Dynamic Power Allocation and Routing for Time-Varying Wireless Networks

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***Abstract*—We consider dynamic routing and power allocation for a wireless network with time-varying channels. The network consists of power constrained nodes that transmit over wireless links with adaptive transmission rates. Packets randomly enter the system at each node and wait in output queues to be transmitted through the network to their destinations. We establish the capacity region of all rate matrices (λ_{ij}) that the system can stably support—where λ_{ij} represents the rate of traffic originating at node i and destined for node j. A joint routing and power allocation policy is developed that stabilizes the system and provides bounded average delay guarantees whenever the input rates are within this capacity region. Such performance holds for general arrival and channel state processes, even if these processes are unknown to the network controller. We then apply this control algorithm to an ad hoc wireless network, where channel variations are due to user mobility. Centralized and decentralized implementations are compared, and the stability region of the decentralized algorithm is shown to contain that of the mobile relay strategy developed by Grossglauser and Tse (2002).**

***Index Terms*—Capacity, control, optimization, queueing.**

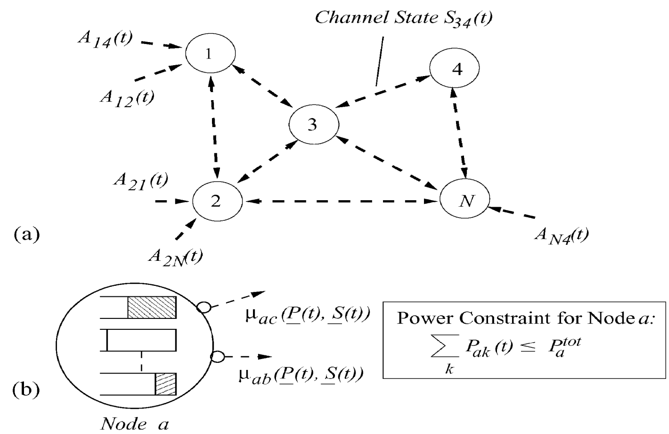

Fig. 1. (a) Wireless network with multiple input streams. (b) Close-up of one node, illustrating the internal queues.

I. Introduction

WIRELESS systems have emerged as a ubiquitous part of modern data communication networks. Demand for these systems continues to grow as applications involving both voice and data expand beyond their traditional wireline service requirements. In order to meet the increasing demand in data rates that are currently being supported by high-speed wired networks composed of electrical cables and optical links, it is important to fully utilize the capacity available in wireless systems, as well as to develop robust strategies for integrating these systems into a large scale, heterogeneous data network. Emerging microprocessor technologies are enabling wireless units to be equipped with the processing power needed to implement adaptive coding techniques and to make intelligent decisions about packet routing and resource management. It is expedient to take full advantage of these capabilities by designing efficient network control algorithms.

In this paper, we develop algorithms for dynamic routing and power allocation in a wireless network consisting of N power constrained nodes. Time is slotted, and in every time slot, the channel conditions of each link randomly change (due to external effects such as fading, user mobility, and/or time-varying weather conditions). Multiple data streams $A_{ij}(t)$ randomly enter the system, where $A_{ij}(t)$ represents an exogenous process of packets arriving to node i destined for node j. Packets are dynamically routed from node to node over multihop paths using wireless data links.

Nodes can transmit data over multiple links simultaneously by assigning power to the link for each node pair (a, b) according to a power matrix $\underline{P}(t) = (P_{ab}(t))$, subject to a total power constraint at each node. Transmission rates over all links are determined by the power allocation matrix $\underline{P}(t)$ and the current channel state $\underline{S}(t)$ according to a rate-power curve $\underline{\mu}(\underline{P}, \underline{S})$. Each node contains $N - 1$ internal queues for storing data according to its destination (Fig. 1). A controller allocates power and schedules the data to be routed over the links in reaction to channel state and queue backlog information. The goal of the controller is to stabilize the system and thereby achieve maximum throughput and maintain acceptably low network delay.

We establish the *network capacity region*: The set of all input rate matrices (λ_{ij}) that the system can stably support (where λ_{ij} represents the rate of data entering node i destined for node j). This region is determined by considering all possible routing and power allocation strategies, and can be expressed in terms of the steady-state channel probabilities, the node power constraints, and the rate-power function $\underline{\mu}(\underline{P}, \underline{S})$. We emphasize that this is a *network layer* notion of capacity, where $\underline{\mu}(\underline{P}, \underline{S})$ is a general function representing the rate achievable on the wireless links under a given physical layer modulation and coding strategy. This is distinct from the *information theoretic* capacity of the wireless network, which includes optimization over all

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possible modulation and coding schemes, and involves many of the unsolved problems of network information theory. We do not address the information theoretic capacity in this work, and use the term *capacity* to represent *network layer capacity*.

We present a joint routing and power allocation policy that stabilizes the system and provides bounded average delay guarantees whenever the input rates are strictly inside the network capacity region. Such performance holds for general ergodic arrival and channel state processes, even if the specific channel probabilities and packet arrival rates are unknown to the network controller. The strategy involves maximizing a stochastic drift metric every time slot. We implement centralized and decentralized versions of the algorithm for an ad hoc wireless network, where channel variations are due to user mobility. Further, we show that this dynamic control strategy can be viewed as a generalization of a corresponding static optimization procedure, establishing a fundamental relationship between network optimization and stochastic control.

Previous work on capacity, optimization, and control of wireless networks is found in [1]–[27]. Connectivity and asymptotic capacity analysis for large static networks is presented in [4] and [5], and for mobile networks in [6]. The exact capacity of a wireless uplink and downlink with multiple users is developed in [7]–[9], where it is assumed that all users have infinite backlog.

Optimization approaches to network resource allocation are developed in [10]–[14] and [29]–[32]. In [10], static routing and power allocation is treated using convex optimization. In [11], various cost metrics are formulated as geometric programs to address resource allocation and quality-of-service. Game theoretic approaches for wireless downlinks are developed in [12] and [13], and for flow networks in [29]–[31], where pricing schemes are considered for achieving a static equilibrium with respect to some utility metric. The equilibrium computed in [31] is shown to be within a constant factor of the maximum utility. Similar constant factor bounds are developed in [32] for shortest-path routing in static networks with simplified models of network delay. Optimal power allocation for minimizing energy expenditure in a network with given rate requirements is considered in [14] under the assumption that transmission rates are linear functions of the signal to interference ratio on each link. In this case, although the network channels and rate requirements are constant, the optimal solution is not static but requires the computation of a periodic transmission schedule to achieve optimality. Simple approximations to optimal scheduling are developed in [15], and schedules for one-dimensional networks are developed in [16].

These optimization approaches do not consider the practical issues of *network control*, where queue management, scheduling, and resource allocation decisions must be made in the presence of stochastic packet arrivals and time-varying channel conditions. Control problems are addressed in [3] and [18]–[27]. In [18], a stabilizing power allocation and routing strategy is developed for a multibeam satellite downlink with random inputs and time-varying channels. Related problems of scheduling users over a single server downlink are considered in [19]–[23]. In [24], a delay optimal strategy is developed for a multiaccess uplink in systems with symmetric user parameters. Asymptotically optimal strategies using heavy traffic limits are developed in [25] and [28] for scheduling multiple users over a shared resource. Transmitter scheduling and power control for one-hop static networks are considered in [26], and one-hop networks with time-varying topology are considered in [27]. Our work is inspired by the approach of Tassiulas in [3], where a Lyapunov drift technique is used to develop a throughput optimal link scheduling policy for a multihop packet radio network. Further work on Lyapunov analysis is found in the switching and scheduling literature [35]–[37] and in [38], [39].

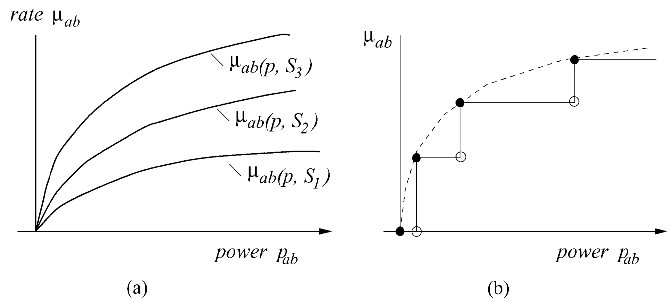

Fig. 2. (a) Set of rate-power curves for improving channel conditions S_1, S_2, S_3. (b) Curve restricted to a finite set of operating points corresponding to full packet transmissions. Curves illustrate behavior on link (a, b) when the single power parameter P_{ab} is increased, in which case the concave increasing profiles are typical.

The main contributions in this paper are the formulation of a general power control problem for time-varying wireless networks, the characterization of the network layer capacity region, and the development of capacity achieving routing and power allocation algorithms that offer delay guarantees and consider the full effects of queueing. These algorithms hold for systems with general arrival and channel processes, including ad hoc networks with mobility. This work unifies notions of *network capacity*, *network optimization*, and *network control*.

In the next section, we introduce the power allocation problem for wireless networks. In Section III, we define network stability and establish the capacity region for wireless networks. In Sections IV and V, a dynamic control strategy is developed and shown to stabilize the network and provide average delay guarantees. Distributed implementations are considered in Section VI, where optimal distributed control is established for networks with independent channels, and a distributed approximation algorithm is developed for networks with interference. We implement the distributed approximation for an ad hoc mobile network and show through analysis and simulation that the algorithm offers higher data rates and lower delay than the Grossglauser–Tse relay algorithm of [6]. Finally, in Section VII, a perspective on *dynamic optimization* is provided by relating the optimal network control algorithms developed here to a classic iterative solution of a static convex program.

II. System Model

Consider an N node network with time-varying channels, as in Fig. 1. Let $\underline{S}(t) = (S_{ab}(t))$ denote the matrix process of channel states, where $S_{ab}(t)$ represents the current state of channel (a, b) (representing, for example, attenuation values and/or noise levels). Time is slotted with slots normalized to integral units $t \in \{0, 1, 2, \ldots\}$. We assume that channels hold their state for the duration of a time slot, and are known to the network controller at the beginning of each slot. Such information can be obtained either through direct measurement (where time slots are assumed to be long in comparison to the required measurement time) or through a combination of measurement and channel prediction. The channel process $\underline{S}(t)$ takes values on a finite-state space, and is ergodic with time average probabilities $\pi_{\underline{S}}$ for each state $\underline{S}$.

Every time slot, a controller determines transmission rates on each link by allocating a power matrix $\underline{P}(t) = (P_{ab}(t))$ subject to a total power constraint $\sum_{b\neq i} P_{ib}(t) \leq P_i^{\text{tot}}$ for all nodes i. Additional power constraints can be introduced, such as constraints on the number of outgoing links that can be allocated power when a node is transmitting or receiving. It is, therefore, useful to represent the power constraint in the form $\underline{P}(t) \in \Pi$, where Π is a compact set of acceptable power allocations that includes the power limits for each node.

