraday Volatility Pattern, Bi-power Variation, Tri-power Quarticity, Quad-power Quarticity, BN-S Jump Test, Jiang-Oomen Jump Test, Finite Sampling Bias

1. Introduction

Conventional financial theory assumes asset returns follow a diffusion process with some underlying variable volatility series $\sigma(t)$. One important quantity in this theory is the integrated power variance for some day $\int_{t-1}^t \sigma(s)^p ds$, and this is important in price modeling and pricing derivatives. However, the integrated power variance cannot be computed directly, because we cannot finite at infinitesimal interval, and for $p = 2$, one important estimator is the

realized variance $RV = \sum_{i=1}^j r_{t,j}^2$, which computes the sum of squares of log returns of intervals throughout the day.

Recently, researchers have added jumps into the model. It turns out that RV is not robust to jumps in the return series. So researchers use the estimator BV , which uses the product of absolute returns of consecutive intervals to estimate integrated variance, and this is supposed to be robust with regard to jumps. Analogously, one can define Tri-power quarticity TP and quad-power quarticity QP as jump-robust estimators for the integrated quarticity. Barndorff-Nielsen and Shephard proposed [1, 2] a jump test using these estimators, and this initiated a recent hot line of research in financial econometrics for designing new jump tests [4, 3]. Most of these jump test use estimators based on multiplying consecutive absolute returns.

Another property of financial series is a pronounced U-shaped daily pattern in volatility, with higher volatility in the beginning and end of a trading day. The original theory does not account for these intraday patterns, and recently, Ronglie [5, 6] discovered that BV , TP , QP are significantly biased downwardly because of the intraday pattern, with the bias increasing as the sampling interval increases.

But in practice, these estimators must be computed for sampling intervals of at least a few minutes, due to market microstructure noise which dominates at higher sampling frequencies. Hence, all practical uses of these estimators are done at a frequency susceptible to the downward bias due to the daily pattern, thus invalidating theories such as jump testing. Because of how widely these estimators are used, it is imperative to correct this bias.

This is a first step in this direction. We propose bias-corrected versions of these estimators, and use them to bias-correct the $BN-S$ and $Jiang-Oomen$ jump tests. In Section 2, we first explain the intuition behind the downward bias in, examine the typical intraday pattern, and propose a simple scaling model that seems to fit observations. In Section 4, we propose bias corrected estimators that satisfy some desirable properties, and support our results using simulation and real data. In Sections 5 and 6, we use these estimators to bias-correct the $BN-S$ jump test, as well as the $Jiang-Oomen$ jump test. We show that many jumps detected by the current test are simply due to the daily pattern in intraday volatility. This matches other research, partially explaining the dual puzzle of over-abundance of jumps detected by jump tests, and dependence on sampling interval [7].

2. Intuition on Why Current Estimators are Downwardly Biased

Define intraday returns as $r_{t,j} = \log p(t + j\delta) - \log p(t + (j - 1)\delta)$, where t correspond to days and j corresponds to $\delta = 1/M$ -sized intraday intervals. The standard theory assumes $r_{t,j} \propto N(0, \sigma(t, j)^2)$ for some underlying volatility series $\sigma(t, j)$. The conventional estimators RV_t and BV_t defined in Section 1 satisfy

$$\frac{1}{M}E[RV_t] = \frac{1}{M} \sum_{j=1}^M \sigma(t, j)^2$$

$$\frac{1}{M}E[BV_t] = \frac{1}{M-1} \sum_{j=2}^M \sigma(t, j)\sigma(t, j-1)$$

Now as $\delta \rightarrow 0$, $E[BV] \rightarrow E[RV]$, because

$$\frac{\sigma(t, j)^2 + \sigma(t, j-1)^2}{2\sigma(t, j)\sigma(t, j-1)} \rightarrow 1$$

This is true because this is simply the arithmetic-mean (AM) of $\sigma(t, j)^2$ and $\sigma(t, j-1)^2$, divided by their geometric mean (GM), and as $\delta \rightarrow 0$, $\sigma(t, j)^2/\sigma(t, j-1)^2 \rightarrow 1$.

However, the AM-GM inequality implies that $GM \leq AM$, so that $E[BV] < E[RV]$, and the gap is heightened when $\sigma(t, j)^2/\sigma(t, j-1)^2$ departs from 1, which occurs when there is a daily pattern in intraday volatility, explained in Section 2.1

By a similar argument, estimators of power variation such as tri-power quarticity TP and quad-power quarticity QP,

$$TP = \left(\frac{\mu_{4/3}^{-3} M^2}{\mu_{4/3}^{-3} M^2}\right) M - 2 \sum_{j=3}^M |r_{t,j}|^{4/3} |r_{t,j-1}|^{4/3} |r_{t,j-2}|^{4/3}$$

$$nQP = \left(\frac{1}{M} \sum_j b_j^4\right) \frac{\mu_{-4}^{-4} M^2}{M-2} \sum_{j=4}^M |r_{t,j}| |r_{t,j-1}| |r_{t,j-2}| |r_{t,j-3}|$$

are downwardly biased, and the effect is more pronounced because of the use of more number of consecutive return terms.

