Assignment 2

Due: October 23, 2014

I. For a 3D isotropic medium, a wave equation can be given as

$$\rho \begin{pmatrix} \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial^2 w}{\partial t^2} \end{pmatrix} = (\lambda + 2\mu) \begin{pmatrix} \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \\ \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \\ \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \end{pmatrix} - \mu \begin{pmatrix} \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \frac{\partial}{\partial z}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \end{pmatrix}. \tag{1}$$

Solve problems below.

1) Start from equation 1 and show a wave equation which is for P-SV waves.

2) For the wave equation in problem 1), we can write a plane-wave solution in medium 1 in Figure 1 as

$$u = \epsilon e^{-i\omega(t - px - \gamma z)}, \; w = e^{-i\omega(t - px - \gamma z)}, \tag{2}$$

where $p = \sin\theta_1/\alpha_1$. Find ϵ and γ for P and SV waves.

3) Let us consider reflection and transmission at the free surface (medium 2 is air and the boundary is the free surface, called *halfspace*). Use the solutions in problem 2), and show the reflection and transmission coefficients at the free surface are given by

$$\begin{pmatrix} \acute{P}\grave{P} & \acute{S}\grave{P} \\ \acute{P}\grave{S} & \acute{S}\grave{S} \end{pmatrix} = \begin{pmatrix} \dfrac{-(\eta_1^2 - p^2)^2 + 4p^2\xi_1\eta_1}{(\eta_1^2 - p^2)^2 + 4p^2\xi_1\eta_1} & \dfrac{\beta_1}{\alpha_1}\dfrac{4p\eta_1(\eta_1^2 - p^2)}{(\eta_1^2 - p^2)^2 + 4p^2\xi_1\eta_1} \\ \dfrac{\alpha_1}{\beta_1}\dfrac{4p\xi_1(\eta_1^2 - p^2)}{(\eta_1^2 - p^2)^2 + 4p^2\xi_1\eta_1} & \dfrac{(\eta_1^2 - p^2)^2 - 4p^2\xi_1\eta_1}{(\eta_1^2 - p^2)^2 + 4p^2\xi_1\eta_1} \end{pmatrix}, \tag{3}$$

where the P and S indicate P and SV waves, respectively, and accents on top of them show the direction of waves (e.g., $\acute{P}$ is upgoing P waves). Vertical slownesses are given by $\xi_1 = \cos\theta_1/\alpha_1$ and $\eta_1 = \cos\phi_1/\beta_1$.

4) To solve the reflection coefficients at the boundary in Figure 1 when medium 2 is also an elastic material, describe the boundary condition we need to consider.

5) Back to the problem in halfspace (medium 2 is air and the boundary is the free surface). Different from Love waves, Rayleigh waves can exist in medium 1 (note that for Love waves, we need a layer with thickness H). For the solution of Rayleigh waves, what are the conditions for the vertical slownesses (ξ_1 and η_1) (*hint* do surface waves propagate vertically?)?

6) Why do not Love waves exist in a half-space (no layer)?

II. You have an earthquake record observed at 103 m below the free surface in a horizontal component (Figure 2). When you assume the incident angle of the wave is almost vertical, the waves are dominated by S waves. The borehole profile is given by logging data (Figure 3). Make a reasonable two-layer velocity model from the borehole profile and estimate the maximum amplitude at the surface given by this earthquake. Note that because of the slower layer near the surface, some amplification occurs, and let us assume the medium has no intrinsic attenuation (*hint* you can find the peak frequency from the right panel of Figure 2 and maximum amplitude from the left panel.)

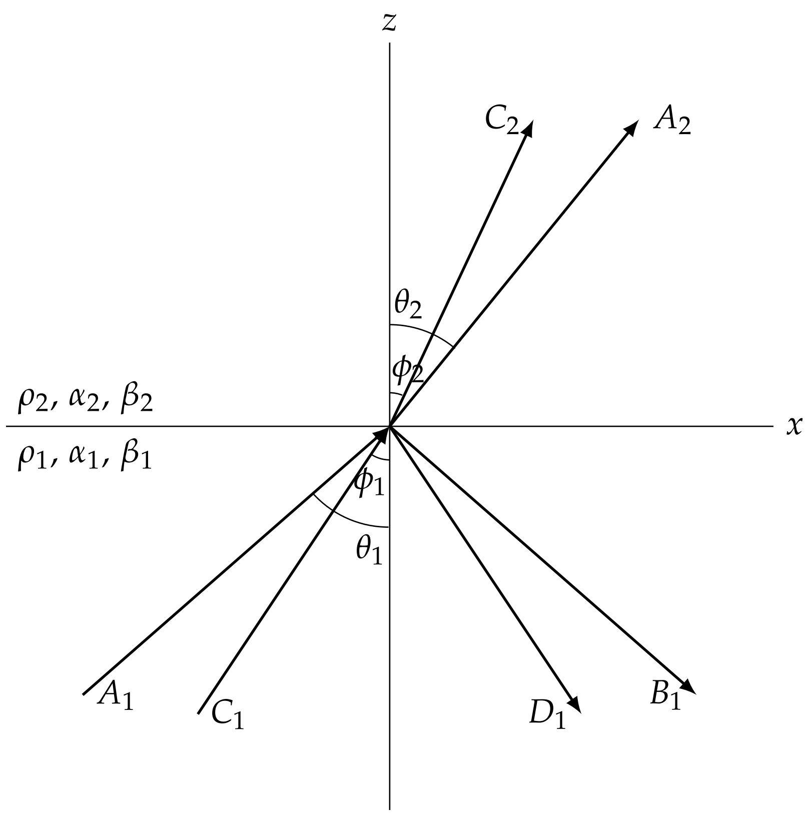

Figure 1: Geometry of reflection and transmission for P-SV waves. A_i, B_i, C_i, and D_i are the amplitude of upgoing P, downgoing P, upgoing SV, and downgoing SV waves in medium i. Waves with amplitudes A_1 and C_1 are incoming waves.

