Blocking Probability and Channel Assignment in Wireless Networks

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***Abstract*— We consider a multi-hop wireless network with a connection-oriented traffic model and multiple transmission channels that can be spatially re-used. In such a network the blocking probability of a call that makes a channel request depends on (a) the channel assignment scheme and (b) the transmission radius of the nodes which affects the network link structure. In this work, we study these two aspects for simple wireless networks. Specifically, we develop blocking probability analysis for a wireless line and grid network and explore the tradeoff between transmission radius and blocking probability for multi-hop calls. We show that for a line network a larger transmission radius can substantially reduce the blocking probability of calls, while for a grid network with a more dense node topology using a smaller transmission radius is better. We, then, investigate various channel assignment schemes and present a novel non-rearranging channel assignment algorithm for multi-hop calls in a general network. Our algorithm efficiently incorporates spatial channel re-use and significantly reduces call blocking probability when compared to other algorithms.**

***Index Terms*— Blocking probability, dynamic channel assignment, transmission radius, wireless networks, connection-oriented traffic, multi-hop calls, wireless interference, quality of service.**

I. Introduction

A MULTI-HOP wireless network is a cooperative network where data streams may be transmitted over multiple wireless hops to reach the destination. The network link structure depends on the transmission radius of the nodes and can be adjusted by varying the transmission power. In this work, we consider such a network without node mobility and with a connection-oriented traffic model. We consider multiple channels that can be spatially re-used and each new call is assigned a channel, if available, for the entire duration the call is active[1]. The wireless interference and traffic models are explained in detail in the next section. Finally for the above network, our goal is to investigate the effect of transmission radius of the nodes and the channel assignment scheme on steady state call blocking probability. Some of the recent work on other QoS issues such as routing in multi-hop networks includes [1], [2], [3], [5].

The effect of transmission radius can be understood as follows. A smaller transmission radius of the nodes causes less interference on each hop but the calls have to hop through many nodes to reach the destination. As the same call is served by many nodes along the route, multi-hopping increases the internal load in the network. In contrast, a larger transmission radius reduces the number of hops of a call but increases the interference constraints at each hop. One of the contributions of this paper is to examine this tradeoff in relation to its effect on blocking probability. For analytical simplicity we focus on two topologies: the line and the grid network. First, we present an exact blocking probability analysis for a single channel wireless line network. We, then, construct a model to compute the blocking probability in the multiple channel case for the random channel assignment policy. Using the formulas obtained, we show that in a line network a larger transmission radius reduces the blocking probability of calls; whereas, for a grid network with an underlying more dense node topology we show that it is more desirable to use a smaller transmission radius. This suggests that for sparse networks the increase in the internal load due to multi-hopping contributes significantly to call blocking whereas for denser networks the increase in the interfering neighboring nodes due to a larger radius is a significant limiting factor. Recently there has been work on throughput analysis addressing the issues of interference, multi-hop transmissions and node mobility [14], [16]. The work in [14], [16] allows queueing of data and analyzes the throughput of the network under different interference and mobility models. However, these objectives and the traffic characteristics are very different from what we consider in this paper.

Finally we also address the issue of dynamic channel assignment to the incoming calls in a general network for a fixed network link structure (i.e. given the transmission radii of the nodes). For multi-hop calls a channel must be allocated on each hop such that the wireless constraints are satisfied. Here, we develop a novel non-rearranging algorithm that spatially re-uses the channels in an efficient manner. Using simulations we compare its performance with other algorithms.

The analysis of blocking probability and dynamic channel assignment has been extensively considered in the context of cellular networks [11], [12], [13]. However there are significant differences between a multi-hop wireless network, the focus of our work, and a cellular network. For example, in a cellular network the communication is with the nearest base-station over a single wireless link; whereas in a multi-hop wireless network, calls hop through various links to reach the destination. This imposes additional complexity as

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[1] A channel, here, refers to a time slot for a TDMA system and a distinct frequency band for a FDMA system.

non-conflicting channels must be allocated on the wireless links along the source-destination path. Another difference between the two networks is that a cellular network has a regular structure which makes the set of interfering cells fixed; whereas in a multi-hop wireless network the set of interfering nodes depends on the node topology and their transmission radii.

Steady state blocking probability is an important and widely studied performance metric for communication networks in general. In wireless cellular networks the work includes [10], [13] and focuses on a cellular network architecture. In all-optical networks the work includes [17], [18] where the goal is to study the blocking probability behavior with limited number of available wavelengths. Finally, there has also been extensive work on blocking probability in traditional wire-line networks with limited link capacities, [8], [9].

The rest of the paper is organized as follows. In Section II, we describe the system model. Section III presents blocking probability analysis for a line network. Section IV considers the effect of transmission radius on blocking probability in a line and a grid network. In Section V, we present channel assignment algorithms and simulation results that compare their performance. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

We consider a wireless network whose node topology does not change over time and the nodes in the network transmit with equal power using an omnidirectional antenna.

Interference Model: We assume a disk model of interference. Let the transmission radius of a node, say T, be defined as the radius of a circle centered at T such that, (a) outside this circle there is no interference from the signal transmitted by T and (b) within this circle there is complete interference of the signal transmitted by T with other ongoing signal reception. We also assume that without any interference from other nodes, the signal transmitted by node T can be perfectly received within its transmission radius. A direct *wireless link* (or simply *link*) exists between any two nodes if they lie within each other's transmission radius. We say that node R is a neighbor of node T if R lies within the transmission radius of T. As the nodes have equal transmission radius, T is also a neighbor of R. Let the set of neighbors of T and R be denoted as $\mathcal{N}_T$ and $\mathcal{N}_R$ respectively. Consider the uni-directional data transfer, $T \rightarrow R$, in channel γ. For this call to be successfully serviced the following criteria need to be satisfied.

1) Nodes T and R must not be involved in any other call transmission/reception in channel γ. This criterion ensures that a node cannot simultaneously serve two calls in channel γ.
2) Neighbors of T ($P \in \mathcal{N}_T$, excluding R) must *not receive* from any other node in channel γ. Otherwise the transmission from T will interfere at P.
3) Neighbors of R ($Q \in \mathcal{N}_R$, excluding T) must *not transmit* to any other node in channel γ. Otherwise the transmission from Q will interfere at R.

Next, consider a bi-directional call between nodes T, R; i.e. data transfer in both directions $T \rightarrow R$ and $R \rightarrow T$. In this case, a node can both transmit and receive data in the reserved channel and thus all the three conditions stated above must be satisfied at both T and R. This means that for $T \leftrightarrow R$ to be successful, neighbors of node T and node R must neither transmit nor receive in channel γ. A node is labeled *inactive* in channel γ if it is not involved in transmission/reception in that channel and *active* otherwise. With this notation, we get the following spatial channel re-use constraint. *For a bi-directional call $T \leftrightarrow R$ to be successful in γ, neighbors of node T excluding R and neighbors of node R excluding T must be inactive*. This "idealized" model approximates realistic interference assumptions and is commonly used in the study of wireless networks [1], [14], [15]. Figure 1 illustrates a single hop bi-directional data transfer between nodes T and R in channel γ. Nodes T and R cannot service any other call in channel γ. Neighbors of node T (T_1, T_2) and neighbors of R (R_1, R_2) must be inactive while call $T \leftrightarrow R$ is active. In the figure, all data transfers marked '$\times$' must not take place for call $T \leftrightarrow R$ to be successful.

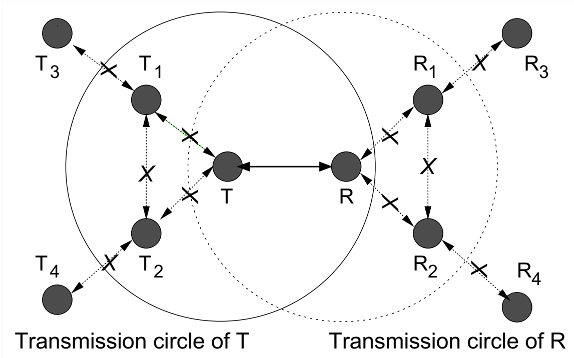

Fig. 1. Interference model for a bi-directional transmission ($T \leftrightarrow R$).