2.1. Intraday Pattern in Volatility

Volatility of returns is known to form a U shape for each day, with greater activity closer to the opening and closing times of the stock market. Figure 1 plots the quantiles and the mean of the intraday pattern in 6-minute absolute

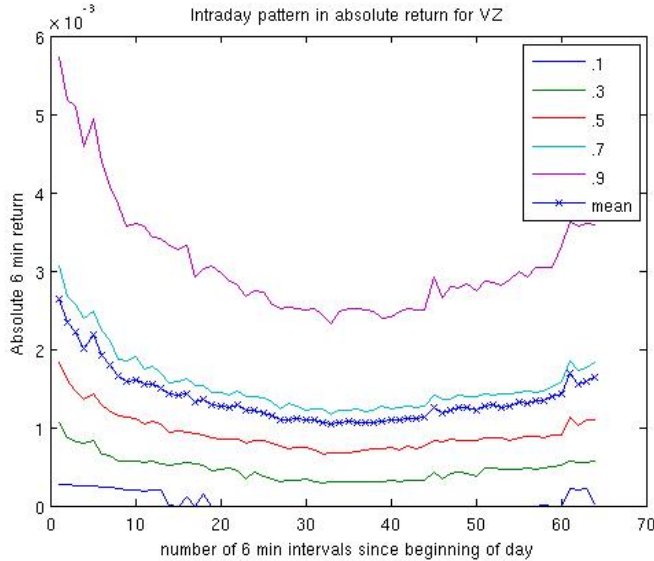


Figure 1: The intraday pattern in 6-minute absolute returns for VZ. For each j , we compute the absolute return in the j th 6-minute interval for all days, and plot the quantiles as well as the mean.

returns for VZ, and similar patterns are observed for all stocks. For each j , we compute the absolute return in the j th 6-minute interval for all days, and plotted the quantiles as well as the mean.

Figure 1 suggests that the distribution of the returns for the j th interval is simply scaled across the day: the quantile lines seem to preserve the same proportions. We check this observation by plotting Figure 2, which has the quantile lines scaled by the mean.

As shown in Figure 2, the quantile lines are roughly horizontal, suggesting that it's possible to account for the daily pattern by some type of simple scaling for each day.

2.2. Simple Scaling Model

While conventional theory assumes returns $r_{t,j} \propto N(0, \sigma'(t, j)^2)$, we formalize the observation in Section 2.1 and assume the model

$$r_{t,j} \propto N(0, (b_j \sigma(t, j))^2)$$

In which the original underlying volatility series $\sigma'(t, j)$ is separable into a persistent term $\sigma(t, j)$ and a daily pattern b_j .

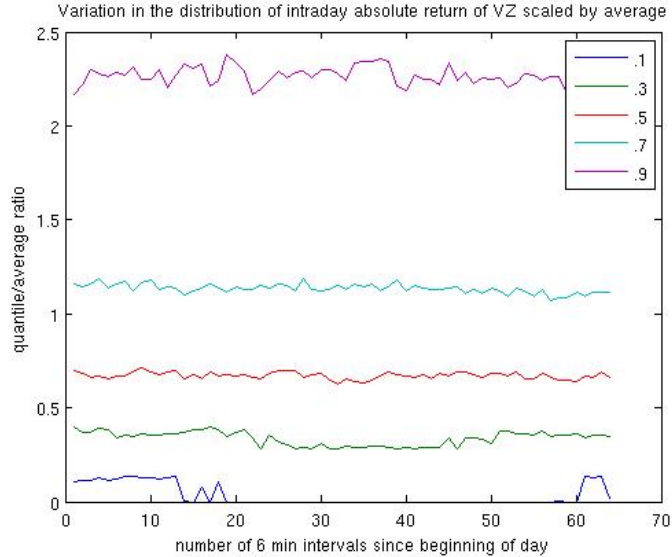


Figure 2: The intraday pattern in 6-minute absolute returns for VZ, after scaling for the mean pattern. Essentially, we scale the quantile lines in Figure 1 by the mean.

One easy check for this is that if this separability assumption is true, then for each j , $r_{t,j}/E_t[|r_{t,j}|]$ should be distributed as $Z/E[|Z|]$, where Z is a normal. We compute this for the stock T and $j = 1, 21, 41, 61$, using 6-minute sampling interval. The results are plotted in Figure 3.

As Figure 3, disregarding noisy results near 0, probably due to market microstructure, we have a reasonable fit in all 4 cases. Hence, our model passes this basic test.

While we may in the future seek theoretical justifications on this model, it seems to satisfy the observations for now, and we try to use it to account for the bias due to intraday pattern.

3. Simple Monte Carlo Simulation

To test the later results, we define a simple Monte Carlo simulation, following [5, 6]. We extract the daily pattern of stock T by computing the average 1-minute absolute return across all days, and for each minute. We use this as the underlying volatility series and take i.i.d. draws from $Z_{t,j} \propto N(0, 1)$, defining $r_{t,j} = b_j Z_{t,j}$. Here $\delta = 1/M = 1/384$ because there are 384 1-minute intervals in a trading day. Note that apart from the daily pattern,

