Loop-Free Backpressure Routing Using Link-Reversal Algorithms

Anurag Rai, Chih-ping Li, Georgios Paschos, and Eytan Modiano, *Fellow, IEEE*

***Abstract*— The backpressure routing policy is known to be a throughput optimal policy that supports any feasible traffic demand, but may have poor delay performance when packets traverse loops in the network. In this paper, we study loop-free backpressure routing policies that forward packets along directed acyclic graphs (DAGs) to avoid the looping problem. These policies use link reversal algorithms to improve the DAGs in order to support any achievable traffic demand. For a network with a single commodity, we show that a DAG that supports a given traffic demand can be found after a finite number of iterations of the link-reversal process. We use this to develop a joint link-reversal and backpressure routing policy, called the loop free backpressure (LFBP) algorithm. This algorithm forwards packets on the DAG, while the DAG is dynamically updated based on the growth of the queue backlogs. We show by simulations that such a DAG-based policy improves the delay over the classical backpressure routing policy. We also propose a multicommodity version of the LFBP algorithm and via simulation show that its delay performance is better than that of backpressure.**

***Index Terms*— Loop free, throughput optimal, backpressure, link reversal.**

I. Introduction

THROUGHPUT and delay are two common metrics used to evaluate the performance of communication networks. For networks that exhibit high variability, such as mobile ad hoc networks, the dynamic backpressure routing policy [1] is a highly desirable solution, known to maximize throughput in a wide range of settings. However, the delay performance of backpressure is poor [2]. The high delay is attributed to a property of backpressure that allows the packets to loop within the network instead of moving towards the destination. In this paper we improve the delay performance of backpressure by constraining the routing to loop free paths.

Manuscript received March 16, 2016; revised April 17, 2017; accepted June 7, 2017; approved by IEEE/ACM TRANSACTIONS ON NETWORKING Editor K. Psounis. Date of publication July 3, 2017; date of current version October 13, 2017. This work was supported by the NSF under Grant CNS-1524317 and Grant CNS-1116209, by the ONR under Grant N00014-12-1-0064, and by DARPA I2O and Raytheon BBN Technologies under Contract No. HROO 1 l-l 5-C-0097. The work of G. Paschos was supported by the National and Community Funds (European Social Fund) through the WiNC Project of the Action: Supporting Postdoctoral Researchers. This paper is an extended version of [15], which appeared in the Proceedings of ACM MobiHoc, 2015. *(Corresponding author: Anurag Rai.)*

A. Rai and E. Modiano are with the Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: rai@mit.edu; modiano@mit.edu).

C.-P. Li was with the LIDS, Massachusetts Institute of Technology, Cambridge, MA 02139 USA. He is now with Qualcomm Research, San Diego, CA 92121 USA (e-mail: cpli@qti.qualcomm.com).

G. Paschos was with the Massachusetts Institute of Technology, Cambridge, MA 02139 USA, and also with CERTH-ITI, Thessaloniki 57001, Greece. He is now with the Mathematical and Algorithmic Sciences Laboratory, Huawei Technologies Co., Ltd., 92100 Boulogne–Billancourt, France (e-mail: georgios.paschos@huawei.com).

Digital Object Identifier 10.1109/TNET.2017.2715807

To eliminate loops in the network, we assign directions to the links such that the network becomes a directed acyclic graph (DAG). Initially, we generate an arbitrary DAG and use backpressure routing over it. If the initial DAG has max-flow smaller than the traffic demand, parts of the network become overloaded. By reversing the direction of the links that point from non-overloaded to overloaded nodes a new DAG with a lower overload is obtained. Iterating over this process, our distributed algorithm gradually converges to a DAG that supports any traffic demand feasible in the network. Hence the loop-free property is achieved without the loss of throughput.

Prior work identifies looping as a main cause for high delays in backpressure routing and proposes delay-aware backpressure techniques. Backpressure enhanced with hop count bias is first proposed in [3] to drive packets through paths with smallest hop counts when the load is low. An alternative backpressure modification that utilizes shortest path information is proposed in [8]. A different line of works proposes to learn the network topology using backpressure and then use this information to enhance routing decisions. In [7] backpressure is constrained to a subgraph which is discovered by running unconstrained backpressure for a time period and computing the average number of packets routed over each link. Learning is effectively used in scheduling [9] and utility optimization [13] for wireless networks. In our work we aim to eliminate loops by restricting backpressure to a DAG, then dynamically improving the DAG by reversing the links in a distributed manner.

The link-reversal algorithms were introduced in [4] as a means to maintain connectivity in networks with volatile links. These distributed algorithms react to any topological changes to obtain a DAG such that each node has a loop-free path to the destination. In [5], one of the link-reversal algorithms was used to design a routing protocol (called TORA) for multihop wireless networks. Although these algorithms provide loop free paths and guarantee connectivity from the nodes to the destination, they do not maximize throughput. Thus, the main goal of this paper is to create a new link-reversal algorithm and combine it with the backpressure algorithm to construct a distributed throughput optimal algorithm with improved delay performance. The main contributions of this paper are as follows:

- For a single commodity network, we study the lexicographic optimization of the queue growth rate. We develop a novel link-reversal algorithm that reverses link direction based on the overload conditions to form a new DAG with lexicographically smaller queue growth rates.
- We propose the loop free backpressure (LFBP) algorithm, a distributed routing scheme that eliminates

loops and retains the throughput optimality property. This is achieved by exploiting the properties of backpressure regarding the queue growth rates in an overloaded network.
- Our simulation results of LFBP show a significant delay improvement over backpressure in static and dynamic networks.
- We extend the LFBP algorithm to networks with multiple commodities, and provide a simulation result to show its delay improvement over backpressure.

II. System Model and Definitions

A. Network Model

We consider the problem of routing single-commodity data packets in a network. The network is represented by a graph $G = (N, E)$, where N is the set of nodes and E is the set of undirected links $\{i, j\}$ with capacity c_{ij}. Packets arrive at the source node s at rate λ and are destined for a receiver node d. Let $f^{\max}$ denote the maximum flow from node s to d in the network G. The quantity $f^{\max}$ is the maximally achievable throughput at the destination node d. In this paper, we do not solve the link scheduling problem, i.e. we assume that all links can be scheduled at the same time.

To avoid unnecessary routing loops, we restrict forwarding along a directed acyclic graph (DAG) embedded in the graph G. An optimal DAG exists to support the max-flow $f^{\max}$ and can be found by: (i) computing a feasible flow allocation (f_{ij}) that yields the max-flow $f^{\max}$ in G (e.g. using [11]); (ii) trimming any positive flow on directed cycles; (iii) defining an embedded DAG by assigning a direction for each link $\{i, j\}$ according to the direction of the flow f_{ij} on that link. Since backpressure achieves the max-flow of a constrained graph [14], performing backpressure routing over the optimal DAG supports λ.

This centralized approach is unsuitable for communication networks, especially when the link capacities are time-varying or when the network undergoes frequent topology changes. In such situations, the optimal embedded DAG also changes with time, which requires constantly repeating the above offline process. Instead, it is possible to use a distributed adaptive mechanism that reverses the direction of links until a DAG that supports the current traffic demand is found. In this paper we propose an algorithm that reacts to the traffic conditions and changes in network topology by switching the direction of some links. To understand the properties of the link-reversing operations, we first study the fluid level behavior of a network under overload conditions.

B. Flow Equations

Consider an *embedded DAG* $D_k = (N_k, E_k)$ in the network graph G, where $N_k = N$ is the set of network nodes and E_k is the set of directed links.[1] For each undirected link $\{i, j\} \in E$, either (i, j) or (j, i) belongs to E_k (but not both). Each directed link (i, j) has the capacity of the undirected counterpart $\{i, j\}$, which is c_{ij}. Let $f_k^{\max}$ be the maximum flow of the DAG D_k from the source node s to the destination node d. Any embedded DAG has smaller or equal max-flow with respect to G, $f_k^{\max} \leq f^{\max}$.

For two disjoint subsets A and B of nodes in D_k, we define $\mathrm{cap}_k(A, B)$ as the total capacity of the directed links going from A to B, i.e.,

$$\mathrm{cap}_k(A, B) = \sum_{(i,j)\in E_k : i\in A,\, j\in B} c_{ij}. \tag{1}$$

A cut is a partition of nodes (A, A^c) such that $s \in A$ and $d \in A^c$. A cut (A_k, A_k^c) is a min-cut if it minimizes the expression $\mathrm{cap}_k(A_k, A_k^c)$ over all cuts. By the max-flow min-cut theorem $f_k^{\max} = \mathrm{cap}_k(A_k, A_k^c)$, where (A_k, A_k^c) is a min-cut of the DAG D_k. We remark that a cut in D_k is also a cut in G or another embedded DAG. However, the value of $\mathrm{cap}_k(A, A^c)$ for partition (A, A^c) depends on the graph considered (see summation in (1)), and thus the min-cuts may differ substantially per DAG.

We consider the network as a time-slotted system, where slot t refers to the time interval $[t, t+1)$, $t \in \{0, 1, 2, \ldots\}$. Each network node n maintains a queue $Q_n(t)$, where $Q_n(t)$ also denotes the queue backlog at time t. We have $Q_d(t) = 0$ for all t since any packet reaching the destination is removed from the network immediately. Let $A(t)$ be the number of exogenous packets arriving at the source node s in slot t. Under a routing policy that forwards packets over the directed links defined by the DAG D_k, let $F_{ij}(t)$ be the number of packets that are transmitted over the directed link $(i, j) \in E_k$ in slot t; the link capacity constraint states that $F_{ij}(t) \leq c_{ij}$ for all t. The queues $Q_n(t), n \neq d$, are updated over slots according to

$$Q_n(t) = Q_n(t-1) + 1_{[n=s]}A(t) + \sum_{i:(i,n)\in E_k} F_{in}(t) - \sum_{j:(n,j)\in E_k} F_{nj}(t), \tag{2}$$

where $1_{[\cdot]}$ is an indicator function.

To study the overload behavior of the system we define the queue overload (i.e., growth) rate at node n as

$$q_n = \lim_{t\to\infty} \frac{Q_n(t)}{t}. \tag{3}$$

Additionally, define the exogenous packet arrival rate λ and the flow f_{ij} over a directed link (i, j) as

$$\lambda = \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} A(\tau), \quad f_{ij} = \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} F_{ij}(\tau),$$

where the above limits are assumed to exist almost surely (see [6] for details). Using the recursion (2), taking time averages and letting $t \to \infty$, we have the fluid-level equation:

$$q_n = 1_{[n=s]}\lambda + \sum_{i:(i,n)\in E_k} f_{in} - \sum_{j:(n,j)\in E_k} f_{nj}, \quad \forall n \in N\backslash\{d\} \tag{4}$$

$$0 \leq f_{ij} \leq c_{ij}, \quad \forall (i, j) \in E_k. \tag{5}$$

Equations (4) and (5) are the flow conservation and link capacity constraints, respectively. A network node n is said to be overloaded if its queue growth rate q_n is positive, which

[1]The notation D_k of an embedded DAG is useful in the paper; it will denote the DAG that is formed after the kth iteration of the link-reversal algorithm.

implies that $Q_n(t) \to \infty$ as $t \to \infty$ (see (3) and [10]). Summing (4) over $n \in N$ yields

$$\sum_{i:(i,d)\in E_k} f_{id} = \lambda - \sum_{n\in N} q_n, \tag{6}$$

where $\sum_{i:(i,d)\in E_k} f_{id}$ denotes the throughput received at the destination d. Therefore, equation (6) states that the received throughput is equal to the exogenous arrival rate λ less the sum of queue growth rates $\sum_{n\in N} q_n$ in the network.

