13. Summary

- Convexity, algebra, and duality.
- A synergy of algebraic and numerical methods.
- Development of relationship between duality, algebra, and geometry.
- Constructive methodology for practically relevant questions.
- A useful *computational framework* for semialgebraic problems.
- Numerous *applications*, same new basic *tools*.
- Sum of squares methods provide easily computable, certified solutions.
- A broad generalization of known successful techniques.
- Tradeoff between accuracy vs. computation time.
## Certificates

<table>
<thead>
<tr>
<th>Degree \ Field</th>
<th>Complex</th>
<th>Real</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td><em>Range/Kernel</em></td>
<td><em>Farkas Lemma</em></td>
</tr>
<tr>
<td></td>
<td>Linear Algebra</td>
<td>Linear Programming</td>
</tr>
<tr>
<td>Polynomial</td>
<td><em>Nullstellensatz</em></td>
<td><em>Positivstellensatz</em></td>
</tr>
<tr>
<td></td>
<td>Bounded degree: LP</td>
<td>Bounded degree: SDP</td>
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<td></td>
<td>Groebner bases</td>
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</tbody>
</table>
Conclusions

Methodological aspects

- Constructive and algebraic aspects of duality.
- The lifting/relaxation/duality general scheme
- Hierarchies of relaxations, certificate complexity.
- Completely algorithmic methodology.
- Unification of many particular cases.

Concrete applications

- Continuous and combinatorial optimization
- Lyapunov and density function computation
- Many others: quantum entanglement, geometric theorem proving, etc.
Open research topics

- Practicalities. Implementation, numerical conditioning, etc.
- How big are the problems that we can solve?
- General theory for \textit{a priori} or approximation guarantees?
- How can we exploit the problem structure for more efficient solutions?
- What are the computational complexity implications?

Lots of things to do!