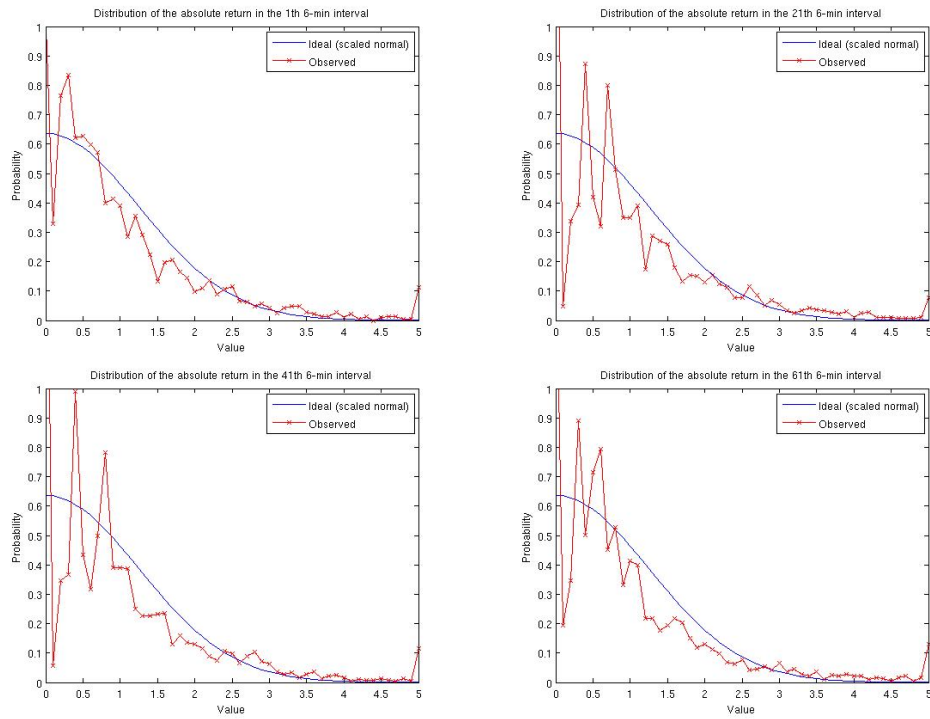


Figure 3: The distribution of $|r_{t,j}|/E_t[|r_{t,j}|]$, for stock T and $j = 1, 21, 41, 61$. (The 1st, 21st, 41st and 61st 6-minute interval respectively.) If the scaling model holds then this should match $Z/E[|Z|]$ for normal Z .

this assumes a constant volatility series $\sigma(t, j)$. We want estimates of power variation, as well as the jump test, to match the proposed theories at least under this simple test.

4. Bias Correcting Various Estimators of Power Variance

4.1. Theoretical Derivations

While there might be many possible ways to change definitions of BV , TP , and QP to account for the bias due to the daily pattern b_j , we want methods that satisfy the following 4 intuitive properties:

1. Scale invariance: Scaling the intraday pattern b_j by some constant factor should not change the estimator.
2. Collapse to old definitions: When b_j 's are set to some constant, our new estimators should collapse to the old estimators.
3. Robust to finite sampling bias: The estimators should be unbiased in finite interval sampling, and this should be true for all sampling intervals. (The old defitions do not satisfy this.)
4. Asymptotic correctness: When $\sigma \rightarrow 0$, our estimators should converge the same underlying integrated power variation as before.

In this section, we propose estimators that satisfy properties 1,2, *roughly* satisfy 3 (correcting for first order effect of daily pattern but still susceptible to downward bias due to other variations in intraday volatility), and satisfy 4 *on average*. (Instead of being asymptotically unbiased and consistent measure of integrated power variation for each day, our estimators are asymptotically correct when averaged over a longer interval, assuming the volatility series $\sigma(t, j)$ is uncorrelated with the daily pattern b_j .)

Recall from Section 2 that $E[BV] \rightarrow E[RV]$ when $\sigma(t, j)^2/\sigma(t, j-1)^2 \rightarrow 1$, so to create a bias-corrected BV , we want to scale these consecutive volatility to be equal before computing sum of consecutive terms. We can do this by computing BV on scaled returns $\frac{r_{t,j}}{b_j}$, but to make BV an estimator for intergrated variance we need to scale back, and many scaling factors are possible. To make sure that properties 3 and 4 hold, we approach this in the following way.

Define new estimators (recall that $\mu_a = E[|Z|^a]$)

$$\begin{aligned} nRV &= \left(\frac{1}{M} \sum_{j=1}^M b_j^2\right) \sum_{j=1}^M M \left(\frac{r_{t,j}}{b_j}\right)^2 \\ nBV &= \left(\frac{1}{M} \sum_{j=1}^M b_j^2\right) \left(\frac{\mu_{1-2M}}{M-1}\right) \sum_{j=2}^M M \left|\frac{r_{t,j}}{b_j}\right| \left|\frac{r_{t,j-1}}{b_{j-1}}\right| \end{aligned}$$

Lemma 1. *Both nRV and nBV satisfy properties 1 and 2 defined above.*

Proof. This can be checked using straightforward algebra □

Lemma 2. *Both nRV and nBV roughly satisfy property 3, in the sense that it corrects for the first-order term in the previous bias due to intraday pattern.*

Proof. This follows from the fact that due to scaling $\sigma(t, j)^2/\sigma(t, j-1)^2 \approx 1$, so assuming low changes in consecutive volatility $\sigma(t, j)$, both $\sum_{j=1} M(\frac{r_{t,j}}{b_j})^2$ and $(\frac{\mu_1^{-2}M}{M-1}) \sum_{j=2} M|\frac{r_{t,j}}{b_j}| |\frac{r_{t,j-1}}{b_{j-1}}|$ are unbiased estimators of M times the average variance $\sigma(t, j)^2$. Note that the intraday pattern changes by a factor of 2 throughout the day, so it is the first order effect and even though there is still bias due to changes in $\sigma(t, j)$, we have corrected for most of the bias. □

Lemma 3. *Assuming that the daily pattern dominates the intraday changes in volatility, nBV is a (roughly) unbiased estimator of nRV .*

Proof. As shown in Section 2, $E[BV] \approx E[RV]$ when $\sigma(t, j)^2/\sigma(t, j-1)^2 \approx 1$, which is true when after our simple scaling. □

Lemma 4. *Assuming that the scaled variance series $\sigma(t, j)^2$ is on average uncorrelated with the daily pattern b_j^2 , then on average, nRV is an unbiased estimator for RV .*

Proof. One can check by straightforward algebra that

$$E[RV] - E[nRV] = \sum_{i < j} (\sigma(t, i)^2 - \sigma(t, j)^2)(b_i^2 - b_j^2)$$

Hence, when $\sigma(t, j)^2$ is uncorrelated on average with b_j^2 , this difference is 0 on average. (Note that while $\sigma(t, j)$ are deterministic variables, we can still average across days according to its distribution.) □

The assumption in Theorem 4 seems reasonable because computing the daily pattern b_j in effect take out the correlation between $\sigma(t, j)$ and b_j on average.