Link rates are determined by a corresponding rate-power curve $\underline{\mu}(\underline{P}, \underline{S}) = (\mu_{ab}(\underline{P}, \underline{S}))$ (see Fig. 2). It is assumed that data can be split continuously, so that each time slot the transmission rate $\mu_{ab}(\cdot)$ determines the number of bits that can be transferred over the wireless link (a, b). Such an assumption is valid if variable length packets can be split and repackaged with new headers for resequencing at the destination (we neglect the extra bits due to such headers in this analysis). Alternately, splitting and relabeling can be avoided altogether if all packets have fixed lengths and the transmission rates are restricted to integral multiples of the packet-length/time slot quotient [see Fig. 2(b)].

Note that, in general, the transmission rate over a link (a, b) of the network depends on the full matrix of power allocation decisions. This is because communication rates over the link may be influenced by interference from other channels. For example, achievable data rates could be approximated by using the signal-to-interference ratio (SIR) in the capacity formula for a white Gaussian noise channel, where the SIR over link (a, b) is defined as the attenuated signal power divided by the total interference at node b.

Example: Rate-Power Curve:

$$\mu_{ab}(\underline{P}, \underline{S}) \triangleq \log\left(1 + \frac{\alpha_{ab} P_{ab}}{N_b + \alpha_{ab}\sum_{j\neq b} P_{aj} + \sum_{i\neq a}\alpha_{ib}\sum_j P_{ij}}\right) \quad (1)$$

where N_b and α_{ab} represent noise and fading coefficients associated with the particular channel state $\underline{S}$.

Alternatively, the $\mu_{ab}(\cdot)$ curves could represent rates for a specific set of coding schemes designed to achieve a sufficiently low probability of error. Note that practical systems rely on a finite databank of codes and, hence, may be restricted to a finite set of feasible operating points. In this case, rate-power curves are piecewise constant [see Fig. 2(b)]. In general, we assume only that $\underline{\mu}(\underline{P}, \underline{S})$ is *upper semicontinuous*[1] in the power matrix $\underline{P}$ for all states $\underline{S}$, and hence at points of discontinuity the function takes its largest limiting value [40].

[1] That is, $\limsup_{k\to\infty} \mu_{ab}(\underline{P}_k, \underline{S}) \leq \mu_{ab}(\underline{P}^*, \underline{S})$ for all (a, b), $\underline{P}^*$, $\underline{S}$, and any sequence $\{\underline{P}_k\}$ such that $\underline{P}_k \to \underline{P}^*$.

A. Control Decision Variables and the Queueing Equation

Each network node i maintains a set of output queues for storing data according to its destination. For convenience, we classify all data flowing through the network as belonging to a particular *commodity* $c \in \{1, \ldots, N\}$, representing the destination node for the data. A network control algorithm makes decisions about power allocation, routing, and scheduling. As a general algorithm might schedule multiple commodities to flow over the same link on a given slot, we define $\mu_{ab}^{(c)}(t)$ as the rate offered to commodity c traffic along link (a, b) during time slot t. The controller, thus, makes the following decisions.

Power Allocation: Choose $\underline{P}(t)$ such that $\underline{P}(t) \in \Pi$.

Routing/Scheduling: Choose $\mu_{ab}^{(c)}(t)$ such that

$$\sum_c \mu_{ab}^{(c)}(t) \leq \mu_{ab}(t) \triangleq \mu_{ab}(\underline{P}(t), \underline{S}(t)). \quad (2)$$

Note that in the special case where there is no power allocation, the $\mu_{ab}(t)$ process is purely determined by the dynamic channel states of the network, and any network control algorithm reduces to pure routing and scheduling.

Let $A_i^{(c)}(t)$ represent the amount of commodity c bits that arrive exogenously to the network at node i during slot t. Let $U_i^{(c)}(t)$ represent the current backlog of bits in node i destined for node c. The $U_i^{(c)}(t)$ processes evolve according to the following queueing dynamics:

$$U_i^{(c)}(t+1) \leq \max\left[U_i^{(c)}(t) - \sum_b \mu_{ib}^{(c)}(t), 0\right] + \sum_a \mu_{ai}^{(c)}(t) + A_i^{(c)}(t). \quad (3)$$

The expression above is an inequality instead of an equality because endogenous arrivals may be less than $\sum_a \mu_{ai}^{(c)}(t)$ if neighboring nodes have little or no commodity c data to transmit. The goal of the controller is to maintain low backlog and thereby stabilize the system. For the first part of this paper, we assume that centralized control is possible, so that the network controller has access to the full backlog and channel state matrices $\underline{U}(t)$ and $\underline{S}(t)$ every time slot. Distributed implementations where each node makes independent control decisions using only local information are considered in Sections VI and VII.

III. Stability and the Network Capacity Region

Here, we develop the capacity region of all data rates stabilizable by a wireless network. We begin with a precise definition of stability.

A. Stability of Queueing Systems

Consider a single queue in isolation, with an input process $A(t)$ and a time-varying server process $\mu(t)$. Let the unfinished work function $U(t)$ represent the amount of unprocessed bits remaining in the queue at time t, which is determined by the stochastics of the input and server processes. As a measure of the fraction of time the unfinished work is above a certain value V, we define the following "overflow" function $g(V)$:

$$g(V) \triangleq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \Pr[U(\tau) > V].$$

The above limit[2] always exists, so that $0 \leq g(V) \leq 1$.

Definition 1: A single server queueing system is *stable* if $g(V) \to 0$ as $V \to \infty$. A *network* of queues is stable if all individual queues are stable.

B. Network Capacity

Assume input processes $A_i^{(c)}(t)$ are stationary and ergodic with rates λ_{ic}, so that $\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} A_i^{(c)}(\tau) = \lambda_{ic}$ with probability 1. Let (λ_{ic}) represent the corresponding $N \times N$ *rate matrix*, having all diagonal entries (λ_{ii}) equal to zero. Assume the channel process $\underline{S}(t)$ is stationary and ergodic with steady-state channel probabilities $\pi_{\underline{S}}$.

Definition 2: The *network capacity region* Λ is the closure of the set of all rate matrices (λ_{ic}) that can be stably supported over the network, considering all possible algorithms (possibly those with full knowledge of future events).

Remarkably, we show that stabilizing policies do not require knowledge of future events and, hence, such knowledge does not expand the region of stabilizable rates. To build intuition about the set Λ, we first consider the capacity region of a traditional wireline network with no time variation, defined on a weighted graph with N nodes, E edges, and edge capacities given by a link matrix (R_{ab}). The link matrix describes the rate at which node a can deliver data to node b, so that $R_{ab} = 0$ if there is no directed edge from node a to node b, and is equal to the positive transmission rate for that link, otherwise. The network capacity region is described implicitly as the set of all arrival rate matrices (λ_{ic}) for which there exist *multicommodity flow variables* $f_{ab}^{(c)}$ (for $a, b, c \in \{1, \ldots, N\}$) that satisfy a set of flow conservation equations, and which additionally satisfy the link constraint $\sum_c f_{ab}^{(c)} \leq R_{ab}$ for all links (a, b). This constraint ensures that the total flow over any link does not exceed the link capacity.

The capacity region of a wireless network differs from that of a wireline network only in the description of the link constraint. Indeed, first note that the time-varying channel conditions of a wireless network require link capacities to be defined in a time average sense, where the resulting transmission rate over a given link (a, b) is averaged over all possible channel states. Second, the resulting time average link rates are not fixed, but depend on the (potentially nonergodic) power allocation policy. Thus, instead of describing the network as a single weighted graph (R_{ab}) of link rates, the network is described by a *collection* of graphs, or a *graph family* Γ. We define the graph family Γ as the following set of node-to-node transmission rate matrices:

$$\Gamma = \sum_{\underline{S}} \pi_{\underline{S}} \text{ Convex\_Hull } \{\underline{\mu}(\underline{P}, \underline{S}) \mid \underline{P} \in \Pi\} \quad (4)$$

where addition and scalar multiplication of sets is used,[3] and the convex hull of a set A is defined as the set of all convex combinations $p_1 a_1 + p_2 a_2 + \cdots + p_k a_k$ of elements $a_i \in A$ (where $\{p_i\}$ are probabilities summing to 1).

Thus, a transmission rate matrix $\underline{R} = (R_{ab})$ is in graph family Γ if and only if $\underline{R}$ can be represented as $\underline{R} = \sum_{\underline{S}} \pi_{\underline{S}} \underline{R_{\underline{S}}}$ for some set of matrices $\underline{R_{\underline{S}}}$, each one being inside the convex hull of the set of node-to-node transmission rates achievable by power allocation under channel state $\underline{S}$. In the following theorem, it is shown that graph family Γ can be viewed as the set of all long-term transmission rates (R_{ab}) that the network can be configured to support on the single-hop wireless links connecting node pairs (a, b).

Theorem 1: Capacity Region for a Wireless Network—The capacity region Λ is the set of all input rate matrices (λ_{ic}) for which there exist multicommodity flow variables $\{f_{ab}^{(c)}\}$ satisfying

$$f_{ab}^{(c)} \geq 0, \quad f_{aa}^{(c)} = f_{ab}^{(a)} = 0 \quad \forall a, b, c \quad (5)$$

$$\lambda_{ic} \leq \sum_b f_{ib}^{(c)} - \sum_a f_{ai}^{(c)} \quad \forall i, c \quad \text{where } i \neq c \quad (6)$$

$$\sum_c f_{ab}^{(c)} \leq R_{ab} \quad \forall a, b, \quad \text{and for some } (R_{ab}) \in \Gamma. \quad (7)$$

Thus, a rate matrix (λ_{ic}) is in the capacity region Λ if there exists a matrix $(R_{ab}) \in \Gamma$ that defines link capacities in a traditional graph network, such that there exist multicommodity flow variables $\{f_{ab}^{(c)}\}$, which support the λ_{ic} rates with respect to this graph. Note that inequalities (5) constrain flow variables to be nonnegative and to be "efficient," in that no node routes data to itself, and no node reinjects delivered data back into the network. Inequality (6) is a conservation constraint that ensures the total flow of commodity c data into a given node i is less than or equal to the total flow out of that node, provided that node i is not the destination.

The proof of Theorem 1 involves showing that $(\lambda_{ic}) \in \Lambda$ is necessary for stability, and that (λ_{ic}) interior to Λ is sufficient. The necessary condition is proven in Appendix A, where it is shown that no control algorithm can achieve stability beyond the set Λ, even if the entire set of future events is known in advance. Sufficiency is shown in the next section, where a stabilizing control policy is constructed.

