Cooperative Routing in Static Wireless Networks

Amir Ehsan Khandani, Jinane Abounadi, Eytan Modiano, and Lizhong Zheng

Abstract—**We study the problem of transmission-side diversity and routing in a static wireless network. It is assumed that each node in the network is equipped with a single omnidirectional antenna and that multiple nodes are allowed to coordinate their transmissions in order to obtain energy savings. We derive analytical results for achievable energy savings for both line and grid network topologies. It is shown that the energy savings of 39% and 56% are achievable in line and grid networks with a large number of nodes, respectively. We then develop a dynamic-programming-based algorithm for finding the optimal route in an arbitrary network, as well as suboptimal algorithms with polynomial complexity. We show through simulations that these algorithms can achieve average energy savings of about 50% in random networks, as compared to the noncooperative schemes.**

Index Terms—**Cooperative transmission, energy efficiency, network reliability, outage probability, routing, wireless networks.**

I. INTRODUCTION

WE study the problem of routing, cooperation, and energy efficiency in static wireless ad hoc networks. In these networks, the nodes often spend most of their energy on communication [1]. In many applications, the nodes are small and have limited and nonreplenishable energy supplies. For this reason, energy conservation is critical for extending the lifetime of these networks, and it is not surprising that the problem of energy efficiency and energy-efficient communication in ad hoc networks has received a lot of attention in the past several years. This problem, however, can be approached from two different angles: energy-efficient route selection algorithms at the network layer or efficient communication schemes at the physical layer. While each of these two areas has received a lot of attention separately, not much work has been done on jointly addressing these two problems. Our analysis in this paper tackles this less studied area.

The amount of energy required to establish a link between two nodes is usually assumed to be proportional to the distance between the nodes raised to a constant power. This fixed exponent, referred to as the path-loss exponent, is usually assumed to be between 2 to 4. Due to this relationship, it is beneficial, in terms of energy saving, to relay the information through a multihop route. Multihop routing extends the coverage by allowing a node to communicate with nodes that would have otherwise been outside of its transmission range. The problem of finding a minimum energy route becomes more interesting

Paper approved by T. T. Lee, the Editor for Switching Architecture Performance of the IEEE Communications Society. Manuscript received February 1, 2006; revised February 16, 2007. This work was supported in part by DARPA/AFOSR through the University of Illinois Grant F49620-02-1-0325, in part by the NSF ITR Grant CCR-0325401, and in part by a grant from Draper Laboratory.

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Digital Object Identifier 10.1109/TCOMM.2007.908538

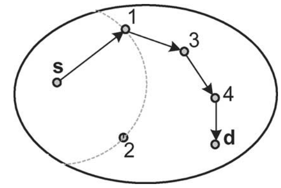

Fig. 1. Multihop routing and WBA.

once some special properties of the wireless medium are taken into account. In particular, in this work, we exploit the wireless broadcast property and the benefit of transmission side diversity to achieve energy savings.

When omnidirectional antennas are used for communication, the signal transmitted by a node is received by all nodes within a certain radius. For example, in Fig. 1, the signal transmitted by s is received by both nodes 1 and 2. This property, usually referred to as the *wireless broadcast advantage* (WBA), was first studied in a network context in [2]. Clearly, this property of the wireless physical medium significantly changes many network layer route selection algorithms. The problem of finding the minimum energy multicast and broadcast tree in a wireless network is studied in [2] and [3]. This problem is shown to be NP-Complete in [4] and [5]. Maric and Yates [6] look at the problem of efficient broadcasting when signal energy accumulation over multiple transmissions is possible. WBA also adds substantial complexity to route selection algorithms even in nonbroadcast scenarios. For example, this model is used in [7] in the context of selecting the minimum energy link and node disjoint paths in a wireless network.

Another interesting property of the wireless medium is the benefit of space diversity at the physical layer. This type of diversity is achieved by employing multiple antennas on the transmitter or the receiver side. It is well known that transmission and receiver space diversity can result in lower error probability or higher transmission capacity [9]–[11]. An overview of different transmission diversity techniques is given in [12]. In our paper, we assume that each node is only equipped with a single antenna. However, we allow for the possibility that several nodes can cooperate with each other in transmitting the information to other nodes, and through this cooperation effectively achieve similar energy savings as a multiple antenna system. An architecture for achieving the required level of coordination among the cooperating nodes is discussed in [8]. We shall refer to the energy savings due to cooperative transmission by several nodes as the *wireless cooperation advantage* (WCA).