C. Properties of Queue Overload Vector

If the traffic arrival rate λ is strictly larger than the maximum flow $f_k^{\max}$ of the DAG D_k, then from (6),

$$\sum_{n\in N} q_n = \lambda - \sum_{i:(i,d)\in E_k} f_{id} \geq \lambda - f_k^{\max} > 0, \tag{7}$$

which implies that $q_n > 0$ for some node $n \in N$. Let $\boldsymbol{q} = (q_n)_{n\in N}$ be the queue overload vector. A queue overload vector $\boldsymbol{q}$ is feasible in the DAG D_k if there exist overload rates $(q_n)_{n\in N}$ and flow variables $(f_{ij})_{(i,j)\in E_k}$ that satisfy (4) and (5). Let $\mathcal{Q}_k$ be the set of all feasible queue overload vectors in D_k. We are interested in the *lexicographically smallest* queue overload vector in set $\mathcal{Q}_k$. Formally, given a vector $\boldsymbol{u} = (u_1, \ldots, u_N)$, let $\overline{u}_i$ be the ith maximal component of $\boldsymbol{u}$. We say that a vector $\boldsymbol{u}$ is *lexicographically smaller* than a vector $\boldsymbol{v}$, denoted by $\boldsymbol{u} <_{\text{lex}} \boldsymbol{v}$, if $\overline{u}_1 < \overline{v}_1$ or $\overline{u}_i = \overline{v}_i$ for all $i = 1, \ldots, (j-1)$ and $\overline{u}_j < \overline{v}_j$ for some $j = 2, \ldots, N$. If $\overline{u}_i = \overline{v}_i$ for all i, then the two vectors are lexicographically equal, represented by $\boldsymbol{u} =_{\text{lex}} \boldsymbol{v}$.[2] The above-defined vector comparison induces a total order on the set $\mathcal{Q}_k$, and hence the existence of a lexicographically smallest vector is always guaranteed [12].

Lemma 1 [6]: Let $\boldsymbol{q}_k^{min}$ be the lexicographically smallest vector in the queue overload region $\mathcal{Q}_k$ of the DAG D_k. We have the following properties:

1) *The vector $\boldsymbol{q}_k^{min}$ is unique in the set $\mathcal{Q}_k$.*
2) *The vector $\boldsymbol{q}_k^{min}$ minimizes the sum of queue overload rates, i.e., it is a solution to the optimization problem:*

$$\textit{minimize} \sum_{n\in N} q_n, \quad \textit{subject to } \boldsymbol{q} \in \mathcal{Q}_k$$

(direct consequence of Theorem 1 in [6]). Due to (6)*, the corresponding throughput is maximized.*

3) *A feasible flow allocation vector $(f_{ij})_{(i,j)\in E_k}$ induces $\boldsymbol{q}_k^{min}$ if and only if over each link $(i,j) \in E_k$ the following holds:*

$$\textit{if } q_i < q_j, \quad \textit{then } f_{ij} = 0, \tag{8}$$
$$\textit{if } q_i > q_j, \quad \textit{then } f_{ij} = c_{ij}. \tag{9}$$

In general, there are many flow allocations that yield the maximum throughput. Focusing on those that additionally induce $\boldsymbol{q}_k^{\min}$ has two advantages. First, these allocations lead to link-reversal operations that improve the max-flow of the DAG D_k. Second, the backpressure algorithm can be used to preform the same reversals and improve the max-flow. We will use these observations to combine link-reversal algorithms with backpressure routing.

[2]Lexicographic order is also known as dictionary order. Two vectors $\boldsymbol{u} = (3,2,1,2,1)$ and $\boldsymbol{v} = (1,2,3,2,2)$ satisfy $\boldsymbol{u} <_{\text{lex}} \boldsymbol{v}$ because $\overline{u}_1 = \overline{v}_1 = 3$, $\overline{u}_2 = \overline{v}_2 = \overline{u}_3 = \overline{v}_3 = 2$, and $\overline{u}_4 = 1 < \overline{v}_4 = 2$.

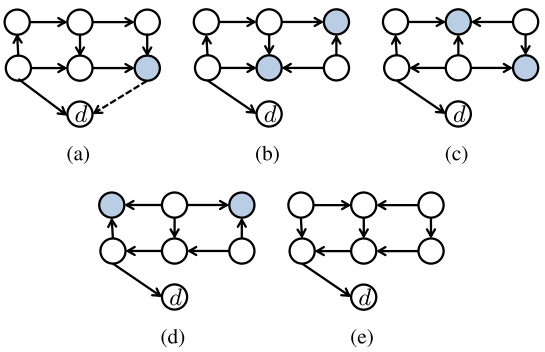


Fig. 1. Illustration of the Gafni-Bertsekas link reversal when the dashed link in Figure 1(a) is lost. At every iteration, the algorithm reverses all the links incident to the nodes with no outgoing link, here represented by the colored nodes. (a) Original DAG. (b) First iteration. (c) Second iteration. (d) Third iteration. (e) Fourth iteration.

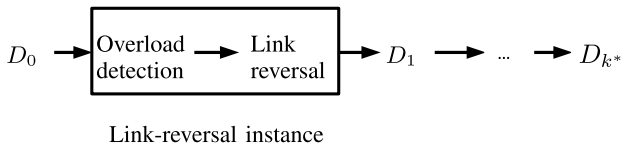


Fig. 2. Iterative process to find the DAG D_{k^*} that supports the throughput λ.

III. Link-Reversal Algorithms

The link-reversal algorithms given in [4], henceforth called the Gafni-Bertsekas link reversal, were designed to maintain a path from each node in the network to the destination. One algorithm relevant to this paper is the *full reversal method*. This algorithm is triggered when some nodes $n \neq d$ lose all of their outgoing links. At every iteration of the algorithm, nodes n, that have no outgoing link, reverse the direction of all their incoming links. This process is repeated until all the nodes other than the destination have at least one outgoing link. When the process stops these nodes are guaranteed to have a path to the destination. The example in Figure 1, taken from [4], illustrates this algorithm at work.

Although the full reversal algorithm guarantees connectivity, the resulting throughput may be significantly lower than the maximum possible. Hence, in this paper we shift the focus from connectivity to maximum throughput. Specifically, we propose a novel link-reversal algorithm that produces a DAG which supports the traffic demand λ, assuming $\lambda \leq f^{\max}$.

We propose an iterative algorithm which begins with an embedded DAG and at each iteration produces a new embedded DAG that supports an *improved* lexicographically smallest overload vector. Within each iteration, an implied routing policy operates on the current DAG yielding the lexicographically optimal flow allocation. A sketch of this process is shown in Figure 2.

A. Initial DAG D_0

We assume that each node in the network has a unique ID. We use these IDs as a topological ordering to the nodes. So the

initial DAG can be created simply by directing each link to go from the node with the lower ID to the node with the higher ID. If the unique IDs are not available, the initial DAG can be created by using a strategy such as the one given in [5].

B. Overload Detection

Given a DAG D_k, $k = 0, 1, 2, \ldots$, we suppose that there is a routing policy π that yields the lexicographically minimal queue overload vector $\boldsymbol{q}_k^{\min}$. [3] Then we use the vector $\boldsymbol{q}_k^{\min}$ to detect node overload and decide whether a link should be reversed.

If the data arrival rate λ is less than or equal to the maximum flow $f_k^{\max}$ of the DAG D_k, then there exists a flow allocation (f_{ij}) that supports the traffic demand and yields zero queue overload rates $q_n = 0$ at all nodes $n \in N$. By the second property of Lemma 1 and nonnegativity of the overload vector, the queue overload vector $\boldsymbol{q}_k^{\min}$ is zero. Thus, the throughput under policy π is λ according to (6), and the current DAG D_k supports λ.

On the other hand, if the arrival rate λ is strictly larger than the maximum flow $f_k^{\max}$, by the second property in Lemma 1 the maximum throughput is $f_k^{\max}$ and the queue overload vector $\boldsymbol{q}_k^{\min} = (q_{k,n}^{\min})_{n \in N}$ is nonzero because we have from (7) that

$$\sum_{n \in N} q_{k,n}^{\min} > \lambda - f_k^{\max} > 0.$$

We may therefore detect the event "DAG D_k supports λ" by testing whether the overload vector $\boldsymbol{q}_k^{\min}$ is zero or non-zero.

The next lemma shows that if DAG D_k does not support λ then it contains at least one under-utilized link (our link-reversal algorithm will reverse the direction of such links to improve network throughput).

Lemma 2: Suppose that the traffic demand λ satisfies

$$f_k^{\max} < \lambda \leq f^{\max}.$$

where $f_k^{\max}$ is the max-flow of the DAG D_k and $f^{\max}$ is the max-flow of the undirected network G. Then there exists a link $(i, j) \in E_k$ such that $q_{k,i}^{min} = 0$ and $q_{k,j}^{min} > 0$.

Proof of Lemma 2: Let A_k be the set of overloaded nodes under a flow allocation that induces the lexicographically minimal overload vector $\boldsymbol{q}_k^{\min}$ in the DAG D_k; the set A_k is nonempty due to $\lambda > f_k^{\max}$ and (7). It follows that the partition (A_k, A_k^c) is a min-cut of D_k (see Lemma 7 in the Appendix).[4] By the max-flow min-cut theorem, the capacity of the min-cut (A_k, A_k^c) in D_k satisfies $\mathrm{cap}_k(A_k, A_k^c) = f_k^{\max} < f^{\max}$.

The proof is by contradiction. Let us assume that there is no directed link that goes from the set A_k^c to A_k in the DAG D_k. It follows that $\mathrm{cap}_k(A_k, A_k^c)$ is the sum of capacities of all undirected links between the sets A_k and A_k^c, i.e.,

$$\mathrm{cap}_k(A_k, A_k^c) = \sum_{i \in A_k,\, j \notin A_k} c_{ij},$$

which is equal to the value of the cut (A_k, A_k^c) in graph G. Since the value of any cut is larger or equal to the min-cut, applying the max-flow min-cut theorem on G we have

$$f^{\max} \leq \sum_{i \in A_k,\, j \notin A_k} c_{ij} = \mathrm{cap}_k(A_k, A_k^c) = f_k^{\max},$$

which contradicts the assumption that $f_k^{\max} < \lambda \leq f^{\max}$. □

C. Link Reversal

We consider the link-reversal algorithm (Algorithm 1) that reverses all the links that go from an overloaded node to a non-overloaded node. Lemma 2 shows that such links always exist if the DAG D_k has insufficient capacity to support the traffic demand $\lambda \leq f^{\max}$ under the lexicographically minimum overflow vector $\boldsymbol{q}_k^{\min}$. This reversal yields a new directed graph $D_{k+1} = (N, E_{k+1})$.

