Physics GRE 1777 Solutions

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Abbreviations

CM: Classical Mechanics
EM: Electricity and Magnetism
WO: Waves and Optics
AP: Atomic Physics
SR: Special Relativity
QM: Quantum Mechanics
TD: Thermodynamics
LM: Lab Methods
ST: Special Topics

1 CM: Forces

Answer: (E)
$F_A = m_A a_A,\ F_B = m_B a_B = 2\ m_A 2\ a_A = 4F_A.$

2 CM: Momentum

Answer: (E)
$\vec{P_i} = \vec{P_f} \implies 0.5 * 2\hat{i} + 1 * 1\hat{j} = 1.5\vec{v_f}$. So, $\vec{v_f} = \frac{1}{1.5}(\hat{i} + \hat{j}) \implies v_f = \frac{2\sqrt{2}}{3}$ m/s. So, $KE_f = \frac{1}{2}(1.5)(\frac{2\sqrt{2}}{3})^2 = \frac{2}{3}$ J, and $KE_i = \frac{1}{2}(0.5)(2)^2 + \frac{1}{2}(1)(1)^2 = \frac{3}{2}$ J. So $KE_i - KE_F = \frac{3}{2} - \frac{2}{3} = \frac{9-4}{6} = \frac{5}{6}$J.

3 CM: SHM

Answer: (E)
$\omega = \sqrt{\frac{g}{L}}$, so $T = \frac{2\pi}{\omega} = 2\pi\frac{L}{g}$. Therefore, $\frac{T_A}{T_B} = \sqrt{\frac{L_A}{L_B}} \implies \frac{L_A}{L_B} = \frac{1}{4} \implies L_B = 4L_A$.

4 EM: Circuits

Answer: (B) We can combine the two rightmost resistors in parallel into $R_{eq} = \frac{1*2}{1+2} = \frac{2}{3}\Omega$. So the total current in the circuit is $I_{tot} = \frac{20}{1+\frac{2}{3}} = 12A$ from V = IR. Then we wanna divide this current between two branches for the 1 and 2 Ω. We can do that by knowing they share the same potential: $I_1 * 1 = I_2 * 2$ and $I_1 + I_2 = 12$, leading to $2I_2 + I_2 = 12 \implies I_2 = 4$A.
Alternatively, we could write Kirchoff's law for the two loops:
$-20 + I_1(1) + (I_1 - I_2)(1) = 0$ and $(I_2 - I_1)(1) + 2I_2 = 0$ and solve for I_2: $3I_2 = I_1 \implies -20 + 6I_2 - I_2 = 0 \implies I_2 = 4$A.

5 EM: Maxwell

Answer: (E)
Remember one of Maxwell's equations $\vec{\nabla} \times \vec{B} = \mu_0(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t})$, where the second term is the displacement current density. So we see that if we integrate over the area of S, the displacement current will be proportional to the rate of change of the electric flux through S.

6 EM: EM Waves

Answer: (B)
In EM waves, $\vec{E}$ is perpendicular to $\vec{B}$ and they are both perpendicular to the direction of propagation. Here the direction of propagation is + $\hat{e_z}$ (from sin(kz - ωt)), so the only choice that is perpendicular to both $\hat{e_z}$ and to $(\hat{e_x}+\hat{e_y})$ is $(-\hat{e_x}+\hat{e_y})$. (Exact direction can be calculated from $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$).

7 TD: Types of Processes

Answer: (C)
A reversible process must have $\Delta S = 0$.

8 TD: Types of Processes

Answer: (B)
In general, $\Delta U = Q - W$. So if $\Delta U = Q$, then W = 0. But W = $\int PdV$, so $\Delta V = 0$.

9 TD: Distribution of Speeds

Answer: (D)
$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$ is one of the main equations you should know for the GRE.

10 ST: Photoelectric Effect

Answer: (D) The equation for the photoelectric effect is $h\nu = \phi + KE$ if $\nu > \nu_{cuttoff}$, where ϕ is the metal work function. And then the stopping potential should cancel the KE, so $e|V_{stop}| = h\nu - \phi$.

11 ST: Characteristic X-Rays

Answer: (C)
Characteristic X-Rays are emitted when outer-shell electrons fill a vacancy in the inner shell of the atom.

12 QM: Angular Momentum

Answer: (E)
For any l, there are (2l+1) possible values for m from -l to +l. So we have 5 possible m values here: -2, -1, 0, 1, 2.

13 QM: Particle in a Box

Answer: (D)
The energies for a particle in a box are $E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$, so $E_1 = \frac{\hbar^2\pi^2}{2ma^2}$.

