Reliability and Route Diversity in Wireless Networks

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***Abstract*—We study the problem of communication reliability of wireless networks in a fading environment based on the outage probability formulation. The exact expression for the disconnect probability, the probability that a transmission by a node is not received correctly by any other node in the network, is obtained for one and two dimensional random networks. We obtain the end-to-end reliability of multi-hop transmission using the outage probability metric and develop algorithms for finding the most reliable route subject to power constraints as well as the minimum energy route subject to a reliability constraint. Finally, we study the tradeoff between outage probability and transmission power, with and without route diversity.**

I. Introduction

In this paper, we develop a new model for route reliability in a wireless network. A wireless link is subject to multi-path fading which makes communication unreliable. We consider the *outage probability* formulation to model individual link reliability, which is more appropriate than the *ergodic capacity* formulation when the channel is slowly changing, or when the data is delay-sensitive and cannot tolerate the delays associated with coding over the fading channel [1]. Our approach is novel in two respects. First, we extend the outage concept to a multi-hop setting, we consider the end-to-end reliability of a multi-hop route and propose algorithms for selecting the optimal route based on outage probability as the reliability metric. In addition, we study the tradeoff between power consumption and end-to-end reliability and show, through simulation, how route diversity can change this tradeoff.

We assume a network where the geographic locations of wireless nodes and thus the distance and the path-loss between nodes are known at the transmitters. We also assume that the multi-path fading environment changes fast enough and/or the data is delay-sensitive such that it is difficult to obtain the instantaneous fading state information at the transmitter. Furthermore, we assume that ARQ or coding over a long period of time to correct for errors are not practical due to delay or hardware constraints. Under these assumptions, the outage probability is a good metric to characterize reliability. We study the reliability of a single route for one session, ignoring the effects of interference from other sessions in the network. In a practical setting, interference between multiple sessions must be dealt with through various medium access techniques at the MAC layer, and our approach may be most relevant in situations where only a small amount of information must be relayed through the network. These assumptions make the problem simple enough to arrive at some guiding theoretical results that capture many of the important element relevant to more complicated settings. For example, we will be able to formulate various route selection algorithms and study the trade off between reliability, as given by the outage probability, and power consumption under various conditions. In a more complicated setting, an additional amount of energy must be spent on exchanging node locations and to coordinate transmissions, but the essential element, which is the trade-off between reliability and power consumption will remain the same.

We start our analysis by looking at the reliability of a point-to-point communication link in Section II, where we develop the mathematical formulation for the *probabilistic link* model. We then turn to the issue of reliability in a network setting. In Section III, we look at the probability that a message transmitted by a node is not received by *any* other node in the network. Defining this event as the *node disconnect event*, we find the exact expression for this probability in one and two dimensional random networks. Moreover, we examine the tradeoff between the disconnect probability and power consumption and quantify the value of space diversity. In Section IV, the reliability framework is extended to a multi-hop route. We look at optimal routing algorithms under various constraints and study the trade-off between route reliability and power consumption. We also briefly explore the idea of *route diversity* as a way to improve the route reliability-power tradeoff.

II. Probabilistic Link Model

Using the outage probability model, the probability of successful reception for a Rayleigh fading link with fixed distance is given by the following simple expression[1]:

$$\mathsf{P}_{\mathsf{Succ}}(\mathsf{d},\mathsf{snr}) = \exp\left(-\frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}\right). \tag{1}$$

where d is distance between nodes, snr is the signal to noise ratio at the transmitter (it is the transmitted power level if the noise level is 1), and k is the path-loss exponent. Similarly, the probability of an un-successful transmission, or *outage*, is given by: $\mathsf{P}_{\mathsf{Outage}}(\mathsf{d},\mathsf{snr}) = 1-\exp\left(-\frac{\mathsf{d}^{\mathsf{k}}}{\mathsf{snr}}\right)$. It can be shown that the outage probability in the high-snr regime

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[1] This formula is obtained using AWGN channel with Rayleigh fading assuming the transmitter does not have information about the fading state. See [6] for more details.

is approximately given by: $\mathsf{P}_{\mathsf{Outage}}(\mathsf{d},\mathsf{snr}) \approx \frac{\mathsf{d}^k}{\mathsf{snr}}$. This type of approximation is typically used to analyze the trade-off between outage probability and transmission power level. In general, for a point-to-point link with a single transmitting and receiving antenna, it is known that the outage probability decays as snr^{-1}. Detailed analysis of this topic is given in [4]. Assuming the noise level is constant, the relationship between snr and the transmitted power is linear. We may refer to snr as the transmitted power in this paper, but the reader should keep in mind that we are simply ignoring the constant multiplicative factor.

