Supporting materials for "Stochastic characterization of mesoscale seismic velocity heterogeneity in Long Beach, California"

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Different models

In the supplemental materials, we examine different ACFs: the Gaussian, self-affine, and Kummer ACFs, for fitting the data shown in Figure 2a. Based on *Klimeš* [2002], these ACFs including von Kármán can be generalized in the form:

$$P(\mathbf{k}) = \zeta \left[\frac{1}{1/\mathbf{a}^2 + \mathbf{k}^2}\right]^{d/2+\kappa} \exp\left(-\frac{\mathbf{a}_{\mathrm{G}}{}^2\mathbf{k}^2}{4}\right), \tag{S1}$$

where $\mathbf{a}_{\mathrm{G}}$ is the Gaussian correlation length and ζ is a parameter to control the intensity of the randomness, and for equation 2,

$$\zeta = \frac{\varepsilon^2 2^d \pi^{d/2}\Gamma(d/2+\kappa)}{\Gamma(|\kappa|)\mathbf{a}^{2\kappa}}. \tag{S2}$$

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When $\mathbf{a} = \infty$ and $\mathbf{a_G} = 0$, equation S1 indicates the self-affine PSDF [*Feder*, 1988] given as

$$P_{\mathrm{sa}}(k_x, k_y, k_z) = \frac{\varepsilon^2 2^d \pi^{d/2} \Gamma(d/2+\kappa)(2/L)^{2\kappa}}{\Gamma(|\kappa|)\left(k_x^2 + k_y^2 + k_z^2\right)^{d/2+\kappa}}, \tag{S3}$$

where L is a reference distance for preserving a physical unit [*Klimeš*, 2002]. We use $L = 1$ km in Figure S1, and for the data fitting discussed below, we invert L as a parameter. Parameters $\mathbf{a}$ and $\mathbf{a_G}$ control how the PSDF deviates from a power law at low and high wavenumber regions, respectively. Hence, $\mathbf{a}$ behaves as a low-cut wavenumber filter, and $\mathbf{a_G}$ as a high-cut filter. When we define ζ directly for equation S1, we obtain a band-limited fractal random model.

Special cases of equation S1 are often used for geophysical applications. For the von Kármán random model, we consider $\mathbf{a_G} = 0$ (equation 2; the black line in Figure S1). For the Gaussian random model, $\kappa = -d/2$:

$$P_{\mathrm{G}}(k_x, k_y, k_z) = \varepsilon^2 \pi^{d/2} a_{\mathrm{G}x} a_{\mathrm{G}y} a_{\mathrm{G}z} e^{-\frac{a_{\mathrm{G}x}^2 k_x^2}{4} + \frac{a_{\mathrm{G}y}^2 k_y^2}{4} + \frac{a_{\mathrm{G}z}^2 k_z^2}{4}}. \tag{S4}$$

Therefore, the Gaussian model does not follow a power law (the dashed black line in Figure S1). A low-pass power-law random model is given by Kummer PSDF ($\mathbf{a} = \infty$; the red line in Figure S1):

$$P_{\mathrm{Ku}}(k_x, k_y, k_z) = \frac{\varepsilon^2 2^{d+2\kappa} \pi^{d/2} \Gamma(d/2) e^{-\frac{a_{\mathrm{G}x}^2 k_x^2}{4} + \frac{a_{\mathrm{G}y}^2 k_y^2}{4} + \frac{a_{\mathrm{G}z}^2 k_z^2}{4}}}{\Gamma(|\kappa|)\left(k_x^2 + k_y^2 + k_z^2\right)^{d/2}\left(a_{\mathrm{G}x}^2 k_x^2 + a_{\mathrm{G}y}^2 k_y^2 + a_{\mathrm{G}z}^2 k_z^2\right)^{\kappa}}. \tag{S5}$$

When we consider that a medium is infinitely large and the velocities of the medium are purely random at all scales, the self-affine or Kummer PSDFs are suitable for the velocity model estimated by seismic tomography. We have limits on the spatial resolution for tomography caused by finite frequencies and finite array aperture, and hence it may be difficult to resolve very small or very large heterogeneity (e.g., deviation from a fractal model).

