

# Survivable Path Sets: A New Approach to Survivability in Multilayer Networks

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**Abstract**—We consider the problem of survivability in multilayer networks. In single-layer networks, a pair of disjoint paths can be used to provide protection for a source–destination pair. However, this approach cannot be directly applied to layered networks where risk-disjoint paths may not always exist. In this paper, we take a new approach, which is based on finding a *set of paths* that may not be disjoint but together will survive any single risk. We start with two-layered communication networks, where the risks are fiber failures. We prove that in general, finding the minimum survivable path set (MSPS) is NP-hard, whereas if we restrict the length of paths the problem can be solved in polynomial time. We formulate the problem as an integer linear program (ILP), and use this formulation to develop heuristics and approximation algorithms. Moreover, we study the minimum cost survivable path set problem, where the cost is the number of fibers used, and thus, nonadditive. Finally, we generalize the survivability problem to the networks with more than two layers. By applying our algorithms for survivable path set, we assess the survivability of communication networks that operate relying on power from a power grid.

**Index Terms**—Approximation algorithms, minimum survivable path set, multilayer networks.

## I. INTRODUCTION

ONE of the most important advances in modern communication networks is embedding multilayer network architectures such as IP-over-WDM.<sup>1</sup> In these layered networks, a logical topology is embedded onto a physical topology such that each logical link is routed using a path in the physical topology. While such a layering approach makes it possible to take advantage of the flexibility of upper layer technology (e.g., IP) and the high data rates of lower layer technology (e.g., WDM), it raises a number of challenges for efficient and reliable operations. One challenge is providing protection for this layered network where

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<sup>1</sup>Wavelength-division multiplexing (WDM) is a technology that allows multiple signals path through a single optical fiber by multiplexing different wavelengths.

loss of a single fiber may cause failure of multiple logical links using it. In this paper, we consider the protection problem in the two layered communication network, and extend it to the case of three layers consisting of a physical-logical communication network and a power grid.

The main objective in the protection of a communication network is to guarantee its connectivity in the case of a failure. Since in reality the probability of two simultaneous failures is very low, we assume that only one failure occurs at a time. The protection problem in single-layer networks is rather straightforward; namely, providing a pair of disjoint paths (one for primary and one for backup) guarantees a route between two nodes against any single link failure.

This approach, however, cannot be directly applied to layered networks, because a pair of seemingly disjoint paths at the logical layer may share a physical link and thus simultaneously fail in the event of a physical link failure. The notion of shared risk link group (SRLG) disjoint paths, i.e., two paths between the source and destination nodes that do not share any risk (e.g., fiber and conduit) was introduced in [1] and formulated in [2]. Nearly all the previous works in the context of layered network protection have focused on finding SRLG-disjoint paths ([3]–[9] among others).

Although the SRLG-disjoint paths problem has been well studied, there are networks in which SRLG-disjoint paths do not exist between a source and a destination. There has been some efforts to address this challenge by choosing a pair of maximally disjoint SRLG paths; i.e., a pair of paths that share the minimum number of risks [2], [10], [11]. Clearly, this cannot survive any single failure; thus, we take an alternative approach that is based on finding a *set* of paths that together will survive any single failure. Thus, in the case that SRLG-disjoint paths do not exist, we may find three or more paths such that in the event of a failure, at least one of the paths remains connected. This notion of *survivable path set* generalizes the traditional notion of SRLG-disjoint paths, enables us to provide protection for a broader range of scenarios and increases the survivability of the network.

The concept of multiple survivable paths has been studied in the single-layer setting [12], [13], where they split data over multiple paths and ensure that the delay over all paths are limited. They select the paths so that in the case of failure of any path, all or a fraction of traffic is guaranteed to survive. However, this problem remains largely unexplored in the multilayer setting. We say that a pair of nodes  $(i, j)$  is survivable if in the case of any single physical failure, nodes  $i$  and  $j$  remain connected. Moreover, we say that a network is survivable if in the case of any single physical failure, the logical layer remains connected

[14]. Clearly, this requires the survivability of every pair of nodes in the network; i.e., if there exists a pair of nodes that is not survivable, then the network is not survivable as well. Therefore, we define the general metric of “survivability” as the fraction of the total pairs of nodes that remain connected after a failure. A network is survivable if survivability is equal to one.

Our contributions in this paper can be summarized as follows:

- 1) we introduce a new notion of survivable path set to provide pairwise protection even for the case where SRLG-disjoint paths do not exist;
- 2) we prove the NP-hardness of the minimum survivable path set (MSPS) problem;
- 3) we show that under certain practical restrictions, the MSPS problem can be solved in polynomial time;
- 4) we develop heuristics and approximation algorithms for the MSPS problem;
- 5) we introduce the new problem of minimum fiber survivable path set (MFSPS) to minimize the number of fibers used in the survivable path set;
- 6) we prove the NP-hardness of the MFSPS problem, and provide heuristics to solve it;
- 7) finally, we generalize the survivability problem to more than two layers where failures can occur in any layer, and by applying our algorithms for survivable path set, we assess the survivability of a communication network that relies on power from a power grid.

The rest of this paper is organized as follows. In Section II, we present the two-layered communication network model. In Section III, we study the problem of finding a minimum set of paths that will survive any single fiber failure and develop several approximation algorithms. We also compare the performance of our approximation algorithms through numerical evaluation. In Section IV, we study the problem of minimizing the backup fibers in the survivable path set and develop new heuristics. In Section V, we generalize the survivability problem to networks with more than two layers. In particular, we show how the power grid affects the survivability of a communication network. The conclusion is presented in Section VI.

## II. COMMUNICATION NETWORK MODEL

We consider a layered network that consists of a logical topology  $G_L = (V_L, E_L)$  built on top of a physical topology  $G_P = (V_P, E_P)$ , where  $V$  and  $E$  are the sets of nodes and links, respectively, and  $V_L \subset V_P$ . Each logical link  $(i, j)$  in  $E_L$  is mapped onto an  $i - j$  path in the physical topology. This is called lightpath routing. Different lightpaths may use the same fiber (physical link), therefore when a fiber fails, all the lightpaths using that fiber will fail. Hence, a logical path survives the failure of any fiber that it does not use.

As mentioned earlier, we generalize the traditional notion of SRLG-disjoint paths to account for the case where there does not exist a pair of SRLG-disjoint paths. In a layered network, a set of logical paths is said to be *survivable* if at least one of the paths remain connected after any single physical link failure. Hence, a survivable set consisting of two paths is a pair of SRLG-disjoint paths. Note that, there may exist a survivable path set even if SRLG-disjoint paths do not exist. For example, consider the

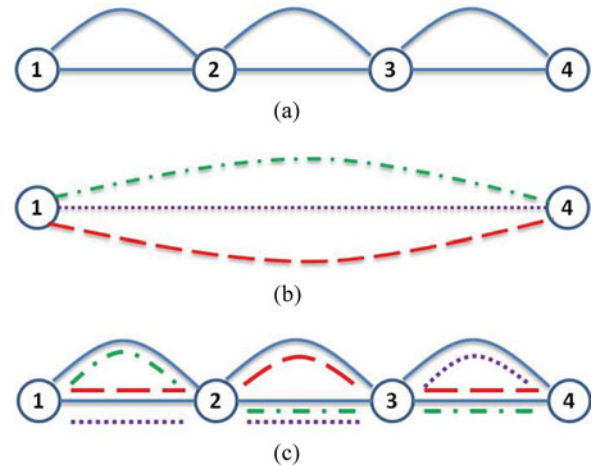


Fig. 1. Topologies in multilayer networks. (a) Physical Topology. (b) Logical Topology. (c) Mapping.

physical and logical topologies in Fig. 1. Each dashed line in Fig. 1(c) shows the lightpath routing of each logical link over the physical topology. Under this lightpath routing, each pair of logical paths between nodes 1 and 4 shares some fiber.