The above model of reliability of a single link will be the building block as we turn to the discussion of reliability in a network setting. Since reliability is a function of the transmitted power, an interesting question in this context is whether the reliability-power tradeoff changes in the presence of multiple intermediate relay nodes. We will see later in this paper that the trade-off remains of the same form for a multi-hop path, even under optimal route selection and power allocation schemes.

III. Network Disconnect Probability

Consider a random two dimensional network as shown in Figure 1. We are interested in the probability that a message transmitted by the source is not received by *any* other node in the network. This is motivated by the desire for measuring asymptotic connectivity as motivated by work such as [3]. This event, which we refer to as the *disconnect event*, is related to the outage event on all links shown in light color in Figure 1. The interesting question is the tradeoff between the disconnect probability and the transmitted power level. Any gain achieved here is purely due to the space diversity created by the presence of multiple receiving nodes and gives us a sense for space diversity gain in a network. We will look at this probability in one and two dimensional Poisson networks[2] with density parameter λ. As will be shown shortly, λ has a significant impact on disconnect probability. Our interest in studying such networks is motivated by our desire to understand the fundamental limits on the performance gains that can be obtained due to diversity. While infinite size networks do not exist in practice, their analysis can be used to gain valuable insight into the potential benefit of space diversity in a network. Similar studies were used in the past (e.g., [3]) to obtain significant insights on wireless network performance and connectivity.

Consider a one dimensional network. Let's fix the source node and measure all distances relative to that node, as shown in Figure 2. Let $\mathsf{Disconnect}(\mathsf{x},\mathsf{d_x},\mathsf{snr})$ be the event that the source node is not connected to *any* node in the segment of $(\mathsf{x},\mathsf{x}+\mathsf{d_x}]$. An infinitesimally short line segment of length $\mathsf{d_x}$ either contains no node or contains only a single node, which happens with probability $\lambda \mathsf{d_x}$. Using the independence of the fading from the event that the line segment contains a node, one can bound this probability by: $1-\mathsf{P}_{\mathsf{Succ}}(\mathsf{x},\mathsf{snr})\lambda\mathsf{d_x} \le \mathsf{P}_{\mathsf{Disconnect}}(\mathsf{x},\mathsf{d_x},\mathsf{snr}) \le 1-\mathsf{P}_{\mathsf{Succ}}(\mathsf{x}+\mathsf{d_x},\mathsf{snr})\lambda\mathsf{d_x}$, where $\mathsf{P}_{\mathsf{Succ}}(\mathsf{d},\mathsf{snr})$ is given by (1). Define the event $\mathsf{Disconnect}(\mathsf{L},\mathsf{snr})$ as the event that the source node is not connected to any node located within distance L from it in one direction. The probability of this event is given by: $\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{L},\mathsf{snr}) = \prod_{i=0}^{\frac{L}{d_x}} \mathsf{P}_{\mathsf{Disconnect}}(\mathsf{i}\mathsf{d_x},\mathsf{d_x},\mathsf{snr})$. Taking the logarithm of both sides, we have:

$$\mathsf{Log}\left(\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{L},\mathsf{snr})\right) = \sum_{i=0}^{\frac{L}{d_x}} \mathsf{Log}\left(\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{i}\mathsf{d_x},\mathsf{d_x},\mathsf{snr})\right).$$