Our aim in this paper is to take advantage of the wireless broadcast property and the transmission side diversity created through cooperation to reduce the end-to-end energy consumption in routing the information between two nodes. To make it clear, consider a simple example. For the network shown in

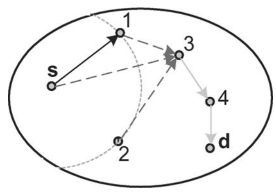

Fig. 2. Cooperative Transmission.

Fig. 2, assume that the minimum energy route from s to d is determined to be as shown using the shaded line. As discussed previously, the information transmitted by node s is received by nodes 1 and 2. After the first transmission, nodes s, 1, and 2 have the information and can cooperate in getting the information to 3, as shown in Fig. 2.

Our goal is to quantify the energy savings that can be achieved through cooperation and to find the optimal cooperative route in order to maximize energy savings. We do not consider other issues such as the level of coordination among the cooperating nodes required, and simply assume that the required coordination can take place by employing appropriate hardware architecture at each node and using a low-bandwidth control channel. We assume that this coordination consumes an amount of energy that is negligible in comparison to the energy required for relaying the actual data. Moreover, we realize that the use of cooperative relaying may require additional transmissions and, hence, be inefficient in terms of bandwidth utilization. However, again, our goal in this paper is merely to quantify the energy savings achievable through cooperation, and hence, we do not concern ourselves with the issues of bandwidth consumption. Potentially, the mechanisms proposed in this paper are applicable to situations where bandwidth is plentiful but energy is scarce (e.g, communications in space). Of course, many of our assumptions are idealized, but allow for analytical tractability of the problem at hand.

We formulate the problem of finding the minimum energy route as two separate minimization problems. First, we look at the problem of optimal transmission of information between two sets of nodes; a set of nodes with the information and a set that must receive the information in a single use of the wireless medium. A separate problem is how to decide which nodes must receive the information in each transmission such that the information is routed to the final destination with minimum *overall* energy. We use dynamic programming (DP) to solve this second minimization problem. We show that the problem of optimal route selection can be mapped to finding the shortest path between two nodes in a graph. We present analytical results on the lower bound of savings in networks with regular line or grid topology. In Section IV, we propose two computationally efficient suboptimal algorithms for finding the optimal path in arbitrary networks and present simulation results for the average energy savings achieved based on these algorithms.

II. Cooperative Transmission

Consider a wireless ad hoc network consisting of an arbitrarily distributed set of nodes where each node has a single omnidirectional antenna. We assume that each node can dynamically adjust its transmitted power to control its transmission radius. It is also assumed that multiple nodes cooperating in sending the information to a single receiver node can precisely time their transmitted signal to achieve perfect phase synchronization at the receiver. Under this setting, the information is routed from the source node to the destination node in a sequence of transmission slots, where each transmission slot corresponds to one use of the wireless channel. In each transmission slot, either a node is selected to broadcast the information to a group of nodes, or a subset of nodes that have already received the information cooperate to transmit that information to another group of nodes. As explained shortly, under our assumptions, it is only reasonable to restrict the size of the receiving set to a single node in the case that multiple nodes are cooperating in the transmission. Thus, each transmission is either a broadcast transmission where a single node is transmitting the information, and the information is received by multiple nodes, or a cooperative transmission where multiple nodes simultaneously send the information to a single receiver. We refer to the first case as the *broadcast mode* and the second case as the *cooperative mode*. The *broadcast mode* takes advantage of the known *WBA*, while the *cooperative mode* utilizes the recently introduced concept of *WCA*.

The routing problem can be viewed as a multistage decision-making problem, where at each stage, the decision is to pick the transmitting and the receiving set of nodes as well as the transmission power levels among all nodes transmitting in that stage. The objective is to send the information to the destination with minimum energy. The set of nodes that have the information at the kth stage is referred to as the kth-stage *reliable set* S_k, and the routing solution may be expressed as a sequence of expanding reliable sets that starts with only the source node and terminates as soon as the reliable set contains the destination node. We denote the transmitting set by S and the receiving set by T. The link cost between S and T, i.e., $\mathrm{LC}(S,T)$, is the minimum total power needed for transmitting from S to T.

In this paper, we make several idealized assumptions about the physical layer model. The wireless channel between any transmitting node labeled s_i and any receiving node labeled t_j is modeled by two parameters: its magnitude attenuation factor α_{ij} and its phase delay θ_{ij}. We assume that the channel phase delay is estimated by the receiver and sent back to the transmitter. This assumption is reasonable for slowly varying channels, where the channel coherence time is much longer than the block transmission time. We also assume a free space propagation model where the power attenuation α_{ij}^2 is proportional to the inverse of the square of the distance between the communicating nodes s_i and t_j. For the receiver model, we assume that the desired minimum transmission rate at the physical layer is fixed and nodes can only decode based on the signal energy collected in a single channel use. We also assume that the received information can be decoded with no errors if the received SNR level is above a minimum threshold $\mathrm{SNR}_{\min}$ and that no information is received otherwise. Note that with this assumption, we do not consider soft information or signal energy accumulation over multiple transmissions. This problem is discussed extensively in the literature [13]–[16]. Extending our route selection approach to include the possibility of soft-information accumulation is an interesting problem, which we hope to study in the future.