The above lemmas add together to show that

Theorem 1. *nBV is a bias-corrected version of BV that satisfies properties 1,2, roughly satisfies 3, and satisfies 4 on average.*

Similarly, define nTP , nQP

$$\begin{aligned} nTP &= \left(\frac{1}{M} \sum_j b_j^4\right)^{\frac{\mu_{4/3}^{-3} M^2}{M-2}} \sum_{j=3}^M \left| \frac{r_{t,j}}{b_j} \right|^{4/3} \left| \frac{r_{t,j-1}}{b_{j-1}} \right|^{4/3} \left| \frac{r_{t,j-2}}{b_{j-2}} \right|^{4/3} \\ nQP &= \left(\frac{1}{M} \sum_j b_j^4\right)^{\frac{\mu_1^{-4} M^2}{M-2}} \sum_{j=4}^M \left| \frac{r_{t,j}}{b_j} \right| \left| \frac{r_{t,j-1}}{b_{j-1}} \right| \left| \frac{r_{t,j-2}}{b_{j-2}} \right| \left| \frac{r_{t,j-3}}{b_{j-3}} \right| \end{aligned}$$

Theorem 2. nTP and nQP are bias-corrected versions of TP and QP that satisfy properties 1,2, roughly satisfy 3, and satisfy 4 on average.

Proof. This is analogous to the proof for BV , except that we start with the estimator $nPV4 = \left(\frac{1}{M} \sum_{j=1}^M b_j^4\right) \sum_{j=1}^M \left(\frac{r_{t,j}}{b_j}\right)^4$, relate this to 4th power variance $PV4 = \sum_{j=1}^M r_{t,j}^4$, and relate $nPV4$ with each of nTP and nQP . The same lemmas can be shown completely analogously. \square

This is intended only as a first step at deriving bias-corrected versions of BV , TP , and QP . Note that we do not satisfy the 4 properties perfectly, especially for the 4th property, and one direction in future research is finding estimators that better satisfy property 4.

4.2. Simulation Results

We generate a simulated return series using the simulation described in Section 3, using the intra-day pattern in 1-minute absolute returns for the stock T. We simulate this for 2924 days, matching the real data available for T.

In Figure 4, we compute the RV_t and nRV_t for each day of the simulated returns series, using some sample interval size δ , and compute the average across all days. We graph the average as we change the interval size δ . We also plot the true integrated variance, which we know because we generated the returns series.

As seen in the graph, both RV and nRV are unbiased estimators of IV , and they remain unbiased even as sample interval size changes. This plot provides some support for our use of nRV as a substitute for RV in our theoretical derivations in Section 4.1.

In Figure 5, we compute the same average plot versus sampling interval for BV_t and nBV_t . Again we also plot the true integrated variance.

As Figure 5 confirms, BV is a biased estimator of the integrated variance in finite interval sampling, with the bias increasing as the sampling interval increases. This bias is about 5% around 10-minute sampling interval, and

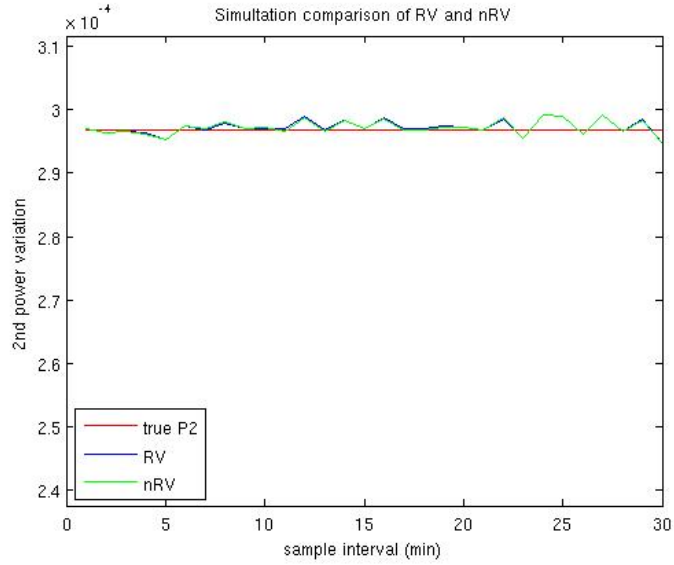


Figure 4: The average RV and nRV in the Monte Carlo simulation, as the sample interval size changes. The horizontal line is the true integrated variance IV .