This last expression can be bounded using bounds for $\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{x},\mathsf{d_x},\mathsf{snr})$. As $\mathsf{d_x}\to 0$, the sum can be replaced by an integral and the lower and the upper bounds converge, see [6] for more details. In the limit, using the fact that $\ln(1-p)\approx -p$ for small values of p, we have: $\mathsf{Log}(\mathsf{P}_{\mathsf{Disconnect}}(\mathsf{L},\mathsf{snr})) = -\int_0^L \mathsf{P}_{\mathsf{Succ}}(\mathsf{x},\mathsf{snr})\lambda \mathsf{d_x} = -\int_0^L \exp(-\frac{x^k}{\mathsf{snr}})\lambda \mathsf{d_x}$. To find the disconnect probability for an asymptotically large network, we are interested in the limit of the above quantity as $L\to\infty$. Using known integral forms given in [5], it can be shown that the disconnect probability is given by the following limit: $\mathsf{Log}(\mathsf{P}^{\mathsf{One-sided}}_{\mathsf{Disconnect}}(\mathsf{snr})) = -\int_0^\infty \exp(-\frac{x^k}{\mathsf{snr}})\lambda\mathsf{d_x} = -\lambda\sqrt[k]{\mathsf{snr}}\frac{1}{k}\Gamma\left(\frac{1}{k}\right)$, where Γ is the well known Gamma function. The above expression only measure the probability that the node is disconnected from all nodes in one direction. The two-sided disconnect probability follow immediately, as given in the following theorem.

Theorem 1: In a Poisson line network with density λ and path-loss exponent k, the probability that a node is disconnected from the network is $\exp\left(-2\lambda\sqrt[k]{\mathsf{snr}}\frac{1}{k}\Gamma(\frac{1}{k})\right)$, where Γ is the well known Gamma function.

Similarly, in the two dimensional case we obtain the following result.

Theorem 2: In a two dimensional Poisson network with density λ and path-loss exponent k, the probability that a node is disconnected from the network is $\exp\left(-2\lambda\pi\sqrt[k]{\mathsf{snr}^2}\frac{1}{k}\Gamma(\frac{2}{k})\right)$, where Γ is the well known Gamma function.

The above results do not require use of the high-snr approximation in their derivation, i.e. the expressions given in theorem 1 and theorem 2 are valid for any snr value. These

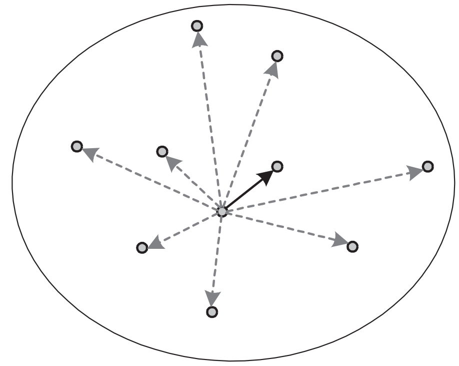

Fig. 1. Disconnect with and without Diversity

[2]In one dimension, this network is constructed by uniformly placing N nodes on a line of length L as both N and L approaches infinity, while keeping the $\frac{N}{L}$ constant at λ. The two dimensional counterpart is constructed similarly by placing N nodes uniformly on a disk with area A while keeping $\frac{N}{A}$ constant at λ.

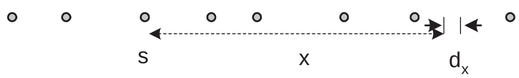

Fig. 2. Random Line Network Disconnect Probability

expressions show that with diversity, the disconnect probability decays *exponentially* with $\sqrt[k]{\mathsf{snr}}$ and $\sqrt[k]{\mathsf{snr}^2}$, in one and two dimensional networks respectively.

IV. Reliability at the Network Layer

We now look at the problem of route selection under reliability and power constraints in a network. A multi-hop route is a sequence of nodes through which the information is relayed from a source node, S, to a destination node, D, i.e. $\mathsf{Route} = (\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$, where, $\mathsf{r}_0 = \mathsf{S}$, $\mathsf{r}_\mathsf{h} = \mathsf{D}$, and h is the number of hops. We assume the network operates based on a time division protocol under which successive transmissions along a route happen in consecutive transmission slots. Route $(\mathsf{s}, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{d})$ is identical to a sequence of h point-to-point links, where for the i^{th} link, relay $\mathsf{i}-1$ is the transmitter and relay i is the receiver, $\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}$ is the transmitted signal-to-noise power, and $\mathsf{d}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}$ is the corresponding distance. We define the event of successful end-to-end transmission as the event that all h transmissions are successful, and assume that fading for different links are i.i.d Rayleigh random variables. Based on this assumption and using (1), the *End-to-End Reliability*, i.e the probability of successful end-to-end transmission, can be written as: $\mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} = \prod_{\mathsf{i}=1}^{\mathsf{h}} \exp\left(-\frac{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right) = \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)$. The total amount of power spent for end-to-end transmission is: $\mathsf{SNR}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})}_{\mathsf{Total}} = \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}$. The end-to-end reliability is a function of the total power consumption, and the problem of optimal route selection, either in reliability or in power sense, only becomes well defined if the other quantity is constrained. This restriction can be applied either at nodal level or at an end-to-end level. We will consider each case next.