14 LM: Dimensional Analysis

Answer: (A)
The only choice with the correct units is (A). This is a repeated question from one of the previous GRE exams.

15 WO: Refraction

Answer: (C)
The index of refraction is related to the dielectric constant by n = $\sqrt{k}$. So, n = 2, therefore v = $\frac{c}{n}$ = 1.5 $\times 10^8$ m/s.

16 WO: Refraction

Answer: (D)
Fermat's principle tells us how light chooses a path to move between two points, so it is how we prove how light moves when hits a boundary, like in reflection and refraction.

17 CM: Orbits

Answer: (D)
From Kepler's third law: $T^2 \propto r^3$, so, $(\frac{T_1}{T_2})^2 = (\frac{r_1}{r_2})^3 = (\frac{1}{4})^3 = \frac{1}{64}$. Therefore, $\frac{T_1}{T_2} = \frac{1}{8}$.

18 CM: Orbits

Answer: (C)
For circular orbits, $\frac{mv^2}{R} = \frac{GmM}{R^2} \implies v = \sqrt{\frac{GM}{R}}$. But the angular momentum L = mvR, so L = $m\sqrt{GMR}$ and $\frac{L_A}{L_B} = \sqrt{\frac{R_A}{R_B}} = \sqrt{2}$.

19 CM: Energy

Answer: (C)
$E_i + W = E_f \implies \frac{1}{2}mv_i^2 + Fd = \frac{1}{2}mv_f^2$. So, $\frac{1}{2}(10)((2)^2 - (1)^2) = 5F \implies 15 = 5F \implies F = 3N$.

20 EM: Sources of B

Answer: (D)
One of the equations to know for the GRE is the magnetic field of a current loop at the center (can be derived from Biot-Savart Law) $B_{center} = \frac{\mu_0 I}{2R}$.

21 EM: Magnetic Force

Answer: (E)
For cyclotrons, $\frac{mv^2}{R} = qvB \implies m\omega = qB$, so $\omega = \frac{qB}{m}$. For a proton, q = e, m = m_p. For a deutron, q = e, m = $2m_p$. For an alpha particle, q = 2e, m = $4m_p$. Therefore, $\omega_p : \omega_d : \omega_\alpha = 1 : \frac{1}{2} : \frac{1}{2}$. Therefore, $\omega_p > \omega_d = \omega_\alpha$. (Here, we approximate the mass of the neutron to be equal to the mass of the proton).

22 AP: Bohr Model

Answer: (A)
In the Bohr model, $E_n \propto Z^2$. Therefore, $\frac{E_{Li}}{E_H} = \frac{3^2}{1^2} = 9$. But $E \propto f \propto \frac{1}{\lambda} \implies \frac{\lambda_{Li}}{\lambda_H} = \frac{1}{9}$.

23 AP: Bohr Model

Answer: (B)
Bohr model has E $\propto \mu$ (the reduced mass). For the positronium, the reduced mass is $\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}$ which is just half the mass for the Hydrogen atom. Therefore, $\frac{E_{pos}}{E_H} = \frac{1}{2}$.

24 AP: Electronic Configuration

Answer: (C)
The Nitrogen atom has the electronic configuration of $1s^2\ 2s^2\ 2p^3$. The three electrons in $2p^3$ all have parallel spins since p has a capacity of 6, and when filling it, you fill it with three parallel spins first then you couple the extra electrons in an anti-parallel sense (known as Hund's Rule). Therefore, the total spin number for the Nitrogen atom is $3 \times \frac{1}{2} = \frac{3}{2}$.

25 QM: Operators

Answer: (B)
For hermitian operators, the eigenvalues are real. So only choices (A) and (B) survive. Also, the eigenvalues have to satisfy $a^4 = 1$, so answer must be (B).

26 QM: Commutation Relations

Answer: (E)
$[T, P] = [\frac{P^2}{m}, P] = 0$ since $[P^2, P] = P^2P - PP^2 = P^3 - P^3 = 0$. So, T and P are compatible observables. All the other choices will result in commutators that relate position to momentum, which will not be zero because X, P are incompatible observables, $[X, P] = i\hbar$.

27 ST: Nuclear Physics

Answer: (A)
Well, we know that nuclei decay via either α^{+2}, β^{+1} or γ. So, γ rays are a very likely product from a nucleus.

28 TD: Cycles

Answer: (C)
In a cycle, $\Delta U = 0$ so Q = W = area enclosed by the cycle. Here, we have a clockwise cycle which is the condition for positive work and heat. So, Q = W = $\frac{1}{2}(6000 - 2000)(0.03 - 0.01) = 40$J.