Suppose that we want to find a set of logical paths between nodes 1 and 4 that can survive any single physical link failure. Clearly, there does not exist a pair of SRLG-disjoint paths as each pair of logical paths shares a fiber. However, it is straightforward to check that the set of three paths can survive any single fiber cut, although they are not SRLG-disjoint. This example shows that the traditional protection schemes based on SRLG-disjoint paths (such as the ones in [2]) may fail to provide protection against single physical link failures, while there exists a set of paths that can together provide protection. Our goal in this paper is to address the problem of finding a set of survivable paths that together will survive any single fiber failure.

## III. MSPS

We start with the problem of finding a MSPS, i.e., the minimum number of paths between a pair of nodes  $s$  and  $t$  that survive any single physical link (fiber) failure. We present a path-based integer linear program (ILP) formulation for this problem, assuming that the entire set of  $s - t$  paths with their routings over fibers is given. For each path  $j$ , let  $P_j$  be a binary variable which takes the value 1 if path  $j$  is selected, and 0 otherwise. The matrix  $A \in \mathbb{R}^{m \times n}$  refers to the mapping of all  $n$  paths over the  $m$  fibers such that  $a_{ij} = 0$  if path  $j$  uses fiber  $i$  and  $a_{ij} = 1$  otherwise. Let  $e$  be a  $m \times 1$  vector of ones. The MSPS problem can be expressed as follows:

$$\text{minimize } \sum_{j=1}^n P_j \quad (1a)$$

$$\text{subject to } A \times P \geq e \quad (1b)$$

$$P_j \in \{0, 1\}, \quad j = 1, \dots, n. \quad (1c)$$

In the aforementioned, the objective function is the number of selected paths. Each row  $i \in \{1, \dots, m\}$  in constraint (1b)

requires that at least one selected path survives the failure of fiber  $i$ , i.e., the selected path set should be survivable. Hence, the optimal solution to the above optimization problem gives a MSPS whenever one exists. Although this formulation requires the knowledge of every path (which is possibly exponential in the number of fibers), the compact and clean expression of the path-based formulation enables us to analyze the useful properties of survivable path sets. Later, we will use this formulation to develop heuristics and approximation algorithms for finding an MSPS. Note that the MSPS problem can also be described by link-based formulation using a polynomial number of constraints and variables without enumerating all of the paths [15].

### A. MSPS in General Setting

In this section, we show that the MSPS problem is NP-hard in general and discuss some algorithms that can be used to solve the problem. In Section III-B, we will study the MSPS problem under a practical constraint. Our first result pertains to the complexity of the MSPS problem as stated in Theorem 1.

*Theorem 1:* Computing the minimum survivable paths in multilayer networks is NP-complete.

*Proof:* The Minimum survivable paths problem can be reduced from the minimum set cover problem, which is known to be NP-complete. Given an instance of minimum set cover problem with set of elements  $E$  and set of subsets  $R$ , we construct a physical topology  $E = \{f_1, \dots, f_m\}$  containing  $m$  elements, where each  $f_i$  corresponds to fiber  $i$  and a logical topology  $R = \{P_1, \dots, P_n\}$  containing  $n$  sets, where each  $P_j$  corresponds to the set of fibers that are *not* used by path  $j$ ; i.e., all fibers that survive the failure of path  $j$ . It follows that the minimum number of logical paths that survives any of the physical fibers is equal to the size of a minimum set cover.

As the last step of the proof, we need to show we can construct a physical topology with the given routing. Given set  $R = \{P_1, \dots, P_n\}$ , we can obtain set  $\bar{R} = \{\bar{P}_1, \dots, \bar{P}_n\}$ , where each  $\bar{P}_j$  corresponds to the set of fibers that are *used* by path  $j$ . We assume that each path is made of only one  $s-t$  lightpath; thus, the logical topology is made of  $n$  parallel  $s-t$  logical links. Given sets  $E$  and  $R$ , the physical topology and lightpath routing in [16] can be used. ■

Since the problem is computationally hard to solve, we consider heuristics and approximation algorithms that give a set of survivable paths in polynomial time. Owing to the similarity to the set cover problem, the heuristics that have been developed for set cover problems can be used here. In particular, a common approach to solve the set cover problem is the greedy algorithm. In order to apply the greedy algorithm to our setting, one needs to enumerate all of the paths with their routings on the fibers. In general, the number of paths in a multilayer network is exponential in the total number of fibers. Moreover, in each iteration, the greedy algorithm tries to find a path that survives the maximum number of fibers; i.e., the path that uses the minimum number of fibers. Since the total number of paths is exponential in the number fibers, it is not possible to search through all paths. In fact, it was shown that even this subproblem is equivalent to the minimum color path problem, which is known to be NP-hard [17].

Another approach that can be used to approximate the set cover problem is randomized rounding, which is based on solving the linear program (LP) relaxation of the original ILP formulation, and rounding the solution randomly. Randomized rounding gives an  $O(\log m)$  approximation, where  $m$  is the number of fibers [18]. This is the best possible approximation for the MSPS problem, which is due to the fact that the minimum set cover problem cannot be approximated within better than a  $\log m$  factor in polynomial time [19].

Fortunately, practical systems impose certain physical constraints that make the survivable path-set problem easier to solve. Due to physical impairments and delay constraints, paths are typically limited in length, and we show that this physical limitation makes the MSPS problem tractable.

### B. Path Length Restricted Version

In this section, we assume that each logical path is restricted to using at most  $K$  fibers. Restricting the length of paths (i.e., number of fibers on each path) is a realistic assumption, because each logical link is typically constrained in the number of fibers that it may use (due to budget and/or delay constraints), and each logical path is constrained in the number of logical links. We would like to remind the difference between logical path and logical link in this paper. A logical link  $(i, j)$  is a link in the logical layer of network, which is mapped onto a path from node  $i$  to node  $j$  in the physical topology. This is called the lightpath routing. A logical path between  $s$  and  $t$  is a set of logical links that form a path from node  $s$  to node  $t$  in the logical layer of network.

*Lemma 1:* Under the path length restriction, the optimal number of survivable paths is at most  $K + 1$ .

*Proof:* By the assumption, each path uses at most  $K$  fibers, and thus, at least  $m - K$  fibers are survived by a path. Suppose that we have selected an arbitrary path, and want to add other paths to form a survivable path set. In the worst case, each of the newly selected paths can survive only a single fiber, which is not survived by the previously selected paths. Since there are at most  $K$  fibers that are not survived by the first path, we need at most  $K$  additional paths to survive the rest of the fibers. Therefore, the total number of paths will not exceed  $K + 1$ . ■

*Lemma 2:* In the path length restricted version of MSPS, the total number of logical paths is polynomial in the number of fibers  $m$ , and can be enumerated in polynomial time.