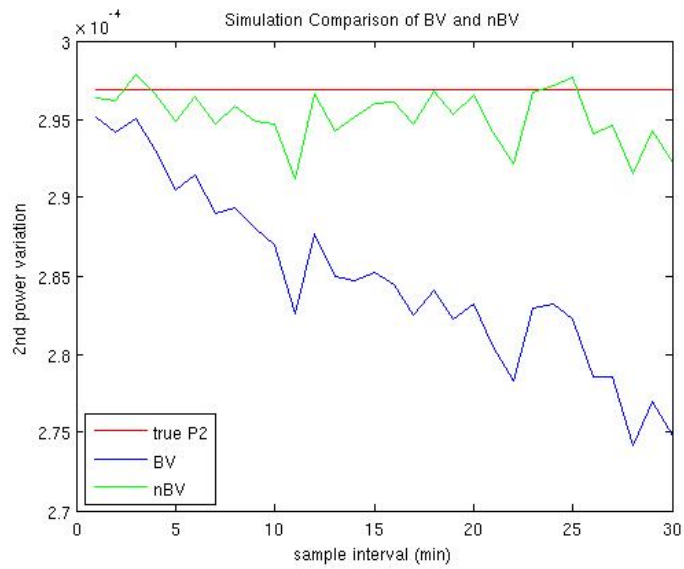


Figure 5: The average BV and nBV in the Monte Carlo simulation, as the sample interval size changes. The horizontal line is the true integrated variance IV .

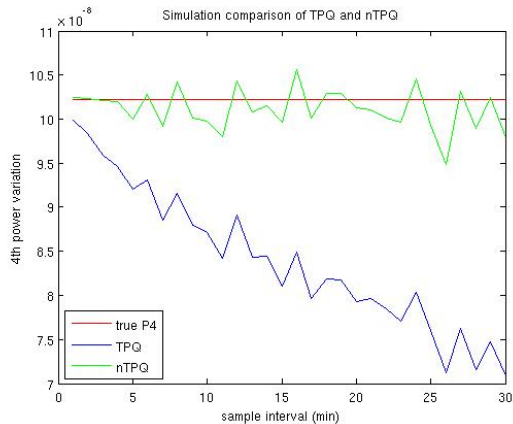


Figure 6: The average TP and nTP in the Monte Carlo simulation, as the sample interval size changes. The horizontal line is the true quarticity.

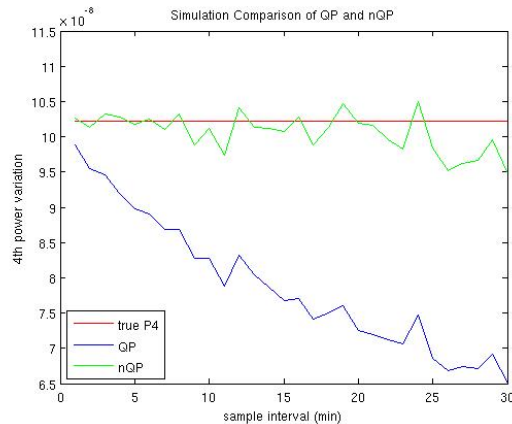


Figure 7: The average QP and nQP in the Monte Carlo simulation, as the sample interval size changes. The horizontal line is the true quarticity.

this magnitude is significant. However, confirming our theory, nBV seems to be an unbiased estimator even as sample interval increases.

In Figures 6 and 7, we plot the same average versus sampling interval plot for TP and QP . As shown by theory, the bias is more pronounced here, reaching around 20% at 10-minute sampling intervals because of greater number of consecutive terms used. On the other hand, our estimators nTP and nQP remain unbiased.

4.3. Results on Real Returns Series

We perform the same analysis as in Section 4.2 except using real returns series. The data available for T are 2924 days of 1-minute returns from 9 April 1997 to 7 January 2009, starting at 9:35am and ending at 4:00pm. In each case, both the original estimators (BV , TP , QP) our estimators (nBV , nTP , nQP) decrease on average as sampling interval increases, probably due to market microstructure noise, which is more pronounced at lower sampling frequency. Nevertheless, the results show the same types of divergence between the measures as in simulation. This suggests that the our estimators, while not immune to microstructure noise, can remove the first-order downward bias in the current estimators.

In Figure 8, we plot the average RV and nRV for various sampling frequencies. This plot shows again that nRV is a good proxy for RV , at least

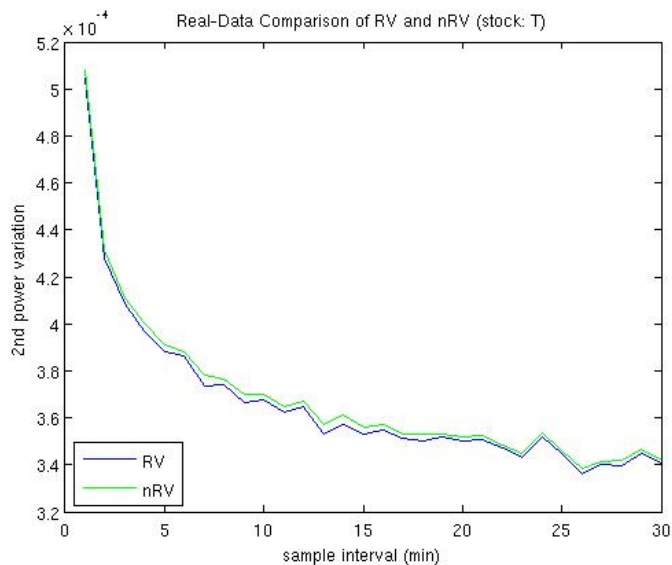


Figure 8: The average RV and nRV found in the actual return series of stock T, as the sample interval changes.

on average, and this is robust with respect to sampling interval size.

In Figure 9, we plot the average BV and nBV for various sampling frequencies. This plot shows the same 5% divergence at 10-minute intervals, confirming simulated results in Section 4.2.