Without loss of generality, we assume that the information is encoded in a signal $\phi(t)$ that has unit power $P_\phi = 1$ and that we are able to control the phase and magnitude of the signal arbitrarily by multiplying it by a complex scaling factor w_i before transmission. The transmitted power by node i is $|w_i|^2$. The noise at the receiver is assumed to be additive, and the noise signal and power are denoted by $\eta(t)$ and P_η, respectively. This simple model allows us to develop a clean formulation for the optimal route selection and find analytical results for achievable energy savings in some simple network topologies.

A. Link Cost Formulation

In this section, our objective is to understand the basic problem of optimal power allocation required for successful transmission of information from a set of source nodes $S = \{s_1, s_2, \ldots, s_n\}$ to a set of target nodes $T = \{t_1, t_2, \ldots, t_m\}$. In order to derive expressions for the link costs, we consider three distinct cases described as follows.

1) Point-to-Point Link: The simplest case is the case where only one node is transmitting within a time slot to a single target node. In this case, $n = 1$ and $m = 1, S = \{s_1\}$, and $T = \{t_1\}$. The channel parameters may be simply denoted by α and θ, and the transmitted signal is controlled through the scaling factor w. Although, in general, the scaling factor is a complex value that absorbs both power and phase adjustment by the transmitter, in this case we can ignore the phase as there is only a single receiver. The model assumptions made in Section II imply that the received signal is simply

$$r(t) = \alpha e^{j\theta} w\phi(t) + \eta(t)\cdot$$

where $\phi(t)$ is the unit-power transmitted signal, and $\eta(t)$ is the receiver noise with power P_η. The total transmitted power is $P_T = |w|^2$, and the SNR ratio at the receiver is $\frac{\alpha^2|w|^2}{P_\eta}$. For complete decoding at the receiver, the SNR must be above the threshold value $\mathrm{SNR}_{\min}$. Therefore, the minimum power required is $\widehat{P_T}$, and hence, the point-to-point link cost $\mathrm{LC}(s_1, t_1)$ is given by

$$\mathrm{LC}(s_1, t_1) \equiv \widehat{P_T} = \frac{\mathrm{SNR}_{\min} P_\eta}{\alpha^2}. \tag{1}$$

In (1), the point-to-point link cost is proportional to $1/\alpha^2$, which is the power attenuation in the wireless channel between s_1 and t_1 and is, therefore, proportional to the square of the distance between s_1 and t_1 under our propagation model.

2) Point-to-Multipoint Broadcast Link: In this case, $n = 1, m > 1, S = \{s_1\}$, and $T = \{t_1, t_2, \ldots, t_m\}$; hence, m simultaneous SNR constraints must be satisfied at the receivers. Assuming that the omnidirectional antennas are being used, the signal transmitted by the node s_1 is received by all nodes within a transmission radius proportional to the transmission power. Hence, a broadcast link can be treated as a set of point-to-point links, and the cost of reaching a set of nodes is the maximum over the costs for reaching each of the nodes in the target set. Thus, the minimum power required for the broadcast transmission, denoted by $\mathrm{LC}(s_1, T)$, is given by

$$\mathrm{LC}(s, T) = \max\{\mathrm{LC}(s_1, t_1), \mathrm{LC}(s_1, t_2), \ldots, \mathrm{LC}(s_1, t_m)\} \tag{2}$$

where $\mathrm{LC}(s_i, t_j)$ is given by (1).

3) Multipoint-to-Point Cooperative Link: In this case, $n > 1, m = 1, S = \{s_1, s_2, \ldots, s_n\}$, and $T = \{t_1\}$. This case corresponds to the setting in which multiple nodes cooperate to transmit the same information to a single receiver node. We will assume that coherent reception is possible, i.e., the transmitters are able to precisely time their transmitted signals such that all the signals arrive in phase at the receiver. In this case, the signals simply add up at the receiver, and complete decoding is possible as long as the received SNR is above the minimum threshold $\mathrm{SNR}_{\min}$. Here, we do not address the feasibility of precise phase synchronization. This is an idealization made for the simplicity and theoretical tractability provided by it. A discussion of mechanisms for achieving this level of synchronization is provided in [8].