29 TD: Types of Processes

Answer: (D)
1 has constant pressure so it's isobaric. And we (should) know that the work done by an isothermal process is always bigger than the work done by an adiabatic process starting from the same state ($W_{isothermal} > W_{adiabatic}$). Therefore 2 must be the isothermal process, 3 the adiabatic.

30 WO: Doppler Effect

Answer: (E)
Here we have a double Doppler shift because first the wall receives a shifted frequency from the source (the moving car), then it reflects it and now the moving car becomes the observer and it's moving. So in total $f' = \frac{v+v_0}{v-v_0} f_0 = \frac{350+3.5}{350-3.5}\, 600 = \frac{1+0.01}{1-0.01}\, 600$.
A useful approximation you might use and it will always make your life easier in Doppler problems (especially relativistic ones) is that $\frac{1}{1-x} \approx 1+x$ if x $\ll$1. Hence, $f' \approx (1+0.01)^2\, 600 = (1 + 0.02 + 0.0001) \times 600 \approx 1.02 * 600 = 612$ Hz.

31 WO: Standing Waves

Answer: (E)
For open-open, we have $f_m = \frac{mv}{2L}$. Using one of the given modes, $50 = \frac{v}{2L} \implies \frac{v}{L} = 100$. So if we close one end, the modes become $f_m = \frac{mv}{4L}$ where m is odd. So, $f_m = \frac{m100}{4} = 25m = 25,\ 75,\ 125,\ 175, \cdots$ So none survive!

32 WO: Telescopes

Answer: (A)
For a telescope: $f_o + f_e = L$ and $|M| = \frac{f_0}{f_e}$. So $20 + f_o = 100 \implies f_0 = 80$ cm. Therefore, M $= \frac{80}{20} = 4$.

33 ST: Lasers

Answer: (A)
Dye lasers are very tunable to cover the visible wavelength spectrum.

34 SR: Length Contraction

Answer: (B)
$L = \frac{L_0}{\gamma} \implies \gamma = \frac{1}{0.8} \implies \beta = 0.6$. (Remember the fact that β and $\frac{1}{\gamma}$ are Pythagorean pairs with a hypotenuse of 1).

35 SR: Time Dilation

Answer: (D)
$\beta = 0.6 \implies \frac{1}{\gamma} = 0.8$. Therefore, $\Delta t = \gamma \Delta t_0 = \frac{2\times10^{-6}}{0.8}$. So, d = vt $= 0.6 \times 3 \times 10^8 \times \frac{2\times10^{-6}}{0.8} = \frac{9}{2}\, 100 = 450m$.

36 SR: Energy-Momentum

Answer: (D)
$m^2 = E^2 - p^2$ (setting c = 1). So, $m^2 = (5)^2 - (5-0.1)^2 = (5)^2 - [(5)^2 + (0.1)^2 - 2 \times 5 \times 0.1] = 1 - 0.01 \approx 1$. So, m = 1 GeV/$c^2$.

37 EM: Gauss's Law

Answer: (D)
The charge makes a total flux around it of $\frac{Q}{\epsilon_0}$ by Gauss's law. But the charge is above the yz plane and half the field lines produced by the charge will go down and hit the yz plane while the other half will go up and will not be felt by the yz plane. So, the total flux through the plane is half the total flux of the charge = $\frac{Q}{2\epsilon_0}$.

38 EM: Potential

Answer: (E)
Our reference for V = 0 is the usual reference and it allows us to think of the whole setup from outside as a sphere potential, so V$(r > b) = \frac{kQ_{tot}}{r} = \frac{k(Q+q)}{r}$. And potential should be continuous, so V at the boundary of r = b is equal to $V(b) = \frac{k(Q+q)}{b}$. But the field inside the conductor is zero, and hence the potential should be constant between r=a and r=b, so in this whole region we just have $V(a < r < b) = \frac{k(Q+q)}{b}$.

39 CM: Forces

Answer: (C)

1. $F_1 = F_d + F_D$.
2. $F_2 = F_d - F_D$.
3. $F_3 = \sqrt{F_d^2 + F_D^2}$.

(here F_D and F_d are just the positive magnitudes). Clearly, F_2 is the smallest. We know that $F_d + F_D > \sqrt{F_d^2 + F_D^2}$ since if we square the LHS, we get $(F_d + F_D)^2 = F_d^2 + F_D^2 + 2F_D F_d > F_d^2 + F_D^2$. Therefore, we see that $F_1 > F_3 > F_2$.