In Figures 10 and 11, we plot the average versus sampling interval plot for TP and QP . The higher powers make the plots more noisy, but there is still the noticeable approximately 20% divergence at 10-minute sampling interval, with the divergence increasing at lower sampling frequency. This confirms the simulation results.

Hence, nBV , nTP , nQP seem promising alternatives to BV , TP and QP in estimating the true quadratic and quartic variance, as they correct for the downward bias due to intraday pattern. Note that as the true volatility in a day may change, causing our estimators to be downwardly biased as well, but the intraday pattern is the first order effect, so our estimators probably correct for most of the bias.

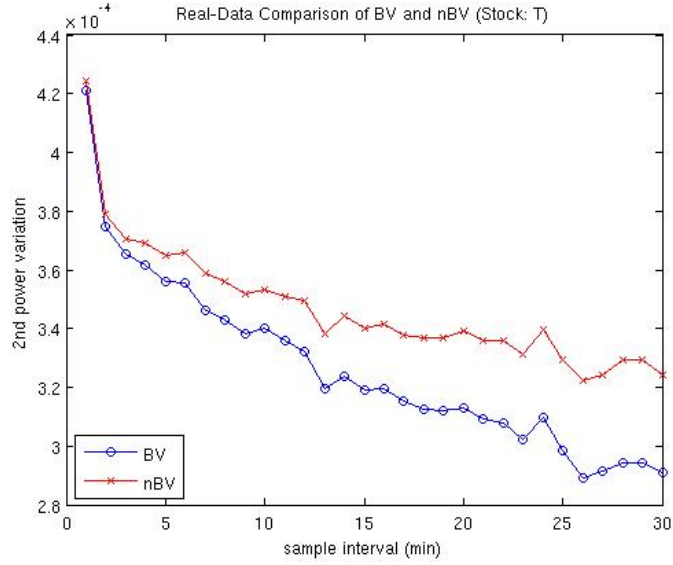


Figure 9: The average BV and nBV found in the actual return series of stock T, as the sample interval changes.

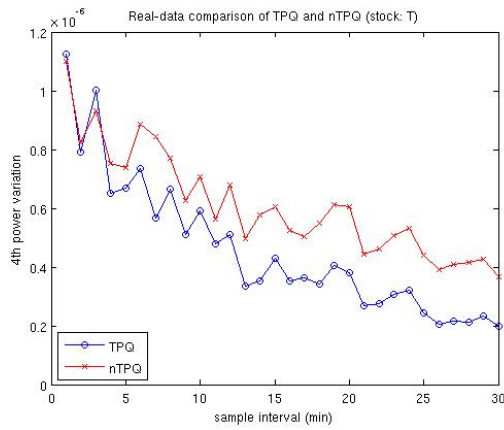


Figure 10: The average TP and nTP found in the actual return series of stock T, as the sample interval changes.

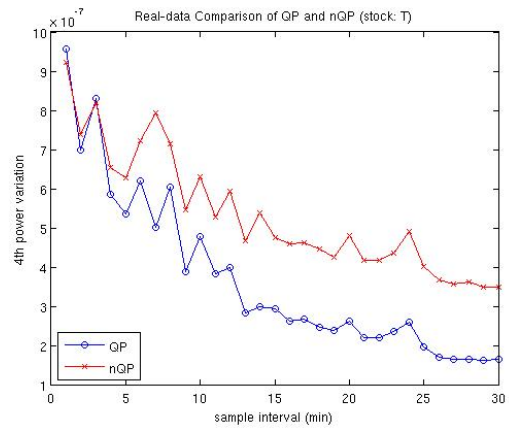


Figure 11: The average QP and nQP found in the actual return series of stock T, as the sample interval changes.

5. Bias Correcting the BN-S Jump Test

5.1. Defining the Bias-Corrected BN-S Test

Using the bias-corrected estimators for quadratic and quartic variation produced in Section 4, we seek to correct the bias in the BN-S jump test due to finite sampling.

The max-adjusted BN-S jump test [1, 2] using tri-power quarticity is a z -statistic defined as

$$z_{TP} = \frac{\frac{RV_t - BV_t}{RV_t}}{\sqrt{\left(\left(\frac{\pi^2}{2} + \pi - 5\right)\left(\frac{1}{n}\right)\max\left(1, \frac{TP_t}{BV_t^2}\right)\right)}}$$

Similarly, we can define the test using quad-power quarticity by replacing TP_t with QP_t [1, 2].

While z_{TP} is an asymptotically correct test for jumps under the original theory, the original theory does not account for the intraday pattern. As BV and TP are both downwardly biased in finite sampling due to the intraday pattern, the z_{TP} becomes upwardly biased.

While we can try to correct this by using nBV , nTP and nQP , our theoretical guarantees for unbiasedness of these estimators are only true on average, and not for each day. In fact, they will be biased if the daily volatility pattern $\sigma(t, j)$ is correlated with the intraday pattern b_j (See section 4.1).

A more theoretically sound approach is to run the original test on the scaled returns series $\frac{r_{t,j}}{b_j}$, which assuming our simple scaling model in Section 2.2, corresponds to a diffusion process with underlying volatility series $\sigma(t, j)$. This is exactly the starting point of the original theory, and we simply pre-clean the data with daily scaling.