*Proof:* Under the assumption of restricted path length, a logical path can consist of up to  $K$  fibers, and thus, at most  $K$  logical links. In a graph with  $n$  nodes, there can be  $O(n^K)$  paths of length up to  $K$ . Since the number of nodes is at most  $2m$ , the total number of logical paths of length up to  $K$  is  $O(m^K)$ . A simple exhaustive search can be used to enumerate all the logical paths.

*Theorem 2:* The path length restricted version of the MSPS problem can be solved in polynomial time.

*Proof:* By Lemma 1, MSPS needs at most  $K + 1$  paths to survive any single failure. Therefore, one can find the exact solution by searching through all subsets of paths with sizes  $2, 3, \dots, K + 1$ . This will take  $O(P^{K+1})$  iterations, where  $P$  is the total number of paths. On the other hand, by Lemma 2, the



total number of paths is  $O(m^K)$ . Therefore, the total running time of exhaustive search is  $O(m^{K(K+1)})$ , which is polynomial in the total number of fibers. ■

Although this exhaustive search returns an optimal solution, its running time can be prohibitive for large values of  $m$  and  $K$ . This motivates us to study heuristics and approximation algorithms with better running time. First, we consider a greedy algorithm, followed by a randomized algorithm based on  $\varepsilon$ -net, which is a well-known technique in the area of computational geometry.

1) *Greedy Algorithm*: The first heuristic we consider is a greedy algorithm, which is similar to the greedy algorithm for the minimum set cover problem. The input to the greedy algorithm is the set of paths with the set of fibers used by each path and the set of all fibers. The greedy algorithm is an iterative algorithm that works as follows. In the first step, it selects a path that uses the minimum number of fibers. In the second step, it selects the path that survives the maximum number of fibers that are not survived yet. The second step is repeated until the selected path set survives all of the fibers. Following the proof of Lemma 1, it can be shown that the greedy algorithm also finds a survivable path set with size at most  $K + 1$ .

As discussed in Section III-A, the greedy algorithm generally gives an  $O(\log m)$  approximation to the MSPS set. However, under the assumption of restricted path length, it provides a better approximation as stated in Theorem 3.

*Theorem 3*: The greedy algorithm provides an  $O(\log K)$  approximation in polynomial time for the path length restricted version of MSPS.

*Proof*: Let  $\xi$  be the size of MSPS. Let  $n_i$  be the number of fibers that are not survived after the  $i$ th iteration of the greedy algorithm. Clearly, we have  $n_1 \leq K$ . Now, note that there is a path that survives at least  $\frac{n_1}{\xi}$  of the remaining  $n_1$  fibers, because otherwise the size of the optimal path set would be larger than  $\xi$ . Hence, in the second iteration, the greedy algorithm would select a path that survives at least  $\frac{n_1}{\xi}$  of fibers. Thus

$$n_2 \leq n_1 - \frac{n_1}{\xi} \leq K \left(1 - \frac{1}{\xi}\right). \quad (2)$$

Similarly

$$n_3 \leq n_2 - \frac{n_2}{\xi} \leq K \left(1 - \frac{1}{\xi}\right)^2 \quad (3)$$

and in general

$$n_i \leq K \left(1 - \frac{1}{\xi}\right)^i. \quad (4)$$

The greedy algorithm will terminate when  $n_t < 1$ , and this condition is satisfied when

$$K \left(1 - \frac{1}{\xi}\right)^t < 1 \quad (5)$$

where  $t$  is the total number of iterations. Since  $1 - x < e^{-x}$  for  $x > 0$ , inequality (5) is satisfied when

$$K e^{-\frac{t}{\xi}} \leq 1 \Leftrightarrow t \leq \xi \times \log K. \quad (6)$$

Therefore, the greedy algorithm provides an  $O(\log K)$  approximation.

To prove the polynomial time complexity, note that in each iteration of the greedy algorithm, the best path can be found in  $O(m^K)$  by searching through all the paths (see the proof of Theorem 2). Furthermore, as mentioned earlier, the greedy algorithm terminates in at most  $K + 1$  iterations. Therefore, the computational complexity of the greedy algorithm is  $O(Km^K)$ . ■

Although the greedy algorithm runs significantly faster than the exhaustive search algorithm, its running time can still be prohibitive for large  $K$  and  $m$ . Hence, we develop a novel randomized algorithm that has a considerably better running time. This algorithm builds upon solutions to the closely related set cover and hitting set problems [20]. In particular, the algorithm is based on  $\varepsilon$ -net, a concept in computational geometry, which provides an approximation algorithm for the hitting set problem.

2)  *$\varepsilon$ -Net Algorithm*: Our  $\varepsilon$ -net algorithm is an iterative algorithm, which selects each path with some probability. If all the fibers are survived by the selected path set in the first iteration, the algorithm terminates. Otherwise, it changes the probability of selecting each path and selects a new set of paths using the new probabilities, until all fibers are survived.

Let  $W_j$  be the weight of path  $j$ , initialized as  $W_j = 1$ . Define the weight of each fiber  $i$  to be the sum of the weights of paths surviving fiber  $i$ , i.e.,

$$W(f_i) = \sum_{j:a_{ij}=1} W_j. \quad (7)$$

*Definition 1*: A fiber is said to be  $\varepsilon$ -Survivable if

$$W(f_i) \geq \varepsilon \sum_{j=1}^n W_j \text{ for some } \varepsilon \in (0, 1) \quad (8)$$

where  $n$  is the total number of paths.

Note that when all of the paths have the same weight of 1, a fiber is  $\varepsilon$ -Survivable if it is survived by at least  $\varepsilon \times n$  paths. Hence, if a fiber is  $\varepsilon$ -Survivable with large  $\varepsilon$ , then it is likely to be survived by randomly selected paths. This observation is exploited in our  $\varepsilon$ -net algorithm as discussed below.

By applying the randomized algorithm for the hitting set problem from [21] and [22], we can obtain a path-selection algorithm for selecting a random subset of paths that will survive all of the  $\varepsilon$ -Survivable fibers, with high probability. In particular, the algorithm finds a set of paths via  $s$  independent random draws, such that in each draw, a path is selected from the entire path set according to the probability distribution  $\mu(P_j) = \frac{W_j}{\sum_{j=1}^n W_j} \forall j$ . Results from [23] and [24] showed that  $s = c \frac{d}{\varepsilon} \log \frac{1}{\varepsilon}$ , where  $d$  is the VC-dimension of our problem set and  $c$  is a constant.

*Definition 2*: Let  $R$  be a set of subsets of  $X$ . A subset  $A \subset X$  is shattered by  $R$  if every subset of  $A$  can be obtained as the intersection of some  $S \in R$  with  $A$ .

The VC-dimension of  $R$ , denoted by  $\dim(R)$ , is defined as the supremum of the sizes of all finite shattered subsets of  $X$ . If arbitrarily large subsets can be shattered, the VC-dimension is  $\infty$  [22].

Consider the following setting for our survivability problem.