Traffic Model: We consider a connection-oriented model wherein the arriving calls require a dedicated channel on each hop along the path. These channels are held up while the call is in progress and released at the end of the call. The main purpose of such a model is to study systems in which allocated channels are not re-assigned very often (e.g. for streaming/voice traffic that may require dedicated channels). Such streaming applications are of increased interest for both military and commercial applications. One can also view a call as an aggregation of various packet data flows which are served on a packet basis but over a single channel. The assumption of connection-oriented traffic simply translates into the fact that a channel once allocated is held for some duration, these channel requests are stochastic with some average rate and there is no queueing of the requests. As such there are no restrictions on these average values other than finiteness. We also assume that all calls require a *single* channel for service on each hop. For length considerations we consider only bi-directional calls in this paper. The reader is referred to [4] for an analysis of uni-directional calls.

III. BLOCKING PROBABILITY ANALYSIS IN A WIRELESS LINE NETWORK

We, now, develop an analysis for the blocking probability of single hop bi-directional calls in a line network. We first analyze a single channel network for which an exact solution

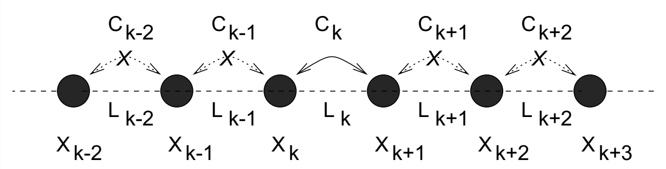

Fig. 2. Constraints representing the simultaneous service of calls in a WLN-1 network.

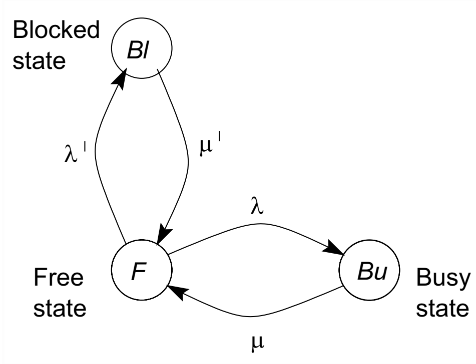

Fig. 3. Three state Markov process model of the channel on a link.

is obtained and then extend it to multiple channels. The expressions derived here form the basis for the study in Section IV. In addition to facilitating elegant solutions, a line network is an important network in practice and serves as a good starting point in understanding network tradeoffs.

A. Single Channel

Consider a wireless line network with nodes located unit distance apart at positions $x = -m, -m+1, ..., m$. We label these nodes as $X_{-m}, X_{-m+1}, ..., X_m$. Let there be a single channel that can be spatially re-used subject to interference constraints. Let each node have a transmission radius of r, where $r \geq 1$ and $r \in Z^+$, a positive integer. Let all calls in the network be bi-directional with the source and destination nodes r units apart, i.e. between nodes X_k and $X_{k+r}, \forall k$. The calls are single hop as each node can communicate directly with a node r units apart. Calls $X_k \leftrightarrow X_{k+r}, \forall k$ arrive according to an independent Poisson process of rate λ. The holding period of each call is independent and identically distributed as an Exponential distribution[2] with mean $1/\mu$. If a call cannot be accepted then it is dropped otherwise it occupies the channel while in progress. We call this network a wireless line network with radius r or WLN-r for short. A WLN-1 network is depicted in Figure 2.

Theorem 1: The blocking probability of a call in an infinite length ($m \to \infty$) WLN-r, $r \in Z^+$, network and $\nu = \lambda/\mu$ $(0 \leq \nu < \infty)$ is,

$$P_B = 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}} \tag{1}$$

where, x is the unique root in (0,1] of $\nu x^{2r+1} + x = 1$.

Proof: See Appendix I. ■

The limiting argument in Theorem 1 helps eliminate edge effects and yields the above simple expression that closely agrees with simulation results for finite length line networks (see [4]).

B. Multiple Channels

In this section, we extend the analysis of WLN-r to the case of multiple channels. We consider the random policy for assigning channels to the incoming calls. In this policy the new call on a link is assigned a channel randomly from among the free channels on that link. Free channels refer to those channels such that the acceptance of a call in these channels does not violate the interference constraints. The random policy is easy to implement practically. However, its exact analysis is complicated by the fact that to make a channel allocation decision we must have knowledge of the channels already occupied by the ongoing calls. This makes the state space for this system very large and an analysis of the steady state probability distribution intractable. Interestingly, since the random policy does not differentiate between the channels an approximate model can be constructed based on an effective load concept. We proceed as follows. From Section III-A, we know the exact value of the blocking probability of a call for the single channel case. Using this result we first construct a single channel tractable markovian model whose parameters are chosen to match the result of (1). This markovian model is then extended with some approximations to incorporate multiple channels. Finally, simulations results are presented that verify that the theoretical values from this model closely agree with the numerical results.

Consider the link L_k $(X_k \leftrightarrow X_{k+r})$ of the line network. For now, assume that there is only a single channel γ in the network and denote its state on link L_k as S_k. We model S_k as a three state process, the free state ($\mathcal{F}$), the busy state ($\mathcal{B}u$) and the blocked state ($\mathcal{B}l$) as shown in Figure 3. The link L_k is said to be in the blocked state if the channel is occupied by a call on an interfering link making it unavailable on link L_k. It is in the busy state if there is a call in progress. Let $Y_{\mathcal{F}\to\mathcal{B}l}$ be the random variable that denotes the transition time from state $\mathcal{F} \to \mathcal{B}l$. The distribution of $Y_{\mathcal{F}\to\mathcal{B}l}$ can be computed through its complicated dependence on the various states of the other links. However, a good approximation is to simply assume it to be exponentially distributed with some rate λ'. The random variable $Y_{\mathcal{B}l\to\mathcal{F}}$ can have a general distribution with mean $1/\mu'$. Figure 3 shows the transition rates of S_k. Using the detailed balance equation (see [6]) of the three state Markov process and letting, $\nu' = \lambda'/\mu'$, $\nu = \lambda/\mu$, we get,

$$\nu' + \nu = \frac{P_B}{1 - P_B} \tag{2}$$

where P_B is known from (1). Thus, the value of ν' that gives the correct P_B value can be obtained from the above equation. Define an *effective load*, $\tilde{\nu} \triangleq \nu' + \nu$, then, we can interpret the load $\tilde{\nu}$ as consisting of two components; the external load ν and the load ν' seen by the link that makes the channel blocked. The effect of interference constraints on blocking probability can, thus, be viewed as an additional load ν'.

[2]The result applies even for general service distributions as the product form solution in the analysis holds here as well.

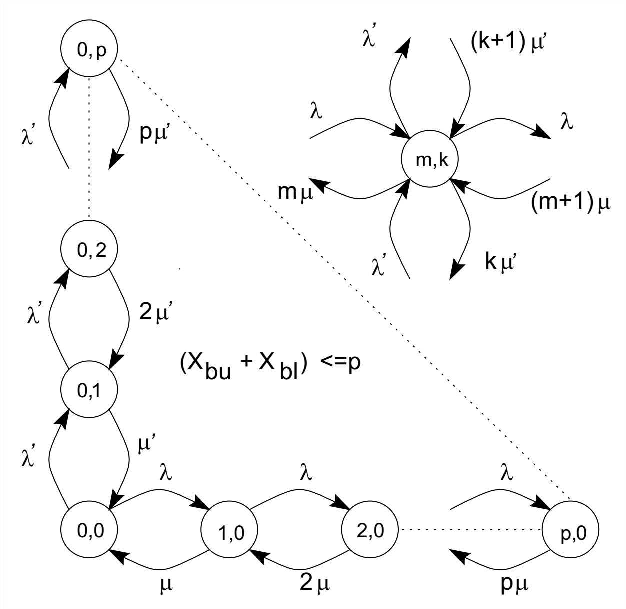

Fig. 4. State transition diagram for the random assignment policy.

