Computer Simulation of a Robotic Golfer

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Abstract -- The analysis of a golf swing based on comparison with an "ideal" swing requires the definition of the "ideal" and a tool for comparison. Jack Nicklaus' description of his own technique is used as a guide to define the "ideal" golf swing. A robotic model is developed to represent the motion of a professional golfer. Using a defined performance measurement, an initial estimation of the model's swing is refined. Eventually, a computer program will graphically simulate the swing of the robotic golfer.

SUMMARY

Introduction

This computer simulation of a robotic golfer is being developed to analyze a golf swing. The swing performed by the robotic model is based on Jack Nicklaus' description in *Golf My Way* [1]. This study presents the procedure used to develop the definition of the "ideal" golf swing. The initial estimation of the swing is explained and followed by the refinement procedure.

Model Description

The design includes seven degrees of freedom, all rotational joints (see Fig. 1), which represent an approximation of the human movement during a golf swing. For the purposes of this analysis, it is sufficient to study the golf swing only through the moment of impact. Therefore, the follow through is not thoroughly addressed and, in our analysis, is not part of the "golf swing." Several simplifying assumptions follow. It is the left arm (of a right-handed golfer) which primarily defines and controls the golf swing. Because the right arm merely accommodates this motion, it is omitted. Also, the elbow joint is excluded since the left arm stays relatively straight. *

Angles θ_1 and θ_2 represent the movement in the hips and waist of the body, respectively. The complete shoulder movement requires three angles. These are: θ_3, the dip of the shoulders, θ_4, the arm's backward and forward motion with respect to the body, and θ_5, the arm's side to side motion with respect to the body. Finally, θ_6 and θ_7 represent the wrist's roll and pitch, respectively. Link lengths and parameters are chosen as:

$L_1 = 1.422$ m,
$L_4 = 0.229$ m,
$L_6 = 0.686$ m,
$L_{8a} = 0.813$ m,
$L_{8b} = 0.038$ m,
$\Phi = 20^o$.

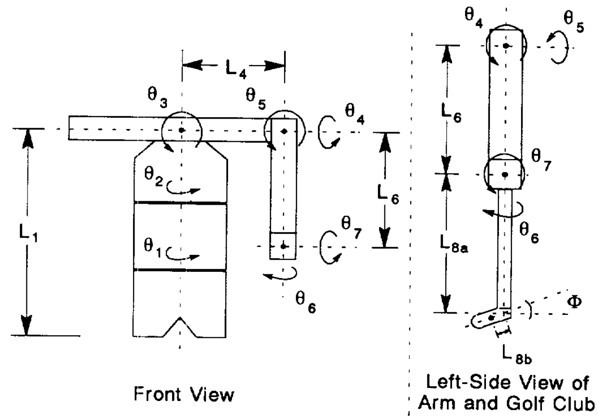

Fig. 1: Robot Model

*During the follow through, it is the *right* arm which primarily defines and controls the swing, while the left arm bends at the elbow to accommodate the motion. Thus, for the model to accurately reproduce the human motion in this part of the swing, the right arm and the elbow joints are necessary.

Kinematics

Frame Assignments: On the basis of the chosen angles and their axes of rotation, the frame assignments are found using the Denavit-Hartenburg method [2].

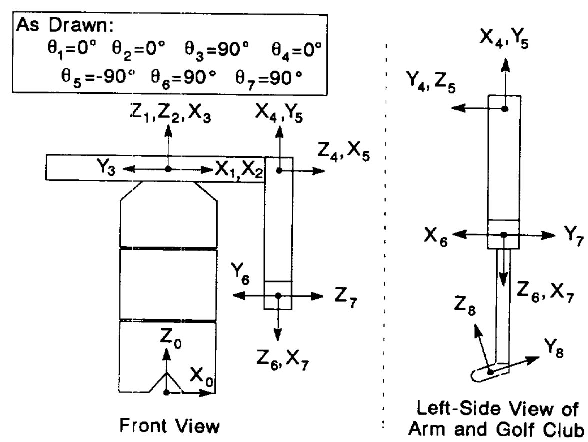

Fig. 2: Frame Assignments

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Each joint angle, θ_i, is defined as the angle between $\mathbf{X}_{i-1}$ and $\mathbf{X}_i$ measured about $\mathbf{Z}_i$. The frame-to-frame matrix transformations based on these frame assignments are ⊕