*Example:* Let  $X = \{P_1, \dots, P_n\}$  be the set of all  $s - t$  paths in the logical layer. Moreover, let  $R = \{f_1, \dots, f_m\}$  be the set of subsets of  $X$ , where each set  $f_i$  denotes the set of paths that survive the failure of fiber  $i$  and  $m$  is the number of fibers. Let  $A$  be an arbitrary subset of  $X$ . According to Definition 2, set  $A$  is shattered by  $R$  if the intersection of  $A$  with sets  $f_1, \dots, f_m$  produces all of the subsets of  $A$ . For instance, set  $A = \{P_1, P_2, P_3\}$  is shattered by  $R$  if there exist at least eight sets  $\{f_1, \dots, f_8\}$  such that

$$\begin{aligned} A \cap f_1 &= P_1, & A \cap f_2 &= P_2, & A \cap f_3 &= P_3 \\ A \cap f_4 &= \{P_1, P_2\}, & A \cap f_5 &= \{P_1, P_3\}, & A \cap f_6 &= \{P_2, P_3\} \\ A \cap f_7 &= A, & A \cap f_8 &= \emptyset. \end{aligned}$$

In the following, we use Definition 2 to find an upper bound on the VC-dimension of set  $R$ .

*Lemma 3:* In path length restricted version of MSPS, the VC-dimension  $d$  is  $O(\log K)$ .

*Proof:* Let  $d$  denote the VC-dimension of our setting. Then, by Definition 2, there exists a subset of paths denoted by  $A$  such that  $|A| = d$  and the intersection of  $A$  with the elements of  $R$  (i.e.,  $\{f_1, \dots, f_m\}$ ) generates all the subsets of  $A$ .

Let  $M$  be the set of all subsets of  $A$ ; therefore,  $|M| = 2^{|A|} = 2^d$ . It is easy to see that each element  $P_j \in A$  is absent exactly in half of the elements of  $M$ , which is equal to  $2^{d-1}$  subsets of  $A$ . As a result, there exists at least  $2^{d-1}$  sets  $f_i$  so that  $P_j \notin f_i$ ; i.e., path  $j$  does not survive the failure of at least  $2^{d-1}$  fibers. On the other hand, by the assumption of path length restriction, each path has at most  $K$  fibers; therefore, each path does not survive the failure of at most  $K$  fibers.

Hence, The inequality  $2^{d-1} \leq K$  holds. This gives  $d \leq 1 + \log K$ , and thus,  $d = O(\log K)$ . ■

*Corollary 1:* Our path-selection algorithm selects  $s = c \frac{\log k}{\varepsilon} \log(\frac{1}{\varepsilon})$  independent paths to find an  $\varepsilon$ -net with high probability, where  $c$  is a constant.

Using the techniques in [24], we design an iterative  $\varepsilon$ -net algorithm as follows. The algorithm is initialized by setting  $\varepsilon = \frac{1}{2}$ . In each iteration, it applies the random path selection and checks the survivability of the selected path set. If not all fiber failures are survived, the algorithm doubles the weight of all paths that survive the failure of fibers in  $\tilde{S}$ , where  $\tilde{S}$  is the set all the fibers that are not survived yet (so that such fibers are more likely to be survived by the new selected paths). This random path selection is repeated  $\frac{2}{\varepsilon} \log(2\varepsilon n)$  times<sup>2</sup> with the new probability distribution (but the repetition is terminated immediately if all the fibers are survived). If all fibers are not survived yet, the algorithm decreases the value of  $\varepsilon$  by half; and repeats the previous steps.

Let  $\xi$  be the optimal value of the MSPS problem. Using Corollary 1 and applying the results in [23] and [24], the following theorem can be proved.

<sup>2</sup>[24, Lemma 3.4] states that if there is a hitting set of size  $c$ , the doubling process cannot iterate more than  $4c \log(\frac{n}{c})$  times. Thus, in our case, if there is a hitting set (defined as a subset of paths such that every fiber can be survived by at least one of the paths in the subset) of size  $\frac{1}{2\varepsilon}$ , we can find it in at most  $\frac{2}{\varepsilon} \log(2\varepsilon n)$  iterations.

TABLE I  
PERFORMANCE BOUNDS UNDER PATH LENGTH RESTRICTED VERSION:  
EXS-EXHAUSTIVE SEARCH, T-TYPE, D-DETERMINISTIC, P-PROBABILISTIC

Method	Approximation	Running Time	T
ExS	Exact Solution	$O(m^{K(K+1)})$	D
Greedy	$O(\log K)$	$O(Km^K)$	D
$\varepsilon$ -net	$O(\log K \log \xi)$	$O(K \log(K) \log(m) \log(\log(K)))$	P

*Theorem 4:* The  $\varepsilon$ -net algorithm finds a set of survivable paths of size  $O(\log K \log \xi)\xi$ , with high probability.

Moreover, it can be shown that the computational complexity of the  $\varepsilon$ -net algorithm is  $O(K \log(K) \log(m) \log(\log(K)))$ .

Finally, we propose another algorithm, which we call the random-sweep greedy (RSG) algorithm. Although, we could not quantify the performance of this algorithm analytically, it performs near optimally in many scenarios as will be shown in Section III-C.

3) *RSG Algorithm:* RSG algorithm is a modified version of the greedy algorithm. Here, the RSG removes a path (from the selected path set), which survives the fibers covered by other selected paths so that the size of the selected path set can be further reduced while maintaining the survivability.

The RSG algorithm also requires the knowledge of the set of paths and associated fibers. Let  $P_i$  be the set of selected paths in the first  $i$  iterations, and  $S_j$  be the set of fibers that are survived by path  $j$ . The first two iterations of RSG are the same as the greedy algorithm. That is, in each iteration, it selects a path that survives the maximum number of fibers that are not survived yet. If the first two paths survive all of the fibers, the algorithm terminates. Otherwise, it continues as follows.

Suppose the RSG algorithm is in the  $i$ th iteration. First, find a path, say  $i$ , that survives maximum number of fibers. Then, pick a path, say  $j$ , randomly from the previously selected path set  $P_{i-1}$ , and find  $S^* = \cup_{k \in P_i, k \neq j} S_k$ , which is the set of fibers that are survived by any of the selected paths other than path  $j$ . If  $S_j \subset S^*$ , remove path  $j$  from the set  $P_i$ . Note that removing such a path does not affect the survivability of the selected path set, i.e., the same set of fibers are still survived after the removal. However, it will decrease the number of selected paths by one. Repeat this procedure for all paths  $j$ , and check if they can be removed from the selected path set  $P_i$ .

Table I summarizes the performance of each algorithm under the path length restriction.

### C. Numerical Comparison of Algorithms

We compare the performance of our algorithms using large-scale random network topologies, as well as the US backbone network topology. In particular, we compare the following algorithms.

- 1) ILP-based optimal algorithm computed by CPLEX; denoted by OPT.
- 2) Simple Greedy algorithm from Section III-B1; denoted by MSPSG.
- 3) Random-Sweep Greedy algorithm from Section III-B3; denoted by RSG.

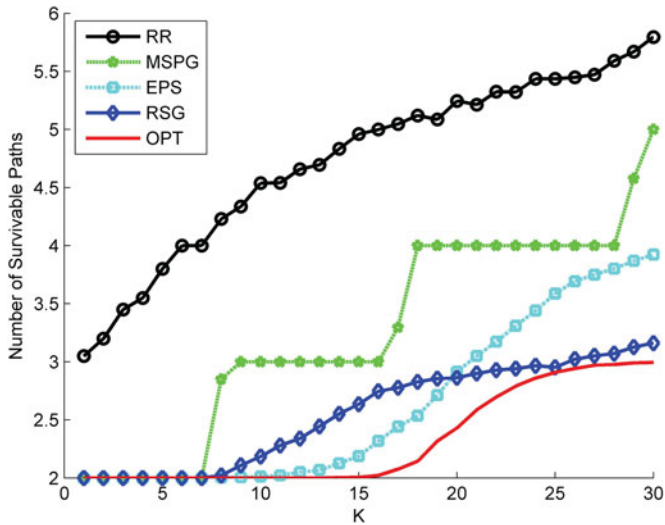


Fig. 2. Comparison of algorithms for MSPS under path length restriction.