40 EM: Circuits

Answer: (A)
The only element that dissipates energy is R. So, we should expect a result P $\propto I^2 R$. The exact equation for power in AC is $P = V_{rms} I_{rms} cos\phi$. But $V_{rms} = I_{rms} Z$. So, $P = I_{rms}^2 Z cos\phi$. But $Zcos\phi$ is just R. (Remember Z = $\sqrt{R^2 + (X_L - X_C)^2}$ and tan $\phi = \frac{X_L - X_C}{R}$, so if you construct that right angle triangle of R, $(X_L - X_C)$ and Z, you see that $Zcos\phi$ = R.). Hence $P = I_{rms}^2 R$ which we could have guessed anyway.

41 LM: Error Analysis

Answer: (E)
The average number of photons N = 0.1 (100) = 10. A decaying process follows Poisson distribution where $\sigma = \sqrt{N} = \sqrt{10} \approx 3$. So N = 10 ± 3.

42 LM: Error Analysis

Answer: (D)
Given $\frac{\sigma_v}{v} = 0.1$, we want to calculate $\frac{\sigma_f}{f}$ where f = $\frac{1}{2}mv^2$. Applying the propagation of error formula, $\sigma_f = \frac{\partial f}{\partial v}\sigma_v = mv\sigma_v = \frac{mv^2 \ \sigma_v}{v} = 0.1mv^2$. Therefore, $\frac{\sigma_f}{f} = \frac{\sigma_f}{0.5mv^2} = \frac{0.1}{0.5} = 0.2$.

43 QM: Hydrogen Atom

Answer: (C)
2s is spherically symmetric, so there should be no dependence on θ or ϕ. Only choices (B), (C) remain. But (B) is the ground state of the hydrogen atom (1s). So the answer must be (C).

44 QM: States

Answer: (A)
The normalization condition is $\sum \text{Prob} = 1 \implies A^2 + A^2 + A^2 = 1 \implies A = \frac{1}{\sqrt{3}}$.

45 QM: Incidence on V

Answer: (A)
Remember that $\frac{\hbar^2 k^2}{2m} = E - V \implies k = \frac{1}{\hbar}\sqrt{2m(E-V)}$. Since regions 1 and 3 have the same E and V, they will definitely have the same k. There is also no reason for the solution to have the special forms of sines or cosines, so generally it would be just Ae^{ikx}. The incident wave oscillates with k in region 1, oscillates with a different k′ in region 2, then back to oscillating with k in region 3.

46 CM: Springs

Answer: (A)
F = kΔx and $\Delta U = \frac{1}{2}k\Delta x^2 = \frac{1}{2}F\Delta x$. So $\frac{\Delta x_1}{\Delta x_2} = \frac{k_1}{k_2} \implies \Delta x_1 < \Delta x_2$. And $\frac{\Delta U_1}{\Delta U_2} = \frac{\Delta x_1}{\Delta x_2} \implies \Delta U_1 < \Delta U_2$.

47 CM: Fluid Dynamics

Answer: (A)
Before: mg = $\rho_W g(0.5V_B)$. After: mg = $\rho_W g(V_{stone} + V'_B)$. Since mg is the same, we see that 0.5 $V_B = V_{stone} + V'_B$. Therefore, $V'_B < 0.5V_B$.

48 CM: Rigid Bodies

Answer: (B)
$I_D = \frac{1}{2}MR^2$. So, $KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}MR^2\frac{v^2}{R^2} = \frac{1}{4}mv^2 = \frac{1}{2}KE_{trans}$. Therefore, $\frac{KE_{rot}}{KE_{tot}} = \frac{KE_{rot}}{KE_{rot}+KE_{trans}} = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{1}{3}$.

49 CM: Projectile

Answer: (B)
$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_x^2 + mgh \implies \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2\cos^2\theta + mgh \implies h = \frac{1}{2g}v_0^2(1-\cos^2\theta) = \frac{1}{2g}v_0^2 sin^2\theta$. Therefore, $\frac{h_1}{h_2} = (\frac{\sin\theta_1}{\sin\theta_2})^2$. So, $\frac{\sin\theta_1}{\sin\theta_2} = \sqrt{2}$.

50 WO: Lenses

Answer: (C)
For the second lens, $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \implies \frac{1}{20} = \frac{1}{10} + \frac{1}{s'} \implies s' = -20$ cm. So, it is a virtual image (since s' is negative) and it is inverted relative to the original object, but upright relative to the first image.

51 WO: Resolution

Answer: (B)
Resolving power is $\frac{\lambda}{\Delta\lambda} = \frac{500}{502-500} = 250$.