$$ {}^{0}_{1}\mathbf{T} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1a} $$

$$ {}^{1}_{2}\mathbf{T} = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1b} $$

$$ {}^{2}_{3}\mathbf{T} = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1c} $$

$$ {}^{3}_{4}\mathbf{T} = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ 0 & 0 & -1 & -L_4 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1d} $$

$$ {}^{4}_{5}\mathbf{T} = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1e} $$

$$ {}^{5}_{6}\mathbf{T} = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & -1 & -L_6 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1f} $$

$$ {}^{6}_{7}\mathbf{T} = \begin{bmatrix} c_7 & -s_7 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1g} $$

For convenience, the end-effector's frame is assigned to the center of the club head so that its orientation coincides with the universal frame at time, t = 0 s, when the golfer addresses the ball. Therefore, the Denavit-Hartenburg method is not used to find the transformation matrix from Frame {7} to Frame {8}. Instead, this matrix is obtained from its basic definition: a combination of the 3x3 rotation matrix and 3x1 position vector of Frame {8} with respect to Frame {7} which yields

$$ {}^{7}_{8}\mathbf{T} = \begin{bmatrix} 0 & -s_\Phi & -c_\Phi & L_{8b}s_\Phi + L_{8a} \\ 0 & c_\Phi & -s_\Phi & -L_{8b}c_\Phi \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1h} $$

Forward Kinematics: The transformation describing the end-effector (Frame {8}) relative to the universal frame (Frame {0}) is found by

$$ {}^{0}_{8}\mathbf{T} = {}^{0}_{1}\mathbf{T}\, {}^{1}_{2}\mathbf{T}\, {}^{2}_{3}\mathbf{T}\, {}^{3}_{4}\mathbf{T}\, {}^{4}_{5}\mathbf{T}\, {}^{5}_{6}\mathbf{T}\, {}^{6}_{7}\mathbf{T}\, {}^{7}_{8}\mathbf{T}\,. \tag{2} $$

When expressed as

$$ {}^{0}_{8}\mathbf{T} = \left[\begin{array}{ccc|c} & {}^{0}_{8}\mathbf{R} & & {}^{0}\mathbf{P}_{8org} \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \tag{3} $$

R is the 3x3 rotation matrix and

$$ {}^{0}\mathbf{P}_{8org} = \begin{bmatrix} {}^{0}x_8 \\ {}^{0}y_8 \\ {}^{0}z_8 \end{bmatrix} \tag{4} $$

is the position vector mapping the origin of Frame {8} to Frame {0}. This vector is defined by the following forward kinematic expressions: ⊕

$$ \begin{aligned} {}^{0}x_8 = {} & c_{12}s_3L_4+[s_5(c_{12}c_3c_4+s_{12}s_4)+c_{12}s_3c_5]L_6 \\ & +\{c_7[c_6(c_5(c_{12}c_3c_4+s_{12}s_4)-c_{12}s_3s_5)-s_6(c_{12}c_3s_4-s_{12}c_4)] \\ & +s_7[s_5(c_{12}c_3c_4+s_{12}s_4)+c_{12}s_3c_5]\}(L_{8a}+s_\Phi L_{8b}) \\ & -\{c_7[s_5(c_{12}c_3c_4+s_{12}s_4)+c_{12}s_3c_5]-s_7[c_6(c_5(c_{12}c_3c_4+s_{12}s_4) \\ & -c_{12}s_3s_5)-s_6(c_{12}c_3s_4-s_{12}c_4)]\}c_\Phi L_{8b} \end{aligned} \tag{5a} $$