- 4)  $\epsilon$ -net algorithm from Section III-B2; denoted by EPS.
- 5) Randomized rounding algorithm from Section III-A; denoted by RR.

1) *Performance in Large-Scale Random Topologies With Path Length Restriction:* We first consider a random layered network, where the logical topology consists of ten paths between nodes  $s$  and  $t$ . This layer is mapped onto the physical topology containing 100 fibers, using the mapping structure shown in [16]. In the  $K$  restricted version of the problem, each path consists of at most  $K$  fibers. For each value of  $K$ , we generate 1000 random topologies each with ten paths routed on the physical topology in a way that each path can select up to  $K$  fibers at random, uniformly and independently. We then apply our algorithms to each network in order to find a MSPS (i.e., to solve the MSPS problem). Note that the performance of randomized rounding and  $\epsilon$ -net algorithms depends on the survivability guarantee of the algorithms, which are 99.9% and 100%, respectively, for the results shown below.

Fig. 2 compares the average number of survivable paths found by each algorithm. It can be seen that as the value of  $K$  increases, the number of paths increases. This is due to the fact that when  $K$  is large, logical paths consist of more fibers; therefore, more logical paths are needed since they can share more fibers. Fig. 3 compares the logarithm of the running time of the algorithms. It can be seen that the randomized rounding algorithm is the fastest, while the RSG algorithm and the  $\epsilon$ -net algorithm have larger running times. Note also that the running times are nearly independent of  $K$ .

2) *Performance in Real Networks:* Next, we examine the performance of the approximation algorithms over the US backbone topology shown in Fig. 4(a). We have designed the logical layer and lightpath routing of logical layer on top of the physical layer. For every pair of nodes in the logical layer, we find the MSPS both using CPLEX to find the exact solution and our approximation algorithms. We constrain the length of paths to four logical links, and find the average size of the MSPS over the pairs of nodes.

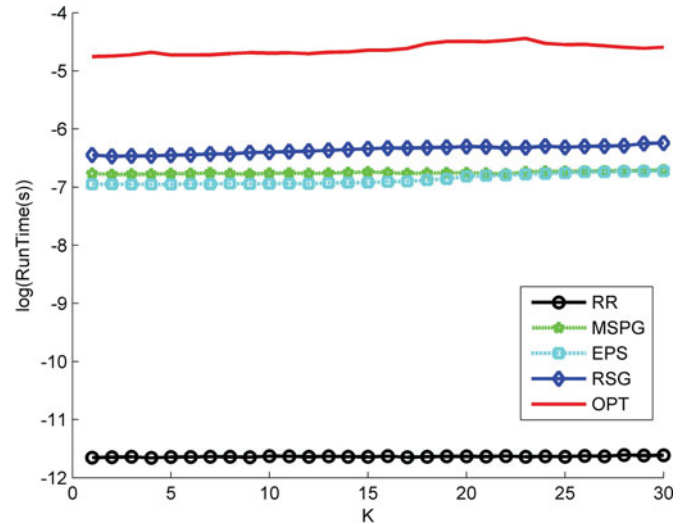


Fig. 3. Run time comparison of different heuristics and optimal algorithm.

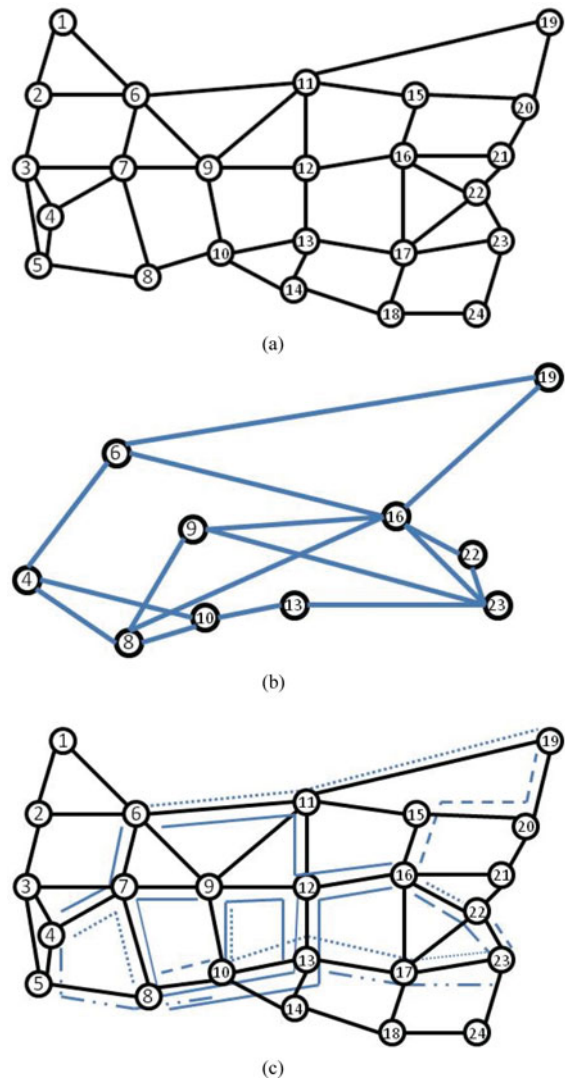


Fig. 4. US backbone topology. (a) US Backbone Topology (Source: [25]). (b) Logical Topology. (c) LightPath routing.



TABLE II  
COMPARISON OF ALGORITHMS FOR MSPS IN US BACKBONE

Method	Number of Paths	Running Time (ms)
ILP	2.113	6.4
RSG	2.196	0.59
MSPSG	2.25	0.24
RR	2.336	0.047
EPS	3.177	0.33

Table II shows the average number of paths and average running time of each algorithm. It can be seen that the RSG is nearly optimal, and furthermore, the randomized rounding gives a solution almost instantly. We also note that the survivability guarantees of the randomized rounding and  $\varepsilon$ -net algorithms are 98% and 97%, respectively, for the results shown in the table.

As explained in Section I, we define the metric of survivability as the fraction of pair of nodes that can survive any single failure. Moreover, the survivability of network has significantly increased from 86.66% to 97.77% due to allowing a *set* of survivable paths instead of only two physically disjoint paths. In particular, simulation results show that 5 out of 45 pairs need to have three paths to survive any single failure. Those pairs are (6, 22), (8, 13), (9, 13), (10, 13), (9, 22). One reason for requiring three paths is that two SRLG-disjoint paths do not exist under the mapping in Fig. 4(c). For example, for pair (9, 13), paths (9 – 8 – 10 – 13) and (9 – 23 – 13) are logically disjoint. However, they share fiber (9 – 10), and thus, are not physically disjoint. Another reason is that two SRLG-disjoint paths use more than four logical links in one of the paths, and thus, they are not used. For example, for pair (8, 13), one can find paths (8, 10, 13) and (8, 4, 6, 16, 22, 23, 13) that are physically disjoint. However, the second path contains six logical links, and is not selected since we only use logical paths containing no more than four logical links.