Algorithm 1 Link-Reversal Algorithm

```
for all (i, j) ∈ E_k do
    if q^min_{k,i} = 0 and q^min_{k,j} > 0 then
        (j, i) ∈ E_{k+1}
    else
        (i, j) ∈ E_{k+1}
    end if
    k ← k + 1
end for
```

The rest of the section focuses on proving that this algorithm obtains a DAG that supports the traffic demand λ within a finite number of iterations. We begin by showing that every intermediate graph produced by this algorithm is a DAG.

Lemma 3: The directed graph D_{k+1} is acyclic.

Proof of Lemma 3: Recall that A_k is the set of overloaded nodes in the DAG D_k under the lexicographically minimum queue overload vector $\boldsymbol{q}_k^{\min}$. Let $L_k \subseteq E$ be the set of undirected links between A_k and A_k^c. Algorithm 1 changes the link direction in a subset of L_k. More precisely, it enforces the direction of all links in L_k to go from A_k to A_k^c.

We complete the proof by construction in two steps. First, we remove all links in L_k from the DAG D_k, resulting in two disconnected subgraphs that are DAGs themselves. Second, consider that we add a link in L_k back to the network with the direction going from A_k to A_k^c. This link addition does not create a cycle because there is no path from A_k^c to A_k, and the resulting graph remains to be a DAG. We can add the other links in L_k one-by-one back to the graph with the direction from A_k to A_k^c; similarly, these link additions do not create cycles. The final directed graph is D_{k+1}, and it is a DAG. See Fig. 3 for an illustration. □

The next lemma shows that the new DAG D_{k+1} supports a lexicographically *smaller* optimal overload vector (and therefore potentially better throughput) than the DAG D_k.

Lemma 4: Let D_k be a DAG with the maximum flow $f_k^{\max} < \lambda \leq f^{\max}$. The DAG D_{k+1}, obtained by performing Algorithm 1 over D_k, has the lexicographically minimum queue overload vector satisfying $\boldsymbol{q}_{k+1}^{min} <_{\mathrm{lex}} \boldsymbol{q}_k^{min}$.

[3] Such a policy π can simply solve an optimization problem offline to compute the required flow allocation. In Section IV, we develop a distributed algorithm using backpressure that does not require the computation of the lexicographically optimal overload vector. We use this vector only to prove the properties of our link-reversal algorithm.

[4] The set A_k^c contains the destination node d and is nonempty.

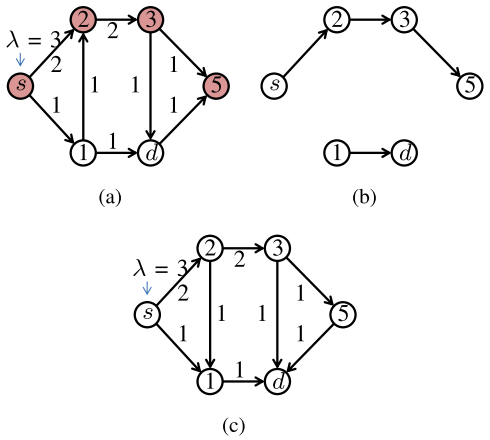


Fig. 3. Illustration for the proof of Lemma 3. (a) The DAG D_k with $A_k = \{s, 2, 3, 5\}$. (b) Two disconnected DAGs formed by removing all links between A_k and A_k^c. (c) The DAG D_{k+1} formed by adding all links in L_k back to the graph with the direction going from A_k to A_k^c.

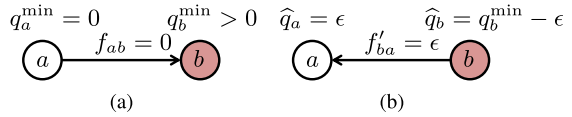


Fig. 4. A link $\{a, b\}$ in the network in Fig. 3 before and after link reversal. Before the reversal, the flow f_{ab} is zero on (a, b). After the reversal, an ϵ flow can be sent over (b, a) so that $(\widehat{q}_a, \widehat{q}_b) <_{\text{lex}} (q_{k,a}^{\min}, q_{k,b}^{\min})$, while the rest of the flow allocation remains the same. (a) Link (a, b) before the link reversal. (b) Link (b, a) after the link reversal.

Proof of Lemma 4: Consider a link $(a, b) \in E_k$ such that $q_{k,a}^{\min} = 0$ and $q_{k,b}^{\min} > 0$; this link exists by Lemma 2. From the property (8), any feasible flow allocation (f_{ij}) that yields the lexicographically minimum overload vector $\boldsymbol{q}_k^{\min}$ must have $f_{ab} = 0$ over link (a, b). The link-reversal algorithm reverses the link (a, b) so that $(b, a) \in E_{k+1}$ in the DAG D_{k+1}. Consider the following feasible flow allocation (f'_{ij}) on the DAG D_{k+1}:

$$f'_{ij} = \begin{cases} \epsilon & \text{if } (i,j) = (b,a) \\ 0 = f_{ji} & \text{if } (i,j) \neq (b,a) \text{ but } (j,i) \text{ is reversed} \\ f_{ij} & \text{if } (i,j) \text{ is not reversed} \end{cases}$$

where $\epsilon < q_{k,b}^{\min}$ is a sufficiently small value. In other words, the flow allocation (f'_{ij}) is formed by reversing links and keeping the previous flow allocation (f_{ij}) except that we forward an ϵ-amount of overload traffic from node b to a. Let $\widehat{\boldsymbol{q}} = (\widehat{q}_n)_{n \in N}$ be the resulting queue overload vector. We have

$$\widehat{q}_b = q_{k,b}^{\min} - \epsilon < q_{k,b}^{\min},\ \widehat{q}_a = \epsilon > q_{k,a}^{\min} = 0, \text{ and}$$
$$\widehat{q}_n = q_{k,n}^{\min},\ n \notin \{a, b\}.$$

Therefore, $\widehat{\boldsymbol{q}} <_{\text{lex}} \boldsymbol{q}_k^{\min}$ (see Fig. 4 for an illustration). Let $\boldsymbol{q}_{k+1}^{\min}$ be the lexicographically minimal overload vector in D_{k+1}. It follows that $\boldsymbol{q}_{k+1}^{\min} \leq_{\text{lex}} \widehat{\boldsymbol{q}} <_{\text{lex}} \boldsymbol{q}_k^{\min}$, completing the proof. □

Theorem 1: Suppose the traffic demand is feasible in G, i.e., $\lambda \leq f^{\max}$, and the routing policy induces the overload vector $\boldsymbol{q}_k^{\min}$ at every iteration k. Then, the link-reversal algorithm will find a DAG whose maximum flow supports λ in a finite number of iterations.

Proof of Theorem 1: The link-reversal algorithm creates a sequence of DAGs $\{D_0, D_1, D_2, \ldots, D_{k^*}\}$, where $q_{k^*}^{\min} = \mathbf{0}$. From Lemma 4, we know that a strict improvement in the lexicographically minimal overload vector is made after each iteration, i.e.,

$$\boldsymbol{q}_0^{\min} >_{\text{lex}} \boldsymbol{q}_1^{\min} >_{\text{lex}} \boldsymbol{q}_2^{\min} >_{\text{lex}} \cdots.$$

The lexicographically minimal overload vector is unique in a DAG by Lemma 1, the DAGs $\{D_0, D_1, D_2, \ldots, D_{k^*}\}$ must all be distinct. Since there are a finite number of unique embedded DAGs in a network, in a finite number of iterations the link-reversal algorithm will find a DAG D_{k^*} that has the lexicographically minimal overload vector $\boldsymbol{q}_{k^*}^{\min} = \mathbf{0}$ and the maximum flow $f_{k^*}^{\max} \geq \lambda$. Note that such a DAG D_{k^*} exists because the undirected graph G has the maximum flow $f^{\max} \geq \lambda$. □

Hence, when $\lambda \leq f^{\max}$, the process of obtaining the lexicographically smallest overload vector and using Algorithm 1 to produce new DAGs eventually finds that DAG that supports the arrival rate.

D. Arrivals Outside Stability Region

We show that even when $\lambda > f^{\max}$, the link reversal algorithm will stop reversing the links in a finite number of iterations, and it will obtain the DAG that supports the maximum throughput $f^{\max}$. We begin by examining the termination condition of our algorithm and show that if the algorithm stops at iteration k, then the DAG D_k supports the max-flow of the network.

Lemma 5: Consider the situation when $\lambda > f_k^{\max}$. If there is no link (i, j) such that $q_{k,i}^{\min} = 0$ and $q_{k,j}^{\min} > 0$, then $f^{\max} = f_k^{\max}$ and $\lambda > f^{\max}$. That is, if there are no links to reverse at iteration k, and $\mathbf{q}_k^{\min} > 0$, then the throughput of D_k is equal to $f^{\max}$.

Proof of Lemma 5: Let A_k be the set of overloaded nodes under a flow allocation that induces the lexicographically minimal overload vector $\mathbf{q}_k^{\min}$ in the DAG D_k. We know that (A_k, A_k^c) is a min-cut of the network from Lemma 7 (in the appendix), so

$$cap_k(A_k, A_k^c) = f_k^{\max}.$$

Suppose the link reversal algorithm stops after iteration k, i.e. at iteration k there are no links to reverse. In this situation, there is no link (i, j) such that $q_{k,i}^{\min} = 0$ and $q_{k,j}^{\min} > 0$, so by property (9), all the links between A_k and A_k^c go from A_k to A_k^c. The capacity of the cut (A_k, A_k^c) is given by

$$cap_k(A_k, A_k^c) = \sum_{i \in A_k, j \in A_k^c} c_{ij}.$$

This is equal to the capacity of the cut (A_k, A_k^c) in the undirected network G. So $f^{\max} \leq cap_k(A_k, A_k^c) = f_k^{\max}$. Because $f_{\max}^k$ cannot be greater than f^{max}, $f_{\max}^k = f^{\max}$. By assumption $\lambda > f_k^{\max}$, so $\lambda > f^{\max}$. □

When $\lambda > f^{\max}$, this lemma shows that the link reversal algorithm stops only when the DAG achieves the maximum

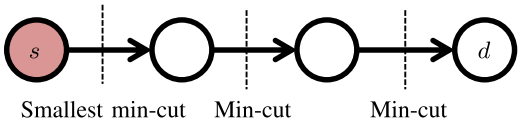


Fig. 5. Different min-cuts in a unit capacity line network. While every min-cut creates a bottleneck for the network, the smallest min-cut is the first bottleneck and has the smallest number of nodes in the source side of the cut.

throughput of the network. Hence, if the DAG doesn't support the maximum throughput, then there exists a link that can be reversed. After each reversal, Lemma 3 holds, so the directed graph obtained after the reversal is acyclic. We can modify Lemma 4 to show that every reversal produces a DAG that supports an improved lexicographically optimal overload vector. We can combine these results to prove the following theorem.

Theorem 2: Suppose the traffic demand is not feasible in G, i.e., $\lambda > f^{\max}$, and the routing policy induces the overload vector $\mathbf{q}_k^{\min}$ at every iteration k. Then, the link-reversal algorithm will find a DAG whose maximum flow supports f^{max} in a finite number of iterations.