IV. Dynamic Control Policy

Here, we develop a control policy that stabilizes the network whenever the input rate matrix is interior to the capacity region Λ. The policy is inspired by the maximum differential backlog algorithms developed by Tassiulas and Ephremides in [3] for stable server scheduling in a multihop radio network, and generalizes the Tassiulas–Ephremides algorithm by considering power allocation and addressing networks with general interference properties and time-varying channel characteristics. We then develop a bound on end-to-end delay by relating performance to that of a stationary policy based on the multicommodity flow variables of Theorem 1.

[2]Where the $\limsup$ of a function $f(t)$ always exists, and is defined: $\limsup_{t\to\infty} f(t) = \lim_{t\to\infty}[\sup_{\tau\geq t} f(\tau)]$.

[3]For sets A, B and scalars α, β, the set $\alpha A + \beta B$ is defined as $\{\gamma \mid \gamma = \alpha a + \beta b$ for some $a \in A, b \in B\}$.

Every time slot the network controller observes the channel state $\underline{S}(t)$ and the matrix of queue backlogs $\underline{U}(t) = (U_i^{(c)}(t))$ and performs routing and power control as follows.

Dynamic Routing and Power Control (DRPC) Policy:

1) For all links (a,b), find commodity $c_{ab}^*(t)$ such that

$$c_{ab}^*(t) = \arg\max_{c\in\{1,\ldots,N\}} \left\{U_a^{(c)}(t) - U_b^{(c)}(t)\right\}$$

and define

$$W_{ab}^*(t) = \max\left[U_a^{(c_{ab}^*(t))}(t) - U_b^{(c_{ab}^*(t))}(t), 0\right]. \quad (8)$$

2) *Power Allocation:* Choose a matrix $\underline{P}(t)$ such that

$$\underline{P}(t) = \arg\max_{\underline{P}\in\Pi} \sum_{a,b} \mu_{ab}(\underline{P}, \underline{S}(t)) W_{ab}^*(t). \quad (9)$$

3) *Routing:* Define transmission rates as follows:

$$\mu_{ab}^{(c)}(t) = \begin{cases} \mu_{ab}(\underline{P}(t), \underline{S}(t)), & \text{if } c = c_{ab}^*(t) \text{ and } W_{ab}^*(t) > 0 \\ 0, & \text{otherwise} \end{cases}. \quad (10)$$

For each link (a,b), transmit commodity c_{ab}^* data according to the rate offered by the power allocation. If any node does not have enough bits of a particular commodity to send over all its outgoing links requesting that commodity, *null bits* are delivered.

Note that the W_{ab}^* values represent the *maximum differential backlog* between nodes a and b (maximized over all commodities c). The policy thus uses backpressure in an effort to *equalize differential backlog*. This strategy is most effective when power is allocated to maximize the rate-backlog product in (9). We emphasize that this scheme does not require knowledge of the arrival rates or channel statistics, and does not use any prespecified set of routes. The route for each unit of data is found dynamically.

A. Network Parameters

In order to analyze the stability and delay properties of the DRPC algorithm, we first specify the following network parameters. Define $\mu_{\max}^{\text{out}}$ and $\mu_{\max}^{\text{in}}$ as the maximum transmission rate out of any node and into any node, respectively, under the best channel condtions

$$\mu_{\max}^{\text{out}} \triangleq \max_{\{i,\underline{S},\underline{P}\in\Pi\}} \sum_b \mu_{ib}(\underline{P}, \underline{S})$$
$$\mu_{\max}^{\text{in}} \triangleq \max_{\{i,\underline{S},\underline{P}\in\Pi\}} \sum_a \mu_{ai}(\underline{P}, \underline{S}).$$

Further assume that the second moment of exogenous arrivals to any node is bounded every time slot by some finite maximum value $A_{\max}^2$, regardless of past history. Specifically, define $H(t) = (\underline{S}(\tau); \underline{A}(\tau))|_{\tau=0}^{t-1}$. Then, for all nodes i and all t, we have

$$\mathbb{E}\left\{\left(\sum_c A_i^{(c)}(t)\right)^2 \,\middle|\, H(t)\right\} \le A_{\max}^2.$$

Note that $\mathbb{E}\{A\} \le \sqrt{\mathbb{E}\{A^2\}}$ for any random variable A, so that we also have $\mathbb{E}\{\sum_c A_i^{(c)}(t) \,|\, H(t)\} \le A_{\max}$.

Define $T_{\underline{S}}(t_0, K)$ as the set of time slots at which $\underline{S}(t) = \underline{S}$ during the interval $t_0 \le \tau \le t_0 + K - 1$, and define $\|T_{\underline{S}}(t_0, K)\|$ as the total number of such slots. For a given value $\delta > 0$, we define the *convergence interval* K to be the smallest number of time slots such that for any t_0, any (i,c), and regardless of past history, we have

$$\left|\lambda_{ic} - \frac{1}{K}\sum_{\tau=t_0}^{t_0+K-1} \mathbb{E}\left\{A_i^{(c)}(\tau) \,|\, H(t_0)\right\}\right| \le \delta \quad (11)$$

$$\sum_{\underline{S}} \left|\pi_{\underline{S}} - \frac{\mathbb{E}\{\|T_{\underline{S}}(t_0,K)\| \,|\, H(t_0)\}}{K}\right| \le \frac{\delta}{\max\{\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}\}}. \quad (12)$$

Such a value K must exist for any stationary and ergodic channel and arrival processes with arrival rates (λ_{ic}) and channel probabilities $\pi_{\underline{S}}$, respectively. This convergence interval represents the time period over which the network is expected to reach steady-state, regardless of past history. More generally, we assume a finite interval size K exists for any given $\delta > 0$, and define arrival processes and channel processes for which this assumption holds to be *rate convergent* and *channel convergent*, respectively. We note that for systems with independent identically distributed (i.i.d.) arrivals and channel states, steady-state is exactly achieved every time slot, so that $K = 1$ even when $\delta = 0$. Below, we develop a bound on the end-to-end delay of the DRPC policy in terms of the parameters $\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}, A_{\max}^2$, and K.

B. Stability and Delay Performance

Theorem 2: Stability of DRPC—Suppose an N-node wireless network has capacity region Λ and rate matrix (λ_{ic}) such that $(\lambda_{ic} + \epsilon) \in \Lambda$ for some $\epsilon > 0$. Then, jointly routing and allocating power according to the above DRPC policy stabilizes the system and guarantees bounded average congestion satisfying

$$\overline{\sum_{i,c} U_i^{(c)\text{DRPC}}} \le \frac{KBN}{\epsilon} + \frac{(K-1)\tilde{B}N}{\epsilon} \quad (13)$$

for K corresponding to $\delta \triangleq \epsilon/6$ in (11) and (12), and where

$$B \triangleq \left(A_{\max} + \mu_{\max}^{\text{in}}\right)^2 + \left(\mu_{\max}^{\text{out}}\right)^2 \quad (14)$$
$$\tilde{B} \triangleq 2\max\left[\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}\right]\left(\mu_{\max}^{\text{out}} + \mu_{\max}^{\text{in}} + A_{\max}\right) \quad (15)$$

and the overbar notation of (13) is defined

$$\overline{\sum_{i,c} U_i^{(c)}} \triangleq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\left[\sum_{i,c}\mathbb{E}\left\{U_i^{(c)}(\tau)\right\}\right].$$

The theorem is proven in Section V. Note that the average congestion bound grows asymptotically like $1/\epsilon$ as the data rates are increased, where ϵ can be viewed as the "distance" measure of the rate matrix to the boundary of the capacity region. Consider now an input matrix (λ_{ic}), where each user i sends at total rate λ, so that $\sum_c \lambda_{ic} = \lambda$. Suppose the matrix satisfies

$(\lambda_{ic}+\epsilon)\in\Lambda$, and define $R\triangleq\lambda+N\epsilon$ as the row sum of the $(\lambda_{ic}+\epsilon)$ matrix. Let $\rho\triangleq\lambda/R$ represent an effective loading on each user (assumed independent of N). Note that $\lambda=\rho R$, and $\epsilon N=R(1-\rho)$. From Little's Theorem, the average bit delay satisfies $\overline{D}_{\rm bit}=(1/N\lambda)\sum_{i,c}\overline{U}_i^{(c)}$ and, hence

$$\overline{D}_{\rm bit}\leq\frac{KBN+(K-1)\tilde{B}N}{\epsilon N\lambda}=\frac{KBN+(K-1)\tilde{B}N}{\rho(1-\rho)R^2}.$$

If $R=O(1/\sqrt{N})$ (as in static Gupta–Kumar networks [4], [5]), then $\overline{D}_{\rm bit}\leq O((KN^2)/(1-\rho))$. If $R=O(1)$ (as in mobile Grossglauser–Tse networks [6]), then $\overline{D}_{\rm bit}\leq O(KN)/(1-\rho)$.

Note that when the network is lightly loaded, there is very little information contained in the differential backlog values. Hence, packets might take many false turns, which could lead to significant delay for large networks (consider the above delay bound for ρ small). Performance can often be improved by using the DRPC algorithm with a restricted set of desirable routes for each commodity. However, restricting the routes may reduce network capacity, and may be harmful in time-varying situations, where networks change and links fail. Alternatively, we can keep the full set of routes, but program a *bias* into the DRPC algorithm so that in low loading situations, nodes are inclined to route packets in the direction of their destinations. We use this idea in the following enhanced DRPC algorithm, defined in terms of constants $\theta_a^c>0$ and $V_a^c\geq 0$.

Enhanced DRPC Algorithm: For all links (a,b), and all commodities c, define $W_{ab}^{(c)}\triangleq\theta_a^c(U_a^c(t)+V_a^c)-\theta_b^c(U_b^c(t)+V_b^c)$, and define c_{ab}^* as the maximizer of $W_{ab}^{(c)}$ over all $c\in\{1,\ldots,N\}$. Power allocation and routing is done as before, solving the optimization (9) with respect to $W_{ab}^*\triangleq W_{ab}^{(c_{ab}^*)}$.

The V_a^c parameters can be chosen as scaled hop count estimates between nodes a and c, so that, in the absence of backlog information, data is routed to reduce the remaining distance to the destination. The θ_a^c values are any weights for prioritizing commodity c service in node a. This enhanced DRPC algorithm can be shown to stabilize the network for any constants $\theta_i^c>0$ and $V_i^c\geq 0$. We note that the weight $\theta_a^c(V_a^c+U_a^c)$ can be used in the same manner as a routing table, and the unfinished work quantities can be updated each time slot by having neighboring nodes transmit their backlog changes over a low bandwidth control channel. As each wireless link transmits only a single commodity every time slot, the number of such backlog increments required to be transmitted over the control channel by any user is on the order of the number of neighboring nodes.