A. Fixed Maximum Transmission Power Per Link

Assume the transmitted signal-to-noise ratio at each link is limited to $\mathsf{SNR}_{\mathsf{Link-Max}}$. This formulation may be appropriate if the maximum transmitted power per node is limited due to hardware constraint. For a fixed route, $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$, the end-to-end reliability is given by: $\mathsf{Reliability}^{(\mathsf{r}_0,\mathsf{r}_1,\cdots,\mathsf{r}_{\mathsf{h}-1},\mathsf{r}_\mathsf{h})} = \exp\left(-\frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{SNR}_{\mathsf{Link-Max}}}\right)$. The end-to-end reliability is a monotonically decreasing function of $\sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}$, which can be treated as the cost metric for route selection. We refer to route selection algorithm based on this cost metric as the **M***inimum* **O***utage* **R***oute* (*MOR*).

B. End-to-End Power/Reliability Optimized Route

Alternatively, one can look at the problem of optimizing transmission power levels and route selection on an end-to-end basis. Consider the problem of minimizing the end-to-end power for a fixed route, $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$, subject to a fixed minimum end-to-end reliability, $\mathsf{Reliability}_{\mathsf{Min}}$. This can be formulated by the following constrained optimization problem:

$$\begin{aligned} \min \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} \\ \text{s.t.} \quad & \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right) \geq \mathsf{Reliability}_{\mathsf{Min}}. \end{aligned} \tag{2}$$

Since exp is a monotonically increasing function, the constraint must be satisfied with equality at the optimal solution. So, the optimization problem is equivalent to:

$$\begin{aligned} \min \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} \\ \text{s.t.} \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}} = -\ln(\mathsf{Reliability}_{\mathsf{Min}}). \end{aligned} \tag{3}$$

This problem can be solved using Lagrangian multiplier technique. The amount of power allocated to link $\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}$ under the optimal power allocation scheme is given by: $\widehat{\mathsf{snr}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} = \frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}}{-\ln(\mathsf{Reliability}_{\mathsf{Min}})} \sqrt{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}$. The corresponding total consumed power is given by $\widehat{\mathsf{SNR}}_{\mathsf{Total}} = \sum_{\mathsf{i}=1}^{\mathsf{h}} \widehat{\mathsf{snr}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} = \frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)^2}{-\ln(\mathsf{Reliability}_{\mathsf{Min}})}$. The last expression can be used as the basis for an optimal route selection scheme, which we refer to as **M***inimum* **P***ower* **R***oute* (*MPR*).

Theorem 3: For a fixed route $(\mathsf{r}_0, \mathsf{r}_1, \cdots, \mathsf{r}_{\mathsf{h}-1}, \mathsf{r}_\mathsf{h})$, the minimum required total power to guarantee the end-to-end reliability of $\mathsf{Reliability}_{\mathsf{min}}$ is $\widehat{\mathsf{SNR}}_{\mathsf{Total}} = \frac{\left(\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right)^2}{-\ln(\mathsf{Reliability}_{\mathsf{Min}})}$, which is achieved when power at each link is allocated such that $\widehat{\mathsf{snr}}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} = \frac{\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}}{-\ln(\mathsf{Reliability}_{\mathsf{Min}})} \sqrt{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}$. Hence, the minimum power route between nodes S and D subject to a guaranteed end-to-end reliability is the route that minimizes $\sum_{\mathsf{i}=1}^{\mathsf{h}} \sqrt{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}$,