Because of the same scaling factor of $\frac{1}{M} \sum_{j=1}^M b_j^2$ in nRV and nBV , we can define

$$nTP2 = \left(\frac{1}{M} \sum_{j=1}^M b_j^2\right)^2 \mu_{4/3}^{-3} \frac{M^2}{M-2} \sum_{j=3}^M |r_{t,j}|^{4/3} |r_{t,j-1}|^{4/3} |r_{t,j-2}|^{4/3}$$

and similarly change the scaling factor in nQP from $\frac{1}{M} \sum_{j=1}^M b_j^4$ to $\left(\frac{1}{M} \sum_{j=1}^M b_j^2\right)^2$ to define $nQP2$. We use nRV , nBV , $nTP2$ and $nQP2$ in the BN-S test, and the original theory implies that it correctly detect jumps in the modified returns theory $\frac{r_{t,j}}{b_j}$. Note that a jump exists in the original series $r_{t,j}$ iff it exists in the scaled series $\frac{r_{t,j}}{b_j}$, so the jump test remains correct, and this accounts for the intraday pattern.

TPQ simulated Sample interval	p-value	Ratio of jump days	
		Old test	New test
1-min	0.05	0.073	0.051
	0.01	0.019	0.011
	0.001	0.0035	0.0019
6-min	0.05	0.088	0.056
	0.01	0.028	0.014
	0.001	0.0056	0.0019
12-min	0.05	0.089	0.56
	0.01	0.028	0.014
	0.001	0.056	0.0019

Table 1: BN-S max-adjusted jump test using TP_t , using 50000 days of data simulated using intraday pattern of T. Both the old and bias corrected tests are shown.

5.2. Simulation Results

We simulate 50000 days of returns using the intraday pattern for T, following the technique explained in Section 3. We run both the original and bias-corrected BN-S jump tests on this, for sampling intervals of 1,6,12 minutes, and p-values of 0.01, 0.05 and 0.001. The test threshold at these p-values are 1.64, 2.33 and 3.09 respectively. Figure 1 shows the result for the test based on tri-power quarticity, and Figure 2 shows it for quad-power quarticity.

As Figures 1 and 2 show, the BN-S test is indeed upwardly biased due to the intraday pattern. (There are no jumps by definition so the ratio of jump days should be close to the p-values, allowing for simulation error.) Depending on the p-value, the bias is a factor of from 3.5 to 6. This confirms the results found by Ronglie [5].

However, our bias-corrected tests reasonably match the theoretical p-values. Note that all tests are done with the same generated return series. Note that at 6,12 minute sampling frequency, bias adjusting the test reduces the number of jumps by a factor of 2-3.

5.3. Running the Bias-Corrected BN-S Test on Real Data

Using real data on T and VZ, we run the old BN-S max-adjusted jump test using tri-power quarticity, and also the bias-corrected version. The data from T is as before and the data from VZ is 2117 days of 1-minute returns from 5 July 2000 to 7 January 2009. The results are shown in Tables 3 and 4.

QP simulated Sample interval	p-value	Ratio of jump days	
		Old test	New test
1-min	0.05	0.075	0.051
	0.01	0.020	0.011
	0.001	0.0037	0.0015
6-min	0.05	0.089	0.057
	0.01	0.029	0.014
	0.001	0.0060	0.0020
12-min	0.05	0.091	0.057
	0.01	0.028	0.015
	0.001	0.0059	0.0020

Table 2: BN-S max-adjusted jump test using QP_t , using 50000 days of data simulated using intraday pattern of T. Both the old and bias corrected tests are shown.

T real data Sample interval	p-value	Ratio of jump days	
		Old test	New test
1-min	0.05	0.80	0.83
	0.01	0.66	0.70
	0.001	0.50	0.54
6-min	0.05	0.23	0.19
	0.01	0.11	0.084
	0.001	0.047	0.029
12-min	0.05	0.18	0.13
	0.01	0.08	0.049
	0.001	0.031	0.014

Table 3: BN-S max-adjusted tri-power quarticity jump test using real data for T. Both the old and bias corrected tests are shown.

VZ real data Sample interval	p-value	Ratio of jump days	
		Old test	New test
1-min	0.05	0.65	0.70
	0.01	0.46	0.51
	0.001	0.29	0.33
6-min	0.05	0.18	0.14
	0.01	0.077	0.055
	0.001	0.031	0.012
12-min	0.05	0.15	0.094
	0.01	0.058	0.032
	0.001	0.019	0.0057

Table 4: BN-S max-adjusted tri-power quarticity jump test using real data for VZ. Both the old and bias corrected tests are shown.

Note that in both cases the microstructure noise at 1-minute sampling interval, by exaggerating the difference between RV and BV , blows up the z -statistics, so we should disregard these results. For 6 and 12 minute intervals, bias-correcting the test decreases the number of jumps by a factor of 2-3, exactly as in simulation results. This suggests that the majority of the jumps found using the BN-S test can be explained by the intraday pattern.

6. Bias-correcting the Jiang-Oomen Jump Test

As for the BN-S test, We try to correct bias in the Jiang-Oomen jump test [3] caused by the intraday pattern. These are preliminary results because we have not found how to bias adjust the swap variance, and we cannot use the original theory to justify the new test.