V. Performance Analysis

We analyze the performance of the DRPC algorithm by comparing it to a stationary algorithm that makes scheduling decisions according to the multicommodity flow variables $\{f_{ab}^{(c)}\}$ of Theorem 1. Indeed, suppose the rate matrix (λ_{ic}) and the channel probabilities $\pi_{\underline{S}}$ were known in advance, and suppose there is a value $\epsilon>0$ such that $(\lambda_{ic}+\epsilon)\in\Lambda$. Then, a set of multicommodity flow variables $\{f_{ab}^{(c)}\}$ and a link rate matrix $(R_{ab})\in\Gamma$ must exist that satisfy the constraints (5)–(7) with respect to the rate matrix $(\lambda_{ic}+\epsilon)$. In particular

$$(\lambda_{ic}+\epsilon)\leq\sum_b f_{ib}^{(c)}-\sum_a f_{ai}^{(c)},\quad\text{for } i\neq c \tag{16}$$

$$\left(\sum_c f_{ab}^{(c)}\right)\leq(R_{ab}). \tag{17}$$

Let $\mu_{ab}(t)\triangleq\mu_{ab}(\underline{P}(t),\underline{S}(t))$ represent the transmission rate offered over link (a,b) on slot t. We first show that it is possible to allocate power in reaction to the current channel state so that the time average of $\mu_{ab}(t)$ converges to R_{ab}.

Lemma 1: Graph Family Achievability—A *stationary randomized* power allocation policy $\underline{P}^{\rm STAT}(\tau)$ can be implemented so that the resulting $\mu_{ab}^{\rm STAT}(t)$ process satisfies

$$\lim_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\mu_{ab}^{\rm STAT}(\tau)=R_{ab}$$

with probability 1 for all (a,b). The structure of the policy is as follows: Every time slot in which the channel state $\underline{S}(t)=\underline{S}$ is observed, the power matrix $\underline{P}^{\rm STAT}(t)$ is chosen randomly from a finite set of allocations $\{\underline{P}_{\underline{S}}^1,\ldots,\underline{P}_{\underline{S}}^m\}$ according to a set of probabilities $\{q_{\underline{S}}^1,\ldots,q_{\underline{S}}^m\}$.

Proof: Note that $(R_{ab})=\sum_{\underline{S}}\pi_{\underline{S}}\underline{R}_{\underline{S}}$. The proof follows by expressing each matrix $\underline{R}_{\underline{S}}$ as a convex combination of matrices in $\{\underline{\mu}(\underline{P},\underline{S})\,|\,\underline{P}\in\Pi\}$ according to Caratheodory's Theorem [40], and defining the probabilities of the stationary randomized scheme according to the weights of each convex combination. A full proof is given in [1]. □

Note that the above lemma is an existential result and is not offered as a practical means of allocating power. We now define a stationary algorithm STAT that uses the above power allocation result to stabilize the network. While this policy cannot be implemented without complete knowledge of the input rates λ_{ic}, channel probabilities $\pi_{\underline{S}}$, and flow variables $f_{ab}^{(c)}$, it yields a simple delay bound.

Stationary Randomized Policy (STAT)—Power Allocation: Every time slot, observe the channel state $\underline{S}$ and allocate power according to the stationary algorithm $\underline{P}^{\rm STAT}(t)$ of Lemma 1.

Scheduling/Routing: For every link (a,b) such that $\sum_c f_{ab}^{(c)}>0$, transmit the single commodity $\hat{c}_{ab}$, where $\hat{c}_{ab}$ is chosen randomly among $c\in\{1,\ldots,N\}$ with probability $f_{ab}^{(c)}/\sum_d f_{ab}^{(d)}$. However, use only a fraction $\sum_d f_{ab}^{(d)}/R_{ab}$ of the instantaneous link rate, so that

$$\mu_{ab}^{(c)\rm STAT}(t)=\begin{cases}\mu_{ab}^{\rm STAT}(t)\frac{\sum_d f_{ab}^{(d)}}{R_{ab}}, & \text{if } c=\hat{c}_{ab}\\ 0, & \text{otherwise}\end{cases}.$$

If a node does not have enough (or any) bits of a certain commodity to send over its output links, *null bits* are delivered.

Note that

$$\mathbb{E}\left\{\mu_{ab}^{(c)\rm STAT}(t)\,\middle|\,\mu_{ab}^{\rm STAT}(t)\right\}=\mu_{ab}^{\rm STAT}(t)\frac{f_{ab}^{(c)}}{R_{ab}} \tag{18}$$

and, hence, the processes $\mu_{ab}^{(c)\rm STAT}(t)$ are rate convergent with time average rates $f_{ab}^{(c)}$. In order to analyze delay, we consider the rates $\mu_{ab}(t)$ and the input processes $A_i^{(c)}(t)$ averaged over K slot intervals. In particular, for any time t_0, we define

$$\tilde{\mu}_{ab}^{(c)}(t_0) \triangleq \frac{1}{K}\sum_{\tau=t_0}^{t_0+K-1} \mu_{ab}^{(c)}(\tau)$$
$$\tilde{A}_i^{(c)}(t_0) \triangleq \frac{1}{K}\sum_{\tau=t_0}^{t_0+K-1} A_i^{(c)}(\tau).$$

From (16), we have that

$$\sum_b f_{ib}^{(c)} - \sum_a f_{ai}^{(c)} - \lambda_{ic} \geq \epsilon \quad \text{for } i \neq c. \tag{19}$$

It follows that for a suitably large value of K, the expectations of $\tilde{\mu}_{ab}^{(c)}(t_0)$ and $\tilde{A}_i^{(c)}(t_0)$ satisfy a similar inequality, as specified by the next lemma.

Lemma 2: Fix $\delta \triangleq \epsilon/6$, which defines the convergence interval K according to (11) and (12). Then, for any time t_0 and regardless of the past history $H(t_0)$, we have

$$\mathbb{E}\left\{\sum_b \tilde{\mu}_{ib}^{(c)\text{STAT}}(t_0) - \sum_a \tilde{\mu}_{ai}^{(c)\text{STAT}}(t_0) \,\middle|\, H(t_0)\right\} - \mathbb{E}\left\{\tilde{A}_i^{(c)}(t_0) \,\middle|\, H(t_0)\right\} \geq \frac{\epsilon}{2}. \tag{20}$$

Proof: Note that for any variables $x_1, x_2, x_3, \tilde{x}_1, \tilde{x}_2, \tilde{x}_3$ satisfying $x_1 - x_2 - x_3 \geq \epsilon$ and $|x_i - \tilde{x}_i| \leq \epsilon/6$ for $i = 1,2,3$, then $\tilde{x}_1 - \tilde{x}_2 - \tilde{x}_3 \geq \epsilon/2$. The proof follows by applying this fact to inequality (19), and is given in [1]. □

A. Lyapunov Drift Analysis

Our stability and delay analysis relies on the following lemma that specifies a sufficient condition for queueing network stability. Define $L(\underline{U}) \triangleq \sum_{i,c}[U_i^{(c)}]^2$ as a *Lyapunov function* of unfinished work, representing a scalar measure of network congestion. For a given control policy X and a given unfinished work $\underline{U}(t)$ at time t, define the *K-step Lypapunov drift* $\Delta_K^X(\underline{U}(t))$, as follows:

$$\Delta_K^X(\underline{U}(t)) \triangleq \mathbb{E}\{L(\underline{U}(t+K)) - L(\underline{U}(t)) \,|\, \underline{U}(t)\}.$$

Theorem 3: Lyapunov Stability—If there exists a positive integer K such that for all time slots t, the K-step Lyapunov drift satisfies

$$\Delta_K^X(\underline{U}(t)) \leq C - \sum_{i,c}\theta_{ic}U_i^{(c)}(t) \tag{21}$$

for some positive constants $C, \{\theta_{ic}\}$, and if $\mathbb{E}\{L(\underline{U}(t_0))\} < \infty$ for $t_0 \in \{0, 1, \ldots, K-1\}$, then the network is stable and

$$\overline{\sum_{i,c}\theta_{ic}U_i^{(c)}} \triangleq \limsup_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\left[\sum_{i,c}\theta_{ic}\mathbb{E}\left\{U_i^{(c)}(\tau)\right\}\right] \leq C. \tag{22}$$

Proof: The proof is given in Appendix B. □

The intuition behind Theorem 3 is that drift is negative whenever backlog is sufficiently large, leading to stability. The fact that Lyapunov drift is compared after K slots (rather than after a single slot) is required for systems with non i.i.d. dynamics. Similar K-slot analysis of Lyapunov drift has been used in [27] to address stability, and similar drift statements for i.i.d. systems where $K = 1$ are found in [35]–[38].

Lemma 3: For any control policy X resulting in decision variables $\mu_{ab}^{(c)}(t)$, the K-step Lyapunov drift at any slot t_0 satisfies

$$\Delta_K^X(\underline{U}(t_0)) \leq K^2BN - 2K\left[\Phi^X(\underline{U}(t_0)) - \beta(\underline{U}(t_0))\right] \tag{23}$$

where B is defined in (14) and

$$\Phi^X(\underline{U}(t_0)) \triangleq \sum_{i,c}U_i^{(c)}(t_0)\mathbb{E}\left\{\left[\sum_b \tilde{\mu}_{ib}^{(c)}(t_0) - \sum_a \tilde{\mu}_{ai}^{(c)}(t_0)\right] \,\middle|\, \underline{U}(t_0)\right\}$$
$$\beta(\underline{U}(t_0)) \triangleq \sum_{i,c}U_i^{(c)}(t_0)\mathbb{E}\left\{\tilde{A}_i^{(c)}(t_0)\,\middle|\,\underline{U}(t_0)\right\}. \tag{24}$$

Proof: Note that $U_i^{(i)}(t_0) \triangleq 0$. For $i \neq c$, the K-step dynamics of unfinished work satisfies

$$U_i^{(c)}(t_0+K) \leq \max\left[U_i^{(c)}(t_0) - \sum_{\tau=t_0}^{t_0+K-1}\sum_b \mu_{ib}^{(c)}(\tau), 0\right] + \sum_{\tau=t_0}^{t_0+K-1}\sum_a \mu_{ai}^{(c)}(\tau) + \sum_{\tau=t_0}^{t_0+K-1} A_i^{(c)}(\tau)$$

where the above expression is an inequality instead of an equality because the total bits arriving to node i from other nodes of the network may be less than $\sum_{\tau=t_0}^{t_0+K-1}\sum_a \mu_{ai}^{(c)}(\tau)$ if these other nodes have little or no data to send, and because some arrivals during the K slot interval may also depart in the same interval. Neglecting the time subscripts t_0 and using the simplified notation $\tilde{\mu}_{ab}^{(c)} \triangleq \tilde{\mu}_{ab}^{(c)}(t_0), \tilde{A}_i^{(c)}(t_0) \triangleq \tilde{A}_i^{(c)}$, we have

$$U_i^{(c)}(t_0+K) \leq \max\left[U_i^{(c)} - \sum_b K\tilde{\mu}_{ib}^{(c)}, 0\right] + \sum_a K\tilde{\mu}_{ai}^{(c)} + K\tilde{A}_i^{(c)}$$