52 WO: Interferometer

Answer: (B)
As the gas is evacuated, the path difference for light is 2d (n-1) (full gas - empty gas). So, 2d (n-1) = m $\lambda \implies 2(0.1)(n-1) = 100\times 632.2\times 10^{-9}$. So n-1 = 316.1 $\times 10^{-6} \implies n = 1.000316$. (If they mean by the optical path length nd, then the above formula is missing a division by n, but as n is very close to 1, it won't really matter).

53 SR: Doppler Shift

Answer: (D)
From Hubble's law, v = H_0 r, v = 75 $\times 10^3 \times 100 = 75 \times 10^5$ m/s. So, $\beta = \frac{v}{c} = 0.025$. Then the Doppler shift follows $\frac{\lambda'}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} \implies (\frac{\lambda'}{\lambda})^2 = \frac{1+0.025}{1-0.025}$. Using the usual approximation for Doppler shifts, ($\frac{1}{1-x} \approx 1 + x$ if x $\ll$1), $\lambda' \approx \sqrt{(1+0.025)^2}\lambda = 1.025\mu$m. So 25 nm longer!

54 LM: Electronics

Answer: (B)
A diode blocks currents in one direction and allows them in the other, hence the sine wave will just exist in its positive half-cycle.

55 ST: Fourier Series

Answer: (B)
f(x) is even (f(x) = f(-x)), so it can't have any sine terms in the expansion, since sine is odd. So, $b_n = 0$ for all n.

56 TD: Distribution of Speeds

Answer: (B)
The total area should be equal to the number of molecules N. So $\frac{1}{2}av_0 + av_0 + \frac{1}{2}av_0 = N \implies 2av_0 = N$ and a = $\frac{N}{2v_0}$.

57 TD: Distribution of Speeds

Answer: (D)
The distribution of speeds at temperature T has a non-zero peak and starts from zero at v=0 and then dies eventually again. Therefore, there is always zero probability of v = 0 (III is correct) and the peak is not at v=0 (II is wrong). Also, the velocities at equilibrium should average to zero once you include the directions of the vectors (I is correct). Hence I and III are correct.

58 TD: Entropy

Answer: (A)
$V_i = V_f \implies W = 0 \implies \Delta U = Q = nc_V \Delta T$. So, $\Delta S = \int \frac{dQ}{T} = \int \frac{nc_V dT}{T} = nc_V \ln(\frac{T_f}{T_i})$. And $c_V = \frac{3}{2}R$, hence $\Delta S = \frac{3}{2}nR\ln(\frac{T_f}{T_i})$.

59 ST: Frank Hertz

Answer: (C)
The Frank Hertz experiment gives peaks at multiples of one transition frequency. So 4.9 Hz, 2 $\times$ 4.9 Hz, 3 $\times$ 4.9 Hz, $\cdots$ Note that no atom has evenly spaced levels anyway, so these evenly spaced energies are just an indicator of one transition.

60 ST: Compton Effect

Answer: (A)
The equation for the Compton effect is exactly choice (A), $\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$.

61 QM: Pauli Exclusion Principle

Answer: (B)
Pauli exclusion principle states that two fermions must have a totally antisymmetric combined wavefunction. So the para case has antisymmetric spin, but symmetric spatial wavefunction. While the ortho has symmetric spin and antisymmetric spatial wavefunction. And two identical particles are on average closer (higher in energy) if their spatial wavefunction is symmetric.

62 QM: Harmonic Oscillator

Answer: (D)
For a 3D harmonic oscillator, E = $(n_x + n_y + n_z + \frac{3}{2})\hbar\omega$. So we need $n_x + n_y + n_z = 2$ which can happen in six different ways: (2,0,0), (0,2,0), (0,0,2), (1,1,0), (1,0,1) and (0,1,1).

63 QM: Angular Momentum

Answer: (A)
$[J^2, J_z] = 0$ since we know they are compatible and share simultaneous eigenstates | jm > such that J^2 | jm > = j (j+1) | jm > and J_z | jm > = m $\hbar$ | jm >.

64 CM: Energy

Answer: (E)
In free fall, $v_f^2 = v_i^2 + 2g\Delta y = 0 + 2 \times 10 \times 100 = 2000$. So W = $KE_f - KE_i = \frac{1}{2}mv_f^2 = \frac{1}{2}1000 \times 2000 = 10^6$J.

65 CM: Rigid Bodies

Answer: (D)
$\tau = I\alpha \implies Mg\frac{L}{2}\sin\theta = \frac{1}{3}ML^2\alpha \implies \alpha = \frac{3}{2L}g\sin\theta$. Here we used the moment of inertia around one end of the rod $I = I_{CM} + Md^2 = \frac{1}{12}ML^2 + M(\frac{L}{2})^2 = \frac{1}{3}ML^2$.