We assume that the n transmitters are able to adjust their phases in such a way that the signal at the receiver is given by

$$r(t) = \sum_{i}^{n} \alpha_{i1} |w_i| \phi(t) + \eta(t).$$

The total transmitted power is $\sum_{i=1}^{n} |w_i|^2$ and the received signal power is $|\sum_{i=1}^{n} w_i \alpha_{i1}|^2$. The power allocation problem for this case is simply

$$\min \sum_{i=1}^{n} |w_i|^2$$
$$\text{s.t.} \frac{|\sum_{i=1}^{n} w_i \alpha_{i1}|^2}{P_\eta} \geq \mathrm{SNR}_{\min}. \tag{3}$$

The Lagrangian multiplier techniques may be used to solve the given constrained optimization problem. The resulting optimal allocation for each node i is given by

$$|\widehat{w_i}| = \frac{\alpha_{i1}}{\sum_i^n \alpha_{i1}^2} \sqrt{\mathrm{SNR}_{\min} P_\eta}. \tag{4}$$

The resulting cooperative link cost $\mathrm{LC}(S, t_1)$, defined as the optimal total power, is, therefore, given by

$$\mathrm{LC}(S, t_1) = \widehat{P}_T = \sum_{i=1}^{n} |\widehat{w_i}|^2 = \frac{1}{\sum_{i=1}^{n} \frac{\alpha_{i1}^2}{\mathrm{SNR}_{\min} P_\eta}}. \tag{5}$$

It is easy to see that (5) can be written in terms of the point-to-point link costs between all the source nodes and the target nodes, given by (1), as

$$\mathrm{LC}(S, t_1) = \frac{1}{\frac{1}{\mathrm{LC}(s_1, t_1)} + \frac{1}{\mathrm{LC}(s_2, t_2)} + \cdots + \frac{1}{\mathrm{LC}(s_n, t_1)}}. \tag{6}$$

A few observations are worth mentioning here. First, from (4), we see that the transmitted signal level is proportional to the channel attenuation. Therefore, in the cooperative mode, *all* nodes in the reliable set cooperate to send the information to a

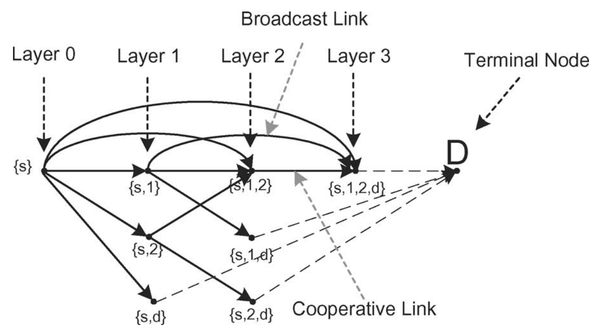

Fig. 3. Cooperation graph for a four-node network.

single receiver. In addition, from (6), we see that the cooperative cost is smaller than each point-to-point cost. This conclusion proves that cooperative transmission always results in energy savings and provides the basic intuition behind the WCA.

Finally, we note that the case of a multipoint-to-multipoint link (i.e., $n > 1$ and $m > 1$) is not a valid option under our assumptions, as synchronizing the transmissions for coherent reception at multiple receivers is not feasible. Therefore, we will not be considering this case.

B. Optimal Cooperative Route Selection

The problem of finding the optimal cooperative route from the source node s to the destination node d, formulated in Section II, can be mapped to a DP problem. The state of the system at stage k is the reliable set S_k, i.e., the set of nodes that have completely received the information by the kth transmission slot. The initial state S_0 is simply $\{s\}$, and the termination states are all sets of nodes that contain d. The decision variable at the kth stage, denoted by U_k, determines the set of nodes that will be added to the reliable set in the next transmission slot. The dynamical system evolves as

$$S_{k+1} = S_k \cup U_k, \qquad k = 1, 2, \ldots. \tag{7}$$

The objective is to find a sequence $\{U_k\}$ or, alternatively, $\{S_k\}$ that minimizes the total transmitted power P_T given by

$$P_T = \sum_k \mathrm{LC}(S_k, U_k) = \sum_k \mathrm{LC}(S_k, S_{k+1} - S_k). \tag{8}$$