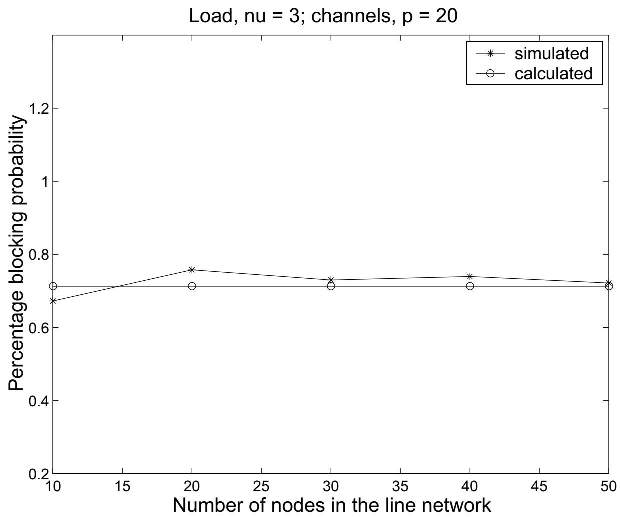

Fig. 5. Comparison of theoretical and simulated values for random policy, $r = 1$, $\nu = 3$ and $p = 20$ channels.

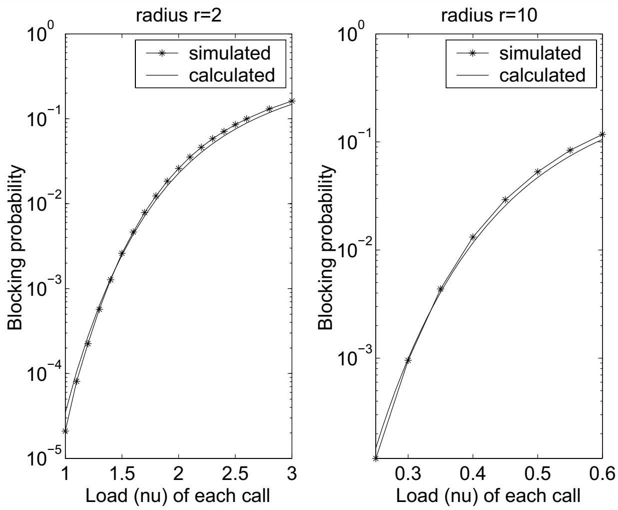

Fig. 6. Comparison of theoretical and simulated values for random policy, $r = 2, 10$ and $p = 20$ channels.

Combining (1) and (2), we get,

$$\tilde{\nu} = \nu' + \nu = \frac{1 + (2r\nu - 1)x^{2r+1}}{x^{2r+1}} \tag{3}$$

The effective load $\tilde{\nu}$ can be understood as follows. If a link of WLN-r is isolated from the network and load $\tilde{\nu}$ applied to it, it would have the same blocking probability as experienced within the line network with symmetrical load ν. An isolated link of a single channel WLN-r is equivalent to a M/M/1/1 system. Thus, the above analogy states that in terms of blocking, a single channel WLN-r network with load ν is identical to a M/M/1/1 system with load $\tilde{\nu}$.

Define the *effective load factor* g as, $g = \tilde{\nu}/\nu$; then g can be expressed as,

$$g = \frac{1 + (2r\nu - 1)x^{2r+1}}{\nu x^{2r+1}} \tag{4}$$

The low load and the high load regimes can be studied by taking the limit $\nu \to 0$ and $\nu \to \infty$ respectively in (4). This yields $\lim_{\nu \to 0} g = 4r + 1$ and $\lim_{\nu \to \infty} g = 2r + 1$. Thus at low loads $\tilde{\nu} \approx (4r + 1)\nu$ and at high loads $\tilde{\nu} \approx (2r + 1)\nu$.

Generalizing to the multiple channel case, define the state of a link as $X(t) \equiv (X_{bu}(t), X_{bl}(t))$ where X_{bu} is the number of busy channels and X_{bl} the number of blocked channels on that link at time t. Let the total number of channels available in the network be p. At any time t, the state $X(t) \equiv (X_{bu}(t), X_{bl}(t))$ must satisfy $X_{bu}(t) + X_{bl}(t) \leq p$. Following the single channel process and the fact that the random policy does not differentiate among the channels we approximate the network as a p server system with rate λ that makes the channels busy and rate λ' that makes the channels blocked. The transition rates among the various states of the process $X(t)$ are shown in Figure 4. Let $\pi(i, j)$ denote the steady state probability that X takes value (i, j). The steady state blocking probability, P_B^{rand}, equals $\sum_{i+j=p} \pi(i, j)$. Solving the detailed balance equations we get,

$$P_B^{rand} = \frac{\frac{\tilde{\nu}^p}{p!}}{1 + \tilde{\nu} + \frac{\tilde{\nu}^2}{2!} + ... + \frac{\tilde{\nu}^p}{p!}} = E(\tilde{\nu}, p) \tag{5}$$

where $E(\nu, p)$ is the Erlang B formula [7] for load ν and p servers. Thus, (5) is same as the blocking probability of an equivalent M/M/p/p system with load $\tilde{\nu}$.

We now present plots that compare the theoretical values obtained from (5) and the simulation results. In all the plots, blocking probability of the center call is computed to minimize edge effects. In Figure 5, we make the comparison by varying the line length. We consider $r = 1$, $p = 20$ channels and $\nu = 3$. The plot shows that for lengths beyond 10 nodes the values closely agree. Thus, (5) derived from the limiting result in (1) holds fairly accurately for finite length line networks. Note that (5) is independent of the line length and hence, in the figure, the curve for calculated values is constant. We next make the comparison by varying r. In Figure 6 we consider

$p = 20$ channels and $r = 2, 10$ for a 50 node line network. The figure shows that even for large r the theoretical values closely agree with simulation results. The above plots are illustrative examples and a similar trend is observed in the simulation results with other network parameters as well.

IV. Effect of Transmission Radius on Blocking Probability

It is clear that if the nodes have a smaller transmission radius then the interference constraints on each hop are fewer but the calls hop through many links to reach the destination. This increases the internal load in the system. In contrast, a larger transmission radius reduces the number of hops of a call but increases the interference constraints at each hop. The effect of this tradeoff on blocking probability is non-trivial and leads to different observations under different node topologies. In this section, we study this tradeoff for two simple node topologies; the line and the grid network.

A. Line Network

We begin by considering the following simple but non-trivial example that lends itself to an exact analysis and also clearly highlights the problem. Consider a line network with *two channels* and with the source-destination nodes of the calls *two* units apart. The arrival process of each call is an independent Poisson process of rate λ and the holding time is i.i.d with mean $1/\mu$. Consider the following two schemes.

Scheme A: The nodes have a transmission radius of unity ($r = 1$) and thus the calls are two hops long. The channels are assigned using the rearrangement channel assignment policy (Section V) as it uses the channel resources optimally.

Proposition 1: The steady state blocking probability for Scheme A in the limit as the length of the line network tends to infinity and $\nu = \lambda/\mu$ $(0 \leq \nu < \infty)$ is,

$$P_B = 1 - \frac{y^3}{1 + 2\nu y^3} \tag{6}$$

where, y is the unique root in $(0, 1]$ of $\nu y^3 + y = 1$.

Proof: See Appendix II. ■

Scheme B: The nodes have a transmission radius of two units ($r = 2$) and hence all calls are single hop. Here, we consider a sub-optimal channel assignment policy that selects a channel randomly from the two channels for each new arriving call. If the channel is free (non-blocked and non-busy) then it is allocated otherwise the incoming call is dropped.