#### IV. MFSPS

Our focus so far has been on providing protection using the minimum number of paths. In this section, our goal is to find a survivable path set that uses the minimum number of fibers; i.e., Minimum Fibers in Survivable Paths Set (MFSPS). This may seem to be equivalent to solving the minimum cost survivable path set (MCSPS) problem, where the cost of each path is the number of fibers used by that path. However, this is not true as the costs in MCSPS are additive, where in MFSPS a fiber that is used by multiple paths should be counted just once. In order to make this point clear, consider Fig. 5. The MCSPS problem will find paths 1 and 2 as the set of survivable paths with total cost 7, while the MFSPS problem will find paths 2, 3, and 4 as the optimal survivable path, which has the total cost 6. In the next section, we will develop an ILP formulation, and analyze the complexity of MFSPS problem.

##### A. ILP Formulation and Complexity

We start with an ILP formulation of the problem. Similar to the MSPS problem, the MFSPS problem can be formulated in

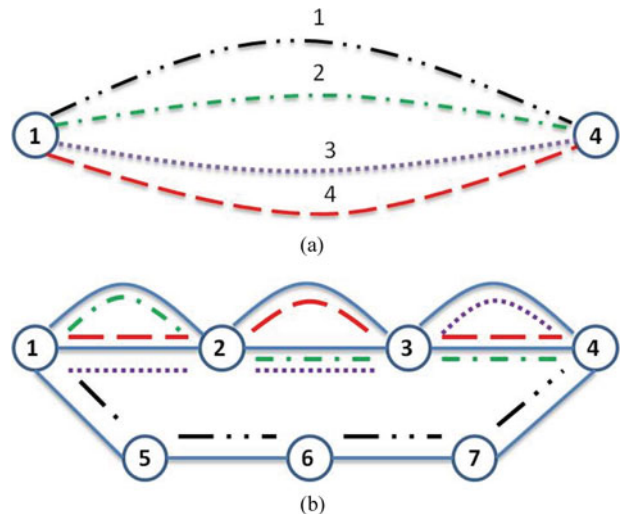


Fig. 5. Routing in multilayer network. (a) Logical Topology. (b) Routing.

several different ways, but here we only present the path-based formulation, which will be used for developing heuristics and approximation algorithms. Given a set of paths and associated fibers, for each path  $j$ , assign a binary variable  $P_j$ , which takes the value 1 if path  $j$  is selected and 0 otherwise. Similarly, for each fiber  $i$ , assign a binary variable  $f_i$ , which takes the value 1 if fiber  $i$  is selected and 0 otherwise. The matrix  $A$  and vector  $e$  are defined in the same way as in the MSPS formulation (1a)–(1c).

$$\text{MFSPS : } \quad \text{minimize} \quad \sum_{i=1}^m f_i \quad (9a)$$

$$\text{subject to} \quad A \times P \geq e \quad (9b)$$

$$f_i \geq P_j \quad \forall f_i \in P_j \quad (9c)$$

$$P_j \in \{0, 1\} \quad \forall P_j. \quad (9d)$$

In the aforementioned, the objective function is the number of fibers used by the selected paths. Again, the constraints in (9b) require the selected path to be survivable. The constraints in (9c) relate the selected paths and fibers, such that a fiber is selected if at least one of the paths using the fiber is selected. Clearly, the optimal solution to the above optimization problem gives a set of survivable paths that use the minimum number of fibers. This MFSPS problem can be shown to be NP-hard.

*Theorem 5:* Computing the set of survivable paths using the minimum number of physical fibers is NP-hard.

*Proof:* We provide a mapping from the minimum three-set cover problem, which is a special version of the set cover problem, where each set has exactly three elements, to the MFSPS problem. The minimum three-set cover problem is NP-hard, and holds all the inapproximability properties of the minimum set cover problem.

Consider an instance of the minimum set cover problem with the ground set  $E$  and a family of subsets  $F$ . Suppose that each subset in  $F$  contains only three elements. To show a mapping, we construct a physical topology as shown in Fig. 6, such that each node on the left corresponds to a subset in  $F = \{C_1, \dots, C_{|F|}\}$  and the nodes on the right are the elements of  $E = \{e_1, \dots, e_m\}$ ,

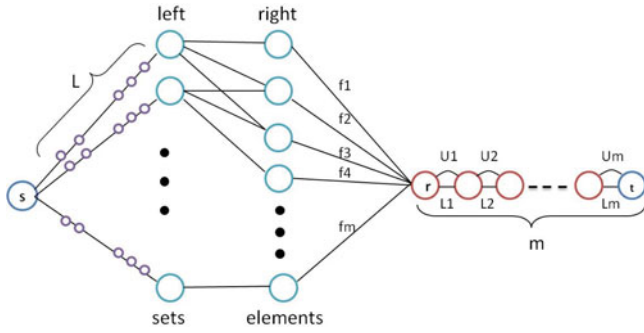


Fig. 6. Physical topology.

where  $m$  is the total number of fibers in our original problem. Node  $j$  on the left is connected to node  $i$  on the right if and only if  $e_i \in C_j$ . Note that a node on the left is connected to only three nodes on the right (i.e., each set contains only three elements). Moreover, there are  $L$  fibers between node  $s$  and every node on the left, where  $L$  is a very large number ( $L \geq 3|E| + 2|F|$ ).

We can construct a logical topology and its lightpath routing over the physical topology such that for protection, we need to have  $m$  paths from  $s$  to  $t$  that pass through all the nodes on the right. Moreover, since each path between  $s$  and the nodes on the left uses a large number of fibers, we should select a survivable path set that uses the minimum number of nodes on the left. Consequently, the minimum fiber survivable path set for the aforementioned layered network gives a minimum set cover for the given instance of  $E$  and  $F$ , which shows the NP-hardness of the MFSPS problem. For the complete proof, see Appendix A. ■

Since the MFSPS problem is a reduction from the minimum three-set cover problem, it does not have an efficient optimal algorithm. If we consider the restricted version of the problem where the length of each path is restricted to  $K$  fibers, then the problem can be solved in polynomial time similar to Section III. However, due to the large runtime of optimal algorithm, we will develop a heuristic in the next section.

### B. MFSPS Greedy Algorithm

Note that the goal here is to find a survivable path set that uses the minimum number of fibers. Hence, it is desired to select a path that uses a small number of new fibers (i.e., fibers not used by already selected paths) while surviving many new fibers (i.e., fibers not survived by already selected paths) as possible. Note that this is clearly different from the MSPS problem, where the number of fibers does not matter. The MFSPS greedy algorithm requires the set of paths and associated fibers as input. We define a new cost metric in order to take into account the two factors simultaneously. The cost  $C_j$  of path  $j$ , which is updated for every iteration, is defined as follows:

$$C_j = \frac{\text{\#newly used fibers by } P_j}{\text{\#newly survived fibers by } P_j}.$$

Note that the nominator (denominator) is the number of fibers used (survived) by path  $j$  and not used (not survived) by the

TABLE III  
COMPARISON OF NUMBER OF FIBERS USED IN DIFFERENT FORMULATIONS

Method	Number of Fibers	Number of Paths
MFSPS ILP	11.2500	2.7727
MFSPS Greedy	11.5455	2.5682
MSPS ILP	17.4545	2.1136
MCSPS ILP	12.1591	2.1136

previously selected paths. Our greedy algorithm selects a path with minimum cost, updates the costs of the remaining paths, and continues until all the fibers are survived.