IV. Distributed Dynamic Algorithm

In the previous sections we developed a link reversal algorithm based on the assumption that we have a routing policy that lexicographically minimized the overload vector $\mathbf{q}_k^{\min}$. The algorithm used $\mathbf{q}_k^{\min}$ to identify the cut (A_k, A_k^c), then reversed all the links that went from the nodes in A_k^c to the nodes in A_k. We then used the properties of lexicographical minimization to show that repeating this process for some iterations results in a DAG that supports the arrival rate λ. We note that any algorithm that can identify this cut (A_k, A_k^c) can be used to perform the reversals, regardless of whether or not it minimizes the overload vector, because it would perform the exact same reversals as the one in Algorithm 1.

In this section, we develop a new method for identifying the cut (A_k, A_k^c) using the backpressure routing algorithm and perform the link-reversals with it. Then we use the results from the previous section to claim that this process also obtains the optimal DAG.

We begin by showing that the cut (A_k, A_k^c) is a unique min-cut of the DAG defined below as the *smallest min-cut*. An example to illustrate this concept is given in Figure 5. We will then develop a threshold based algorithm that uses the queue backlog information of backpressure to identify this min-cut. This will enable us to perform the same reversal that we performed in the previous section without computing the lexicographically minimal overload vector.

Definition 1: We define the smallest min-cut (X^*, X^{*c}) in the DAG D_k as the min-cut with the smallest number of nodes in the source side of the cut, i.e., (X^*, X^{*c}) solves

$$\begin{aligned} &\textit{minimize:} \quad |X| \\ &\textit{subject to:} \quad (X, X^c) \textit{ is a min-cut of } D_k. \end{aligned}$$

The algorithm starts by creating an initial DAG D_0 using the method presented in Section III-A. Then, we use the backpressure algorithm to route the packets from the source to the destination over D_0. Let $Q_n(t)$ be the queue length at node n in slot t. The backpressure algorithm can be written as in Algorithm 2. It simply sends packets on a link (i, j) if node i has more packets than j.

Algorithm 2 Backpressure Algorithm (BP)

```
for all (i,j) ∈ E_k do
    if Q_i(t) ≥ Q_j(t) then
        Transmit min{c_ij, Q_i(t)} packets from i to j
    end if
    Update Q_i(t)
end for
Update Q_j(t)
```

Since backpressure is throughput optimal [1], if the arrival rate is less than $f_0^{\max}$, then all queues are stable. If the arrival rate is larger than $f_0^{\max}$, the system is unstable and the queue length grows at some nodes. In this case, the next lemma shows that if we were using a routing policy that produced the optimal overload vector $\mathbf{q}_k^{\min}$, the set of all the overloaded nodes A_k and the non-overloaded nodes A_k^c form the smallest min-cut of the DAG D_k.

Lemma 6: Let A_k be the set of overloaded nodes under a flow allocation (f_{ij}) that induces the lexicographically minimum overload vector in the DAG D_k. If $|A_k| > 0$, then (A_k, A_k^c) is the unique smallest min-cut in D_k.

Proof of Lemma 6: The proof is in Appendix B. □

Essentially, at every iteration, the link reversal algorithm of Section III discovers the smallest min-cut (A_k, A_k^c) of the DAG D_k and reverses the links that go from A_k^c to A_k. Now the following theorem shows that the backpressure algorithm can be augmented with some thresholds to identify the smallest min-cut.

Theorem 3: Assume that (A_k, A_k^c) is the smallest min-cut for DAG D_k with a cut capacity of $f_k^{\max} = cap\,(A_k, A_k^c) < \lambda$. If packets are routed using the backpressure routing algorithm, then there exist finite constants T and R such that the following happens:

1) *For some $t < T$, $Q_n(t) > R$ for all $n \in A_k$, and*
2) *For all t, $Q_n(t) < R$ for $n \in A_k^c$.*

Proof of Theorem 3: We will prove the two claims separately. The proof will use the fact that the smallest min-cut forms the first bottleneck for the DAG D_k which will overload A^k and prevent backlog to build in A_k^c. The detailed proofs for both claims are given in the Appendix C. □

Each node n has a threshold-based smallest min-cut detection mechanism. When we start using a particular DAG D_k, in each time-slot, we check whether the queue crosses a prespecified threshold R_k. Any queue that crosses the threshold gets marked as overloaded. After using the DAG D_k for T_k timeslots, all the nodes that have their queue marked overloaded form the set A_k. When the time T_k and threshold R_k are large enough, the cut (A_k, A_k^c) is the smallest min-cut as proven in Theorem 3. After determining the smallest min-cut, an individual node can perform a link reversal by comparing its queue's overload status with its neighbor's.

All the links that go from a non-overloaded node to an overloaded node are reversed to obtain D_{k+1}. The complete LFBP algorithm is given in Algorithm 3.

Algorithm 3 LFBP (Executed by Node n)

```
Input: sequences {T_k}, {R_k}, unique ID n
Generate initial DAG D_0 by directing each link {n, j} to
    (n, j) if n < j, to (j, n) if j > n.
Mark the queue Q_n as not overloaded
Initialize t ← 0, k ← 0
while true do
    Use BP to send/recive packets on all links of node n
    if (Q_n(t) > R_k) then
        Mark Q_n as overloaded.
    end if
    t ← t + 1

    T_k ← T_k − 1
    if T_k = 0 then
        Reverse all links (j, n) such that Q_j is not overloaded
            and Q_n is overloaded.
        k ← k + 1
        Mark Q_n as not overloaded
    end if
end while
```

This algorithm simply adds link reversal to BP, hence the complexity of LFBP is just the sum of the BP and link-reversal algorithm. At each time-slot the link reversal algorithm checks whether a node is overloaded, so the computation required is $O(|N|)$. BP requires $O(|E|)$ computation at each time-slot because the algorithm computes the differential backlog for each link. Hence, the total computation required by the network for one time-slot of LFBP is $O(|N| + |E|)$.

Finally we give the following corollary that shows that the LFBP algorithm finds the optimal DAG.

Corollary 1: Suppose the traffic demand is feasible in G, i.e., $\lambda \leq f^{\max}$. Then, the LFBP algorithm (Algorithm 3) will find a DAG, whose maximum flow supports λ, in a finite number of iterations.

Proof of Corollary 1: Theorem 3 shows that LFBP identifies the smallest min-cut (A, A^c) for the DAG D_k. Lemma 6 shows that A is the set of overloaded nodes, and A^c is the set of non-overloaded nodes in a flow allocation that induces the lexicographically minimal overload vector. LFBP reverses the links going from A^c to A, which is also the reversals performed by the link reversal algorithm (Algorithm 1). Hence, by Theorem 1, LFBP obtains the DAG that supports λ. □

Good choices for the thresholds T_k and R_k are topology dependent. When the value of R_k is too small, nodes that are not overloaded might cross the threshold producing a false positive. If the value of R_k is large but T_k is small, the overloaded nodes might not have enough time to develop the backlog to cross R_k which produces false negatives. Hence, a good strategy is to choose a large R_k so that the non-overloaded nodes don't (or rarely) cross this threshold, then chose a large T_k such that the overloaded nodes have enough time to build the backlog to cross R_k. Optimizing these thresholds requires further research. Note that our algorithm performance degrades graciously with false positives/negatives. Even when it detects the smallest min-cut incorrectly, the actions of the algorithm preserve the acyclic structure. Thus, in the subsequent iterations the algorithm can improve the DAG again.

A. Multi-Source Single Destination Networks

There are many scenarios, e.g. sensor networks, when several nodes in the networks need to send data to a central destination. The Gafni-Bertsekas link-reversal algorithm in [4] was also designed for this situation. In such networks, Algorithm 3 obtains a DAG that supports the given arrivals, provided that the arrivals are supportable by the undirected network. We can see this by transforming the multi source network into an equivalent single source network.

Let us consider a network with arrival of rate $\lambda_n \geq 0$ at node n; $\lambda_d = 0$. We can do the following transformation to convert this network into a single-source single-destination network where the result of Corollary 1 holds. We create a fictitious source s' with an arrival rate of $\lambda_{s'} = \sum_n \lambda_n$ then add fictitious links (s', n) with capacity λ_n for al n. We know that Algorithm 3, finds a DAG that supports the arrival rate $\lambda_{s'}$ in this modified network. The only way to stabilize this network is to have an arrival of rate λ_n on each node n. Hence, this DAG must also stabilize the multi-source network.

B. Preventing Dead Ends

There can be several DAGs that support a given arrival rate in a particular undirected network. Some of these DAGs can include dead-ends, i.e. a node that has no path to the destination. When a packet reaches such a node, it gets stuck withing the network forever. Moreover, having dead ends cannot improve the throughput of a network. If we perform a flow allocation, for the optimal throughput, none of the flows can pass through a dead end node. In Algorithm 3, the dead end nodes either never receive any packets because they are unreachable from the source, or they receive minimal packets after some time because they build a high backlog. In either case, this algorithm achieves the required throughput in the long run. Nevertheless having dead ends is an undesired phenomenon, and we would like to avoid it.

To remove dead-ends, we propose to use the Gafni-Bertsekas link reversal once the network is stable. When the source node detects that it is no longer overloaded, it can broadcasts a message to all the nodes informing them to perform such a reversal. We know that the Gafni-Bertsekas link-reversal obtains a DAG where all the nodes have a path to the destination, hence, it results in a dead-end free DAG. Also, from [4, Proposition 2], we also know that any node that has a path to the destination does not perform a reversal at any point of the algorithm. That is, all the existing paths from a node to the destination stay intact during the iterations. This says that the algorithm does not decrease the throughput of the network from the source to the destination. Hence, it will produce a dead end free DAG that also supports the arrivals.

C. Algorithm Modification for Topology Changes

In this section we consider networks with time-varying topologies, where several links of graph G may appear or disappear over time. Although the DAG that supports λ depends on the topology of G, our proposed policy LFBP can adapt to the topology changes and efficiently track the optimal solution. Additionally, the loop free structure of a DAG is preserved under link removals.

To handle the appearance of new links in the network smoothly, we will slightly extend LFBP to guarantee the loop free structure. For a DAG D_k, every node n stores a unique state $x_n(k)$ representing its position in the topological ordering of the DAG D_k. The states are maintained such that they are unique and all the links go from a node with the lower state to a node with the higher state. When a new link $\{i, j\}$ appears we can set its direction to go from i to j if $x_i(k) < x_j(k)$ and from j to i otherwise. Since this assignment of direction to the new link is in alignment with the existing links in the DAG, the loop-free property is preserved.

The state for each node n can be initialized using the unique node ID during the initial DAG creation, i.e. $x_n(0) = n$. Then whenever a reversal is performed the state of node n can be updated as follows:

$$x_n(k) = \begin{cases} x_n(k-1) - 2^k\Delta, & \text{if } n \text{ is overloaded,} \\ x_n(k-1), & \text{otherwise.} \end{cases}$$

Here, Δ is some constant chosen such that $\Delta > \max_{i,j\in N} x_i(0) - x_j(0)$. Note that this assignment of state is consistent with the way the link directions are assigned by the link reversal algorithm. The states for the non-overloaded nodes are unchanged, so the links between these nodes are unaffected. Also, the states for all the overloaded nodes are decreased by the same amount $2^k\Delta$, so the direction of the links between the overloaded nodes is also preserved. Furthermore, the quantity $-2^k\Delta$ is less than the lowest possible state before the kth iteration, so the overloaded nodes have a lower state than the non-overloaded nodes. Hence, the links between the overloaded and non-overloaded nodes go from the overloaded nodes to the non-overloaded nodes.