[where the $\tilde{\mu}_{ab}^{(c)}$ variables are defined after (18)]. By squaring both sides and noting that $\max^2[x, 0] \leq x^2$, we have

$$\left[U_i^{(c)}(t_0+K)\right]^2 - \left[U_i^{(c)}(t_0)\right]^2 \leq K^2\left(\sum_b \tilde{\mu}_{ib}^{(c)}\right)^2 + K^2\left(\sum_a \tilde{\mu}_{ai}^{(c)} + \tilde{A}_i^{(c)}\right)^2 - 2KU_i^{(c)}(t_0)\left[\sum_b \tilde{\mu}_{ib}^{(c)} - \sum_a \tilde{\mu}_{ai}^{(c)} - \tilde{A}_i^{(c)}\right].$$

The expression for Lyapunov drift is obtained from the above inequality by summing over all nodes i and commodities c and taking conditional expectations

$$\Delta_K^X(\underline{U}(t_0)) \leq K^2N\left(\left(\mu_{\max}^{\text{out}}\right)^2 + \left(\mu_{\max}^{\text{in}} + A_{\max}\right)^2\right) -2K\sum_{i,c} U_i^{(c)}(t_0)\mathbb{E}\left\{\sum_b \tilde{\mu}_{ib}^{(c)} - \sum_a \tilde{\mu}_{ai}^{(c)} - \tilde{A}_{ic} \,\middle|\, \underline{U}(t_0)\right\}$$

where we have used the $\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}$, and $A_{\max}^2$ bounds by noting that the sum of squares of positive numbers is less than or equal to the square of the sum (see details in [1]). □

B. Comparing Performance of DRPC and STAT

Using the definitions of $\Phi^X(\underline{U}(t_0))$ and $\beta(\underline{U}(t_0))$ given in (24) together with inequality (20), we have

$$\Phi^{\text{STAT}}(\underline{U}(t_0)) - \beta(\underline{U}(t_0)) \geq \frac{\epsilon}{2}\sum_{i,c} U_i^{(c)}(t_0). \quad (25)$$

Plugging this inequality directly into the Lyapunov drift expression (23) and using the drift theorem (Theorem 3) yields the following bound on network congestion under STAT:

$$\overline{\sum_{i,c} U_i^{(c)\text{STAT}}} \leq \frac{K^2BN}{K\epsilon} = \frac{KBN}{\epsilon}.$$

We now consider another algorithm FRAME, which is a frame-based modification of the DRPC policy: Scheduling, power allocation, and routing under FRAME are done every time slot exactly as in the DRPC algorithm, with the exception that backlog updates are performed only every K slots. Specifically, for any time slot τ within a K slot frame $\{t_0, t_0+1, \ldots, t_0+K-1\}$, power is allocated to maximize $\sum_{ab}\mu_{ab}(\underline{P}, \underline{S}(\tau))W_{ab}^*(t_0)$ subject to $\underline{P} \in \Pi$. Thus, current channel state information but out of date backlog information is used every slot.

Lemma 4: $\Phi^{\text{FRAME}}(\underline{U}(t_0)) \geq \Phi^{\text{STAT}}(\underline{U}(t_0))$.

Lemma 5: Comparing FRAME and DRPC, we have

$$\Phi^{\text{DRPC}}(\underline{U}(t_0)) \geq \Phi^{\text{FRAME}}(\underline{U}(t_0)) - (K-1)N\tilde{B}/2.$$

Lemma 4 is proven at the end of this section, and Lemma 5 is proven in Appendix C. Using these lemmas with (25), it follows that:

$$\Phi^{\text{DRPC}}(\underline{U}(t_0)) - \beta(\underline{U}(t_0)) \geq \frac{\epsilon}{2}\sum_{i,c} U_i^{(c)}(t_0) - \frac{(K-1)\tilde{B}N}{2}.$$

Using this bound in the Lyapunov drift expression (23) yields

$$\Delta_K^{\text{DRPC}}(\underline{U}(t_0)) \leq K^2BN + K(K-1)\tilde{B}N - K\epsilon\sum_{i,c} U_i^{(c)}(t_0).$$

Applying the drift theorem (Theorem 3) proves that DRPC is stable with the congestion bound as given in Theorem 2. □

To complete the analysis, we prove Lemma 4.

Proof of Lemma 4: We have the following identity:

$$\sum_{i,c} U_i^{(c)}(t_0)\left[\sum_b \mu_{ib}^{(c)}(\tau) - \sum_a \mu_{ai}^{(c)}(\tau)\right] = \sum_{abc}\mu_{ab}^{(c)}(\tau)\left[U_a^{(c)}(t_0) - U_b^{(c)}(t_0)\right]. \quad (26)$$

Taking conditional expectations above and summing over τ yields an alternative way to express $\Phi^X(\underline{U}(t_0))$

$$\Phi^X(\underline{U}(t_0)) = \frac{1}{K}\sum_{\tau=t_0}^{t_0+K-1}\mathbb{E}\left\{\sum_{abc}\mu_{ab}^{(c)}(\tau) \times \left[U_a^{(c)}(t_0) - U_b^{(c)}(t_0)\right] \,\middle|\, \underline{U}(t_0)\right\}.$$

For every $\tau \in \{t_0, \ldots, t_0+K-1\}$, the FRAME algorithm is designed to maximize $\sum_{abc}\mu_{ab}^{(c)}(\tau)[U_a^{(c)}(t_0) - U_b^{(c)}(t_0)]$ over all possible algorithms, including algorithm STAT. To see this, note that: $\sum_{abc}\mu_{ab}^{(c)}(\tau)\left[U_a^{(c)}(t_0) - U_b^{(c)}(t_0)\right] \leq \sum_{abc}\mu_{ab}^{(c)}(\tau)W_{ab}^*(t_0) \leq \sum_{ab}\mu_{ab}(\underline{P}(\tau), \underline{S}(\tau))W_{ab}^*(t_0)$, where the first inequality follows by definition of W_{ab}^*, and the second inequality follows from (2). By definition, the final expression is maximized under the FRAME strategy. □

VI. Distributed Implementation

The DRPC algorithm of the previous section involves solving a constrained optimization problem every time slot, where current channel state and queue backlogs appear as parameters in the optimization. Here, we consider decentralized implementations, where users attempt to maximize the weighted sum of data rates in (9) by exchanging information with their neighbors. The *current neighbors* of a node i is defined as the set $\Omega_i(t)$, representing the nodes to which node i can currently transmit and receive. Theoretically, all nodes could be neighbors, as the power transmitted from one node may be detected everywhere. However, to limit implementation complexity, it is practical to restrict neighbors to a fixed set of nearby nodes with the best channel conditions. We assume the neighbor sets $\Omega_i(t)$ are defined according to some such rule, and that nodes have knowledge of the link conditions between themselves and their neighbors and are informed of the queue backlogs of their neighbors via a low bandwidth control channel.

A. Networks With Independent Channels

Consider a network with independent channels, so that the transmission rate on any given link (a,b) depends only on the local link parameters: $\mu_{ab}(\underline{P}, \underline{S}) = \mu_{ab}(P_{ab}, S_{ab})$. Assume that the rate functions $\mu_{ab}(P_{ab}, S_{ab})$ are concave in the single power variable P_{ab} for every channel state S_{ab} (representing diminishing returns in data rate for each incremental increase in power). These assumptions are valid when all links use orthogonal coding schemes, beamforming, and/or when links are spacially separated such that channel interference is negligible.

In this case, the optimization problem (9) has a simple *decoupling property*, where the weighted sum is maximized by separately maximizing each term. This corresponds to nodes making independent power control and routing decisions based only on their local information. Indeed, each node $n \in \{1, \ldots, N\}$ maximizes $\sum_b W_{nb}^*(t)\mu_{nb}(P_{nb}, S_{nb}(t))$ subject to its power constraint $\sum_{b\in\Omega_n(t)} P_{nb} \leq P_n^{\text{tot}}$. This optimization is a standard problem of concave maximization subject to a simplex constraint, and can be solved easily in real-time with any degree of accuracy [41].

B. Distributed Approximation for Networks With Interference

Consider a network with rate-power curves described by the $\log(1+\text{SIR})$ function given in (1). This network has dependent, interfering channels, and the associated optimization problem (9) is nonlinear, nonconvex, and difficult to solve even in a centralized manner. Here, we provide a simple decentralized approximation, where nodes use a portion of each time slot to exchange control information with neighbors

1) At the beginning of each slot, nodes randomly decide to transmit with probability q. All transmitting nodes send a control signal of power γP_{tot}, where γ is some globally known scaling factor designed to limit power expended by the control signal.
2) Define Q as the set of all transmitting nodes. Each node b measures its total resulting interference $\gamma I_b \triangleq \gamma \sum_{i\in Q} \alpha_{ib} P_{\text{tot}}$, and sends this scalar quantity over a control channel to all neighbors.
3) Using knowledge of the interference, attenuation values, and queue backlogs associated with all neighboring nodes, each transmitting user a decides to transmit using full power to the single neighbor b who maximizes the function

$$W_{ab}^* \log\left(1 + \frac{\alpha_{ab}P_{\text{tot}}}{N_b + I_b - \alpha_{ab}P_{\text{tot}}}\right).$$

Note that this allows nodes to receive from multiple transmitters simultaneously, with rates that correspond to the effective SIR of each transmission. The constraint that a transmitting node cannot simultaneously receive can easily be incorporated by setting $I_b = \infty$ (or, equivalently, $\alpha_{b,b} = \infty$), for all transmitting nodes b.

By convexity of the $\log(1 + p/(n - p))$ function, it can be shown that the power allocation that maximizes $\sum_{ab} W_{ab}^* \log(1 + \text{SIR})$ [for SIR as defined in (1)] has the form where each transmitting user sends to the single neighbor as in step 3 above. However, note that there are 2^N possible subsets of transmitting users. The above distributed algorithm is not optimal over all possible power allocation strategies, as it chooses transmitters randomly. However, we note that the algorithm is *optimal over the class of all algorithms that use either zero power or full power, and that choose transmitters randomly with probability* q. This holds because the $\{0, P_{\text{tot}}\}$ restriction can be viewed as another constraint describing the set Π of acceptable power allocations, and because the random transmitter selection can be viewed as a random channel outage. Thus, the above distributed algorithm achieves network capacity over this modified channel model (see [1] for details, including bounds on capacity and delay).