$$ \begin{aligned} {}^{0}y_8 = {} & s_{12}s_3L_4+[s_5(s_{12}c_3c_4-c_{12}s_4)+s_{12}s_3c_5]L_6 \\ & +\{c_7[c_6(c_5(s_{12}c_3c_4-c_{12}s_4)-s_{12}s_3s_5)-s_6(s_{12}c_3s_4+c_{12}c_4)] \\ & +s_7[s_5(s_{12}c_3c_4-c_{12}s_4)+s_{12}s_3c_5]\}(L_{8a}+s_\Phi L_{8b}) \\ & -\{c_7[s_5(s_{12}c_3c_4-c_{12}s_4)+s_{12}s_3c_5]-s_7[c_6(c_5(s_{12}c_3c_4-c_{12}s_4) \\ & -s_{12}s_3s_5)-s_6(s_{12}c_3s_4+c_{12}c_4)]\}c_\Phi L_{8b} \end{aligned} \tag{5b} $$

$$ \begin{aligned} {}^{0}z_8 = {} & L_1-c_3L_4+(s_3c_4s_5-c_3c_5)L_6 \\ & +\{c_7[c_6(s_3c_4c_5+c_3s_5)-s_3s_4s_6]+s_7[s_3c_4s_5-c_3c_5]\}(L_{8a}+s_\Phi L_{8b}) \\ & -\{c_7(s_3c_4s_5-c_3c_5)-s_7[c_6(s_3c_4c_5+c_3s_5)-s_3s_4s_6]\}c_\Phi L_{8b} \end{aligned} \tag{5c} $$

Initial Analysis

Procedure: The angle trajectories are estimated according to Nicklaus' description of his typical swing, supported by photographs of other professional golfers [3]. These trajectories, pictured in Figs. 3a, 3b, and 3c, specify the angles in degrees through the *entire* golf swing. The full swing lasts 2.0 s, with the moment of impact at 1.5 s. The swing through time is divided into three main parts: the back swing occurring from 0 to 1.0 s, the down swing from 1.0 to 1.5 s, and the follow through from 1.5 to 2.0 s.

Using eqs. (5a-5c), the joint trajectory is converted to a Cartesian trajectory. The position and orientation of the club head with respect to the universal frame through time, are shown in Fig. 4a and 4b.

⊕ $c_i = \cos(\theta_i)$, $s_i = \sin(\theta_i)$, $c_{ij} = \cos(\theta_i + \theta_j)$, $s_{ij} = \sin(\theta_i + \theta_j)$