In this scheme, the states x_n decrease unboundedly as more reversals are preformed. In order to prevent this, after a certain number of reversals, we can rescale the states by dividing them by a large positive number. This decreases the value of the state while maintaining the topological ordering of the DAG. The number of reversals k can be reset to 0, and a new Δ can be chosen such that it is greater than the largest difference between the rescaled states.

V. Complexity Analysis

To understand the number of iteration the link-reversal algorithm takes to obtain the optimal DAG, we analyze the time complexity of the algorithm.

Theorem 4: Let C be a vector of the capacities of all the links in E, and let I be the set of indices $1, 2, \ldots, |E|$. Define $\delta > 0$ to be the smallest positive difference between the capacity of any two cuts. Specifically, δ is the solution of the following optimization problem

$$\min_{A,B\subseteq I} \sum_{a\in A} c_a - \sum_{b\in B} c_b$$
$$\textit{subject to:} \quad \sum_{a\in A} c_a > \sum_{b\in B} c_b.$$

The number of iterations taken by the link reversal algorithm before it stops is upper bounded by $\lceil |N| \frac{f^{\max}}{\delta} \rceil$, where $f^{\max}$ is the max-flow of the undirected network.

Proof of Theorem 4: After each iteration of the link-reversal algorithm, either the max-flow of the DAG increases, or the max-flow stays the same and the number of nodes in the source side of the smallest min-cut increases (see Lemma 8 in the Appendix). We can bound the number of consecutive iterations such that there is no improvement in the max-flow. In particular, every such iteration will add at least one node to the source set. So, it is impossible to have more than $|N| - 2$ such iteration. Hence, every $|N|$ iterations we are guaranteed to have at least one increase in the max-flow.

Max-flow is equal to the min-cut capacity, and min-cut capacity is defined as the sum of link capacities. Say, the max-flow of DAG D_{k+1} is greater than that of D_k. Let A be the set of indices (in the capacity vector C) of the links in the min-cut of D_{k+1} , and B be the set of indices of the links in the min-cut of D_k. This choice of A and B forms a feasible solution to the optimization problem given in the theorem statement. Since the optimal solution δ lower bounds all the feasible solutions in the minimization problem, the increase in the max-flow must be greater than or equal to δ.

Every $|N|$ iteration the max-flow increases at least by δ. Hence, the DAG supporting the max-flow $f^{\max}$ is formed within $\lceil |N| f^{\max}/\delta \rceil$ iterations. □

Corollary 2: In a network where all the link capacities are rational with the least common denominator $\mathcal{D} \in \mathbb{N}$, the number of iterations is upper bounded by $(|N|\mathcal{D}f^{\max})$.

Proof of Corollary 2: Since the capacities are rational we can write the capacity of the i^{th} link as $c_i = \frac{\mathcal{N}_i}{\mathcal{D}}$, where $\mathcal{N}_i$ is a natural number. From the definition of δ in Theorem 4, we get δ to be the value of the following optimization problem:

$$\min_{A,B\subseteq I} \frac{1}{\mathcal{D}} \left(\sum_{a\in A} \mathcal{N}_a - \sum_{b\in B} \mathcal{N}_b \right)$$
$$\text{subject to:} \sum_{a\in A} \mathcal{N}_a > \sum_{b\in B} \mathcal{N}_b.$$

All the $\mathcal{N}_{(.)}$ are integers, so to satisfy the constraint we must have the difference $\sum_{a\in A} \mathcal{N}_a - \sum_{b\in B} \mathcal{N}_b \geq 1$. Hence $\delta \geq \frac{1}{\mathcal{D}}$. Using this value of δ in Theorem 4, we can see that the number of iterations is upper bounded by $(|N|\mathcal{D}f^{\max})$. □

Corollary 3: In a network with unit capacity links, the number of iterations the link-reversal algorithm takes to obtain the optimal DAG is upper bounded by $|N||E|$.

Proof of Corollary 3: The max-flow $f^{\max} \leq |E|$. So, by Corollary 2, the number of iterations is upper bounded by $|N||E|$. □

We conjecture that these upper bounds are not tight, and finding a tighter bound will be pursued in the future research. We simulated the link reversal algorithm in

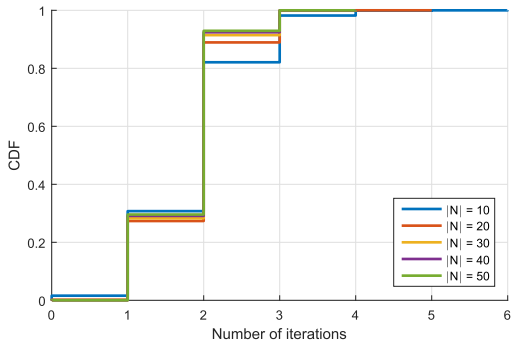


Fig. 6. CDF of the number of iterations taken by the link reversal algorithm to obtain the optimal DAG for Erdős-Rényi networks with $|N|$ nodes.

Erdős-Rényi networks ($p = 0.5$) with $|N| = 10, 20, \ldots, 50$. For each $|N|$ we generated 10^6 different graphs and randomly assigned capacities to the links. The link reversal algorithm started with a random initial DAG. We found that it took less than 2 iterations on average to find the optimal DAG. A plot of the emperical CDF is given in Figure 6. We also performed similar simulations with completely connected graphs with random link capacities. This experiment produced similar results. It took less than 2 iterations on average and a maximum of 5 iterations to find the optimal DAG.

A worst case lower bound for the number of iteration is $|N|$. This lower bound can be achieved in a line network where the initial DAG has all of its links in the wrong direction.

VI. Simulation Results

We compare the delay performance of the LFBP algorithm and the BP algorithm via simulations. We will see that the network with the LFBP routing has a smaller backlog on average under the same load. This shows that the LFBP algorithm has a better delay performance. We consider two types of networks for the simulations: a simple network with fixed topology, and a network with grid topology where the links appear and disappear randomly.

A. Fixed Topology

We consider a network with the topology shown in Figure 7(a). The edge labels represent the link capacities. The undirected network has the maximum throughput of 15 packets per time slot. Figure 7(b) shows the initial DAG D_0. Instead of running the initial DAG algorithm of Section III-A, here we choose a zero throughput DAG to test the worst-case performance of LFBP. The arrivals to the network are Poisson with rate $\lambda = 15\rho$, where we vary $\rho = .5, .55, \ldots, .95$. For the LFBP algorithm, we set the overload detection threshold to $R_k = 60$ for all n, k. To choose this parameter, we observed that the backlog buildup in normal operation rarely raises above 60 at any non-overloaded node. We also choose the detection period $T_1 = 150$ and $T_k = 50$ for all $k > 1$. This provides enough time for buildup, which improve the accuracy of the overload detection mechanism.

We simulate both algorithms for one million slots, using the same arrival process sample path. Figures 7(c) - 7(e) show the various DAGs that are formed by the LFBP algorithm at iterations $k = 1, 2, 3$. We can see that the nodes in the smallest min-cut get overloaded and the link reversals gradually improve the DAG until the throughput optimal DAG is reached.

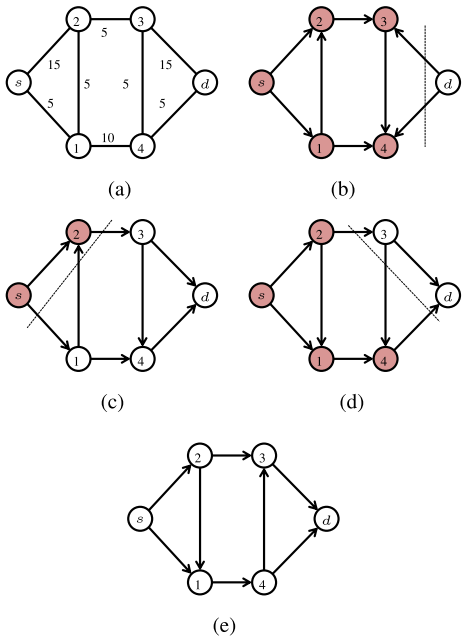


Fig. 7. Figure (a) depicts the original network. Figures (b)-(e) are the various stages of the DAG. The red nodes represent the overloaded nodes, and the dashed line shows the boundary of the overloaded and the non-overloaded nodes.

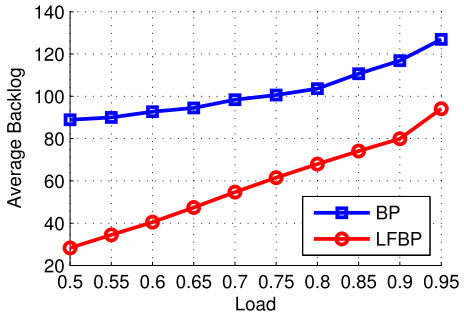


Fig. 8. Average backlog in the network (Fig. 7(a)) with fixed topology for the Loop Free Backpressure (LFBP) and the Backpressure (BP) algorithms.

Figure 8 compares the total average backlog in the network for BP and LFBP, which is indicative of the average delay. A significant delay improvement is achieved by LFBP, for example at load 0.5 the average delay is reduced by 66% We observe that the gain in the delay performance is more pronounced when the load is low. In low load situations, the network doesn't have enough "pressure" to drive the packets to the destination and so under BP the packets go in loops. Figure 9 shows the evolution of the average backlog over time for a specific load of 0.5. We can see that the backlog grows until $t = T_1 = 150$ because the initial DAG has zero throughput. After the reversals start the backlog decreases and converges around the average backlog of 30.

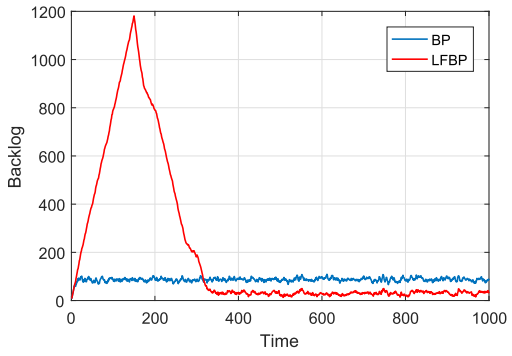


Fig. 9. Evolution of total backlog in the network over time for $\rho = 0.5$. The backlog grows at the rate of $\lambda = 7.5$ until the first reversal at $t = 150$ because the initial DAG was chosen to have zero throughput.

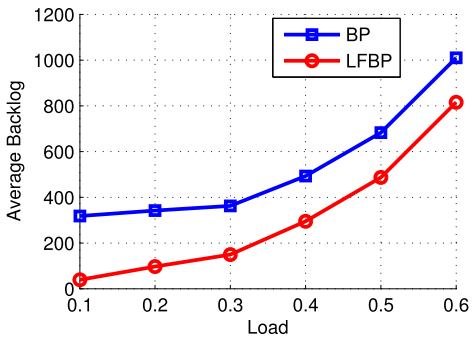


Fig. 11. Average backlog in the network with random link failures (Fig. 10) for the Loop Free Backpressure algorithm and the Backpressure algorithm.