### C. Numerical Comparison of Algorithms

We consider the same topology as in Section III-C2, and the goal is to find the set of survivable paths between every pair of nodes so that minimum number of fibers is used. We solve the optimal MFSPS, optimal MSPS, optimal MCSP, and the greedy algorithm presented for MFSPS in Section IV-B.

Table III compares the average number of paths and number of fibers used in the survivable path sets, for different algorithms, over all sets of node pairs. Table shows that the MFSPS greedy algorithm performs near-optimally in the number of fibers. Moreover, It can be seen that while MFSPS ILP gives the smallest set of fibers and largest set of paths, the MSPS ILP gives the largest set of fibers and smallest set of paths, which shows a tradeoff between these two metrics in this example. This is due to the fact that if there exist SRLG-disjoint paths, MSPS always finds such paths. However, if such paths traverse many fibers, they will not be the optimal solution of MFSPS. In fact, MFSPS may find more paths that use fewer fibers. Note that such tradeoff between the number of paths and fibers does not always exists, and it completely depends on the topology. Finally, it can be seen from Table III that MCSPS ILP has a reasonable performance as it keeps both the number of paths and the number of fibers small.

## V. SURVIVABILITY WITH MORE THAN TWO DEPENDENT LAYERS: INTERACTION WITH THE POWER GRID

In this section, we generalize the problem of survivability to networks with more than two dependent layers, where these layers could belong to one or several infrastructures. It is easy to see that one can always map the failures in different layers to the set of survivable paths through a matrix  $A_G$  similar to matrix  $A$  in Section III, where rows of matrix  $A_G$  correspond to the single failures in all lower layers, and the columns of  $A_G$  correspond to the paths in the top layer. Similar to matrix  $A$ ,  $A_G(i, j) = 1$  if path  $j$  survives, the failure of risk  $i$  and  $A_G(i, j) = 0$  otherwise. Once the matrix  $A_G$  is constructed for a given layered network, all of our algorithms in the previous sections can be readily applied.

In the following, we investigate the survivability problem in a network with three dependent layers, where the two top layers are the logical and physical layers of communication network and the lower layer is the power grid. We show how adding



the third layer of power grid will affect the survivability of the network.

### A. Power Grid and Communication Network

It has been shown that communication networks are dependent on the power grid in the sense that communication nodes receive power from the power grid. Therefore, if a power node fails, all of the corresponding communication nodes that receive power only from that power node will fail as well [26], [27].

Here, we consider a two layered communication network that is dependent on a power grid. The model of communication network is fully explained in Section II. In the following, we explain briefly the power grid model and the dependence between the physical layer of the communication network and the power grid.

1) *Power Grid Model*: The power grid can be modeled as a graph  $G^{\text{power}} = (V^{\text{power}}, E^{\text{power}})$ , where  $V^{\text{power}}$  and  $E^{\text{power}}$  are power nodes and lines, respectively. There are three types of power nodes in a grid: *Generators* that generate power, *Loads* that consume power, and *Substations* that neither generate nor consume power.

Unlike communication networks, the flow in power lines cannot be controlled manually; instead, it is determined based on the principles of electricity. As a consequence, the failure model in the power grid is also different from that in communication networks. When a power node or line fails, its load is shifted to the other elements of the grid. During this process, the flow in one or more lines may be pushed beyond their capacity, which leads to the failure of these overloaded lines. Then, the failure of these lines causes the redistribution of power and may lead to extra failures. This process of failures in the power grid is referred to as ‘cascading failures.’

In summary, unlike communication network, the failure of an element in the power grid can lead to the failure of several elements in that grid. In this paper, we do not discuss the details of the cascading model (it can be found in [28]). Note that the effect of cascading failures for any given set of initial failures can be calculated in polynomial time.

2) *Dependence Model*: We assume that the nodes in the physical layer of the communication network depend on one or multiple loads in power grid. According to this model, when a power line fails, the failure cascades through the power grid due to overloading of some lines. Then, some loads lose their power (power is zero), and the corresponding nodes in the physical layer of the communication network fail as well. Consequently, the fibers attached to those nodes fail, which lead to the failure of links in the logical layer of the communication networks. In this three-layered dependent network, there are risks of losing a power line or a fiber, and the goal is finding a set of survivable paths between every pair of nodes so that they can survive any single failure of any type.

### B. Simulation Results

In this section, we consider the US IP-backbone discussed in Section III-C2 as the communication network and the IEEE

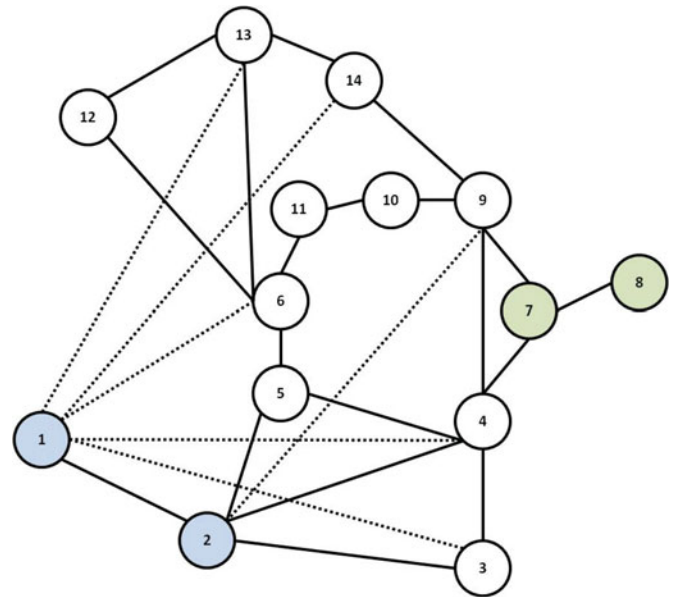


Fig. 7. Augmented IEEE 14-bus benchmark—Dotted lines show the additional lines; nodes 1, 2 are generators and 7, 8 are substations.

14-bus benchmark as the power grid. The information of IEEE 14-bus benchmark can be found in [29].

We tested the robustness of the grid for every single power line failure, and observed that in some cases, loss of a power line leads to the failure of the whole grid. Therefore, we improved the power grid by adding some power lines between the generators and loads. We set the reactance of the new lines to the average of reactance of the old neighbor lines. Fig. 7 shows the augmented topology of IEEE 14-bus benchmark. Dotted lines show the additional power lines. We have also investigated the impact of a single power line on the performance of power nodes. Table IV shows the set of power nodes that fail due to single power line failures considering the cascading effect inside the power grid.

Next, we design the dependence topology of the communication nodes and the loads in power grid. Each communication node can receive power from one or two power nodes. We name these nodes as ‘primary’ and ‘secondary’ power nodes. When a communication node receives power only from the primary nodes, it is a ‘single dependence’ and when it receives power from both primary and secondary nodes, it is a ‘double dependence.’ In the case of double dependence, a communication node fails if both primary and secondary power nodes fail. Table V lists the communication nodes and the corresponding primary and secondary power nodes. In this experiment, for every communication node, we have set the closest power node as the primary power node. However, we have selected two sets of secondary power nodes. The secondary power nodes A have been selected so that in the case of a single power line failure, both the primary and secondary nodes may fail. However, the secondary power nodes B have been selected so that in the case of a single power line failure, at least one of the power nodes survives.