Alternatively, we can formulate an optimal route Section problem where we seek the most reliable end-to-end route subject to a fixed end-to-end power, $\mathsf{SNR}_{\mathsf{Total-Max}}$. The constrained optimization problem in this case is:

$$\begin{aligned} \max \quad & \exp\left(-\sum_{\mathsf{i}=1}^{\mathsf{h}} \frac{\mathsf{d}^\mathsf{k}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}{\mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}}}\right) \\ \text{s.t} \quad & \sum_{\mathsf{i}=1}^{\mathsf{h}} \mathsf{snr}_{\mathsf{r}_{\mathsf{i}-1}\mathsf{r}_\mathsf{i}} \leq \mathsf{SNR}_{\mathsf{Total-Max}} \end{aligned}$$

This problem can be solved using a technique very similar to the approach used to solve (2). Omitting the details of the optimization, we obtain the following result, *Theorem 4:* The most reliable route between nodes S and D in a wireless network where the fading parameters of different links are independent Rayleigh random variables and the maximum end-to-end power is limited to $\mathsf{SNR}_{\mathsf{Total-Max}}$ is the route that minimizes $\sum_{i=1}^{h}\sqrt{d^{k}_{r_{i-1}r_i}}$, and the corresponding end-to-end reliability is given by $\mathsf{Reliability}_{\mathsf{Optimal}} = \exp\left(-\frac{\left(\sum_{i=1}^{h}\sqrt{d^{k}_{r_{i-1}r_i}}\right)^2}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right)$. To achieve this solution the power should be allocated as $\widehat{\mathsf{snr}}_{r_{i-1}r_i} = \mathsf{SNR}_{\mathsf{Total-Max}}\frac{\sqrt{d^{k}_{r_{i-1}r_i}}}{\sum_{i=1}^{h}\sqrt{d^{k}_{r_{i-1}r_i}}}$ for each link $r_{i-1}r_i$.

The optimization problems that we looked at in this Section are dual problems and it should not be surprising that the cost metric in both cases is $\sum_{i=1}^{h}\sqrt{d^{k}_{r_{i-1}r_i}}$. To clarify this point, we present a graphical illustration of the end-to-end route reliability under the optimal power allocation scheme. For any fixed route, different power allocation schemes result in different end-to-end reliability and power consumption. If we were to characterize each power allocation scheme only by the total consumed power and the resulting end-to-end reliability, each allocation scheme could be represented by a point in the two dimensional plot of the end-to-end reliability versus the total power. Certain allocation schemes are optimal, i.e. they either minimize the total power consumed to achieve a guaranteed end-to-end reliability or maximize the end-to-end reliability for a fixed consumed power.

In the first formulation, we found the optimal power allocation that minimized the total power subject to a guaranteed end-to-end reliability. Graphically, this optimization corresponds to moving along the horizontal line in Figure 3 to find the allocation scheme that minimizes the total consumed power while ensuring that the end-to-end reliability is at least $\mathsf{Reliability}_{\mathsf{min}}$. We found that the reliability and power corresponding to the optimal allocation are related by the following relationship: $\widehat{\mathsf{SNR}}_{\mathsf{Total}} = \frac{\left(\sum_{i=1}^{h}\sqrt{d^{k}_{r_{i-1}r_i}}\right)^2}{-\ln(\mathsf{Reliability}_{\mathsf{Min}})}$. In the second formulation, we found the optimal power allocation to maximize the end-to-end reliability for a given end-to-end power. This corresponds to moving along the vertical line in Figure 3 and finding the allocation scheme that maximizes the reliability subject to a constraint on the total consumed power given by $\mathsf{SNR}_{\mathsf{Total-Max}}$. We found that the resulting end-to-end reliability for this optimal allocation is

$$\mathsf{Reliability}_{\mathsf{Optimal}} = \exp\left(-\frac{\left(\sum_{i=1}^{h}\sqrt{d^{k}_{r_{i-1}r_i}}\right)^2}{\mathsf{SNR}_{\mathsf{Total-Max}}}\right) \quad (4)$$

The reliability-power relations specified by above expressions are identical, and the set of optimal power allocations can be represented by a single curve in the this two dimensional space, as shown in Figure 3. We refer to this curve as the *Optimal Reliability-Power Trade-off* curve. The shape of this curve for a route is a function of the number and distance between relays along that route.