Proposition 2: The steady state blocking probability for Scheme B in the limit as the length of the line network tends to infinity and $\nu = \lambda/\mu$ $(0 \leq \nu < \infty)$ is,

$$P_B = 1 - \frac{x^5}{1 + 2\nu x^5} \tag{7}$$

where, x is the unique root in $(0, 1]$ of $x + \frac{\nu}{2}x^5 = 1$.

Proof: The channel assignment policy in Scheme B performs a simple random splitting of the arrival stream into two independent poisson processes of rate $\lambda/2$ applied to each channel. Thus, the blocking probability of a call is equal to the blocking probability with load $\nu/2$ in a single channel WLN-2 network. Plugging $r = 2$ and $\nu/2$ in (1) we get the desired result. ■

It is clear that the channel assignment policy in Scheme B under-utilizes the channels as it rejects a call if the randomly selected channel is not free without considering the state of the other channel. The following theorem shows that even with this inefficient random policy Scheme B with a larger radius has a lower blocking probability as compared to Scheme A. Thus, for any fixed blocking probability threshold β the supportable load ν is higher for Scheme B than Scheme A. The result, thus, highlights that in networks with low node density a larger transmission radius can lead to better network performance.

Theorem 2: The blocking probability for Scheme B is lower than the blocking probability for Scheme A for all load $\nu = \lambda/\mu$ satisfying $0 < \nu < \infty$.

Proof: See Appendix III. ■

Consider, next, a more general setting of a line network with p channels and calls of length $k > 1$, i.e. between nodes k units apart ($k = 1$ is the trivial case). The traffic model is identical to that considered earlier. We consider the random channel allocation policy and use the expressions derived in Section III-B to compare the following transmission schemes.

Scheme 1: The transmission radius of the nodes is $s(< k)$ and hence each call is $n = \frac{k}{s}$ hops long. To simplify the mathematical exposition, we consider those s for which n is an integer. This simplified system gives an indication of the results that would be observed in the general non-integer case. Further, the assumption can be justified for the following cases; (a) $\frac{k}{s}$ is large and hence rounding to an integer would not affect the results significantly; and (b) k is non-prime so that we can always find s that would make n an integer. We also consider the low blocking probability regime as it is practically significant and helps us make simplifying approximations. In this regime almost all calls get served and the average load on each link ($X_m \leftrightarrow X_{m+s}, \forall m$) is $\approx n\nu$, since there are n calls hopping through a link and $n\nu$ is the sum of the loads of all these calls. Assuming this load to be Poisson, the probability P_L that none of the channels on a link are free can be computed by considering this as an equivalent WLN-s system with p channels, load $n\nu$ on each link and $r = s$. Using (3) and (5) we get,

$$\tilde{\nu_1} = \frac{1 + (2ns\nu - 1)x^{2s+1}}{x^{2s+1}}\ ,\ (n\nu x^{2s+1} + x = 1) \tag{8}$$

$$P_L = E(\tilde{\nu_1}, p) \tag{9}$$

Let P_B^1 denote the blocking probability of a n hop call, then, its clear that P_B^1 is greater than the blocking probability on the first hop. As the latter value equals P_L we get, $P_B^1 > P_L = E(\tilde{\nu_1}, p)$.

Scheme 2 : The transmission radius of the nodes is k and hence each call is single hop. For this system Theorem 1 gives the exact blocking probability in the single channel case. Let P_B^2 denote the blocking probability with multiple channels, then, following the methodology of Section III-B we have,

$$\tilde{\nu_2} = \frac{1 + (2k\nu - 1)x^{2k+1}}{x^{2k+1}}\ ,\ (\nu x^{2k+1} + x = 1) \tag{10}$$

$$P_B^2 = E(\tilde{\nu_2}, p) \tag{11}$$

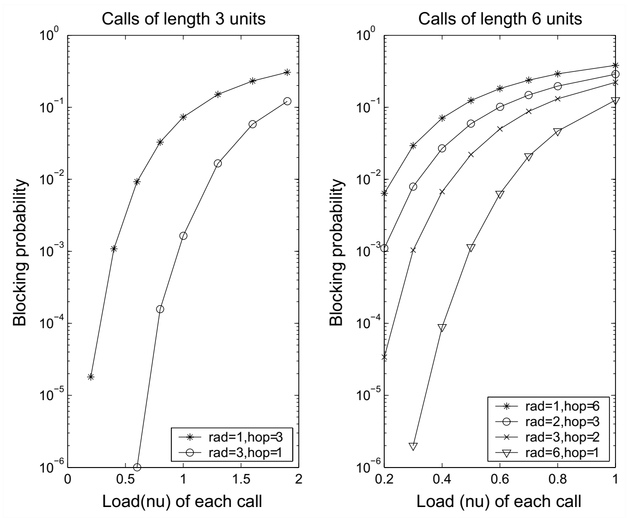

Fig. 7. Line network with calls of length 3 and 6 and 20 channels.

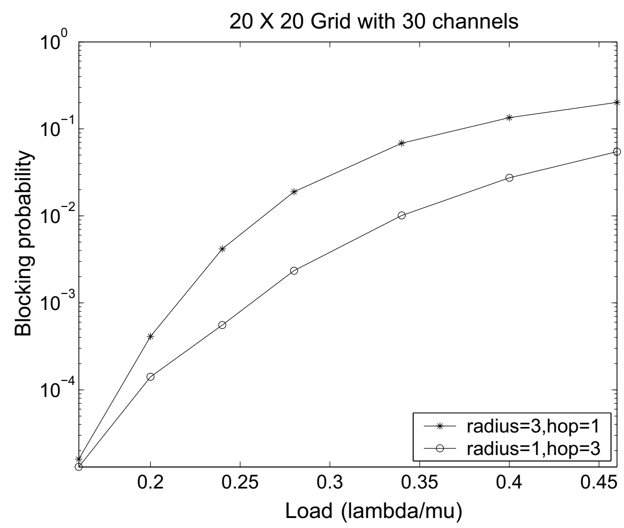

Fig. 8. Grid network with calls of length 3.

It can be shown (see [4]) that $P_B^2 < P_L < P_B^1, \forall \nu > 0, k \in Z^+$. The proof is omitted for brevity but it follows along similar lines as the proof of Theorem 2. Mathematically, the tradeoff in the above two schemes can be understood by examining their polynomial equations given in brackets in (8) and (10). In Scheme 1 we have a higher load $n\nu$ but a smaller exponent $2s+1$ of x due to less interference at each hop; while in Scheme 2 the load is ν but a higher exponent $2k+1$ of x due to more interference.

The intuition behind this result is that for a line network with a sparse node topology the blocking probability increase due to a larger set of interfering nodes (larger radius) is smaller as compared to an increase due to larger effective link load caused by multi-hopping. Figure 7 presents simulation plots verifying this claim. Blocking probability is computed for the center call to minimize edge effects. The first plot has calls of length 3 and two scenarios of radius 1 and 3. The second plot has calls of length 6 with radius 1, 2, 3 and 6. Note that the reduction in blocking probability by using a larger transmission radius is a few orders of magnitude and this difference increases with the length of the calls.

B. Grid Network

We observed in Section III-B that as $\nu \to 0$, $\tilde{\nu} \approx (4r+1)\nu$. A similar observation can be shown to hold for a single-hop single channel general network as well i.e. as $\nu \to 0$, $\tilde{\nu} \approx \alpha\nu$, $\alpha =$ total number of interfering calls + 1 (see [4]). Extending to multiple channels, as in Section III-B, the blocking probability with p channels is $P_B = E(\tilde{\nu}, p)$.

Consider an infinite grid network (to avoid edge effects) with calls of length 3 between nodes $\{x, y\} \to \{x+3, y\}$ and $\{x, y\} \to \{x, y+3\}$ and load ν. The arguments can be easily generalized to longer length calls. In the first scenario, transmission radius of each node is 3 and hence calls are single hop. Here, each link has 134 interfering links all of which carry load ν (see [4]). Thus, $\tilde{\nu} = 134\nu + \nu$ and the blocking probability of a call is $P_B \approx E(135\nu, p)$.