We will refer to the solution to this problem as the optimal transmission policy. The optimal transmission policy can be mapped to finding the shortest path in the state space of this dynamical system. The state space can be represented by a graph with all possible states, i.e., all possible subsets of nodes in the network, as its nodes. We refer to this graph as the *cooperation graph*. The nodes in the cooperation graph are connected with arcs representing the possible transitions between the states. As the network nodes are allowed only to either fully cooperate or broadcast, the graph has a special layered structure as illustrated by Fig. 3. Notice that the nodes in the kth layer are all the reliable sets of size $k+1$. Hence, in a network with $n+1$ nodes, the cooperation graph has n layers, and the kth layer has $\binom{n}{k}$ nodes. The arcs between the nodes in adjacent layers correspond to cooperative links, whereas broadcast links are shown by cross-layer arcs. Fig. 3 shows the cooperation graph corresponding to a four-node network. The nodes are marked as s and d for source and destination nodes, respectively, and 1 and 2 for the two relay nodes. An example of a cooperative link and a broadcast link are marked in Fig. 3. Some of the links are omitted for clarity.

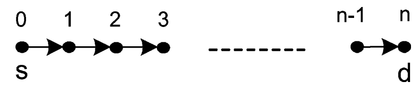

Fig. 4. Regular line topology.

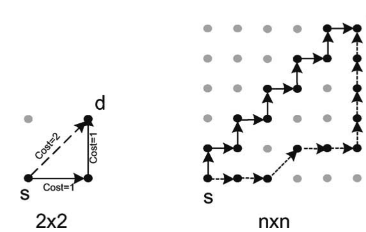

Fig. 5. Regular grid topology.

The costs on the arcs are the link costs defined in Section II-A. All terminal states are connected to a single artificial terminal state, denoted by D, by a zero-cost arc. The optimal transmission policy is simply the shortest path between nodes s and D. There are 2^n nodes in the cooperation graph for a network with $n+1$ nodes. Therefore, standard shortest path algorithms have a complexity of $O(2^{2n})$. However, the special layered structure of the cooperation graph can be utilized to find the optimal route in $O(n2^n)$ using the *sequential scanning algorithm*. This algorithm is based on scanning the cooperation graph from left to right and constructing the shortest path to each node at the kth layer based on the shortest path to the nodes in the previous layers. This is the algorithm for finding the optimal cooperative route in an arbitrary network graph data structure. Initialize all nodes at kth when $k = n+1$. For the kth layer, the shortest path to $\binom{n}{k}$ nodes must be calculated. This operation has a complexity of the order $O(2^n)$. Furthermore, there are n layers in the cooperation graph. Therefore, finding the optimal route is of complexity $O(n2^n)$. Although the *sequential scanning algorithm* substantially reduces the complexity for finding the optimal cooperative route in an arbitrary network, its complexity is still exponential in the number of nodes. Hence, finding the optimal cooperative route in an arbitrary network becomes computationally intractable for larger networks. For this reason, we will focus on developing computationally simpler and relatively efficient suboptimal algorithms and assessing their performance through simulation in Section IV of this paper.

III. Analytical Results For Line and Grid Topologies

In this section, we develop the analytical results for achievable energy savings in line and grid networks. In particular, we consider a *regular line* topology (see Fig. 4) and a *regular grid* topology (see Fig. 5) where the nodes are located at equal distance from each other. Before proceeding further, let us define precisely what we mean by energy savings for a cooperative routing strategy relative to the optimal noncooperative strategy

$$\text{Savings} = \frac{P_T(\text{Noncooperative}) - P_T(\text{Cooperative})}{P_T(\text{Noncooperative})}. \quad (9)$$

where P_T strategy denotes the total transmission power for the strategy. We will make use of the following lemma regarding the existence of the average of terms for a decreasing sequence. The proof is rather simple and omitted for brevity.

Lemma 1: Let a_n be a decreasing sequence with a finite limit c, then $\lim_{m\to\infty} \frac{\sum_{n=1}^{m} a_n}{m} = c$.

A. Line Network-Analysis

Fig. 4 shows a regular line where nodes are located at unit distance from each other on a straight line. The optimal noncooperative routing strategy in this network is to always send the information to the next nearest node in the direction of the destination until the destination node is reached. From (1), the link cost for every stage is $\text{SNR}_{\min}P_\eta/\alpha^2$, where α is the magnitude attenuation between two adjacent nodes located 1-distance unit apart from each other. Under our assumptions, α^2 is proportional to the inverse of the distance squared. Therefore

$$P_T(\text{Noncooperative}) = n\frac{\text{SNR}_{\min}P_\eta}{\alpha^2}. \quad (10)$$