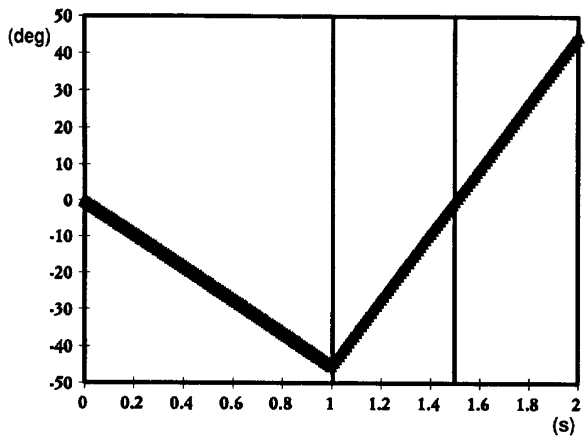

Fig. 3a: $\theta_1 = \theta_2$ vs. Time

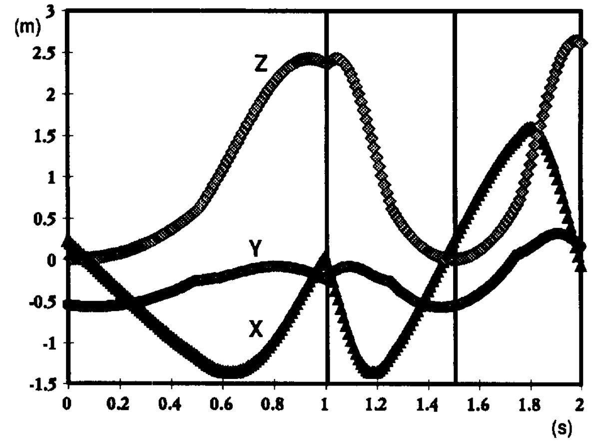

Fig. 4a: Club Head Position vs. Time

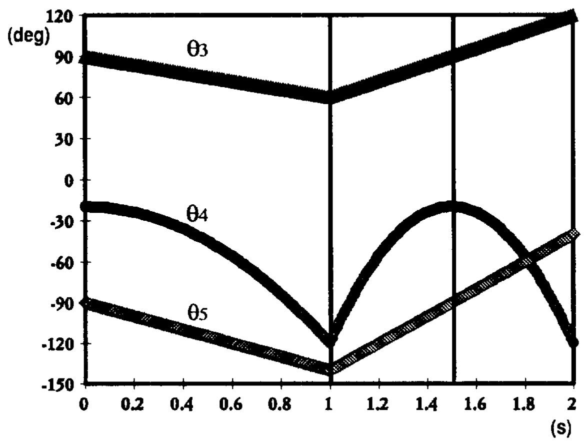

Fig. 3b: θ_3, θ_4, θ_5 vs. Time

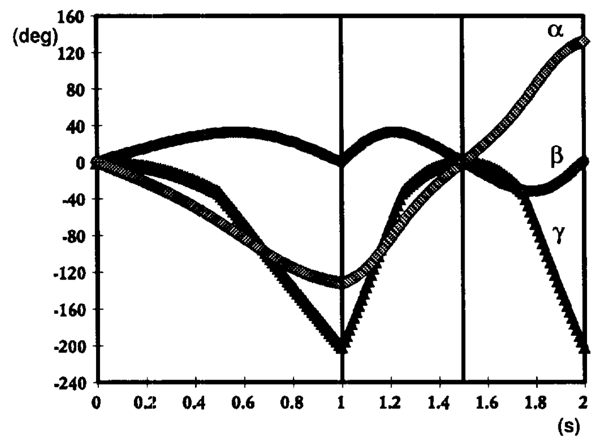

Fig. 4b: Club Head Orientation vs. Time
(X-Y-Z Fixed Angles)

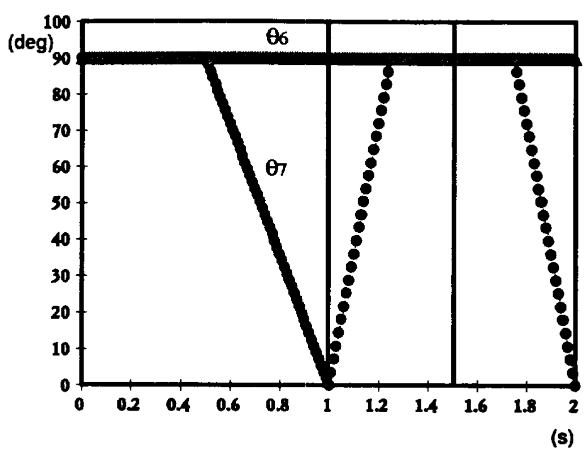

Fig. 3c: θ_6, θ_7 vs. Time

Observations: By inspection of Fig. 4a, the validity of the club head's position can be verified. During the back swing, the club head moves to the right, as indicated by decreasing **X**, slightly backward as indicated by slightly increasing **Y**, and up as indicated by increasing **Z**. The downswing is the reverse of the back swing at twice the speed. During the follow through, the club head moves to the left as indicated by increasing **X**, slightly forward as indicated by increasing **Y**, and up as indicated by increasing **Z**. Note that during the back swing, the down swing, and the follow through, **X** changes its direction as the club head is lifted over the golfer's head, as expected.

Fig. 4b shows that the orientation of the end-effector coincides with the universal frame at t = 0 s and at t = 1.5 s (the moment of impact), as it should. Note that X-Y-Z fixed angles notation [4] is used to describe the orientation of the end-effector's frame.

Performance Analysis

A performance measurement to aid in the refinement of the trajectory is based on Nicklaus' assertion that the path of the club head during the swing should remain in a single plane. This plane of swing passes through the ball and the club head at the beginning of the down swing and aligns with the direction in which the ball should travel. These requirements, applied to the robot model, indicate that the plane should pass through the ball and the origin of Frame {8} at time t = 1.0 s, as well as align with the direction of the $\mathbf{X}_0$ vector. Letting

$$P_0 = {}^0\mathbf{P}_{8org}(t=0.0\ s) \quad \text{and} \quad P_1 = {}^0\mathbf{P}_{8org}(t=1.0\ s),$$

the unit normal vector to the plane of swing is defined as

$$\hat{\mathbf{n}} = \frac{\overrightarrow{P_0P_1} \times \mathbf{X}_0}{\left\| \overrightarrow{P_0P_1} \times \mathbf{X}_0 \right\|} . \tag{6}$$

For the joint trajectories of Figs. 3a, 3b, and 3c, the plane of swing lies from the vertical $\mathbf{Z}_0$ axis at an angle of $\alpha = -7.8^o$ about $\mathbf{X}_0$.