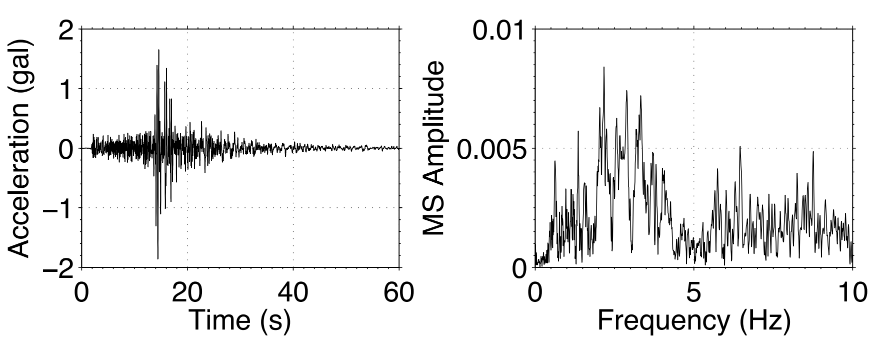

Figure 2: Earthquake record at 103-m depth (left panel is time series and right is the mean-square power spectra of the waveforms).

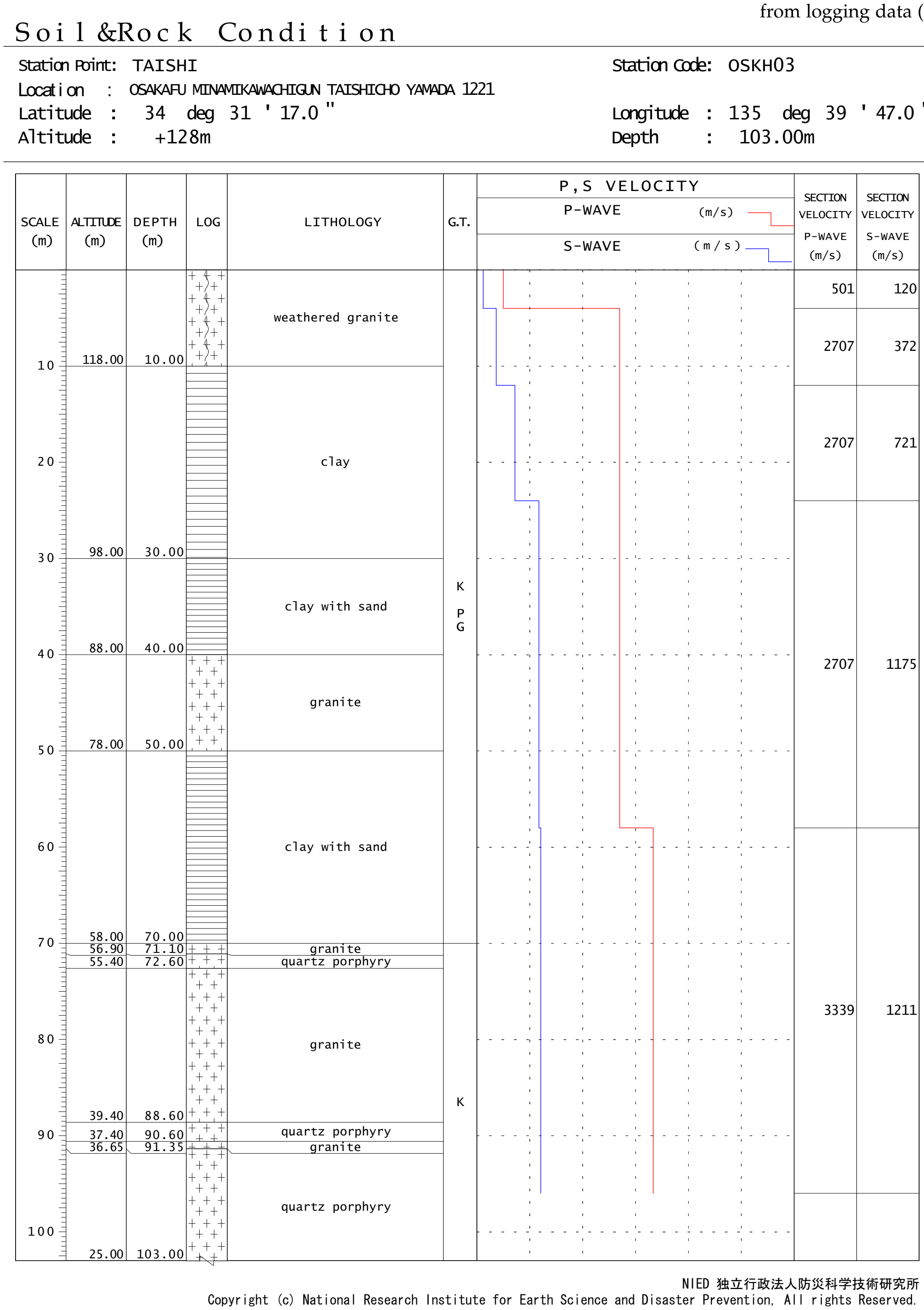

Figure 3: Borehole profile estimated from logging data (in Osaka, Japan).