66 CM: Springs

Answer: (C)
Two springs in series add like resistors in parallel, so $k_{eq} = (\frac{1}{k_1} + \frac{1}{k_2})^{-1} = \frac{k}{2}$. So, $\omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k}{2m}}$.

67 CM: Hamiltonians

Answer: (E)
H = T + U = $\frac{P^2}{2m} + \frac{1}{2}kx^2$.

68 EM: Gauss's Law

Answer: (A)
Using Gauss's law just inside every cavity in the conductor part, we see that E = 0 $\implies q_{in} = 0$, so the inner surface of each cavity develops a charge of minus what's held inside the cavity. So cavity A gets -2q, cavity B gets +4q and cavity C gets 0.

69 EM: Induction

Answer: (D)
$|\text{emf}| = \frac{N\partial\Phi}{\partial t} = \frac{N\Delta BA}{\Delta t} = \frac{250\times0.05\times0.2}{0.25} = 10\text{V}$.

70 QM: Incidence on V

Answer: (D)
If $k_1 = k_2$, then there should be no reflection since the potential is zero in that case. The only non trivial choice that satisfies this condition is (D).

71 QM: Operators

Answer: (E)
We know that $[\sigma_x, \sigma_y] = 2i\sigma_z \implies \sigma_x\sigma_y - \sigma_y\sigma_x = 2i\sigma_z$. So $\sigma_x\sigma_y$ probably won't be zero nor involving just σ_z without an i. You can also check by multiplying the matrices that $\sigma_x\sigma_y = -\sigma_y\sigma_x$.

72 ST: Nuclear Physics

Answer: (C)
The famous nuclear binding energy per nucleon graph of all elements has a peak at ^{56}Fe.

73 ST: Particle Physics

Answer: (E)
The muon, electron and tau particles are all leptons that have the same properties but just different masses.

74 ST: Particle Physics

Answer: (A)
A lepton does not have any quark content. While baryons (like n and p) have three quarks each.

75 ST: Bragg's Law

Answer: (C)
Bragg's law is $2d\sin\theta = m\lambda$. So, we see that d (distance between layers) is in the order of λ. Then as KE $= \frac{p^2}{2m}$ and using De Broglie's wavelength $p = \frac{h}{\lambda} \implies$ KE $= \frac{h^2}{2\lambda^2 m} = \frac{(6.6\times10^{-34})^2}{2\times9.1\times10^{-31}\times(0.4\times10^{-9})^2}$ J $= \frac{(6.6\times10^{-34})^2}{2\times9.1\times10^{-31}\times(0.4\times10^{-9})^2\times1.6\times10^{-19}}$ eV ≈ 10 eV.

76 EM: Magnetic Force

Answer: (E)
When the particle is undeflected, qvB = qE and hence v $= \frac{E}{B}$.

77 EM: Boundary Conditions

Answer: (C)
The boundary conditions for B as seen by Maxwell's equations ensure that the perpendicular component of B is continuous across boundaries. So the perpendicular component of B vanishes also outside and hence B is totally tangential.

78 EM: Radiation

Answer: (B)
An oscillating charge has a far Poynting vector in the $\hat{r}$ direction (radially outward), so it's to the right here.

79 CM: Forces

Answer: (C)
For m_2, Newton's second law is $m_2g - T = m_2a \implies T = m_2(g-a) \implies T < m_2g.$

80 CM: Gravitation

Answer: (D)
Any of the two small masses feels a total radial force of $\frac{GmM}{R^2} + \frac{Gmm}{(2R)^2} = \frac{Gm}{R^2}(M + \frac{m}{4})$. So, they feel an effective big mass $M' = M + \frac{m}{4}$.

81 WO: Thin Films

Answer: (C)
There is a phase shift of π between the two rays reflected from the first and second face (because one of them is reflected by $n_2 > n_1$ and the other is not). So the condition for constructive interference is (m + $\frac{1}{2}$) $\frac{\lambda}{n}$ = 2t. Therefore, (m + $\frac{1}{2}$) λ = 1500 nm. If m =0, λ = 3000 nm. If m =1, λ = 1000 nm. If m =2, λ = 600 nm. Hence 600 nm is one of the strongly reflected wavelengths. (Note that we divided the wavelength by n since when it travels inside the thin film, it has this reduced wavelnegth)

82 WO: Refraction

Answer: (D)
The beam is reflected with an angle of 60° , so the angle of incidence inside is also 60° , and from the right angle triangle, we see that the angle of refraction inside is just 90° - 60° = 30° . And hence our angle outside satisfies Snell's law $1 \sin\alpha = n \sin 30°$. Therefore, $\alpha = \sin^{-1}\frac{n}{2}$.