The performance measurement at each instant, defined as the perpendicular distance from the origin of Frame {8} to the defined plane of swing, is given by

$$PM = \hat{\mathbf{n}} \cdot ({}^0\mathbf{P}_{8org} - P_1) \tag{7}$$

and is shown in Fig. 5.

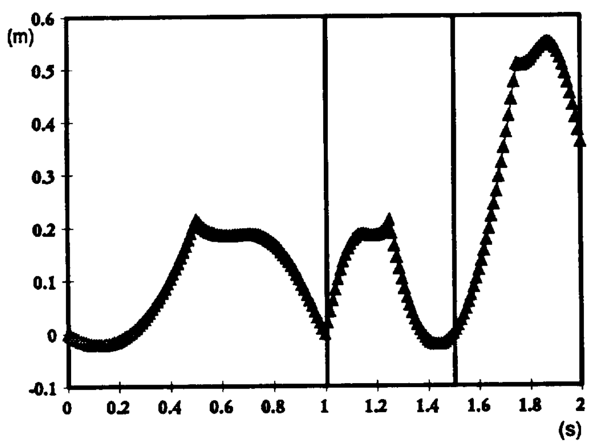

Fig. 5: Performance Measurement vs. Time

Trajectory Refinement

Procedure: The performance measurement is used to obtain the incremental values (in Cartesian coordinates) required to move the club head into the plane at each instant in time by

$$\Delta {}^0\mathbf{P}_{8org} = \begin{bmatrix} 0.0 \\ -PM\cos(\alpha) \\ -PM\sin(\alpha) \end{bmatrix} \tag{8}$$

where α is the angle (about $\mathbf{X}_0$) of the plane of swing.

The Jacobian, used to relate incremental changes in Cartesian coordinates to incremental changes in joint angles is given by

$$\mathbf{J}(\Theta) = \frac{d\,{}^0\mathbf{P}_{8org}}{d\Theta} \tag{9}$$

$$= \begin{bmatrix} \frac{d\,{}^0x_8}{d\theta_1} & \frac{d\,{}^0x_8}{d\theta_2} & \frac{d\,{}^0x_8}{d\theta_3} & \frac{d\,{}^0x_8}{d\theta_4} & \frac{d\,{}^0x_8}{d\theta_5} & \frac{d\,{}^0x_8}{d\theta_7} \\ \frac{d\,{}^0y_8}{d\theta_1} & \frac{d\,{}^0y_8}{d\theta_2} & \frac{d\,{}^0y_8}{d\theta_3} & \frac{d\,{}^0y_8}{d\theta_4} & \frac{d\,{}^0y_8}{d\theta_5} & \frac{d\,{}^0y_8}{d\theta_7} \\ \frac{d\,{}^0z_8}{d\theta_1} & \frac{d\,{}^0z_8}{d\theta_2} & \frac{d\,{}^0z_8}{d\theta_3} & \frac{d\,{}^0z_8}{d\theta_4} & \frac{d\,{}^0z_8}{d\theta_5} & \frac{d\,{}^0z_8}{d\theta_7} \end{bmatrix}$$

where

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_7 \end{bmatrix} . \tag{10}$$

Note that dependence on θ_6, the roll in the wrist, is omitted from eq. (9) and eq. (10) because θ_6 is still not varying.