TABLE IV  
IMPACT OF A SINGLE POWER LINE FAILURES ON THE POWER NODES,  
CONSIDERING THE CASCADING FAILURES EFFECT

Failed Power Line	Failed Power Nodes
(1,2)	$\emptyset$
(1,3)	3
(1,4)	3
(1,5)	11
(1,6)	$\emptyset$
(1,13)	3
(2,4)	5
(2,5)	5,6,10,11,12
(2,9)	$\emptyset$
(3,4)	$\emptyset$
(4,5)	$\emptyset$
(4,7)	9,10,11
(4,9)	$\emptyset$
(5,6)	$\emptyset$
(6,11)	10,11
(6,12)	10,11
(6,13)	10
(7,8)	12
(7,9)	12,13
(9,10)	$\emptyset$
(9,14)	12
(10,11)	14
(12,13)	$\emptyset$
(13,14)	$\emptyset$

TABLE V  
DEPENDENCY TOPOLOGY

Communication Node	Primary Power Node	Secondary Power Node A	Secondary Power Node B
1	3	4	4
2	3	4	4
3	3	4	4
4	4	3	3
5	4	3	3
6	6	5	9
7	6	5	9
8	4	3	3
9	5	6	4
10	9	10	6
11	6	5	9
12	5	6	4
13	13	12	11
14	9	10	6
15	11	10	13
16	11	10	13
17	13	14	11
18	14	13	9
19	12	13	13
20	12	13	13
21	10	9	13
22	10	9	13
23	14	13	9
24	14	13	9

We consider the following five scenarios and check the survivability of the logical communication network under each scenario.

- 1) Communication nodes are only dependent on primary power nodes. The risk is the failure of a single power node. Failures *do not* cascade through the power grid.

TABLE VI  
MSPS IN POWER-COMMUNICATION NETWORK; IN THE FIRST COLUMN NOTATION  $R$  REPRESENTS RISK TYPE; IN THE FIRST ROW, NOTATION  $P$  REPRESENTS THE NUMBER OF PATHS;  $NoP$  MEANS THAT NO SET OF PATHS CAN BE FOUND TO SURVIVE ANY SINGLE RISK

Scenario	$1P$	$2P$	$3P$	$NoP$	Survivability
Single Dependence $R$ :Power Node	3	21	2	19	57.77%
No Cascade					
Single Dependence $R$ :Power Line	4	3	0	38	15.55%
Cascade					
Double Dependence A $R$ :Power Line	4	10	0	31	31.11%
Cascade					
Double Dependence B $R$ :Power Line	45	0	0	0	100%
Cascade					
Double Dependence B $R$ :Power Line & Fiber	0	39	5	1	97.77%
Cascade					

- 2) Communication nodes are only dependent on primary power nodes. The risk is the failure of a single power line. Failures *cascade* through the power grid.
- 3) Communication nodes are dependent on both primary and secondary power nodes of set A. The risk is the failure of a single power line. Failures *cascade* through the power grid.
- 4) Communication nodes are dependent on both primary and secondary power nodes of set B. The risk is the failure of a single power line. Failures *cascade* through the power grid.
- 5) Communication nodes are dependent on both primary and secondary power nodes of set B. The risk is either the failure of a single power line or the failure of a fiber. Power line failures *cascade* through the power grid.

Table VI shows the number of pairs that can survive with 1, 2, or 3 paths and the pairs that cannot survive at all. Note that surviving with only 1 path means that there exists a path that will not fail due to any single failure. Moreover, the total survivability of the network as the percentage of survivable pairs is presented under each scenario. Since we have ten nodes in our logical topology [see Fig. 4(b)], the total number of pairs is 45. Similar to Section III-C2, we consider paths that use fewer than four logical links.

It can be seen that under scenario 1, 57.77% of the pairs can survive any single failure event. This means that in 42.22% of the pairs, all paths connecting each pair share at least one power node. In scenario 2, the survivability has decreased to 15.55%, which is due to large scale cascading failures inside the power grid. We also observed that under scenarios 3 and 4, the network becomes more robust as every communication node is supported by two power nodes. Comparing scenarios 3 and 4 shows that choosing secondary nodes of set B (scenario 4) is indeed more robust and all pairs can survive with only one path. This is because we had selected the primary and secondary power nodes so that under any single power line failure, at least

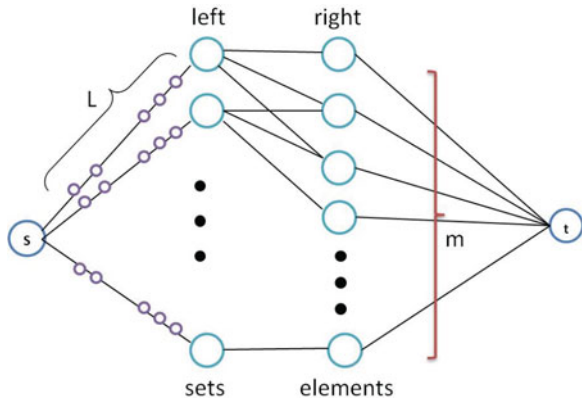


Fig. 8. Logical topology.

one of them can provide power for the communication nodes. Therefore, no failure in power grid can affect the logical layer. Finally, under scenario 5, we have considered two types of risks, which are either fiber or power line failures. It can be seen that the survivability results are similar to results in Section III-C2). The reason is that under double dependency model B, the power line failures do not affect the logical layer, and the failures in this layer are only due to fiber losses.

One can also repeat the same experiment to find the set of survivable paths that use the minimum number of fibers; i.e., MFSPS problem.

## VI. CONCLUSION

We considered the problem of survivability in multilayered networks. The traditional disjoint paths approach for protection cannot be directly applied to layered networks, since physically disjoint paths may not always exist in such networks. To address this issue, we introduced the new notion of survivable path set. We showed that, in general, the problem of finding the MSPS in two-layered networks is NP-hard. However, under practical constraints, we are able to develop both optimal and approximation algorithms for the MSPS problem. We also extended the results to networks with more than two dependent layers and showed that the same results hold in this general setting.

## APPENDIX

### LOGICAL TOPOLOGY OF THEOREM 5

In the logical topology in Fig. 8, from node  $s$  to the nodes on the right, each logical link is routed exactly on one fiber, while from nodes on the right side to node  $t$ , there are  $m$  parallel logical links with a specific routing. The first logical link will be routed on fibers  $f_1$ ,  $U_1$ , and  $L_2$  to  $L_m$ , the second logical link will be routed on fibers  $f_2$ ,  $L_1$ ,  $U_2$ , and  $L_3$  to  $L_m$  and so on. Therefore, logical link  $i$  will use fibers  $f_i$ ,  $U_i$  and all the other  $L_j$ s ( $j \neq i$ ).

To survive any single failure in the fibers from the right nodes to node  $t$ , we need to have at least  $m$  paths, each going through one of the parallel logical links. Clearly, these  $m$  paths will not share any fiber from the left nodes to the right nodes, as well.

Finally, to survive any fiber failure from node  $s$  to the left nodes, it is enough to have at least two logically disjoint paths from node  $s$  to the left nodes.

Consequently, it is enough just to have  $m$  paths from  $s$  to  $t$  that cover all the nodes on the right-hand side. Since  $m > 3$ , it is guaranteed that these  $m$  paths will pass through at least two nodes on the left, as well; i.e., the paths will survive any single fiber failure. The remaining of the proof is explained in the main text.

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