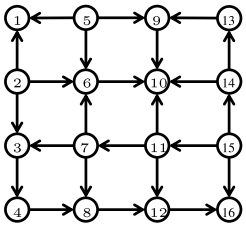


Fig. 10. Initial DAG for the LFBP algorithm chosen so that the LFBP needs several iterations to reach the optimal DAG. All the links have capacity six.

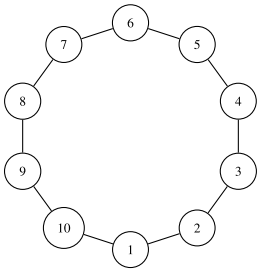


Fig. 12. Ring network topology. All links are bidirectional with unit capacity. Traffic goes from node 1 to node 10.

B. Randomly Changing Topology

To understand the delay performance of the LFBP algorithm on networks with randomly changing topology, we consider a network where 16 nodes are arranged in a 4×4 grid. All the links are taken to be of capacity six. For the LFBP algorithm, we choose a random initial DAG with zero throughput shown in Figure 10. The source is on the upper left corner (node 1) and the destination is on the bottom right (node 16).

In the beginning of the simulations all 24 network links are activated. At each time slot an active link fails with a probability 10^{-4} and an inactive link is activated with a probability 10^{-3}. The maximum throughput of the undirected network without any link failures is 12. Clearly on average, each link is "on" a fraction $\frac{10}{11}$ of the time, and thus the average maximum throughput of the undirected network with these link failure rates is $\frac{10}{11} \times 2 \times 6 = 10.9$. The arrivals to the networks are Poisson with rate $\lambda = 10.9\rho$, where $\rho = .1, .2, \ldots, .6$. For the LFBP algorithm, the detection threshold is set to $R_k = 100$ and the detection period is $T_k = 30$ for all n, k. These parameters were chosen so that there are several reversals before a topology change occurs in the undirected network. The simulation was carried out for a million slots.

Figure 11 compares the average backlog of LFBP and BP. In the low load scenarios LFBP reduces delay significantly (by 85% for load = 0.1) even though the topology changes challenge the convergence of the link-reversal algorithm. As the load increases, both the algorithms begin to obtain a similar delay performance.

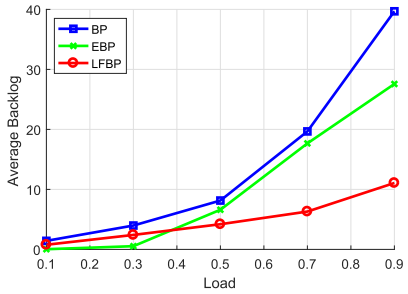
VII. Comparison With Enhanced Backpressure

We compare the performance of LFBP against the Enhanced Backpressure (EBP) algorithm from [3]. EBP aims to reduce the delay by sending more traffic on the shorter paths. This is accomplished by including pre-computed lengths of the shortest path in the weight calculation. EBP is very similar to the algorithm in [8].

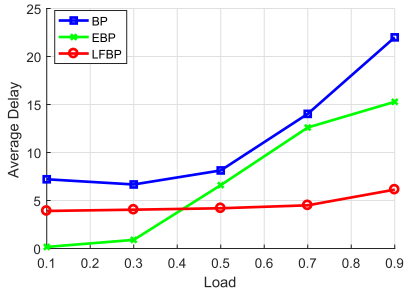
To compare the performance of LFBP and EBP, we simulate these algorithms in a network with a bidirectional ring topology as shown in Figure 12. The network has only one commodity going from node 1 to node 10. To reach the destination, the packets can either traverse the nodes 1, 2, …, 10, or they can use the direct link (1,10).

The results of the experiment is given in Figure 13. We can see that LFBP performs better than EBP for higher loads. In order to support a high load in this topology, the longer path (1,2,…,10) must be used. LFBP uses both long and the short path equally, however EBP tries to send most of its traffic through the link (1,10). Note that under EBP, even a packet that has reached node 3, 4 or 5 prefers to use the path through the link (1,10) as this is the shorter path to node 10. EBP performs better for low loads because these loads can be supported by using just the shortest path which requires only one hop.

It is easy to see that LFBP doesn't always perform better than EBP, even under high load. LFBP spreads the traffic throughout the network whereas EBP concentrates it on the shorter paths. Therefore, if a network can be stabilized without using the longer paths, EBP would perform better. We will see this situation in the next section where we simulate these algorithms on a grid network.



(a)



(b)

Fig. 13. Average backlog and delay in a ring network where the source and the destination nodes are neighbors. (a) Average backlog for different loads. (b) Average delay for different loads.

VIII. Multicommodity Simulation

We extend of the link reversal algorithm to the networks with multiple commodities. The multi-commodity algorithm is identical to the single commodity algorithm, with the exception that we now use the multicommodity backpressure of [1]. Each node n maintains a queue $Q_n^y(t)$ for each commodity y. Each commodity is assigned its own initial DAG. A pseudocode for the multicommodity LFBP that we used is given in Algorithm 4. An important direction for future research is to determine whether the claims proven for a single commodity in the previous sections extend to the multicommodity case.

The link-reversal algorithm checks for the overload for each commodity on each node, so the computation required for the link-reversal in a Y-commodity network at each time-slot is $O(Y|N|)$. Also, multi-commodity BP requires $O(Y|E|)$ computation at each time-slot because it computes the differential backlog for each link for each commodity. So, the computation required by the network for one time-slot of Multicommodity LFBP is $O(Y(|N|+|E|))$.

For the simulation, we consider a network arranged in a 4×4 grid as shown in Figure 10. Each link has a capacity of 6 packets per time-slot. There are three commodities in the network defined by the source destination pairs (1,16), (4,13) and (5,8). For the LFBP algorithm, each commodity starts with the same initial DAG given in Figure 10.

We use the arrival rate vector $\lambda^{\max} = [7.18, 6.96, 9.86]$, which is a max-flow vector for this network computed by solving a linear program. We scale this vector by various load factors ρ ranging from 0.1 to 0.9. The arrivals for each commodity i is Poisson with rate $\rho\lambda_i^{\max}$. In the beginning of the LFBP simulation, $\lfloor 500/\rho \rfloor$ dummy packets are added to the source of each commodity. This is helpful in low load cases because it forces the algorithm to find a DAG with high throughput, and avoids stopping at a DAG that only supports the given (low) load. R_k was chosen to be 50 and $T_k = 50$ for all $k > 0$. For the EBP simulation, the length of the shortest paths were scaled by the maximum link capacity, 6, in order to improve its performance as suggested in [3].

Algorithm 4 Multicommodity LFBP (Executed by n)

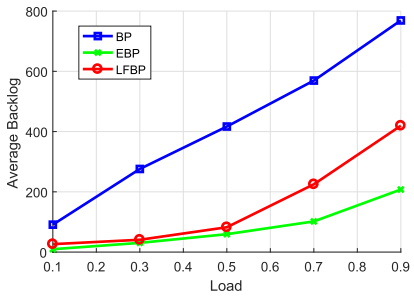
Input: sequences $\{T_k\}, \{R_k\}$, unique ID n
For each commodity y, generate initial DAG D_0^y by directing $\{n, j\}$ to (n, j) if $n < j$, to (j, n) if $j > n$.
Mark all queues Q_n^y as not overloaded
Initialize $t \leftarrow 0$, $k \leftarrow 0$
while true **do**
  Use Multicommodity BP to send/recive packets on all links of node n
  for all y **do**
    if $(Q_n^y(t) > R_k)$ **then**
      Mark this Q_n^y as overloaded.
    end if
  end for
  $t \leftarrow t + 1$

  $T_k \leftarrow T_k - 1$
  if $T_k = 0$ **then**
    for all y **do**
      Reverse links (j, n) in D_k^y if Q_j^y is not overloaded and Q_n^y is overloaded.
    end for
    $k \leftarrow k + 1$
    Mark all queues as not overloaded
  end if
end while

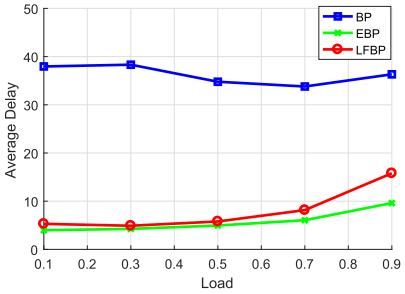
Figure 14 shows the average backlog and delay in the network for different loads under backpressure, enhanced backpressure and multicommodity LFBP. We can see that both LFBP and EBP have significantly improved delay performance compared to backpressure. We can also see that EBP outperforms LFBP. In a grid topology, most of the throughput can be obtained by using short paths. EBP keeps the traffic focused in these paths, whereas LFBP spreads the traffic throughout the network equally which causes higher delay.

IX. Conclusion

Backpressure routing and link reversal algorithms have been separately proposed for time-varying communication networks. In this paper we show that these two distributed schemes can be successfully combined to yield good throughput and delay performance.We develop the Loop-Free Backpressure Algorithm which jointly routes packets in a constrained DAG and reverses the links of the DAG to improve its throughput. We show that the algorithm ultimately results in a DAG that yields the maximum throughput. Additionally, by restricting the routing to this DAG we eliminate loops, thus reducing the average delay. Future investigations involve



(a)



(b)

Fig. 14. Average backlog and in a multicommodity network with fixed topology for BP, EBP and LFBP algorithms. Both EBP and LFBP performs significantly better than BP. In this topology, EBP performs better than LFBP because most of the throughput can be obtained by using the short paths. (a) Average backlog for different loads. (b) Average delay for different loads.

optimization of the overload detection parameters and studying the performance of the scheme on the networks with multiple commodities.

Appendix A
Lemma 7

Lemma 7: Consider a DAG D_k with source node s, destination node d, and arrival rate λ. Let A_k be the set of overloaded nodes under the flow allocation (f_{ij}) that yields the lexicographically minimum overload vector. If $|A_k| > 0$, then (A_k, A_k^c) is a min-cut of the DAG D_k.

Proof of Lemma 7: First we show that (A_k, A_k^c) is a cut, i.e., the source node $s \in A_k$ and the destination node $d \in A_k^c$. The destination node d has zero queue overload rate $q_d = 0$ because it does not buffer packets; hence $d \in A_k^c$. We show $s \in A_k$ by contradiction. Assume $s \notin A_k$. The property (8) shows that there is no flow going from A_k^c to A_k, i.e.,

$$\sum_{(i,j)\in E_k:\, i\in A_k^c,\, j\in A_k} f_{ij} = 0.$$

The flow conservation equation applied to the collection A_k of nodes yields

$$\begin{aligned}\sum_{n\in A_k} q_n &= \sum_{(i,n)\in E_k:\, i\in A_k^c,\, n\in A_k} f_{in} - \sum_{(n,j)\in E_k:\, n\in A_k,\, j\in A_k^c} f_{nj} \\ &= -\sum_{(n,j)\in E_k:\, n\in A_k, j\in A_k^c} f_{nj} \le 0,\end{aligned}$$

which contradicts the assumption that the network is overloaded (i.e., $|A_k| > 0$). Note that in the above equation λ does not appear because of the premise $s \notin A_k$.