The idea of randomly choosing users to transmit is similar to the technique used in the Grossglauser–Tse relay algorithm of [6], designed to achieve $O(1)$ throughput for networks with an arbitarily large number of users N. However, rather than transmitting to the nearest receiver (as in [6]), our algorithm chooses the receiver with the largest backlog-rate metric. As it is optimal over all random $\{0, P_{\text{tot}}\}$ selection algorithms, it supports a set of data rates that contains the set of rates supportable by the Grossglauser–Tse algorithm. In particular, it also achieves $O(1)$ throughput independent of N.

C. Simulation of Centralized and Distributed DRPC

Here, we apply the distributed DRPC policy of the previous subsection to an ad hoc network with mobility and inter-channel interference. Consider a square network with 10 mobile users, with user locations discretized to a 5×5 grid. The stochastic channel process $\underline{S}(t)$ is characterized by the following model of user mobility: Every time slot, users keep their locations with probability $1/2$, and with probability $1/2$ they move one step in either the North, South, West, or East directions (uniformly distributed over all feasible directions). Each user is power constrained to P_{tot}, is restricted to transmitting to only one other user in a given time slot, and cannot transmit if it is receiving. Power radiates omnidirectionally, and signal attenuation between two nodes a and b is determined by the fourth power of the distance between them (as in [15]), so that α_{ab} coefficients are

$$\alpha_{ab} = \begin{cases} 1/[((x_a - x_b)^2 + (y_a - y_b)^2)^2 + 1], & \text{if } a \neq b \\ \infty, & \text{if } a = b \end{cases}$$

where $(x_a, y_a), (x_b, y_b)$ represent user locations within the network. Note that the extra "+1" term in the denominator is inserted to model the reality that attenuation factors α_{ab} are kept below 1 (so that signal power at the receiver is never more than the corresponding power used at the transmitter). The α_{aa} values are set to infinity as a simple way to enforce the constraint that transmitting nodes cannot receive.

Multiuser interference is modeled similarly to the rate-power curve given in (1). However, rather than use the $\log(1 + \text{SIR})$ function, we use a rate curve determined by four different quadrature amplitude modulation (QAM) schemes designed for error probabilities less than 10^{-6}. The rate function is thus

$$\mu_{ab}(\underline{P}, \underline{\alpha}) = f(\text{SIR}_{ab}(\underline{P}, \underline{\alpha}))$$

where $f(\cdot)$ is a piecewise constant function of SIR, and is defined by the QAM coding schemes given in Fig. 3.

We consider the enhanced DRPC algorithm with $\theta_i^c = 1, V_i^c = 1$ for all $i \neq c$, and $V_i^i = 0$, and assume the power/noise coefficient is normalized to $P_{\text{tot}}/N_b = 20\Delta^2$,

modulation	bits/symbol	SIR/symbol
2 PAM	1	$0.25\,\Delta^2$
4 QAM	2	$0.50\,\Delta^2$
16 QAM	4	$1.25\,\Delta^2$
64 QAM	6	$5.25\,\Delta^2$

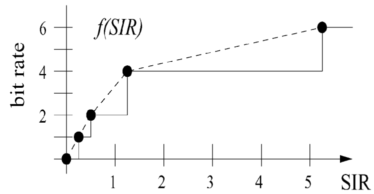

Fig. 3. Piecewise constant rate curve for the four modulation schemes described in the table. Scaled power requirements are shown, where Δ represents the minimum distance between signal points.

where Δ is the minimum distance between signal points in the QAM modulation scheme. The algorithm is approximated using the distributed implementation described in the previous subsection, where each node transmits using full power with probability $q = 1/2$. As the network is small, we simply define the neighbor set $\Omega_i(t)$ for each user i to be the set of all other nodes in the network. A centralized implementation is also considered, where the optimization problem (9) is implemented using a steepest ascent search on the piecewise linear relaxation of the $f(\mathrm{SIR})$ curve (see Fig. 3). The resulting data rates are then "floored" according the threshold levels of the piecewise constant curve $f(\mathrm{SIR})$. Note that the relaxed problem remains nonlinear and nonconvex [because SIR is nonconvex in the power variables, see (1)] and, hence, the result of the steepest ascent search may be suboptimal.

We simulate the centralized and decentralized implementations of DRPC and compare to the performance offered by the two-hop relay algorithm presented in [6]. The relay algorithm restricts routes to two-hop paths, and hence relies on user mobility for delivering data. Note that the relay algorithm was developed to demonstrate nonvanishing capacity for large networks, and was not designed to maximize throughput or achieve low delay. Thus, it is not completely fair to compare performance with the DRPC algorithms. However, the comparison illustrates that significant capacity and delay improvements are possible even among the class of algorithms that choose transmitters randomly.

We set the sender density parameter of the relay algorithm to $q = 1/2$. The relay algorithm was designed for nodes to transmit data at a fixed rate, attainable whenever the SIR for a given wireless link exceeds a threshold value. However, in order to make a fair comparison, we allow the relay algorithm to transmit at rates given by the full $f(\mathrm{SIR})$ curve. Following the scenario of [6], we assume user i desires communication with only one other user (namely, user $(i+1) \bmod N$). Unit length packets arrive according to Poisson processes, where nine of the users receive data at rate λ_1, and the remaining user receives data at rate λ_2. In Fig. 4, we plot the average network delay from simulation of the three algorithms when the rates (λ_1, λ_2) are linearly scaled upwards to the values $(.585, 2.925)$. From Fig. 4, we see that the centralized DRPC algorithm provides stability and bounded delays at more than four times the data rates of the two-hop relay algorithm, and more than twice the data rate of the decentralized DRPC algorithm. This illustrates the advantages of exploiting channel state and queue backlog

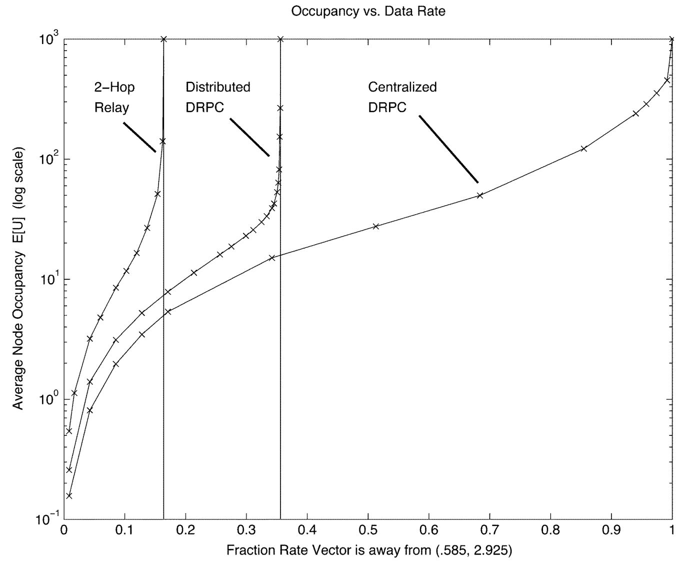

Fig. 4. Simulation results for the DRPC algorithm and the relay algorithm as rates are increased toward $(\lambda_1, \lambda_2) = (.585, 2.925)$.

information. We further note that the two-hop relay algorithm relies on full and homogeneous mobility of all users, while the DRPC algorithms have no such requirement and can be used for networks with arbitrary mobility.

VII. Multicommodity Flows and Convex Duality

The DRPC algorithm stabilizes the network and offers average delay guarantees whenever the input rate matrix is inside the capacity region of the wireless network. Here, we consider a related problem of computing an offline multicommodity flow given a known rate matrix (λ_{ij}). Classical multicommodity flow problems for wired networks can be reduced to linear programs, and fast approximation algorithms are developed in [33]. A distributed algorithm was first given in [34], and game theory approaches are developed in [29].

Here, we consider the flow problem for wireless networks, and compare the dynamic Lyapunov drift approach to a more traditional static optimization technique. A convex optimization problem corresponding to a multicommodity flow in the wireless network is formulated, and it is shown that a classical subgradient search method for solving the problem via convex duality theory corresponds exactly to a deterministic network simulation of the DRPC policy. Notions of duality are also used in [10], [13], [29], and [30] to consider static network optimization, where dual variables play the role of prices charged by the network to multiple users competing for shared network resources in order to maximize their own utility. In our context, the dual variables correspond to *queue backlogs*, rather than network prices. This illustrates a relationship between static optimization and the dynamic DRPC policy and contributes to a growing theory of *dynamic optimization*, suggesting that static algorithms can be modified and applied in dynamic settings while preserving analytical optimality.

We restrict attention to time invariant systems, so that the rate-power curve is only a function of power: $\underline{\mu}(\underline{P}, \underline{S}) = \underline{\mu}(\underline{P})$. Given a rate matrix (λ_{ij}), the problem of finding a multicommodity flow corresponds to the following convex optimization problem:

$$\begin{aligned}&\text{Maximize: } 1\\&\text{Subject to:}\\&\lambda_{ic} + \sum_a f_{ai}^{(c)} \leq \sum_b f_{ib}^{(c)} \quad \forall i, c, \quad \text{such that } i \neq c\\&\left(\left\{f_{ab}^{(c)}\right\}, \{\mu_{ab}\}\right) \in \Theta.\end{aligned} \tag{27}$$

where Θ is the set of all variables $(\{f_{ab}^{(c)}\}, \{\mu_{ab}\})$ such that

$$\begin{aligned}f_{ab}^{(c)} \geq 0, &\quad \text{for all } a, b, c \in \{1, \ldots, N\}\\f_{aa}^{(c)} = f_{ab}^{(a)} = 0, &\quad \text{for all } a, b, c \in \{1, \ldots, N\}\\\left(\sum_c f_{ab}^{(c)}\right) \leq (\mu_{ab}), &\quad \text{for some } (\mu_{ab}) \in \Gamma.\end{aligned} \tag{28}$$