In the second scenario, transmission radius of each node is 1 and hence the calls are three hop in length. In the low blocking regime almost all calls get served and the average load on each link is $\approx 3\nu$. Treating the system as an equivalent network with load 3ν on each link, and with each link having 23 interfering links including itself, the effective load equals $\tilde{\nu} = 23*3\nu$. The probability that no channel is free at a link is $E(69\nu, p)$. Making a further simplification that the links block independently the probability that a 3-hop call is blocked is $\approx 1 - (1 - E(69\nu, p))^3 \approx 3E(69\nu, p)$.

It can be easily checked that for low ν and moderate number of channels we have $E(135\nu, p) > 3E(69\nu, p)$ which suggests that it is preferable to use a smaller transmission radius. The intuitive reason is that a grid network has a denser node topology than a line network. As a result the number of interfering links increase much rapidly with an increase in the transmission radius of the nodes leading to higher blocking probability than using a smaller transmission radius. Figure 8 presents simulation results that justify this conclusion. The plot shows the blocking probability of the center call in a 20$\times$20 grid with 30 channels. All calls are of length 3 and two cases of radius 1 and 3 are considered. As is evident from the plot, blocking probability for radius 1 is lower than that for radius 3.

V. CHANNEL ASSIGNMENT ALGORITHMS

In the earlier sections we studied the effect of transmission radius on call blocking probability. We next address the issue of channel assignment to multi-hop calls. We assume in this section that the transmission radius of the nodes is fixed and study various channel assignment algorithms. We propose a new algorithm called the Local Channel Reuse Algorithm (LCRA) that spatially re-uses the channels in an efficient way thereby reducing the blocking probability of calls as compared to other algorithms such as rearrangement, random and first fit algorithms. Next, we describe these algorithms in detail.

Rearrangement Algorithm: The rearrangement algorithm was first presented in [11] for cellular networks. This policy admits an incoming call even if it requires rearrangement of the allocated channels to the calls in progress. If no such rearrangement is feasible that can accommodate the new incoming call then it is dropped. It is clear that this policy cannot be easily implemented in practice due to the computationally intensive search for feasible assignments. However, as shown in [4] there is a simple characterization of this feasibility for a line network.

Non-rearranging Algorithms: Here we consider algorithms that are not allowed to rearrange the channels allocated to the existing calls. Such algorithms are clearly more practical. The algorithms that we study are the random, first fit and LCRA. These algorithms base their decision on the set of free channels available at a node. Free channels refer to the non-busy and non-blocked channels. For a node N, the set of free channels $\mathcal{F}_N$ contains all those channels in which node N and its neighbors are inactive. The set of free channels for a link $N \leftrightarrow M$ is the set of all those channels that are free at both nodes N and M. We have, $\mathcal{F}_{N\leftrightarrow M} = \mathcal{F}_N \cap \mathcal{F}_M$. The channels are arbitrarily assigned an index number for the implementation of the algorithms.

Single Hop Calls: Consider a single hop call between nodes S and D. Let $\tilde{g}()$ be the decision function that selects a channel from the set $\mathcal{F}_{S\leftrightarrow D}$ then the chosen channel $\gamma_c = \tilde{g}(\mathcal{F}_S \cap \mathcal{F}_D)$. The decision function $\tilde{g}()$ for the various algorithms is as follows.

Random Algorithm: $\tilde{g}()$ chooses a channel randomly from the set of free channels.

First Fit Algorithm: $\tilde{g}()$ chooses a channel that has the lowest index among the set of free channels. This algorithm has been studied earlier in WDM optical networks [19].

Local Channel Re-use Algorithm (LCRA): Consider a link $S \leftrightarrow D$ on which the channel needs to be allocated. Let $\mathcal{N}_S$ and $\mathcal{N}_D$ be the neighbors of nodes S and D respectively. Let the nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ be denoted as $N_1, N_2, .., N_{|\mathcal{N}_S \cup \mathcal{N}_D|}$ and the set of free channels be $\Gamma = \mathcal{F}_S \cap \mathcal{F}_D$. LCRA chooses a channel $\gamma_c \in \Gamma$ such that γ_c minimizes the number of nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ that have γ_c as a free channel in the present state. This leads to blocking of that channel for the least number of neighboring nodes. Mathematically, let $I_{N_i}(\gamma_k) = 1$ if γ_k is free at node N_i and $I_{N_i}(\gamma_k) = 0$ otherwise. Let $\Omega(\gamma_k)$ be the number of nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ with γ_k free, then,

$$\begin{aligned} \Omega(\gamma_k) &= \sum_{N \in \mathcal{N}_S \cup \mathcal{N}_D} I_N(\gamma_k) \\ \gamma_c &= \tilde{g}(\Gamma) = \arg\ \min_{\gamma_k \in \Gamma}\ \ \Omega(\gamma_k) \end{aligned}$$

If there are more than one γ_k that minimize $\Omega()$ then the smallest indexed γ_k is selected.

To understand how this algorithm uses the channels in an efficient manner suppose channel γ_c is chosen. Then, nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ cannot use channel γ_c as long as the allocated call is active. Therefore, all those nodes that had γ_c as a free channel before the call request was made remove γ_c from their set of free channels. *LCRA minimizes the number of nodes that get blocked by the choice of a particular channel*. The fact that some nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ do not have γ_c in their set of free channels also implies that there is presently an active call in their neighborhood but that call does not interfere with the new incoming call on $S \leftrightarrow D$. Choosing such a channel will then lead to a local re-use of the channels. Thus, *LCRA tries to locally re-use the channels*. A more general weight based LCRA can be found in [4].

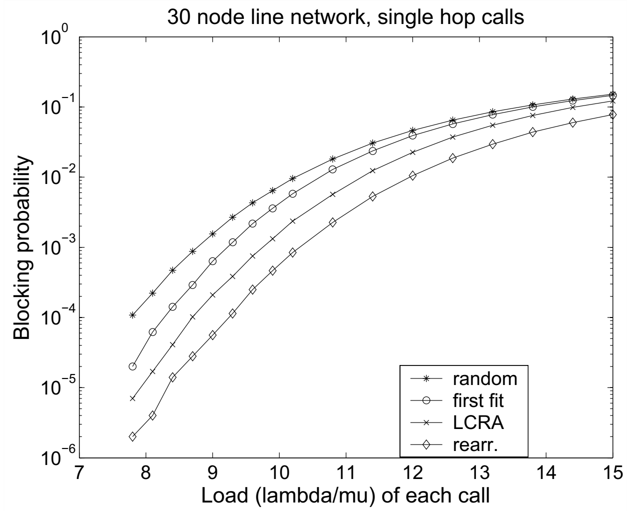

Fig. 9. Line network with unit length calls.

Multihop Calls: A multihop call is regarded as a sequence of single hop calls where the first call arrives on the first link followed by an arrival on the second link and so on until the last link along the path. With this interpretation, we assign channels to a multihop call by repeating the single hop procedure in a sequence over the multihop path. Along the multihop path if at any link there are no free channels available then the call is dropped.

We next present simulation results that compare the performance of the above stated algorithms in a line and a grid network. These plots are illustrative examples of the trend that follows in the extensive simulations we carried out by varying different parameters. In all the cases, we compute the blocking probability of the center call to minimize edge effects. In both networks, the transmission radius of each node is fixed at unity. The arrival process of all the calls is Poisson with rate λ while the departure time is Exponentially distributed with mean $1/\mu = 1$. The load in the plots equals λ/μ. The number of channels in all cases is, $p = 50$.