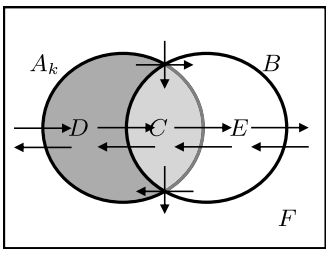


Fig. 15. A partition of the node set N where $A_k = C \cup D$ and $B = C \cup E$.

By the max-flow min-cut theorem, it remains to show that the capacity of the cut (A_k, A_k^c) is equal to the maximum flow $f_k^{\max}$ of the DAG D_k. Under the flow allocation (f_{ij}) that induces the lexicographically minimal overload vector, the throughput of the destination node d is the maximum flow $f_k^{\max}$ (see Lemma 1). It follows that

$$f_k^{\max} = \lambda - \sum_{i\in N} q_i = \lambda - \sum_{i\in A_k} q_i \tag{10}$$

$$= \sum_{(i,j)\in E_k:\, i\in A_k,\, j\in A_k^c} f_{ij} \tag{11}$$

$$= \sum_{(i,j)\in E_k:\, i\in A_k, j\in A_k^c} c_{ij} = \mathrm{cap}_k(A_k, A_k^c). \tag{12}$$

where (10) uses (7) and $q_i = 0$ for all nodes $i \notin A_k$, (11) follows the flow conservation law over the node set A_k, and (12) uses the property (9) in Lemma 1. □

Appendix B
Proof of Lemma 6

Proof of Lemma 6: Lemma 7 shows that (A_k, A_k^c) is a min cut of the DAG D_k. It suffices to prove that if there exists another min-cut (B, B^c), i.e., $A_k \neq B$ and $\mathrm{cap}_k(A_k, A_k^c) = \mathrm{cap}_k(B, B^c)$, then $A_k \subset B$. The proof is by contradiction. Let us assume that there exists another min-cut (B, B^c) such that $A_k \not\subset B$. We have the source node $s \in A_k \cap B$ and the destination node $d \in A_k^c \cap B^c$. Consider the partition $\{C, D, E, F\}$ of the network nodes such that $C = A_k \cap B$, $D = A_k \backslash B$, $E = B \backslash A_k$ and $F = N \backslash (A_k \cup B)$ (see Fig. 15). Since $A_k \not\subset B$ and $A_k \neq B$, we have $|D| > 0$. Also, we have $s \in C$ and $d \in F$. Let (f_{ij}) be a flow allocation that yields the lexicographically minimum overload vector in D_k. Properties (8) and (9) show that

$$f_{ij} = c_{ij}, \quad \forall\, i \in A_k,\ j \in A_k^c, \tag{13}$$

$$f_{ij} = 0, \quad \forall\, i \in A_k^c,\ j \in A_k. \tag{14}$$

The capacity of the cut (B, B^c) in the DAG D_k, defined in (1), satisfies

$$\mathrm{cap}_k(B, B^c) = \mathrm{cap}_k(B, D) + \mathrm{cap}_k(B, F), \tag{15}$$

where $B^c = D \cup F$. Under the flow allocation (f_{ij}), we have

$$\mathrm{cap}_k(B, D) = \sum_{(i,j)\in E_k: i\in B, j\in D} c_{ij} \ge \sum_{(i,j)\in E_k: i\in B, j\in D} f_{ij}. \tag{16}$$

Applying the flow conservation equation to the collection of nodes in D yields

$$\sum_{(i,j)\in E_k: i\in B, j\in D} f_{ij} \geq \sum_{i\in D} q_i + \sum_{(i,j)\in E_k: i\in D, j\in F} f_{ij}. \quad (17)$$

In (17), the first term is the sum of incoming flows into the set D; notice that there is no incoming flow from F to D because of the flow property (14). The second term is the sum of queue overload rates in D. The last term is a partial sum of outgoing flows leaving the set D, not counting flows from D to B; hence the inequality (17). From the flow property (13), the outgoing flows from the set D to F satisfy

$$\sum_{(i,j)\in E_k: i\in D, j\in F} f_{ij} = \sum_{(i,j)\in E_k: i\in D, j\in F} c_{ij}. \quad (18)$$

Combining (15)-(18) yields

$$\begin{aligned} \text{cap}_k(B, B^c) &= \text{cap}_k(B, D) + \text{cap}_k(B, F) \\ &\geq \sum_{i\in D} q_i + \sum_{(i,j)\in E_k: i\in D, j\in F} c_{ij} + \text{cap}_k(B, F) \\ &> \sum_{(i,j)\in E_k: i\in D, j\in F} c_{ij} + \text{cap}_k(B, F) \\ &= \text{cap}_k(A_k \cup B, F), \end{aligned} \quad (19)$$

where the second inequality follows that all nodes in D are overloaded and $q_n > 0$ for all $n \in D$. Inequality (19) shows that there exists a cut $(A_k \cup B, F)$ that has a smaller capacity, contradicting that (B, B^c) is a min-cut in the DAG D_k. Finally, we note that the partition (A_k, A_k^c) is unique because the lexicographically minimal overload vector is unique by Lemma 1. □

APPENDIX C
PROOF OF THEOREM 3

Proof of the First Claim: First we will show that the queue at the source $Q_s(t)$ crosses any arbitrary threshold R_1. We know that for some node $n \in A_k$, $Q_n(t) \to \infty$ as $t \to \infty$ because the external arrival rate to the source $s \in A_k$ is larger than the rate of departure from set A_k, i.e. $\lambda > cap(A_k, A_k^c)$. The backpressure algorithm sends packets on a link (i,j) only if $Q_i(t) > Q_j(t)$. Hence, at any time-slot if a node $b \neq s$ has a large backlog, then one of its parents p must also have a large backlog. Q_p can be slightly smaller than Q_b because Q_b might also receive packets from other nodes at the same time-slot. Specifically, $Q_p(t) > Q_b(t+1) - \sum_i c_{ib}$. Performing the induction on the parent of p we can see that the source node must have a high backlog when any node in A_k develops a high backlog. Note that the network is a DAG and the node n received packets form the source to develop its backlog, so the induction much reach the source node. Hence, when $Q_b(T_1) \gg R_1$, $Q_s(t) > R_1$ for some $t < T_1$.

Now we will show that every node in A_k crosses the threshold R. Let $B_1 \subseteq A_k$ be the set of nodes such that $Q_n(t) > R_1$ for some time $t < T_1$. We showed that $s \in B_1$. We will show that when $B_1 \neq A_k$, there exists some set B_2, such that (i) $B_1 \subset B_2$, and (ii) for every node $n \in B_2$, $Q_n(t) > R_2$ for some $t < T_2$. Here, R_2 and T_2 are large thresholds.

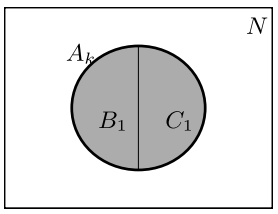


Fig. 16. Let (A_k, A_k^c) be the smallest min-cut. We showed that $s \in B_1$. Say, $c_{C_1A^c} \geq c_{B_1C_1}$ then the cut (B_1, B_1^c) has the capacity of $c_{B_1A^c} + c_{B_1C_1} \leq cap(A_k, A_k^c)$. This contradicts the assumption that (A_k, A_k^c) is the smallest min-cut. So, $c_{C_1A_k^c} < c_{B_1C_1}$.

Assume $B_1 \neq A_k$. Let $C_1 = A_k \backslash B_1$, i.e all nodes in C_1 haven't crossed the threshold R_1 until time T_1. Let $c_{B_1C_1}$ be the total capacity of the links going from B_1 to C_1, and $c_{C_1A_k^c}$ be the total capacity of the links going from C_1 to A_k^c. We have $c_{B_1C_1} > c_{C_1A_k^c}$ because (A_k, A_k^c) is the smallest min-cut (see Figure 16). When the backlogs of the nodes of B_1 are much larger than the nodes of C_1, the nodes in C_1 receive packets from B_1 at the rate of $c_{B_1C_1}$ packets per time-slot, and no packets are sent in the reversed direction. The rate of packets leaving the nodes in C is upper bounded by $c_{B_1A_k^c}$ which is smaller than the incoming rate. Hence, at least one node $n' \in C$ must collect a large backlog, say larger than $R_2 < R_1$. So, each node in the set $B_2 = B_1 \cup \{n'\}$ have a backlog larger than R_2 at some finite time T_2.

Now using induction we can see that for B_m where $m < |A_k|$, $B_m = A_k$ and all the nodes in B_m cross a threshold $R = \min\{R_1, \ldots, R_m\}$ by time $T = \max\{T_1, \ldots, T_m\}$. □

Of the Second Claim: We will use the following fact to prove this claim: for any subset of nodes S, if the number of packets entering S is lower than or equal to the number of packets leaving S on every time-slot, then the total backlog in S doesn't grow. So, the backlog in each node of S is bounded.

Assume a node b develops a backlog $Q_b(t) > R_1$. Here R_1 is a chosen such that

$$R_1 = |A_k^c| \sum_{i,j\in A_k^c} c_{ij} + \max_{n\in A_k^c} Q_n(0).$$

Consider a subset B of A_k^c such that for every node $i \in B$ and $j \in C = A_k^c \backslash B$, $(Q_i(t) - Q_j(t)) > c_{ij}$. The sets B and C must be nonempty because $Q_b(t)$ is large and $Q_d(t)$ is zero, that is $b \in B$ and $d \in C$. Note that backpressure doesn't send any data from C to B.

Let c_{AB} be the capacity of the links going from A to B, and let c_{BC} be the capacity of the links going from B to C. So, the number of packets entering B at timeslot t is upper bounded by c_{AB}. The number of packets leaving B is equal to c_{BC}. Since (A, A^c) is the smallest min-cut, $c_{AB} \leq c_{BC}$ (see Figure 17). Hence, the number of packets entering B is less than or equal to the number of packets leaving it at time t.

Therefore as soon as one of the nodes crosses threshold R_1, the sum backlog becomes bounded. We can choose a threshold $R \gg R_1$ such that this threshold is never crossed by any nodes in A_k^c. □

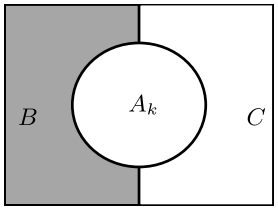


Fig. 17. Let (A_k, A_k^c) be the smallest min-cut. We showed that $d \in C$. Say, $c_{AB} > c_{BC}$ then the cut $(B \cup A_k, (B \cup A_k)^c)$ has the capacity of $c_{BC} + c_{A_kC} < c_{AB} + c_{A_kC} = cap(A_k, A_k^c)$. This contradicts the assumption that (A_k, A_k^c) is the smallest min-cut. So, $c_{AB} < c_{BC}$.

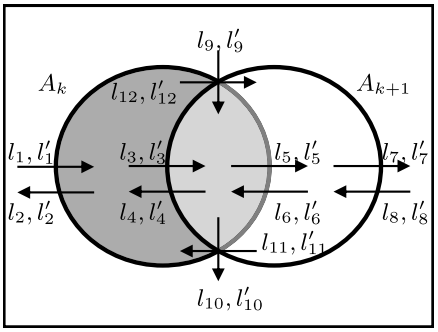


Fig. 18. Here l_i represents the sum of the capacities of the links going from one partition to the next in the DAG D_k, and l'_i represents the sum of the link capacities in the DAG D_{k+1}. For example, $l9$ and $l9'$ represent the links that go from $(A_k \cup A_{k+1})^c$ to $(A_k \cap A_{k+1})$ in DAGs D_k and D_{k+1} respectively.