In our proposed scheme, we restrict the cooperation to nodes along the optimal noncooperative route. That is, at each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the minimum energy noncooperative route. This cooperation strategy is referred to as the *cooperation along the minimum energy noncooperative path* (CAN) strategy.[1] With the CAN strategy, after the mth transmission slot, the reliable set is $S_m = \{s, 1, \ldots, m\}$, and the link cost associated with the nodes in S_m cooperating to send the information to the next node $(m+1)$ follows from (6), and is given by

$$\text{LC}(S_m, m+1) = \frac{\text{SNR}_{\min}P_\eta}{\sum_{i=1}^{m+1}\frac{\alpha^2}{i^2}}. \quad (11)$$

Therefore, the total transmission power for the CAN strategy is

$$P_T(\text{CAN}) = \sum_{m=0}^{n-1}\text{LC}(S_m, m+1) = \frac{\text{SNR}_{\min}P_\eta}{\alpha^2}\sum_{m=0}^{n-1}\frac{1}{C(m+1)} \quad (12)$$

where

$$C(m) = \sum_{i=1}^{m}\frac{1}{i^2}. \quad (13)$$

Theorem 1: For a regular line network as shown in Fig. 4, the CAN strategy results in the energy savings of $(1 - \frac{1}{n}\sum_{m=1}^{n}\frac{1}{C(m)})$. As the number of nodes in the network grows, the energy saving approaches $(1 - \frac{6}{\pi^2}) \approx 39\%$.

[1] It can be shown that this strategy is also the optimal cooperative strategy in a line network.

Proof: Without lose of generality, we assume that $\frac{\text{SNR}_{\min}P_\eta}{\alpha^2} = 1$. Hence, the cost for the minimum energy noncooperative route, as given by (10), simplifies to n. The cost of the CAN cooperation scheme calculated in (12) simplifies to

$$P_T(\text{Cooperative}) = \sum_{m=1}^{n}\frac{1}{C(m)} \quad (14)$$

where $C(m)$ is defined by (13). The energy savings achieved, as defined by (12), is

$$\text{Savings}(n) = \frac{P_T(\text{Noncooperative}) - P_T(\text{Cooperative})}{P_T(\text{Noncooperative})} \quad (15)$$

$$= \frac{n - \sum_{m=1}^{n}\frac{1}{C(m)}}{n} \quad (16)$$

$$= 1 - \frac{1}{n}\sum_{m=1}^{n}\frac{1}{C(m)} \quad (17)$$

where $\frac{1}{C(m)}$ is a decreasing sequence with limit of $\frac{6}{\pi^2}$. So, based on lemma 1, we have

$$\lim_{n\to\infty}\text{Savings}(n) = 1 - \lim_{n\to\infty}\frac{1}{n}\sum_{m=1}^{n}\frac{1}{C(m)} = 1 - \frac{6}{\pi^2}. \quad (18)$$

This establishes the claim and completes the proof.

B. Grid Network

Fig. 5 shows a regular $n \times n$ grid topology with s and d located at the opposite corners. An $n \times n$ grid can be decomposed into many 2×2 grid. Without lose of generality, we assume that a transmission to a neighbor in vertical or horizontal direction has a cost of 1 unit. Under this assumption, in a 2×2 grid, a diagonal transmission has a cost of 2 units, equal to the cost of one horizontal and one vertical transmission. In an $n \times n$ grid, there are many noncooperative routes with an equal cost of $2n$ units. Fig. 5 shows two such routes for an $n \times n$ grid. We will base our analysis for deriving a lower bound for the savings based on the stair-like noncooperative path shown in Fig. 5. The following theorem states that the energy savings achieved by the CAN strategy.

Theorem 2: For a regular grid network as shown in Fig. 5, the energy savings achieved by using the CAN strategy approaches 56% in a large network.

Proof: In general, the cooperative cost of the mth stage of the proposed strategy is

$$C_{\text{grid}}(m) = \text{LC}(\{1, \ldots, m\}, m+1) = \frac{1}{\sum_{i=1}^{m}\frac{1}{\text{LC}(i, m+1)}}. \quad (19)$$

The point-to-point costs can be written as

$$\text{LC}(i, m) = \left(\left\lceil\frac{m-i}{2}\right\rceil\right)^2 + \left(\left\lfloor\frac{m-i}{2}\right\rfloor\right)^2 \quad (20)$$

which gives

$$C_{\text{grid}}(m) = \frac{1}{\sum_{i=1}^{m} \frac{1}{\text{LC}(i,m+1)}} = \frac{1}{\sum_{i=1}^{m} \frac{1}{\left(\left\lceil \frac{m+1-i}{2} \right\rceil\right)^2 + \left(\left\lfloor \frac{m+1-i}{2} \right\rfloor\right)^2}}. \tag{21}$$

The final expression can be manipulated and written as

$$C_{\text{grid}}(m) = \frac{1}{\sum_{k=1}^{\lceil \frac{m}{2} \rceil} \frac{1}{2k^2-2k+1} + \sum_{k=1}^{\lfloor \frac{m}{2} \rfloor} \frac{1}{2k^2}} \tag{22}$$

where $C_{\text{grid}}(m)$ is a decreasing sequence of numbers, and its limit can be analytically calculated to be equal to 0.44.