We fit the estimated velocities using each random model (Figure Figure S2). None of them fits the observed data better than the von Kármán model. The Gaussian model cannot explain the rich short-wavenumber heterogeneity (Figure S2a). The Kummer and self-affine models are very similar because the inversion tends to move to $\mathbf{a_G} = 0$ (Figure S2bc). The estimated κ for Figure S2bc is almost the same as the one obtained for the von Kármán model. The Gaussian correlation length captures the large-scale heterogeneity and does not show ellipsoidal anisotropy in Figure S2ab, suggesting that, in contrast to small-scale heterogeneity, large-scale heterogeneity of the study area is not anisotropic. Also, because no model explains the anisotropic short-wavelength heterogeneity, the models in Figure S2 do not represent the observed data as well as the von Kármán model (the self-affine model has no anisotropy by definition).

References

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Klimeš, L., 2002. Correlation functions of random media, *Pure appl. geophys.*, *159*, 1811–1831.

Sato, H., M. C. Fehler, and T. Maeda, *Seismic wave propagation and scattering in the heterogeneous earth*, 2 ed., Springer, 2012.

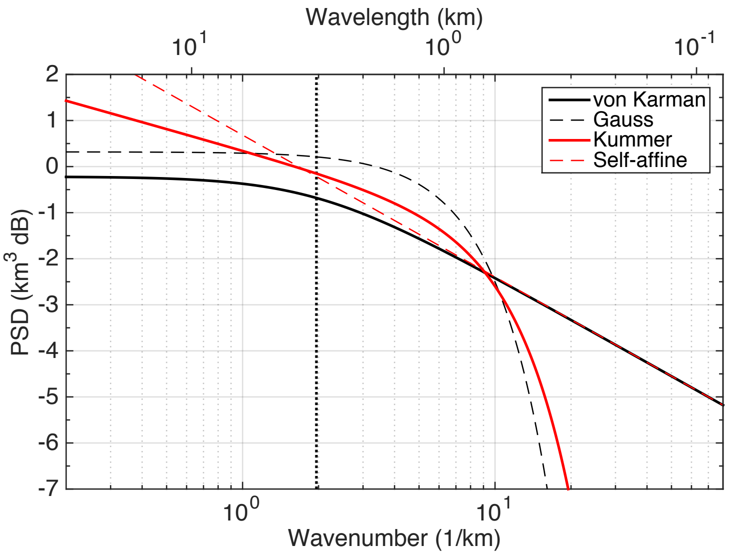

Figure S1. PSDFs of four models [*Klimeš*, 2002; *Sato et al.*, 2012]. The correlation length (for both Gaussian and von Kármán), Hurst exponents, and the intensity of the randomness are identical for all models. The vertical black dotted line indicates the inverse of the autocorrelation length used to compute the PSDFs (1/0.51 km^{-1}).

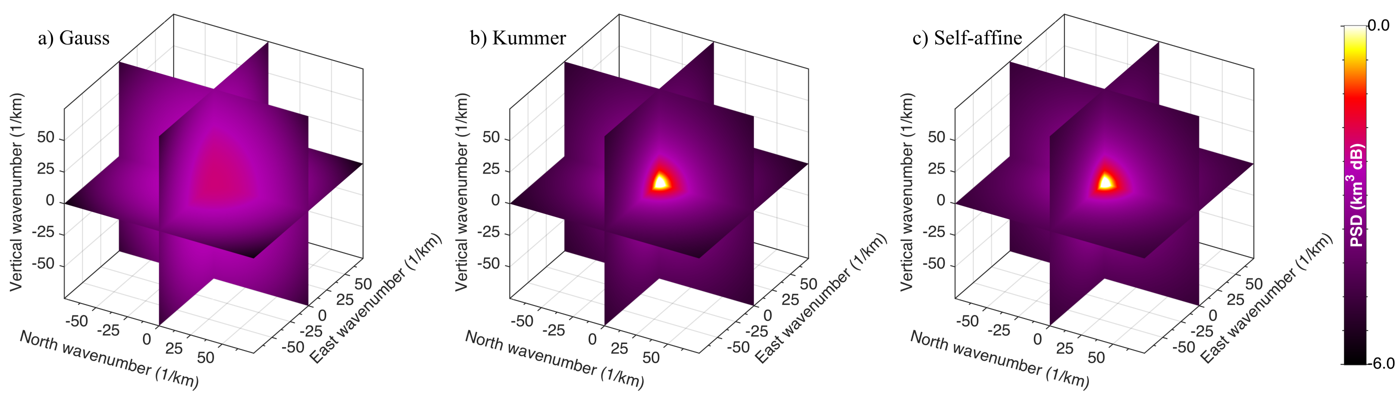

Figure S2. Best-fit PSDFs for the velocity data shown in Figure 2a using (a) Gaussian, (b) Kummer, and (c) Self-affine correlation functions.