Figure 9 compares the blocking probability in a line network with 30 nodes and unit length calls. LCRA performs better than both the random and the first fit algorithms. Observe that if we fix a particular value of blocking probability, then, LCRA can support a higher load per call as compared to random and first fit algorithms. As expected the rearrangement algorithm has the lowest blocking probability. Simulating the rearrangement policy in a grid network is practically difficult. Therefore, in a grid network we compare the blocking probability for the random, the first fit and the LCRA algorithms. Figure 10 shows the comparison plot for a 20×20 grid with unit length calls. LCRA performs better than both the random and the first fit policy. In a grid network, a node has more interfering neighbors than a line network; therefore, an efficient spatial re-use of the channels will have a greater impact on blocking probability. This is evident from the wider spread between the curves in Figure 10 as compared to the plot for the line network. Finally considering multihop calls, Figure 11

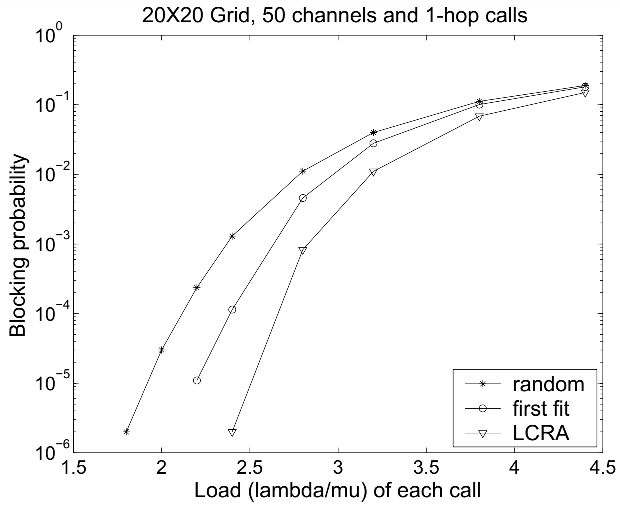

Fig. 10. Grid Network with unit length calls.

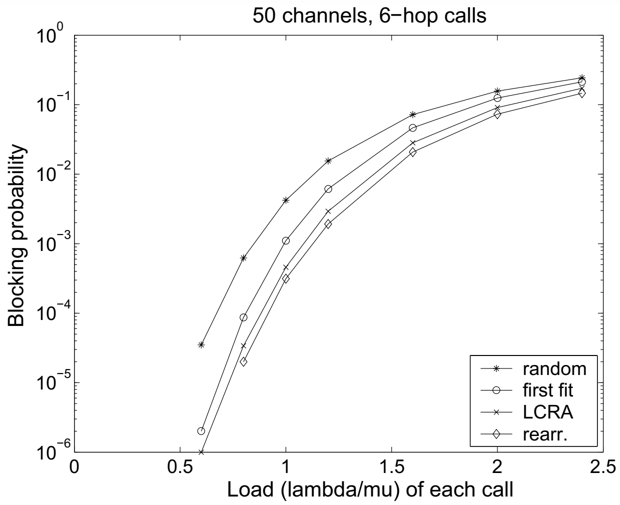

Fig. 11. Line network with 6-hop calls (length 6 units).

compares the blocking probability for random, first fit, LCRA and the rearrangement algorithms in a line network with all calls 6-hop long. Here again, LCRA outperforms both the random and the first fit policies and is fairly close to the rearrangement policy.

VI. Conclusion

We studied the blocking probability behavior of connection oriented traffic for multi-hop wireless line and grid topologies. We focused on two aspects, namely, the effect of transmission radius of the nodes and the dynamic channel assignment algorithm. We presented a blocking probability model and derived formulas that yielded useful insights. For example, we showed that in the line topology using a large transmission radius substantially reduces blocking probability; while the opposite is true in the more dense grid topology. We, then, developed a novel channel assignment algorithm that reduces blocking probability by spatially re-using the channels in an efficient manner. Using simulations, we compared its performance with other algorithms and showed significant improvements. In this work, we considered networks with linear and quadratic increase in the number of interfering nodes; an interesting future research direction is to study the relationship between blocking probability and a general rate of increase of interfering nodes with the transmission radius. It would also be interesting to develop channel assignment algorithms with node mobility.

Appendix I
Proof of Theorem 1

The proof is structured as follows. First, we obtain the local constraints for the successful service of a call. We, then, define a state vector that describes the system evolution over time and obtain its steady state distribution. We evaluate the blocking probability of call C_0 and then derive the result by taking appropriate limits. Before we present the details we state the following lemma that will be useful in the analysis. Its proof is straight forward and omitted for brevity.

Lemma 1: Let $\mathcal{X}$ and $\mathcal{Y}$ be finite disjoint discrete sets and $f(x)$ and $g(y)$ are any two functions defined on $\mathcal{X}$ and $\mathcal{Y}$, then

$$\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} f(x)g(y) = (\sum_{x\in\mathcal{X}} f(x))(\sum_{y\in\mathcal{Y}} g(y)) \qquad (12)$$

Consider a line network as stated in Section III-A. Based on the wireless model outlined in Section II, we can specialize the interference constraints to a line network as follows. Let C_k denote the call between nodes X_k and X_{k+r}. A node X_l is *active* if either call C_{l-r} ($X_{l-r} \leftrightarrow X_l$) or call C_l ($X_l \leftrightarrow X_{l+r}$) is active; otherwise node X_l is *inactive*. From Section II, call C_k can be successfully serviced if neighbors of nodes X_k and X_{k+r} are inactive. These include nodes X_{k-r},..,X_{k+2r}; excluding X_k and X_{k+r} . This implies that calls C_{k-2r},.., C_{k+2r} must be inactive for call C_k to be successful. We refer to this as the *local constraint* on call service. Figure 2 illustrates this constraint for WLN-1 and call C_k.

Next, we define a state vector that describes the system evolution over time. Let $n_k(t)$ denote the number of calls C_k in progress at time t. Let $\nu = \lambda/\mu$ and define the state vector $\mathbf{n}(t) = (n_{-m}(t), \ldots, n_{m-r}(t))$. The vector, $\mathbf{n}(t)$, enlists the number of active calls between all the distinct node pairs that are r units apart. State $\mathbf{n}$ is admissible if $\mathbf{n} \geq 0$ and satisfies the local constraint for all active calls. Let $\mathcal{G}(m)$ denote the set of all admissible states for WLN-r with m nodes. For the network that we consider there are $2m+1$ nodes $(X_{-m}, \ldots, X_m)$ and hence with the above notation this set is $\mathcal{G}(2m+1)$. It is easy to see that the stochastic process $(\mathbf{n}(t), t \geq 0)$ is an aperiodic, irreducible, finite state Markov process and hence has a unique stationary distribution $\pi(\mathbf{n}) = P(\mathbf{n} = [n_{-m}, .., n_{m-r}])$ given by the product form solution,

$$\pi(\mathbf{n}) = \frac{1}{S(2m+1)} \prod_{i=-m}^{m-r} \frac{\nu^{n_i}}{n_i!} \ , \quad \mathbf{n} \in \mathcal{G}(2m+1) \qquad (13)$$

where $S(2m+1)$ is the normalization constant. For a single channel network $n_i = 0$ or 1 and $n_i! = 1$. Let $n_{total} =$

$n_{-m}+..+n_{m-r}$, then simplifying (13) we get,

$$\pi(\mathbf{n}) = \frac{\nu^{n_{total}}}{S(2m+1)}, \quad \mathbf{n}\in\mathcal{G}(2m+1) \quad (14)$$

$$S(2m+1) = \sum_{\mathbf{n}\in\mathcal{G}(2m+1)} \nu^{n_{total}} \quad (15)$$

where the last equation follows from $\sum_{\mathbf{n}\in\mathcal{G}(2m+1)}\pi(\mathbf{n})=1$. Now, in principle, we can obtain the blocking probability of any call by summing $\pi(\mathbf{n})$ over the blocking states; however, evaluating $S(2m+1)$ is not easy. In the rest of the proof we focus on call C_0 of the network and obtain its blocking probability in the limit $m\to\infty$. This eliminates edge effects and yields a simple elegant expression. The non-blocking states for call C_0 are $\{\mathbf{n}:\mathbf{n}\in\mathcal{G}(2m+1)$ and $n_{-2r},..,n_{2r}=0\}$. Denote this set as $\mathcal{G}_N(2m+1)$ and let P^0_{NB} be the probability that in steady state call C_0 is not blocked. We then have,

$$P^0_{NB} = \sum_{\mathcal{G}_N(2m+1)}\pi(\mathbf{n}) \quad (16)$$

$$= \frac{\sum_{\mathbf{n}\in\mathcal{G}_N(2m+1)}\nu^{n_{total}}}{S(2m+1)} \quad (17)$$