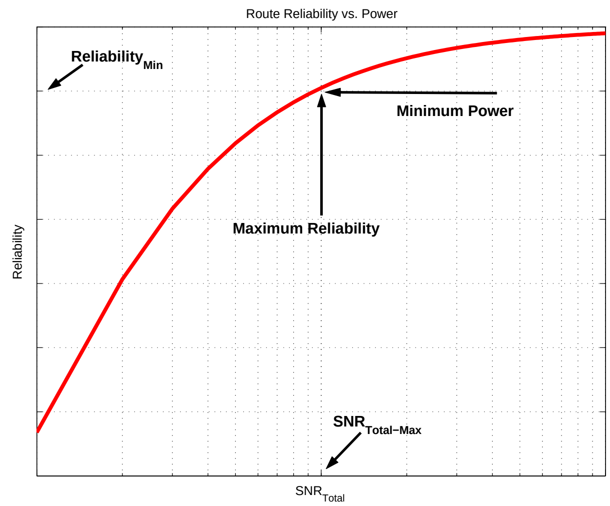

Fig. 3. Route Reliability vs. Power

A. *Outage Probability-Power Trade-off and Diversity*

Similar to the case of a point-to-point link, here we consider the trade-off between route outage and consumed power in a network setting. This type of analysis gives insight to how fast the end-to-end outage decreases as more power is spent on the transmission. First, we look at the case that the maximum transmitted power at each link is fixed to $\mathsf{SNR}_{\mathsf{Link-Max}}$. The outage probability, ρ, is given by : $\rho = 1 - \exp\left(-\frac{\sum_{i=1}^{h} d^{k}_{r_{i-1}r_i}}{\mathsf{SNR}_{\mathsf{Link-Max}}}\right)$.

For high level of $\mathsf{SNR}_{\mathsf{Link-Max}}$, i.e. small values of ρ, we can use the approximation of $\exp(x) \approx 1 + x$ to simplify this relation to:

$$\rho \approx \frac{\sum_{i=1}^{h} d^{k}_{r_{i-1}r_i}}{\mathsf{SNR}_{\mathsf{Link-Max}}}. \quad (5)$$

Similarly, (4) gives relationship between the end-to-end reliability and the total power consumption under the optimal power allocation scheme. Writing (4) in terms of the route outage probability and following a similar approach, we find:

$$\rho \approx \frac{\left(\sum_{i=1}^{h}\sqrt{d^{k}_{r_{i-1}r_i}}\right)^2}{\mathsf{SNR}_{\mathsf{Total-Max}}}. \quad (6)$$

From (5) and (6), we observe that in the high-snr regime, the route outage probability decays as $\mathsf{SNR}^{-1}_{\mathsf{Link-Max}}$ and $\mathsf{SNR}^{-1}_{\mathsf{Total-Max}}$, respectively. This is consistent with what we observed for a single link. It should also be clear that route selection does not have any effect on the form of this trade-off. By selecting the optimal route, we minimized the end-to-end outage probability by minimizing the numerator of (5) and (6). However, that process does not change the form of the denominators, which dictate the outage-power trade-off in the high-snr regime. This shows that as long as we limit our approach to a single transmitter and a single receiver per link, even under optimal power allocation and route selection, the trade-off maintains the same form as in the single link case.

An interesting question is how the above mentioned trade-off can be improved. As expected, the idea of diversity at the route level, or *route diversity*, is key to this relationship as we demonstrate using a simple simulation example. The network that we use for our simulations is constructed by uniformly placing 10 nodes inside a circle with radius 1 with the source and destination nodes placed at the two opposite ends of a diameter of this circle, see Figure 4. We only consider the case that the transmitted power is fixed at $\mathsf{SNR}_{\mathsf{Link-Max}}$, similar to case A in Section IV. In this case, the most reliable route, i.e. the *Minimum Outage Route (MOR)*, is selected based on the algorithm discussed in Section IV. Now consider the possibility that each relay node can receive the information from the last two nodes immediately before it along the selected route as shown in Figure 4. This corresponds to a diversity of two transmitting nodes for each transmission.