Manipulation of eq. (9) yields

$${}^0\dot{\mathbf{P}}_{8org} = \mathbf{J}(\Theta)\dot{\Theta} . \tag{11}$$

Using the generalized inverse expression for a non-square matrix, eq. (11) can be written as

$$\dot{\Theta} = \{[\mathbf{J}^T(\Theta)\mathbf{J}(\Theta)]^{-1}\mathbf{J}^T(\Theta)\}\,{}^0\dot{\mathbf{P}}_{8org} . \tag{12}$$

Substituting incremental velocities for the instantaneous velocities in eq. (12) and canceling Δt, the following approximation is obtained:

$$\Delta\Theta \doteq \{[\mathbf{J}^T(\Theta)\mathbf{J}(\Theta)]^{-1}\mathbf{J}^T(\Theta)\}\,\Delta {}^0\mathbf{P}_{8org} . \tag{13}$$

By substituting eq. (8) into eq. (13) for each instant of time, the required joint increments to move the club head into the plane are found. The results of the trajectory refinement are shown in Figs. 6-8.

Observations: The trajectory refinement, based on the performance measurement, accomplishes its main objective of moving the club head closer to the plane of swing. However, the refined swing will still require adjustment.

Figs. 6a, 6b, and 6c show that the refined joint trajectories are no longer smooth and thus demand unrealistic motion. In Fig. 7a, the **Y** trajectory indicates that the club head does not pass behind the robotic model during the back swing. A realistic golf swing requires positive **Y** values at the top of the back swing and beginning of the down swing. Also in Fig. 7a, the slopes of the **Y** and **Z** curves are not quite zero at the moment of impact. This is undesired because a momentum of the club head in any direction other than positive **X** will hook or slice the ball.