83 ST: Radiation Power

Answer: (C)
Power received = 3 (1000) = 3000 W. For the reflective surface, we have to double the power to look at the total force as the power is received then emitted. So, P = F v $\implies$ 6000 = F $\times 3 \times 10^8 \implies F = 2 \times 10^{-5}$N.

84 SR: Doppler Shift

Answer: (C)
$\frac{\lambda'}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} \implies 9 = \frac{1+\beta}{1-\beta} \implies 9 - 9\beta = 1 + \beta \implies 10\beta = 8 \implies \beta = 0.8.$

85 SR: Energy-Momentum

Answer: (A)
$\beta = 0.6 \implies \gamma = \frac{1}{0.8} = \frac{5}{4}$. And W $= E_f - E_i = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2 = (\frac{5}{4} - 1)mc^2 = \frac{1}{4}mc^2$.

86 ST: Hall Effect

Answer: (E)
Hall effect is always used to find the sign of charge carriers.

87 CM: Energy

Answer: (E)
Energy is conserved here, potential energy gets transferred to kinetic. Also, we can measure g by this setup, by measuring the time the mass drops and from that deduce a and from that calculate g. Momentum conservation, however, is not valid here since gravity acts like an external force that changes the momentum of the two masses.

88 CM: Relative Velocity

Answer: (B)
The velocity of the payload is equal to 100 m/s north + g Δt down = 100 m/s north + 40 m/s down. So relative to the plane, we just subtract the velocity of the plane which is 100 m/s north and we get 40 m/s down.

89 CM: Terminal Speed

Answer: (B)
Terminal speed happens when m g = drag force = c v^2. Hence, $\frac{v_1^2}{v_2^2} = \frac{m_1}{m_2} \implies \frac{v_1}{v_2} = \sqrt{2}$.

90 CM: SHM

Answer: (A)
$E_{tot} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}0.3 \times (0.04)^2 = 0.24$ mJ. Note that the maximum speed is the given 0.04 m/s since it occurs at the equilibrium position.

91 TD: Carnot

Answer: (E)
The Carnot effeciency equation is $\frac{W}{Q_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{295}{305} = \frac{10}{305} \implies \frac{Q_H}{W} = \frac{305}{10} = 30.5$.

If we wanted to be exact and compare to Q_C instead, note that $\frac{Q_H}{Q_C} = \frac{T_H}{T_C} = \frac{295}{305}$ and hence $\frac{Q_C}{W} = \frac{295}{305}\frac{305}{10} = 29.5$, so we are still fine.

92 QM: Particle in a Box

Answer: (B)
$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, so we are clearly in the n = 3 state, which has the probability density function as shown in Fig (1). So from symmetry, we see that the needed probability is $\frac{1}{6}$.

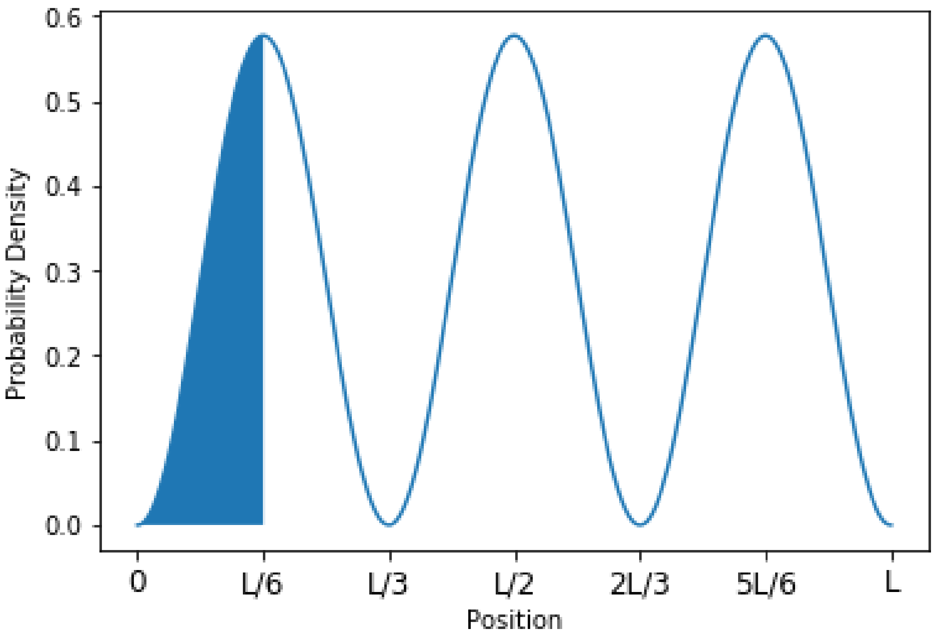

Figure 1: $|\psi|^2$ for the n = 3 wavefunction for a particle in a box of length L

93 SR: 4-Vectors

Answer: (E)
$E^2 - (pc)^2 = (mc^2)^2$ is an invariant.