For each realization of this network, the end-to-end outage probability is a function of the node location (inter-node distance) and power. For the base case, i.e. without diversity, the expression for the end-to-end outage probability was given earlier. For the diversity scheme we outlined above, a similar expression for the end-to-end outage, as a function of inter-relay distances, can be calculated recursively (see [6]). Instead of showing the reliability improvement for any particular network realization, we average the outage probabilities over 1000 realizations of this network. We do this by calculating the end-to-end outage versus power curve for each realization, and then taking the average outage at each snr across the 1000 simulations. Figure 5 shows the resulting *average* outage probability vs. snr curves. Even with this limited diversity, the end-to-end outage decays as snr^{-2}. This type of improvement in the relationship between the end-to-end route outage and power is achieved through route diversity and does not require any coding, ARQ, or transmitter side information.

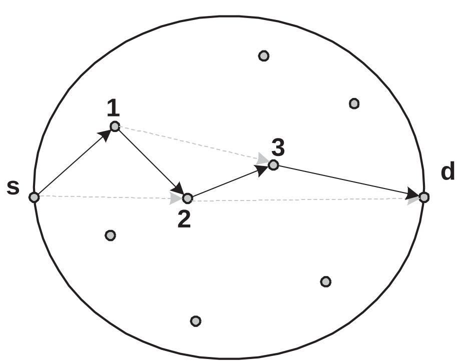

Fig. 4. Simulation Network

V. Summary and Conclusions

The problem of communication reliability and diversity in multi-hop wireless networks was studied based on the outage probability model for wireless channels. We derived the exact expression for the disconnect probability in random one and two dimensional networks. Extending this framework to a multi-hop setting, we studied the end-to-end route reliability in a wireless network. We developed algorithms for finding the optimal route between a source-destination pair under various reliability and power constraints, and showed that the trade-off between the end-to-end reliability and consumed power is similar to that over a single link; as long as each transmission is limited to a single transmitting and receiving node. Our basic simulations showed that route diversity can take advantage of the wireless broadcast property and the independence of fade parameters between different pairs of nodes to fundamentally change this trade-off.

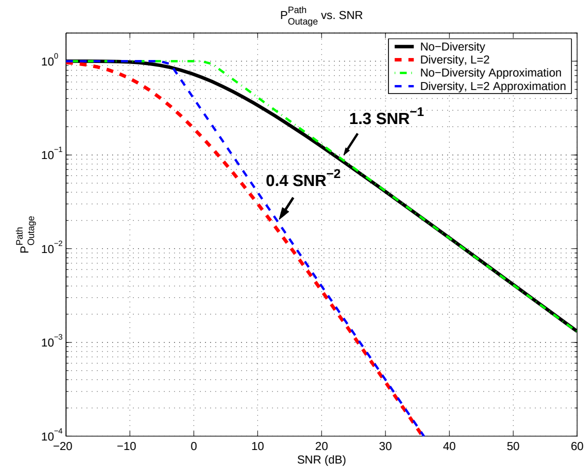

Fig. 5. Average Outage Probability

References

[1] E. Biglieri, J. Proakis and S. Shamai, "Fading channels: information-theoretic and communications aspects," *IEEE Tran. on Info. Theory* vol 44, pp. 2619-2692, Oct. 1998.

[2] J. N. Laneman, "Cooperative diversity in wireless networks: algorithms and architectures," Ph.D. dissertation, MIT, Cambridge, MA, 2002.

[3] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," in *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming,* W.M. McEneany, G. Yin, and Q. Zhang, Eds. Boston, MA: Birkhauser, 1998, pp. 547-566.

[4] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental trade-off in multiple-antenna channels," *IEEE Tran. on Info. Theory* vol. 49, pp. 1073-1096, May 2003.

[5] I. S. Gradshtein, I. M. Ryzhik, A. Jeffrey and D. Zwillinger, *Table of Integrals, Series, and Products* Academic Press 2007.

[6] A. E. Khandani, "Cooperative routing in wireless networks", Master's Thesis, MIT, Cambridge, MA, May 2004.