The total cost for the cooperative route in an $n \times n$ grid is

$$P_T(\text{Cooperative}) = \sum_{m=1}^{2n} C_{\text{grid}}(m) \tag{23}$$

The energy saving, as defined by (9), is

$$\begin{aligned}\text{Savings}(n) &= \frac{P_T(\text{Noncooperative}) - P_T(\text{Cooperative})}{P_T(\text{Noncooperative})} \\ &= \frac{2n - \sum_{m=1}^{2n} C_{\text{grid}}(m)}{2n} \\ &= 1 - \frac{1}{2n}\sum_{m=1}^{2n} C_{\text{grid}}(m).\end{aligned} \tag{24}$$

Since $C_{\text{grid}}(m)$ is a decreasing sequence and $\lim_{m\to\infty} C_{\text{grid}}(m) = 0.44$, by lemma 1, the savings in the case of a regular grid, as calculated in (24), approaches $1 - 0.44 = 56\%$. This establishes the claim and completes the proof for the lower bound of achievable savings in a regular grid.

IV. Suboptimal Algorithms

In this section, we present two possible general suboptimal algorithms and related simulation results. The simulations are over a network generated by randomly placing nodes on a 100×100 grid and randomly choosing a source–destination pair of nodes. For each realization, the minimum energy noncooperative path was found. The proposed suboptimal algorithms were used to find the cooperative paths. The performance results reported are the energy savings of the resulting cooperative strategy with respect to the optimal noncooperative path averaged over 100 000 simulation runs.

The two suboptimal algorithms analyzed are outlined as follows.

CAN-l: In this suboptimal approach, the optimal noncooperative route is first selected. In each step of the cooperative routing, the last *l* nodes along the optimal noncooperative route cooperatively send the information to the next node along the optimal noncooperative route. The only processing needed in this class of algorithm is to find the optimal noncooperative route. For this reason, the complexity of this class of algorithms is the same as finding the optimal noncooperative path in a network or $O(n^2)$, where n is the total number of nodes in the network.

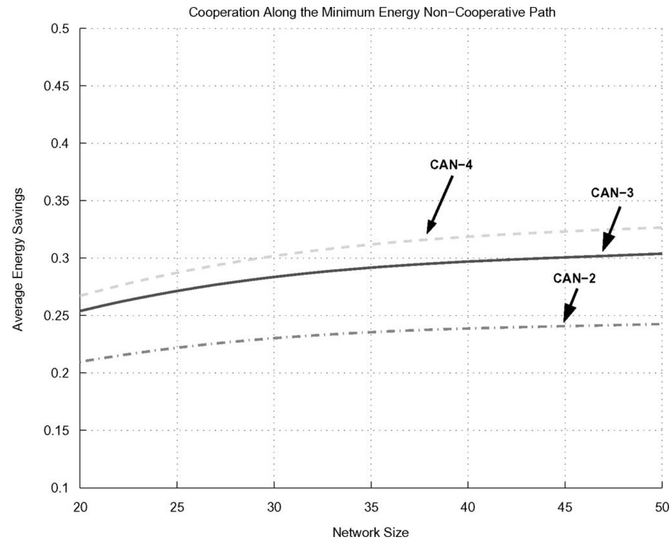

Fig. 6. Performance of CAN.

Progressive cooperation PC-l: In this algorithm, the optimal noncooperative route, denoted by *best path*, is found after each transmission by combining all or a subset of nodes in the reliable set into a single node, denoted by *super node*, and finding the shortest noncooperative route between the *super node* and the destination. A formal description of this algorithm is given as follows.

Initialize best path to the optimal noncooperative route. Initialize the *super node* to contain only the source node.
Repeat: Send the information to the first node along the current *best path*. Update the *super node* to include all past *l* nodes along the current *best path*. Update the link costs accordingly, i.e., by considering the *super node* as a single node and by using (6). Compute the optimal noncooperative route for the new network/graph and update the *best path* accordingly.
Stop: Stop as soon as the destination node receives the information.

For example, algorithm *PC-3* always combines the last three nodes along the current *best route* into a single node, finds the shortest path from that combined node to the destination, and sends the information to the next node along that route. This algorithm turns out to have a complexity of $O(n^3)$ since the main loop is repeated $O(n)$ times and each repetition has a complexity of $O(n^2)$.