APPENDIX D
LEMMA 8

Lemma 8: Consider the case when $\lambda > f_k^{\max}$. The link reversal algorithm is applied on DAG D_k to obtain D_{k+1}. Let (A_k, A_k^c) and (A_{k+1}, A_{k+1}^c) be the smallest min-cuts of D_k and D_{k+1} respectively. Then, either $cap_k(A_k, A_k^c) > cap_{k+1}(A_{k+1}, A_{k+1}^c)$, or $cap_k(A_k, A_k^c) = cap_{k+1}(A_{k+1}, A_{k+1}^c)$ and $|A_{k+1}| > |A_k|$

Proof: Consider the partitioning of the nodes as shown in Figure 18. For $i = 1, \ldots, 12$, l_i represents the sum of the capacities of the links going from one partition to the next in the DAG D_k, and l'_i represents the sum of the link capacities in the DAG D_{k+1}. The capacities of the smallest min-cut, before and after the reversal are given by

$$cap_k(A_k, A_k^c) = l_2 + l_5 + l_{10} + l_{12} \text{ and}$$
$$cap_{k+1}(A_{k+1}, A_{k+1}^c) = l'_4 + l'_7 + l'_{10} + l'_{11}$$

respectively. Note that only the links that are coming into A_k are different in D_k and D_{k+1}. So

$$l_i = l'_i \quad \text{for } i = 3, 4, 7, 8. \tag{20}$$

Because of the reversal there are no links coming into A_k in the DAG D_{k+1}:

$$l'_1, l'_6, l'_9, l'_{11} = 0. \tag{21}$$

After the reversal, the incoming links to A_k become outgoing from A_k,

$$l'_{10} = l_{10} + l_9. \tag{22}$$

(Corresponding equations for l'_2, l'_5 and l'_{12} are omitted because they are not necessary for the proof). Since (A_k, A_k^c) is a min-cut,

$$l_5 \leq l_7. \tag{23}$$

This is true because otherwise the cut $(A_k \cup A_{k+1}, (A_k \cup A_{k+1})^c)$ in the DAG D_k has a smaller capacity then the min cut $(A_k, A_k)^c$. Specifically, let us assume $l_5 > l_7$. Then, we get the contradiction:

$$\begin{aligned} cap_k(A_k \cup A_{k+1}, (A_k \cup A_{k+1})^c) &= l_2 + l_7 + l_{10} \\ &< l_2 + l_5 + l_{10} + l_{12} \\ &= cap_k(A_k, A_k)^c \end{aligned}$$

First we will show that if $A_k \backslash A_{k+1} \neq \phi$, then the capacity of the DAG must have increased. The proof is by contradiction.

Let us assume that the throughput didn't increase. So,

$$\begin{aligned} cap_k(A_k, A_k^c) &\geq cap_{k+1}(A_{k+1}, A_{k+1}^c) \\ &= l'_4 + l'_7 + l'_{10} + l'_{11} \\ &= l_4 + l_7 + l_{10} + 0 && (24) \\ &\geq l_4 + l_5 + l_{10} && (25) \\ &= cap_k(A_k \cap A_{k+1}, (A_k \cap A_{k+1})^c). && (26) \end{aligned}$$

(24) is follows from (20) and (21), and (25) follows from (23). Since $A_k \backslash A_{k+1} \neq \phi$ by assumption, $|A_k| > |A_k \cap A_{k+1}|$. This leads to a contradiction, because in DAG D_k the cut $(A_k \cap A_{k+1}, (A_k \cap A_{k+1})^c)$ is smaller than the smallest min-cut (A_k, A_k^c). Hence, $cap_k(A_k, A_k^c) < cap_{k+1}(A_{k+1}, A_{k+1}^c)$.

Next, we will consider the case $A_k \backslash A_{k+1} = \phi$. Using (23),

$$cap_k(A_k, A_k^c) = l_5 + l_{10} \leq l_7 + l_{10}.$$

In this situation, we again have two cases. First, if $A_k = A_{k+1}$ we know that $l'_{10} > l_{10}$ and $l_7 = 0$. Hence, $cap_k(A_k, A_k^c) < l'_{10} = cap_{k+1}(A_{k+1}, A_{k+1}^c)$.

Second, if $A_k \subset A_{k+1}$, then $|A_k| > |A_{k+1}|$ and

$$l'_{10} \geq l_{10}. \tag{27}$$

Using (20) and (27) $cap_k(A_k, A_k^c) \leq cap_{k+1}(A_{k+1}, A_{k+1}^c)$. □

REFERENCES

[1] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Trans. Autom. Control*, vol. 37, no. 12, pp. 1936–1949, Dec. 1992.

[2] L. X. Bui, R. Srikant, and A. Stolyar, "A novel architecture for reduction of delay and queueing structure complexity in the back-pressure algorithm," *IEEE/ACM Trans. Netw.*, vol. 19, no. 6, pp. 1597–1609, Dec. 2011.

[3] M. J. Neely, E. Modiano, and C. E. Rohrs, "Dynamic power allocation and routing for time-varying wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 89–103, Jan. 2005.

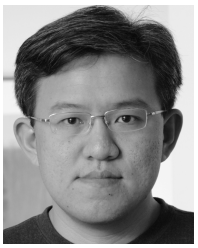
[4] E. Gafni and D. Bertsekas, "Distributed algorithms for generating loop-free routes in networks with frequently changing topology," *IEEE Trans. Commun.*, vol. 29, no. 1, pp. 11–18, Jan. 1981.

[5] V. D. Park and M. S. Corson, "A highly adaptive distributed routing algorithm for mobile wireless networks," in *Proc. INFOCOM*, Apr. 1997, pp. 1405–1413.

[6] L. Georgiadis and L. Tassiulas, "Optimal overload response in sensor networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2684–2696, Jun. 2006.

[7] H. Xiong, R. Li, A. Eryilmaz, and E. Ekici, "Delay-aware cross-layer design for network utility maximization in multi-hop networks," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 5, pp. 951–959, May 2011.
[8] L. Ying, S. Shakkottai, A. Reddy, and S. Liu, "On combining shortest-path and back-pressure routing over multihop wireless networks," *IEEE/ACM Trans. Netw.*, vol. 19, no. 3, pp. 841–854, Jun. 2011.
[9] P.-K. Huang, X. Lin, and C.-C. Wang, "A low-complexity congestion control and scheduling algorithm for multihop wireless networks with order-optimal per-flow delay," *IEEE/ACM Trans. Netw.*, vol. 21, no. 2, pp. 495–508, Apr. 2013.
[10] M. J. Neely, *Stochastic Network Optimization With Application to Communication and Queueing Systems*. San Rafael, CA, USA: Morgan & Claypool, 2010.
[11] L. R. Ford, Jr., and D. R. Fulkerson, "Maximal flow through a network," *Can. J. Math.*, vol. 8, pp. 399–404, Feb. 1956.
[12] L. Georgiadis, P. Georgatsos, K. Floros, and S. Sartzetakis, "Lexicographically optimal balanced networks," *IEEE/ACM Trans. Netw.*, vol. 10, no. 6, pp. 818–829, Dec. 2002.
[13] L. Huang and M. J. Neely, "Delay reduction via Lagrange multipliers in stochastic network optimization," *IEEE Trans. Autom. Control*, vol. 56, no. 4, pp. 842–857, Apr. 2011.
[14] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks," in *Foundations and Trends in Networking*. Breda, The Netherlands: Now, 2006.
[15] A. Rai, C. P. Li, G. Paschos, and E. Modiano, "Loop-free backpressure routing using link-reversal algorithms," in *Proc. ACM MobiHoc*, Jun. 2015, pp. 87–96.

Anurag Rai received the B.S. and M.S. degrees in computer science from Brigham Young University in 2010 and 2012, respectively. He is currently pursuing the Ph.D. degree in electrical engineering and computer science with MIT. His research interests include control of communication networks, distributed algorithms, and inference in communication networks.



Chih-ping Li received the B.S. degree from National Taiwan University in 2001, and the M.S. and Ph.D. degrees from the University of Southern California in 2005 and 2011, respectively, all in electrical engineering. He was a Post-Doctoral Associate with the Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, between 2011 and 2014. Since 2014, he has been with the Corporate Research and Development Department, Qualcomm, where he has been involved in 5G wireless system design and analysis. His research interests include stochastic control, resource allocation, QoS in communication networks, wireless networks, and queueing systems.

Georgios Paschos received the Diploma degree in electrical and computer engineering from the Aristotle University of Thessaloniki, Greece, in 2002, and the Ph.D. degree in wireless networks from the ECE Department, University of Patras, under the supervision of Prof. S. Kotsopoulos, in 2006. From 2007 to 2008, he was an ERCIM Post-Doctoral Fellow with the Team of Prof. Norros, VTT, Finland. From 2008 to 2014, he was with the Center of Research and Technology Hellas, Informatics and Telematics Institute, Greece, working with Prof. L. Tassiulas. He also taught with the Department of Electrical and Computer Engineering, University of Thessaly, as an Adjunct Lecturer, from 2009 to 2011. He spent two years with the Team of Prof. E. Modiano, MIT. He has been a Principal Researcher with Huawei Technologies Co., Ltd., Paris, France, leading the Network Control and Resource Allocation Team, since 2014. He serves as a TPC Member of INFOCOM, WiOPT, and Netsoft. Two of his papers received the best paper award in GLOBECOM '07 and IFIP Wireless Days '09, respectively. He serves as an Associate Editor of the IEEE/ACM TRANSACTIONS ON NETWORKING.

Eytan Modiano (S'90–M'93–SM'00–F'12) received the B.S. degree in electrical engineering and computer science from the University of Connecticut, Storrs, CT, USA, in 1986, and the M.S. and Ph.D. degrees in electrical engineering from The University of Maryland, College Park, MD, USA, in 1989 and 1992, respectively. He was a Naval Research Laboratory Fellow between 1987 and 1992 and a National Research Council Post-Doctoral Fellow during 1992–1993. Between 1993 and 1999, he was with the MIT Lincoln Laboratory. Since 1999, he has been with the Faculty of MIT, where he is currently a Professor with the Department of Aeronautics and Astronautics, Laboratory for Information and Decision Systems. His research is on communication networks and protocols with emphasis on satellite, wireless, and optical networks. He is an Associate Fellow of the AIAA. He was a co-recipient of the MobiHoc 2016 best paper award, the WiOpt 2013 best paper award, and the SIGMETRICS 2006 best paper award. He was the Technical Program Co-Chair of the IEEE WiOpt 2006, the IEEE Infocom 2007, the ACM MobiHoc 2007, and the DRCN 2015. He is the Editor-in-Chief of the IEEE/ACM TRANSACTIONS ON NETWORKING. He served as an Associate Editor of the IEEE TRANSACTIONS ON INFORMATION THEORY and the IEEE/ACM TRANSACTIONS ON NETWORKING. He served on the IEEE Fellows Committee.