Evaluating the numerator in (17): To characterize $\mathcal{G}_N(2m+1)$, we need to determine the feasible state space of the remaining calls C_{-m},..,C_{-2r-1} and C_{2r+1},..,C_{m-r}. Let the feasible state space of calls C_{-m},..,C_{-2r-1} be $\mathcal{L}$ and that of calls C_{2r+1},..,C_{m-r} be $\mathcal{M}$. Then, using our notation we have the following equivalence, $\mathcal{L}\equiv\mathcal{G}(m-r)$ (since C_{-m},..,C_{-2r-1} denote the distinct calls among node pairs within $X_{-m},..,X_{-r-1}$ which is a WLN-r with $m-r$ nodes). Similarly, $\mathcal{M}\equiv\mathcal{G}(m-2r)$. Now, given $\{C_{-2r},..,C_{2r}\}=0$, the state of calls C_{-m},..,C_{-2r-1} is not constrained by the state of calls $C_{2r+1},..,C_{m-r}$, since an active call C_k only affects calls C_{k-2r},.., C_{k+2r}. Therefore, $\mathcal{G}_N(2m+1)$ is simply the cartesian product of $\mathcal{L}$ and $\mathcal{M}$ and we have, $\mathcal{G}_N(2m+1)=\{\mathcal{L}\times\mathcal{M},n_{-2r},..,n_{2r}=0\}$. Let $n_L=n_{-m}+..+n_{-2r-1}$ and $n_M=n_{2r+1}+..+n_{m-r}$, then using Lemma 1 we get,

$$\sum_{\mathbf{n}\in\mathcal{G}_N(2m+1)}\nu^{n_{total}} = \sum_{\mathcal{G}(m-r)\times\mathcal{G}(m-2r)}\nu^{n_L}\nu^{n_M} \quad (18)$$

$$= S(m-r)S(m-2r) \quad (19)$$

Evaluating the denominator in (17): We partition the set $\mathcal{G}(2m+1)$; evaluate $S(2m+1)$ over each partition and then sum up. To do this, condition on the state of calls C_{-r},..,C_{r-1}. For these calls, other than the all zero state, there are $2r$ distinct feasible states corresponding to one call being active and the rest inactive, i.e. $C_p=1, C_{l\neq p}=0$ $\forall p,l\in\{-r,..,r-1\}$. A state with more than one call being active among C_{-r},..,C_{r-1} is infeasible. Let $S()|\{constraint\}$ represent the evaluation of the function $S()$ under the specified $constraint$ and $l\in\{-r,..,r-1\}$. Then,

$$S(2m+1) = S(2m+1)|\{C_{-r},..,C_{r-1}=0\} + \sum_{p=-r}^{r-1}S(2m+1)|\{C_p=1,C_{l\neq p}=0\} \quad (20)$$

To evaluate (20), let $p=r-j$. For a particular term in the summation, the condition $C_p=1$ can be written as $C_{r-j}=1$. The set of calls that must be inactive (for $C_{r-j}=1$) are C_{-r-j},..,C_{3r-j}. This leaves the state of calls $C_{-m},..,C_{-r-j-1}$ independent of the state of calls $C_{3r-j+1},..,C_{m-r}$. Thus for this term, applying Lemma 1, the normalization constant $S(2m+1)$ (with the constraint $C_{r-j}=1, C_{l\neq(r-j)}=0, l\in\{-r,..,r-1\}$) equals $\nu S(m-j)S(m-3r+j)$ [3]. Similarly for the term with the constraint $C_{-r},..,C_{r-1}=0$, the state of calls $\{C_{-m},..,C_{-r-1}\}$ and $\{C_r,..,C_{m-r}\}$ are independent and $S(2m+1)$ evaluates to $S(m)S(m-r+1)$. Thus, we get,

$$S(2m+1) = S(m)S(m-r+1) + \nu\sum_{j=1}^{2r}S(m-j)S(m-3r+j) \quad (21)$$

We have, thus far, evaluated P^0_{NB} in terms of the $S()$ function. It turns out that if we consider the limiting behavior ($m\to\infty$) then an elegant solution is obtained. To see this, let $\lim_{m\to\infty}\frac{S(m-1)}{S(m)}=x$ (the existence of this limit is shown later). As the length of the line network tends to infinity, we have $\lim_{m\to\infty}P^0_{NB}=P_{NB}$, the probability of non-blocking of any call. Combining (17), (19), (21) and taking limits we get,

$$P_{NB} = \frac{\lim_{m\to\infty}\frac{S(m-r)S(m-2r)}{S(m)S(m-r+1)}}{1+\nu\sum_{j=1}^{2r}\lim_{m\to\infty}\frac{S(m-j)S(m-3r+j)}{S(m)S(m-r+1)}} \quad (22)$$

$$= \frac{x^{2r+1}}{1+2r\nu x^{2r+1}} \quad (23)$$

where $\frac{S(\alpha)}{S(\beta)}=\frac{S(\alpha)}{S(\alpha+1)}\frac{S(\alpha+1)}{S(\alpha+2)}\cdots\frac{S(\beta-1)}{S(\beta)}\to x^{\beta-\alpha}$, $\forall\beta\geq\alpha$. To prove the existence of the limit, $\lim_{m\to\infty}\frac{S(m-1)}{S(m)}$, we evaluate $S(m)$ by conditioning on the state of the call associated with the leftmost node. The two conditioning cases are: the call inactive and the call active.

$$S(m) = S(m-1)+\nu S(m-2r-1) \quad (24)$$

$$1 = \lim_{m\to\infty}\left(\frac{S(m-1)}{S(m)}+\frac{\nu S(m-2r-1)}{S(m)}\right) \quad (25)$$

First, note that $S(m)$ is non-negative and non-decreasing for $\nu\in[0,\infty)$ and hence the sequence $\frac{S(m-1)}{S(m)}$ is bounded and satisfies $0<\frac{S(m-1)}{S(m)}\leq 1$, $\forall m$. For every bounded sequence there exists a convergent subsequence, $\{m_i\}$, for which $\frac{S(m_i-1)}{S(m_i)}$ converges to some limit, say x. Take the limit in (25) over this subsequence which gives the following polynomial equation $1=x+\nu x^{2r+1}$ and x is the root of this equation in $(0,1]$. Now, $x+\nu x^{2r+1}=1$, has a unique root in $(0,1]$; to see this re-write it as $\nu x^{2r}+1=1/x$. For $x\in(0,1]$, $1/x$ is a decreasing function taking values in $[1,\infty)$; while $\nu x^{2r}+1$, $r\geq 1$ is a non-decreasing function taking values in $(1,1+\nu]$. Hence, the two curves intersect at a unique $x\in(0,1]$. This means that any convergent subsequence converges to the same limit. In particular, $\liminf_{m\to\infty}\frac{S(m-1)}{S(m)}$ and $\limsup_{m\to\infty}\frac{S(m-1)}{S(m)}$ are the limit points of convergent subsequences and by the above argument they are equal. Thus,

[3]Feasible state space of calls $C_{-m},..,C_{-r-j-1}$ is $\mathcal{G}(m-j)$ and the that of calls $C_{3r-j+1},..,C_{m-r}$ is $\mathcal{G}(m-3r+j)$

$\lim_{m\to\infty} \frac{S(m-1)}{S(m)}$ exists and is the unique root of $x+\nu x^{2r+1} = 1$. Finally, the blocking probability of a call, P_B, is given by,

$$P_B = 1 - \frac{x^{2r+1}}{1+2r\nu x^{2r+1}} \ , \quad x+\nu x^{2r+1} = 1 \tag{26}$$