A variant of this algorithm keeps a window of size *w* of the most recent nodes, and in each step, all subsets of size *l* among the last *w* nodes are examined and the path with the least cost is chosen. We will index this algorithm as *PC-l-w*. For example, in *PC-3-4*, all subsets of size *3* among the last *four* nodes are examined at each stage, and the path with the lowest cost is chosen. This variant has a complexity of $O\left(\binom{w}{l} \times n^3\right)$, where w is the window size, *l* is the cooperation limit, and n is the number of nodes in the network. We refer to this variant as *progressive cooperation with window*.

Figs. 6 and 7 show average energy savings ranging from 20% to 50% for CAN and PC algorithms. It can be seen that PC-2

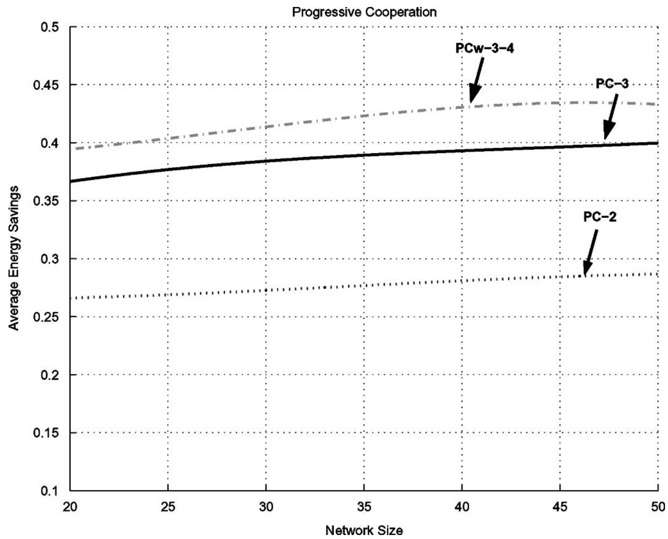

Fig. 7. Performance of PC.

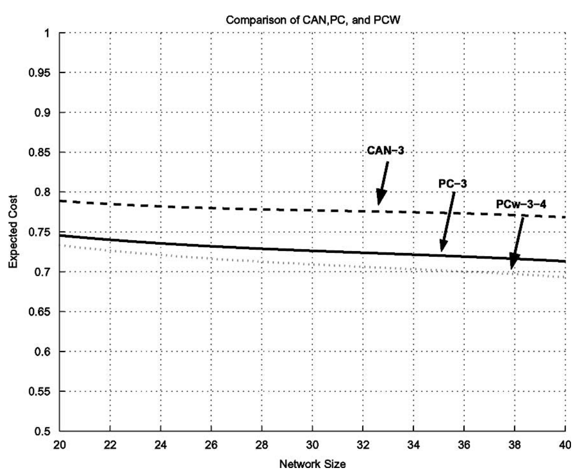

Fig. 8. Comparison of CAN, PC, and PC-w.

performs almost as well as CAN-3, and PC-3 performs much better than CAN-4. This shows that the method for approximating the optimal route is a very important factor in increasing the savings. Fig. 8 compares CAN, PC, and PC-w on the same chart. It is seen that PC-3-4 performs better than PC-3, which performs substantially better than CAN-4. In general, it can be seen that the energy savings increase with l, and that improvements in savings are smaller for larger values of l. As there is a tradeoff between the algorithm complexity and the algorithm performance, these simulation results indicate that it would be reasonable to choose l to be around 3 or 4 for both the CAN and PC algorithms.

V. Conclusion

In this paper, we formulated the problem of finding the minimum energy cooperative route for a wireless network under idealized channel and receiver models. Our main assumption were that the channel state is known at the transmitter and precise power and phase control, to achieve coherent reception, is possible. We focused on the optimal transmission of a single message from a source node to the destination node through sets of nodes that may act as cooperating relays. Fundamental to the understanding of the routing problem was the understanding of the optimal power allocation for a single message transmission between two sets of nodes in a single use of the wireless medium. We presented the solution to this problem and used that as the basis for solving the minimum energy cooperative routing problem. For a regular line and a regular grid topology, we analytically obtained the energy savings due to cooperative transmission, demonstrating the benefits of the proposed cooperative routing scheme. We used DP to formulate the optimal cooperative routing problem as a multistage decision problem. The problem of finding the optimal route was shown to be equivalent to finding the shortest path in the corresponding cooperation graph. The optimal algorithm turns out to be computationally intractable in large networks. For this reason, we proposed two suboptimal algorithms for general networks and confirmed through simulations that even these suboptimal algorithms can achieve energy savings of close to 50% in a general network.

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