The maximization function "1" is used as an artifice to pose this multicommodity flow problem in the framework of an optimization problem. Note that the set Θ is convex and compact (it inherits convexity and compactness from the set $\tilde{\Gamma}$ consisting of all link transmission rate matrices (R_{ab}) entrywise less than or equal to some element of Γ, see [1]). Moreover, the objective function "1" and all inequality constraints are linear. The optimization problem is therefore convex [40], and has a dual formulation, where the optimal solution of the dual problem exactly corresponds to an optimal solution of the original "primal" problem (27). To form the dual problem, we introduce nonnegative Lagrange multipliers $\{U_i^{(c)}\}$ for each of the inequality constraints in (27), and define the dual function

$$L\left(\left\{U_i^{(c)}\right\}\right) \triangleq \max_{(\{f_{ab}^{(c)}\}, \{\mu_{ab}\}) \in \Theta} \left[1 + \sum_{i \neq c} U_i^{(c)} \left(\sum_b f_{ib}^{(c)} - \sum_a f_{ai}^{(c)} - \lambda_{ic}\right)\right]. \tag{29}$$

The dual problem to (27) is

$$\begin{aligned}&\text{Minimize:} \quad L\left(\left\{U_i^{(c)}\right\}\right)\\&\text{Subject to:} \quad U_i^{(c)} \geq 0, \quad \text{for all } i, c \in \{1, \ldots, N\}.\end{aligned}$$

The dual problem is always convex, and the minimizing solution can be obtained using classical subgradient search methods (where the function $-L(\{U_i^{(c)}\})$ is maximized). Consider a fixed stepsize method with stepsize $T = 1$. The basic subgradient search routine starts with an initial set of values $U_i^{(c)}(0)$ for the Lagrange multipliers, and upon each iteration $t = \{1, 2, \ldots\}$ these values are updated by computing a subgradient $\underline{\eta}$ for one time unit, and, if necessary, projecting the result back onto the set of nonnegative values [40]

$$U_i^{(c)}(t+1) = \max\left[U_i^{(c)}(t) + \eta_i^{(c)}, 0\right]. \tag{30}$$

However, it is shown in [40] that a particular subgradient of $-L(\{U_i^{(c)}\})$ is

$$\underline{\eta} = \left(-\sum_b f_{ib}^{*(c)} + \sum_a f_{ai}^{*(c)} + \lambda_{ic}\right)\Bigg|_{(i,c) \in \{1, \ldots, N\}^2} \tag{31}$$

where the $\{f_{ab}^{*(c)}\}$ variables are solutions to the maximization in (29) with $U_i^{(c)} = U_i^{(c)}(t)$. Using (31) in (30) for all $i \neq c$, we find

$$U_i^{(c)}(t+1) = \max\left[U_i^{(c)}(t) - \sum_b f_{ib}^{*(c)} + \sum_a f_{ai}^{*(c)} + \lambda_{ic}, 0\right]. \tag{32}$$

From (32), it is apparent that the Lagrange multipliers $\{U_i^{(c)}(t)\}$ play the role of unfinished work in a multinode queueing system with input rates λ_{ic}, where $U_i^{(c)}(t)$ represents the amount of commodity c bits in node i. In this way, the $f_{ab}^{*(c)}$ values can be viewed as the transmission rates allocated to commodity c traffic on link (a, b). Equation (32), thus, states that the unfinished work at time $t + 1$ is equal to the unfinished work at time t plus the net influx of bits into node i. Thus, the operation of projecting the Lagrangian variables onto the positive orthant acts exactly as an implementation of the standard queueing equation.

It is illuminating to calculate the optimal $f_{ab}^{*(c)}$ values by performing the maximization in (29). To this end, we need to maximize $\sum_{i \neq c} U_i^{(c)}(t)(\sum_b f_{ib}^{(c)} - \sum_a f_{ai}^{(c)})$ subject to the constraints of (28). However, as in the proof of Lemma 4, we can switch the sum to find

$$\sum_{i \neq c} U_i^{(c)} \left(\sum_b f_{ib}^{(c)} - \sum_a f_{ai}^{(c)}\right) = \sum_{abc} f_{ab}^{(c)} \left[U_a^{(c)} - U_b^{(c)}\right].$$

Remarkably, from the right-hand side above, it is apparent that the optimal values $f_{ab}^{*(c)}$ are identical to the resulting link rates $\mu_{ab}^{(c)}(\underline{P})$ that would be computed if the DRPC algorithm were used to calculate routing and power allocation decisions in a network problem with unfinished work levels $U_i^{(c)}(t)$. It follows that the DRPC algorithm can be viewed as a dynamic implementation of a subgradient search method for computing the solution to an optimization problem using convex duality. This suggests a deeper relationship between stochastic network control algorithms and subgradient search methods. It would be interesting to explore how the two interact and build upon each other. For example, there are several known improvements to classical subgradient search routines. Perhaps such improvements could reduce the complexity of optimal and suboptimal dynamic network controllers. Also, note that the optimization problem (27), which maximizes the function "1," can be adjusted to maximize some other performance criteria, which may offer additional quality of service guarantees in the corresponding dynamic network control problem.

VIII. Conclusion

We have formulated a general power allocation problem for a multinode wireless network with time-varying channels and adaptive transmission rates. The problem was formulated at the network layer assuming a given (but arbitrary) set of rate-power functions corresponding to the particular modulation and coding strategy being used at the physical layer. These rate-power functions provide a simple separation between physical layer and network layer concepts while enabling network control algorithms to be adapted to the unique channel characteristics of the wireless links. The network capacity region was established, and a DRPC algorithm was developed and shown to stabilize the network whenever the arrival rate matrix is within the capacity region. Such stability holds for arbitrary ergodic arrival and channel processes, even if these processes are unknown to the network controller. Delay bounds were derived and shown to grow asymptotically in N/ϵ, where N represents the size of the network and ϵ represents a measure of distance between the arrival rates and the capacity region boundary. Distributed implementations of DRPC were considered for ad hoc mobile networks, and the use of rate and backlog information was shown to offer considerable performance gains.

The dynamic operation of the DRPC policy was shown to be fundamentally related to a classical iterative technique for solving a static convex program, where unification of the two problems is achieved through the theory of convex duality. We believe that such *dynamic optimization* contributes to bridging the gap between theoretical optimization techniques and implementable control algorithms.

Appendix A
Network Capacity Region Λ

Here, we establish that $(\lambda_{ij}) \in \Lambda$ is a necessary condition for stability in a wireless network. The proof uses the following preliminary lemma from [1, Ch. 2].

Lemma 6: Necessary Condition for Queue Stability—Let $U_i^{(c)}(t)$ represent the unfinished work values in a slotted time queueing network. If the network is stable, then for any $\delta > 0$, there exists a finite value V for which arbitrarily large times $\tilde{t}$ can be found so that $\Pr[\sum_{ic} U_i^{(c)}(\tilde{t}) \leq V] > 1-\delta$. In particular, for the case $\delta = 1/2$, there exists a value V such that the probability that work in all queues simultaneously drops below V is greater than 1/2 infinitely often.

Theorem 1a: Necessary Condition for Stability—The condition $(\lambda_{ic}) \in \Lambda$ is necessary for network stability.

Proof: Consider a system with rate convergent inputs with rates (λ_{ic}), and let process $X_i^{(c)}(t)$ represent the amount of commodity c bits that exogenously enter the network at node i during the interval $[0,t]$. Suppose the system is stabilizable by some routing and power control policy, perhaps one that bases decisions upon complete knowledge of future arrivals and channel states. We show that multicommodity flow variables must exist that satisfy (5)–(7). Let $U_i^{(c)}(t)$ represent the resulting unfinished work function for commodity c in node i under this stabilizing policy. Further, let $F_{ab}^{(c)}(t)$ represent the total number of bits from commodity c transmitted over the (a,b) link during the first t slots. We have for all time

$$F_{ab}^{(c)}(t) \geq 0, \quad F_{aa}^{(c)}(t) = F_{ab}^{(a)}(t) = 0, \quad \forall a,b,c \tag{33}$$

$$X_i^{(c)}(t) - U_i^{(c)}(t) = \sum_b F_{ib}^{(c)}(t) - \sum_a F_{ai}^{(c)}(t), \quad \forall i \neq c \tag{34}$$

$$\sum_c F_{ab}^{(c)}(t) \leq \sum_{\tau=1}^{t} \mu_{ab}(\underline{P}(\tau), \underline{S}(\tau)), \quad \forall (a,b) \tag{35}$$

where (34) follows because the unfinished work in any node is equal to the difference between the total bits that have arrived and departed. Inequality (35) holds because the total bits transferred over any link (a,b) is less than or equal to the offered transmission rate summed up to time t.

By the necessary condition for network stability specified in Lemma 6, there must exist some finite value V such that at arbitrarily large times $\tilde{t}$, the unfinished work in all queues is simultaneously less than V with probability at least $1/2$. Let $T_{\underline{S}}(t)$ represent the slots of $\{0,\ldots,t\}$ during which the channel is in state $\underline{S}$. Let $\|T_{\underline{S}}(t)\|$ denote the total number of these slots, and note that $\|T_{\underline{S}}(t)\|/t \rightarrow \pi_{\underline{S}}$ with probability 1 for each $\underline{S}$. Fix an arbitrarily small value $\epsilon > 0$. We seek to find a time $\tilde{t} \geq V/\epsilon$ such that all of the following inequalities are satisfied:

$$\frac{U_i^{(c)}(\tilde{t})}{\tilde{t}} \leq \frac{V}{\tilde{t}} \leq \epsilon, \quad \forall i,c \tag{36}$$

$$\frac{X_i^{(c)}(\tilde{t})}{\tilde{t}} \geq \lambda_{ic} - \epsilon, \quad \text{for all } i \neq c \tag{37}$$

$$\frac{\|T_{\underline{S}}(\tilde{t})\|}{\tilde{t}} \leq \pi_{\underline{S}} + \epsilon, \quad \text{for all channel states } \underline{S}. \tag{38}$$

Define $A_{\tilde{t}}$ as the event that (36) is satisfied, and note that arbitrarily large values of $\tilde{t}$ exist such that $Pr[A_{\tilde{t}}] > 1/2$. Define $B_{\tilde{t}}$ as the event that both (37) and (38) are satisfied. Because there are a finite number of input processes and channel states, we have that $\Pr[B_{\tilde{t}}] \rightarrow 1$ as $\tilde{t} \rightarrow \infty$. It follows there is a time $\tilde{t}$ such that $\Pr[A_{\tilde{t}}] > 1/2$ and $\Pr[B_{\tilde{t}}] > 1/2$. Hence, $\Pr[A_{\tilde{t}} \cap B_{\tilde{t}}] = \Pr[A_{\tilde{t}}] + \Pr[B_{\tilde{t}}] - \Pr[A_{\tilde{t}} \cup B_{\tilde{t}}] \geq \Pr[A_{\tilde{t}}] + \Pr[B_{\tilde{t}}] - 1 > 0$. That is, with nonzero probability, all inequalities (36)–(38) are simultaneously satisfied.