Appendix II
Proof of Proposition 1

The blocking probability analysis follows the methodology used in Appendix I. Let C_k denote the call between nodes X_k and X_{k+2} and $n_k(t)$ be the number of calls C_k in progress. Define the state vector as $\mathbf{n}(t) = (n_k(t), k \in -m, ..., m-2)$. Since we use the rearrangement policy for assigning channels the vector $\mathbf{n} = (n_{-m}, .., n_{m-2})$ completely describes the system behavior. The stochastic process $\mathbf{n}(t)$ is an aperiodic, irreducible, finite state Markov process with a product form steady state distribution $\pi(\mathbf{n})$. Let $N(2m+1)$ be the normalization constant and $\mathcal{H}(2m+1)$ denote the admissible state space for the line network (with $2m+1$ nodes). Then,

$$\pi(\mathbf{n}) = \frac{1}{N(2m+1)} \prod_{j=-m}^{m-2} \frac{\nu^{n_j}}{n_j!} \ \ , \ \mathbf{n} \in \mathcal{H}(2m+1) \tag{27}$$

Since the calls are two hops and adjacent hops cannot share the same channel, each call requires two channels to get served. As there are only two channels in the network we get the constraint $0 \le n_j \le 1, \ \forall j$ and $n_j! = 1$. Let $n_{total} = n_{-m} + .. + n_{m-2}$ then (27) simplifies to,

$$\pi(\mathbf{n}) = \frac{\nu^{n_{total}}}{N(2m+1)} \ , \ \mathbf{n} \in \mathcal{H}(2m+1) \tag{28}$$

$$N(2m+1) = \sum_{\mathbf{n}\in\mathcal{H}(2m+1)} \nu^{n_{total}} \tag{29}$$

Suppose call C_k is active with channel γ_1 assigned on $X_k \leftrightarrow X_{k+1}$ and γ_2 assigned on $X_{k+1} \leftrightarrow X_{k+2}$. Call C_{k-3} can be simultaneously active by having an assignment γ_1 on $X_{k-3} \leftrightarrow X_{k-2}$ and γ_2 on $X_{k-2} \leftrightarrow X_{k-1}$. This assignment satisfies the wireless constraints for unit transmission radius. Similarly we can have a feasible assignment for call C_{k+3} while C_k is active. Thus only calls $C_{k-2}, C_{k-1}, C_{k+1}, C_{k+2}$ must be inactive for call C_k to be successfully serviced. These local constraints define the set $\mathcal{H}(2m+1)$. Now, consider call C_0 of the line network. The non-blocking states (denoted $\mathcal{H}^0_N(2m+1)$) for call C_0 are $\{\mathbf{n} : \mathbf{n} \in \mathcal{H}(2m+1)$ and $n_{-2}, .., n_2 = 0\}$. Given $(C_{-2}$,..,$C_2) = 0$, the state of calls C_{-m},..,C_{-3} is not constrained by the state of calls C_3,..,C_{m-2}. Thus, $\mathcal{H}^0_N(2m+1) = \mathcal{H}(m) \times \mathcal{H}(m-2)$ where, by our notation, the feasible state space of calls C_{-m},..,C_{-3} is $\mathcal{H}(m)$ and of C_3,..,C_{m-2} is $\mathcal{H}(m-2)$. From Lemma 1 and $n_{-2}, .., n_2 = 0$ we get,

$$P^0_{NB} = \frac{(\sum_{\mathcal{H}(m)} \nu^{n_{-m}+..+n_{-3}})(\sum_{\mathcal{H}(m-2)} \nu^{n_3+..+n_{m-2}})}{N(2m+1)} \tag{30}$$

$$= \frac{N(m)N(m-2)}{N(2m+1)} \tag{31}$$

where P^0_{NB} is the probability of non-blocking of C_0. We next evaluate $N(2m+1)$ by partitioning $\mathcal{H}(2m+1)$ conditioned on the states of C_{-1}, C_0; evaluating $N(2m+1)$ over each partition and summing up. The steps involved are identical to those in Appendix I and can be found in [4].

$$\begin{aligned} N(2m+1) &= N(m+1)N(m) + \nu N(m)N(m-2) \\ &+ \nu N(m-1)N(m-1) \end{aligned} \tag{32}$$

Let $y = \lim_{m\to\infty} \frac{N(m-1)}{N(m)}$. Combining (31), (32) and $\lim_{m\to\infty} P^0_{NB} = P_{NB}$ we get (33) and (34) on the next page. As in Appendix I, we can prove the existence and uniqueness of y by evaluating $N(m)$ by conditioning on the state of the leftmost call. This gives, $N(m) = N(m-1) + \nu N(m-3)$; divide by $N(m)$ and take the limits. The steps are omitted for brevity. The polynomial equation, thus, obtained is $1 = y + \nu y^3$ and we get,

$$P_B = 1 - \frac{y^3}{1+2\nu y^3}, \quad \nu y^3 + y = 1 \tag{35}$$

Appendix III
Proof of Theorem 2

To compare Schemes A and B, we need to compare (6) and (7) for the same load ν. For load $\nu = 0$, P_B is zero for both schemes. Excluding the $\nu \to \infty$ case, we show that there does not exist a load ν such that the two blocking probabilities are equal (and hence the two P_B curves never cross). The proof is by contradiction. Let $\hat{\nu} > 0$ be such that the blocking probability for Schemes A and B are equal. Equating (6) and (7), we get,

$$\frac{\tilde{x}^5}{1+2\hat{\nu}\tilde{x}^5} = \frac{\tilde{y}^3}{1+2\hat{\nu}\tilde{y}^3} \quad \Rightarrow \quad \tilde{x}^5 = \tilde{y}^3 \tag{36}$$

where $\tilde{x}$ and $\tilde{y}$ are the unique roots in (0,1) of $\frac{\hat{\nu}}{2}x^5 + x = 1$ and $\hat{\nu}y^3 + y = 1$ respectively. Note that as $\hat{\nu}$ is strictly greater than 0, $\tilde{x}$, $\tilde{y}$ are strictly less than 1. Rearranging,

$$\hat{\nu}\tilde{x}^5 = \hat{\nu}\tilde{y}^3 = 1 - \tilde{y} \ \ (\text{as } \hat{\nu}\tilde{y}^3 + \tilde{y} = 1) \tag{37}$$

$$= 1 - \tilde{x}^{5/3} \ \ (\text{From (36)}) \tag{38}$$

Substituting (38) in $\frac{\hat{\nu}}{2}\tilde{x}^5 + \tilde{x} = 1$ we get $2\tilde{x} - \tilde{x}^{5/3} = 1$. However, for $\tilde{x} \in (0,1)$, $2\tilde{x} - \tilde{x}^{5/3}$ is an increasing function taking values in (0,1). Thus, for $\tilde{x} \in (0,1)$, $2\tilde{x} - \tilde{x}^{5/3} \neq 1$ and we arrive at a contradiction. This proves that for finite $\nu > 0$, P_B for Schemes A and B cannot be equal. For a particular value, say $\nu = 1$, it can be easily shown that $P_B(Scheme\ B) < P_B(Scheme\ A)$. Combining this with the fact that the two P_B curves do not cross each other we conclude that for all $\nu \in (0, \infty)$, blocking probability for Scheme B is smaller than that for Scheme A.

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$$P_{NB} = \frac{\lim_{m\to\infty} \frac{N(m)N(m-2)}{N(m+1)N(m)}}{1 + \nu \lim_{m\to\infty} \frac{N(m)N(m-2)}{N(m+1)N(m)} + \nu \lim_{m\to\infty} \frac{N(m-1)N(m-1)}{N(m+1)N(m)}} \quad (33)$$

$$= \frac{y^3}{1+2\nu y^3} \quad (34)$$

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