94 EM: Circuits

Answer: (D)
The initial charge on the capacitor is $Q_0 = CV_0$. This charge is conserved, so in the final situation, it is still the total charge. We can combine the two empty capacitors in series to have a capacitance of $\frac{C}{2}$. Now the charge on this equivalent capacitor and the charge on the first capacitor must add up to the original charge $Q_1 + Q_2 = Q_0 = CV_0$. And also they share the same voltage $V_1 = V_2 = V$. So, the first equation becomes $CV + \frac{C}{2}V = CV_0 \implies V = \frac{2}{3}V_0$.

95 EM: Potential

Answer: (A)
In both cases, the integral for V is very simple V = $\int \frac{kdq}{r} = \frac{kQ}{r}$ since r is constant for both cases. r at the center is the radius R, and r outside is just $\sqrt{R^2 + x^2} = \sqrt{2}$. So, $\Delta U = q\Delta V = kqQ(\frac{1}{R} - \frac{1}{r}) = 9 \times 10^9 \times 5 \times 10^{-6} \times 3 \times 10^{-6}(1 - \frac{1}{\sqrt{2}}) = 135 \times 10^{-3}(1 - 0.707) \approx 39.5 \times 10^{-3}$J.

96 EM: Sources of B

Answer: (C)
Diamagnetic materials lower the magnetic field magnitude (B) but the direction remains the same.

97 CM: Lagrangians

Answer: (A)
L = T - U. Let's have a look at the gravitational potential here. Defining the reference for y = 0 at the top wall, we get $U_g = -mg(h_1 + h_2) = -mga(\cos\theta_1 + \cos\theta_2)$. If the angles are small, we can expand the cosine into $\cos\theta \approx 1 - \frac{\theta^2}{2}$. Therefore, for one of the two masses $U_g = -mga(1 - \frac{\theta^2}{2}) = -mga + \frac{mga\theta^2}{2}$. Now let's get rid of the constant potential that appears (-mga) by shifting the reference down at a distance a below the wall, and hence the gravititional potential becomes $U_g = \frac{mga}{2}(\theta_1^2 + \theta_2^2)$. Therefore, only choice (A) gives a form that is T - U.

98 ST: Bragg's Law

Answer: (D)
m $\lambda = 2d\sin\theta$. So, to have two angles, m must go up to 2. So, $2\lambda = 2d\sin\theta \implies \lambda = d\sin\theta$. Therefore, $\lambda < d$.

99 QM: Selection Rules

Answer: (B)
$\Delta l = 0$ is a forbidden transition, so choice (B) is the only correct option. Only $\Delta l = \pm 1$ are allowed.

100 ST: Mössbauer Spectroscopy

Answer: (C)
The velocity graph we see is a result of some tiny Doppler shifts, so from that we can deduce the shift in the wavelength, and hence the change in energy. And from the uncertainty principle, we can approximate the change in time, which is the lifetime needed.
So, v $\approx$ 1 mm/s means $\beta = \frac{1}{3} \times 10^{-11}$. Since this is a very tiny β, we use the usual approximation in Doppler to get, $\frac{\lambda'}{\lambda} \approx (1 + \beta)^2 \approx 1 + 2\beta$. So, to first order, $\frac{\Delta\lambda}{\lambda} = 2\beta$.
Relating this to the energy shift, note that $E = \frac{hc}{\lambda}$, so $\Delta E = \frac{\partial E}{\partial \lambda}\Delta\lambda = \frac{hc}{\lambda^2}\Delta\lambda = \frac{E}{\lambda}\Delta\lambda$. Hence, $\frac{\Delta E}{E} = \frac{\Delta\lambda}{\lambda} = 2\beta \implies \Delta E = 2\beta E$. And using $\Delta E \Delta t \approx \hbar \implies \Delta t \approx \frac{6.6\times10^{-34}}{2\pi\times\frac{2}{3}\times10^{-11}\times14.4\times10^3\times1.6\times10^{-19}} \approx 10^{-7}$s.