Now, define variables $f_{ab}^{(c)} \triangleq F_{ab}^{(c)}(\tilde{t})/\tilde{t}$. It is clear from (33) that these flow variables satisfy the constraints (5). Using (36) and (37) in (34), it follows that for all $i \neq c$:

$$\lambda_{ic} \leq \sum_b f_{ib}^{(c)} - \sum_a f_{ai}^{(c)} + 2\epsilon \tag{39}$$

and, hence, the flow conservation constraint (6) is arbitrarily close to being satisfied. Applying inequality (35) at time $\tilde{t}$ and considering entrywise matrix inequalities, we have

$$\begin{aligned}\left(\sum_c f_{ab}^{(c)}\right) &\leq \left(\frac{1}{\tilde{t}} \sum_{\tau=1}^{\tilde{t}} \mu_{ab}(\underline{P}(\tau), \underline{S}(\tau))\right) \\ &= \sum_{\underline{S}} \frac{\|T_{\underline{S}}(\tilde{t})\|}{\tilde{t}} \frac{1}{\|T_{\underline{S}}(\tilde{t})\|} \left(\sum_{\tau \in T_{\underline{S}}(\tilde{t})} \mu_{ab}(\underline{P}(\tau), \underline{S})\right) \\ &= \sum_{\underline{S}} \frac{\|T_{\underline{S}}(\tilde{t})\|}{\tilde{t}} \left(\mu_{ab}^{\underline{S}}\right) \end{aligned} \tag{40}$$

where the matrices $(\mu_{ab}^{\underline{S}})$ in (40) are elements of Convex_Hull$\{\underline{\mu}(\underline{P},\underline{S}) \,|\, \underline{P} \in \Pi\}$. Using (38) in (40), we find

$$\left(\sum_c f_{ab}^{(c)}\right) \leq \sum_{\underline{S}} \pi_{\underline{S}} \left(\mu_{ab}^{\underline{S}}\right) + \epsilon\left(\mu_{ab}^{\max}\right) \text{Card}\{\underline{S}\} \tag{41}$$

where Card$\{\underline{S}\}$ represents the number of channel states $\underline{S}$, and $\mu_{ab}^{\max}$ represents the maximum possible transmission rate of the (a,b) link. Hence, the right-hand side of inequality (41) is arbitrarily close to a point in Γ (compare with (7)).

Hence, with nonzero probability, the multicommodity flows $f_{ab}^{(c)}$ (defined in terms of the $F_{ab}^{(c)}(t)$ processes) satisfy (39) and (41). It follows that *there must exist* flow values $f_{ab}^{(c)}$ that satisfy (39) and (41) (otherwise, the inequalities would occur with probability 0). As the multicommodity flow constraints (5)–(7) are arbitrarily close to being satisfied, it follows that they *can* be satisfied if each nonzero entry of the (λ_{ic}) rate matrix is reduced by an arbitrarily small amount. This proves that the input rate matrix (λ_{ic}) is a limit point of the capacity region Λ. In [1] it is shown that the capacity region is compact. Hence, it contains its limit points, so that $(\lambda_{ic}) \in \Lambda$. □

APPENDIX B
LYAPUNOV DRIFT THEOREM

Here, we prove Theorem 3, establishing a sufficient condition for network stability. The proof uses a simple telescoping series argument similar to [35], [38], and [39], together with the machinery of the $g(V)$ overflow function.

Proof of Theorem 3 : Consider (21) at times $t = jK + t_0$, where $t_0 \in \{0, \ldots, K-1\}$. Taking expectations of this inequality over the distribution of $\underline{U}(jK+t_0)$ and summing over j from $j=0$ to $j=J-1$ creates a telescoping series, yielding

$$\mathbb{E}\{L(\underline{U}(JK+t_0))\} - \mathbb{E}\{L(\underline{U}(t_0))\} \leq CJ - \sum_{j=0}^{J-1}\sum_{ic}\theta_{ic}\mathbb{E}\left\{U_i^{(c)}(jK+t_0)\right\}.$$

Dividing by J and using nonnegativity of the Lyapunov function, we have

$$\frac{1}{J}\sum_{j=0}^{J-1}\sum_{ic}\theta_{ic}\mathbb{E}\left\{U_i^{(c)}(jK+t_0)\right\} \leq C + \mathbb{E}\{L(\underline{U}(t_0))\}/J.$$

The above inequality holds for all t_0. Summing over $t_0 \in \{0, \ldots, K-1\}$ yields

$$\frac{1}{J}\sum_{\tau=0}^{JK-1}\sum_{ic}\theta_{ic}\mathbb{E}\left\{U_i^{(c)}(\tau)\right\} \leq KC + \sum_{t_0=0}^{K-1}\mathbb{E}\{L(\underline{U}(t_0))\}/J.$$

Dividing by K and taking the lim sup of the above inequality as $J \to \infty$ yields the performance bound (22).

To prove stability, note the performance bound implies that for any queue (i,c)

$$\limsup_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\left\{U_i^{(c)}(\tau)\right\} \leq C/\theta_{ic}.$$

Now, considering the overflow function $g_{ic}(V)$, we have

$$\begin{aligned} g_{ic}(V) &\triangleq \limsup_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\Pr\left[U_i^{(c)}(\tau) > V\right] \\ &\leq \limsup_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\left\{U_i^{(c)}(\tau)\right\}\Big/V \leq \frac{C}{\theta_{ic}V} \end{aligned}$$

where we have used the fact that $Pr[U > V] \leq \mathbb{E}\{U\}/V$ for any nonnegative random variable U. Taking limits as $V \to \infty$ shows that $g_{ic}(V) \to 0$ and proves stability. □

APPENDIX C
COMPARING DRPC AND FRAME

Proof of Lemma 5: Consider an implementation of the DRPC algorithm, and let $\underline{U}(t_0)$ represent the unfinished work matrix at the start of a frame. Let $\underline{U}(\tau)$ represent the unfinished work at some time τ during the frame $\{t_0, \ldots, t_0+K-1\}$. At any such time τ, the DRPC algorithm selects transmission rates $\mu_{ab}^{(c)}(\tau)$ that maximize $\sum_{a,b,c}\mu_{ab}^{(c)}(\tau)[U_a^{(c)}(\tau) - U_b^{(c)}(\tau)]$ over all other possible control decisions. Hence

$$\sum_{a,b,c}\mu_{ab}^{(c)\text{DRPC}}(\tau)\left[U_a^{(c)}(\tau) - U_b^{(c)}(\tau)\right] \geq \sum_{a,b,c}\mu_{ab}^{(c)\text{FRAME}}(\tau)\left[U_a^{(c)}(\tau) - U_b^{(c)}(\tau)\right]$$

where the values $\mu_{ab}^{(c)\text{FRAME}}(\tau)$ represent the control decisions that would be made by the FRAME algorithm at time τ if the backlog matrix at time t_0 were $\underline{U}(t_0)$. Using (26) to switch the summation, we have

$$\begin{aligned} &\sum_{i,c}U_i^{(c)}(\tau)\left[\sum_b \mu_{ib}^{(c)\text{DRPC}}(\tau) - \sum_a \mu_{ai}^{(c)\text{DRPC}}(\tau)\right] \\ &\quad\geq \sum_{i,c}U_i^{(c)}(\tau)\left[\sum_b \mu_{ib}^{(c)\text{FRAME}}(\tau) - \sum_a \mu_{ai}^{(c)\text{FRAME}}(\tau)\right]. \end{aligned}$$

Defining $\delta_i^{(c)}(\tau) \triangleq U_i^{(c)}(\tau) - U_i^{(c)}(t_0)$ and noting that $\delta_i^{(c)}(t_0) = 0$, it follows that:

$$\begin{aligned} &\sum_{i,c}U_i^{(c)}(t_0)\left[\sum_b \mu_{ib}^{(c)\text{DRPC}}(\tau) - \sum_a \mu_{ai}^{(c)\text{DRPC}}(\tau)\right] \\ &\quad\geq \sum_{i,c}U_i^{(c)}(t_0)\left[\sum_b \mu_{ib}^{(c)\text{FRAME}}(\tau) - \sum_a \mu_{ai}^{(c)\text{FRAME}}(\tau)\right] \\ &\quad\quad - 2\sum_{i,c}\left|\delta_i^{(c)}(\tau)\right|\max\left[\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}\right] \end{aligned} \tag{42}$$

where we used the fact that:

$$\left|\sum_{i,c} \delta_i^{(c)}(\tau) \left[\sum_b \mu_{ib}^{(c)}(\tau) - \sum_a \mu_{ai}^{(c)}(\tau)\right]\right| \leq \sum_{i,c} \left|\delta_i^{(c)}(\tau)\right| \max\left[\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}\right].$$

Summing (42) over $\tau \in \{t_0, \ldots, t_0+K-1\}$ and taking conditional expectations yields [using the definition of $\Phi^X(\underline{U}(t))$ in (24)]

$$\Phi^{\text{DRPC}}(\underline{U}(t_0)) \geq \Phi^{\text{FRAME}}(\underline{U}(t_0)) - \frac{2}{K} \sum_{\tau=t_0+1}^{t_0+K-1} \mathbb{E}\left\{\sum_{i,c} \left|\delta_i^{(c)}(\tau)\right|\right\} \max\left[\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}\right].$$

The expected magnitude of change in unfinished work from time t_0 to time τ is at most $(\mu_{\max}^{\text{out}} + \mu_{\max}^{\text{in}} + A_{\max})(\tau - t_0)$ at any node, which leads to

$$\Phi^{\text{DRPC}}(\underline{U}(t_0)) \geq \Phi^{\text{FRAME}}(\underline{U}(t_0)) - \frac{2\max[\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}]}{K} \times N \sum_{\tau=t_0+1}^{t_0+K-1} (\tau - t_0)\left(\mu_{\max}^{\text{out}} + \mu_{\max}^{\text{in}} + A_{\max}\right).$$

Changing variables to $v \triangleq \tau - t_0$ and using the fact that $\sum_{v=1}^{K-1} v = K(K-1)/2$, we have

$$\Phi^{\text{DRPC}}(\underline{U}(t_0)) \geq \Phi^{\text{FRAME}}(\underline{U}(t_0)) - (K-1)N \max\left[\mu_{\max}^{\text{out}}, \mu_{\max}^{\text{in}}\right]\left(\mu_{\max}^{\text{out}} + \mu_{\max}^{\text{in}} + A_{\max}\right).$$

This proves the lemma [note definition of $\tilde{B}$ in (15)]. □

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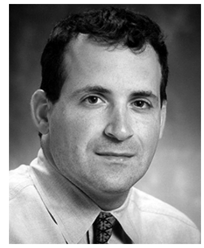

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