The Jiang-Oomen test uses the log return series $r_{t,j}$ defines the swap variance as

$$swV_t = \sum_{j=1}^M e^{r_{t,j}} - 1 - r_{t,j}$$

It further defines Ω_t , an estimator for the integrated sixth power variation, using

$$\Omega_t = \frac{\mu_6 \mu_{3/2}^{-2}}{9} \frac{M^3}{M-3} \sum_{j=4}^M |r_{t,j}|^{3/2} |r_{t,j-1}|^{3/2} |r_{t,j-2}|^{3/2} |r_{t,j-3}|^{3/2}$$

The Jiang-Oomen ratio test is a two-sided test with statistic

$$JO_t = \frac{MBV_t}{\sqrt{\Omega_t}} \left(1 - \frac{BV_t}{swV_t}\right)$$

The same theory in Section 2 shows that the estimator Ω is downwardly biased in finite sampling, because of the variation in the expected values of the consecutive returns caused by the intraday pattern. Since this should be an estimator of the sixth power variation, we use the same method as in Section 4.1 and define

$$n\Omega_t = \left(\frac{1}{M} \sum_{j=1}^M b_j^6\right) \frac{\mu_6 \mu_3^{-2}}{9} \frac{M^3}{M-3} \sum_{j=4}^M \left| \frac{r_{t,j}}{b_j} \right|^{3/2} \left| \frac{r_{t,j-1}}{b_{j-1}} \right|^{3/2} \left| \frac{r_{t,j-2}}{b_{j-2}} \right|^{3/2} \left| \frac{r_{t,j-3}}{b_{j-3}} \right|^{3/2}$$

This can be shown to be on average an unbiased estimator of the sixth power variation, assuming that the scaled returns series $\frac{r_{t,j}}{b_j}$ ($j = \{1, 2, \dots, M\}$) for a day is on average uncorrelated with the intraday pattern b_j ($j = \{1, 2, \dots, M\}$).

6.1. Simulation Results

As with the BN-S test, we create a 50000 day returns series using the method in Section 3 and the pattern for T. We run both the old and the bias-corrected Jiang-Oomen tests, and show the results in Table 5. In fact, the same returns series from the BN-S simulation is used here. The thresholds at p-values of 0.05, 0.01 and 0.001 are 1.96, 2.58 and 3.29 respectively.

As Table 5 shows, while the original Jiang-Oomen test is hugely upwardly biased when sampling interval is large. Moreover, the corrected test accounts for most of the bias. The ratio of jump days does not match the theoretical p-values as well as in the BN-S test. One reason is that we have not theoretically justified the bias-corrected test as before, because the complicated nature of the swap variance. However, note that the bias-corrected test matches the theoretical values much better than the old test, especially for larger p-values of .05 and .01.

6.2. Running the Bias-Corrected J-O Test on Real Data

As with the BN-S test, we run both the old and the bias-corrected Jiang-Oomen test on real return series of T and VZ. The results are tabulated in Tables 6 and 7.

J-O simulated Sample interval	p-value	Ratio of jump days	
		Old test	New test
1-min	0.05	0.041	0.029
	0.01	0.013	0.0081
	0.001	0.003	0.0016
1-min	0.05	0.062	0.037
	0.01	0.027	0.013
	0.001	0.011	0.0043
1-min	0.05	0.080	0.046
	0.01	0.040	0.020
	0.001	0.019	0.0088

Table 5: Jiang-Oomen swap variance ratio jump test, using 50000 days of data simulated using intraday pattern of T. Both the old and bias corrected tests are shown.

J-O on T Sample interval	p-value	Ratio of jump days	
		Old test	New test
1-min	0.05	0.080	0.070
	0.01	0.044	0.035
	0.001	0.022	0.017
6-min	0.05	0.11	0.056
	0.01	0.065	0.027
	0.001	0.040	0.016
12-min	0.05	0.12	0.062
	0.01	0.081	0.034
	0.001	0.056	0.020

Table 6: Jiang-Oomen swap variance ratio jump test on real data for T. Both the old and bias corrected tests are shown.

J-O on VZ Sample interval	p-value	Ratio of jump days	
		Old test	New test
1-min	0.05	0.11	0.081
	0.01	0.068	0.048
	0.001	0.037	0.027
6-min	0.05	0.10	0.049
	0.01	0.057	0.025
	0.001	0.035	0.015
12-min	0.05	0.12	0.059
	0.01	0.080	0.031
	0.001	0.049	0.016

Table 7: Jiang-Oomen swap variance ratio jump test on real data for VZ. Both the old and bias corrected tests are shown.

As Tables 6 and 7 show, Jiang-Oomen test is much more robust to microstructure noise as the BN-S test, as the ratio of jumps at 1-minute interval is not as unreasonably high. However, using the bias-corrected tests decreases the number of jumps flagged by a ratio of 2-3, which is interestingly exactly as in the BN-S test. This suggests that most of the jumps found using the Jiang-Oomen test are actually due to the intraday pattern.

7. Future Work

Some directions for future work include:

- Perform more intricate Monte-Carlo simulation accounting for a persistent volatility series.
- Run the tests on more stocks to confirm findings.
- Find estimators of power variation that is unbiased for each day. The current nBV , nTP , nQP estimators are designed to be the first steps in correcting for the bias due to intraday patten. While they are unaverage unbiased estimators of power variation, for days whose underlying scaled volatility $\sigma(t, j)$ (positively/negatively) correlated with the daily pattern b_j , they are (downwardly/upwardly) biased.
- Re-examine the theory of the Jiang-Oomen test. Unlike our bias-corrected BN-S test, our bias-corrected Jiang-Oomen test is not jus-

tified by the simple scaling model and the original theory. We simply corrected for the first-order bias, but have not shown the statistics to be correct. We will further examine this in the future.

- Bias correct the Lee-Mykland test. The Lee-Mykland jump test [4] also uses an estimator analogous to BV , and therefore is also susceptible to the bias due to intraday pattern. We will explore deriving a bias-corrected version of that test.

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