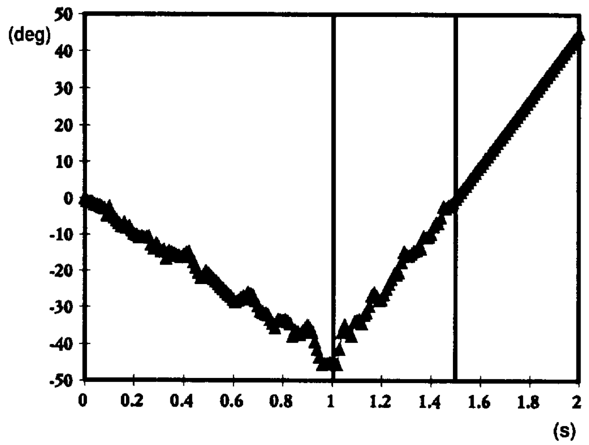

Fig. 6a: Refined $\theta_1 = \theta_2$ vs. Time

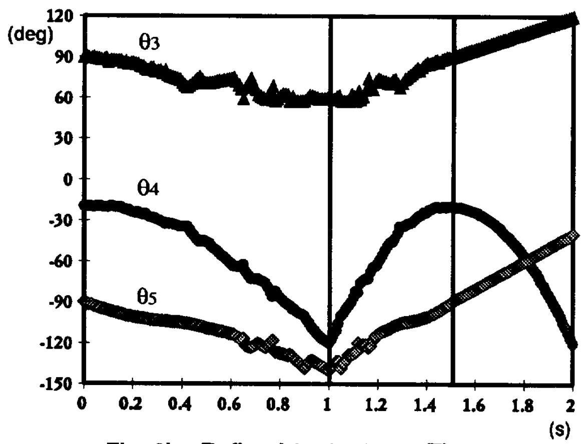

Fig. 6b: Refined θ_3, θ_4, θ_5 vs. Time

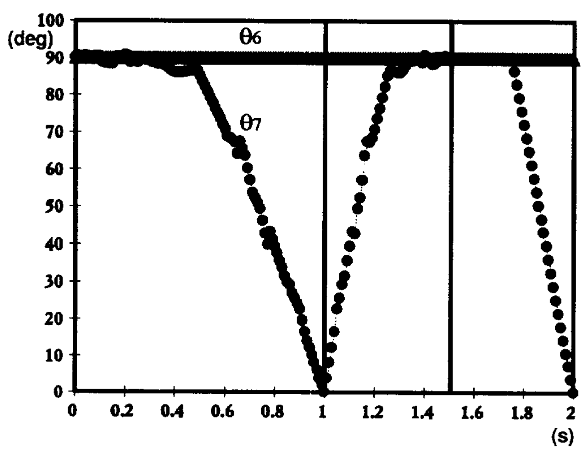

Fig. 6c: Refined θ_6, θ_7 vs. Time

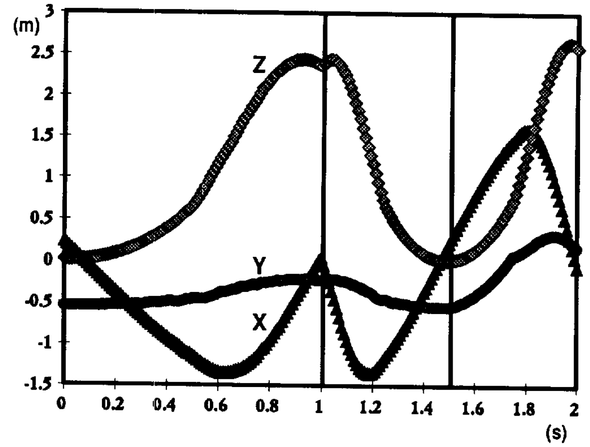

Fig. 7a: Refined Club Head Position vs. Time

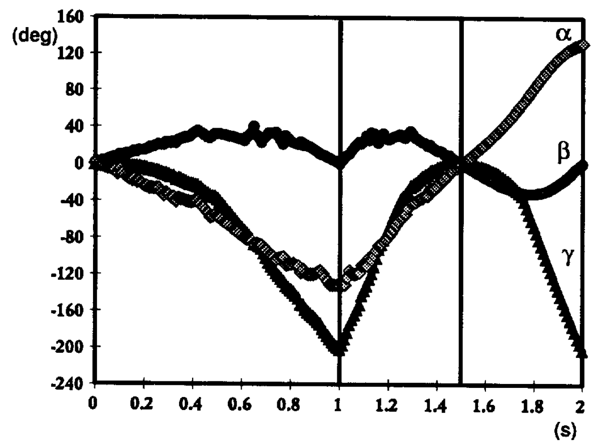

Fig. 7b: Refined Club Head Orientation vs. Time (X-Y-Z Fixed Angles)

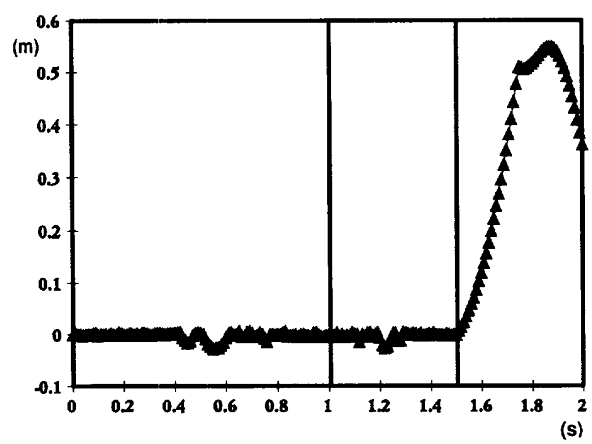

Fig. 8: Refined Performance Measurement vs. Time

Future Plans

The observations gained from the trajectory refinement suggest the need for a redefined plane of swing. By varying θ_6 in Fig. 3c, a less vertical plane of swing (more negative α) will be obtained. The trajectory refinement will then be repeated using the new plane of swing. ✦

The computer program, already used to calculate the forward kinematics and performance measurement, will be enhanced to allow user interaction. The user will enter values for variable parameters and perform a comparative analysis with an already defined "ideal" swing. The final addition to the program will be a visual simulation tool. This will display three views of the robotic golfer through time, i.e., front, left, and top.

REFERENCES

[1] Jack Nicklaus and Ken Bowden, *Golf My Way.* New York: Simon & Schuster, Inc., 1974.

[2] John J. Craig, *Introduction to Robotics, Mechanics and Control*, 2nd ed. Reading: Addison-Wesley Publishing Co.,1989, pp. 74-77.

[3] Peter Kostis with John Huggan, "3 Keys to Distance," *Golf Digest*, vol. 43, No. 7, pp. 50-53, July 1992.

[4] John J. Craig, *Introduction to Robotics, Mechanics and Control*, 2nd ed. Reading: Addison-Wesley Publishing Co.,1989, pp. 45-48.

✦When θ_6 varies, eq. (9) becomes a 3x7 matrix and eq. (10) becomes a 7x1 matrix.