Age Optimal Information Gathering and Dissemination on Graphs

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Abstract—We consider the problem of timely exchange of updates between a central station and a set of ground terminals V, via a mobile agent that traverses across the ground terminals along a mobility graph $G = (V, E)$. We design the trajectory of the mobile agent to minimize average-peak and average age of information (AoI), two recently proposed metrics for measuring timeliness of information. We consider randomized trajectories, in which the mobile agent travels from terminal i to terminal j with probability $P_{i,j}$. For the information gathering problem, we show that a randomized trajectory is average-peak age optimal and factor-$8\mathcal{H}$ average age optimal, where $\mathcal{H}$ is the mixing time of the randomized trajectory on the mobility graph G. We also show that the average age minimization problem is NP-hard. For the information dissemination problem, we prove that the same randomized trajectory is factor-$O(\mathcal{H})$ average-peak and average age optimal. Moreover, we propose an age-based trajectory, which utilizes information about current age at terminals, and show that it is factor-2 average age optimal in a symmetric setting.

Index Terms—Age of information, wireless networks, trajectory optimization, scheduling.

1 Introduction

Many emerging applications depend on the collection and delivery of status updates between a set of ground terminals and a central terminal using mobile agents. Examples include: measuring traffic at road intersections [2], temperature, and pollution in cities [3], ocean monitoring using underwater autonomous vehicles [4], and surveillance using UAVs [5]. All of these applications depend upon regular status updates, that are communicated in a timely manner, so as to keep the central terminal and the ground terminals updated with fresh information.

Age of Information (AoI) is a recently proposed metric that captures timeliness of the received information [6]–[8]. Unlike packet delay, AoI measures the lag in obtaining information at the destination node, and is therefore suited for applications involving gathering or dissemination of time sensitive updates [9]–[11]. Age of information, at a destination, is defined as the time that elapsed since the last received information update was generated at the source. AoI, upon reception of a new update packet, drops to the time elapsed since generation of the packet, and grows linearly otherwise. For detailed surveys of AoI literature, see [9] and [12].

We consider the problem of AoI minimization in gathering and dissemination of information updates, between a set of ground terminals and a central terminal. The information updates can be as small as a single packet containing temperature information or a high fidelity image or a video file. The ground terminals represent locations where information of interest is being generated or needs to be delivered. Thus, they could be sensors in a wireless sensor network equipped with low power transmitters, where a mobile agent is used for real-time monitoring. Another example is a set of locations that needs timely monitoring via a mobile agent, like traffic at intersections in a city. A third application consists of using a mobile agent to disseminate timely updates in remote locations in the event of disasters or an emergency. Further, while we discuss the problem in terms of a physical agent and locations, the model can also describe timely monitoring of dynamic content on the Web using a web-crawler.

We consider two versions of the problem - 1) information gathering in which ground terminals generate new updates whenever needed with no queuing; and 2) information dissemination in which new updates are generated by a central station according to a stochastic process, get queued into FCFS queues and are delivered to ground terminals.

The age or freshness of information gathered and disseminated depends on the trajectory of the mobile agent, whose mobility is constrained to a *mobility graph* $G = (V, E)$. The mobile agent can move from ground terminal i to ground terminal j only if $(i, j) \in E$. This model can be used to capture the fact that the agent may not be able to move between any arbitrary locations due to topological limitations.

The problem of persistent monitoring in dynamic environments has been considered in [13]–[15] using tools from optimal control. These works focus on minimizing uncertainty when source locations are time varying, rather than timely monitoring over a fixed set of locations. There is also a large body of work focused on planning trajectories for a mobile agent to optimize traditional performance metrics in wireless sensor networks; primarily throughput, delay and network lifetime; by leveraging variants of the Travelling Salesman Problem (TSP) [16]–[20]. However, these works do not focus on freshness as a metric. We observe connections to this line of work in our paper, where we establish the optimality of a Hamiltonian cycle trajectory in a symmetric setting.

Closer to our work are [21] and [22], in which some approximation trajectories to minimize maximum latency on metric graphs were proposed. In [23], the authors consider trajectory planning for a mobile agent to minimize AoI. They obtain the best permutation of nodes for the mobile agent to visit in sequence, given Euclidean distances between the nodes. A similar problem is also considered in [24], where average-peak age minimization

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for UAV-assisted IoT networks is considered. However, average age minimization is not considered. In this work, we consider a mobile agent gathering/disseminating information from multiple nodes. Its mobility is constrained by a general graph G, and we seek average-peak and average age optimal trajectories over the space of *all trajectories* allowed on this graph G, not just permutations of nodes. To the best our knowledge, this is the first work to consider both average-peak and average AoI minimization on general mobility graphs G. A preliminary version of this work was presented in [1].

For the information gathering problem, we design trajectories for the mobile agent to minimize average-peak age and average age, two popular metrics of AoI. We first consider the space of randomized trajectories, in which the mobile agent traverses edges according to a random walk on the mobility graph G. We show that a randomized trajectory is in fact average-peak age optimal, and that it can be obtained in polynomial time using the Metropolis-Hastings algorithm. We then prove that solving for the average age optimal trajectory is NP-hard, in a symmetric setting, and propose a heuristic randomized trajectory that is simultaneously average-peak age optimal and factor-$8\mathcal{H}$ average age optimal, where $\mathcal{H}$ is the mixing time of the randomized trajectory on G. The factor $\mathcal{H}$ can scale with the graph size, especially if the graph is not well connected. Thus, we propose an age-based trajectory, in which the mobile agent uses the current AoI to determine its motion, and show that it is factor-2 optimal in a symmetric setting.

In the information dissemination setting, the mobile agent receives update packets in separate first-come-first-served (FCFS) queues, one for each terminal. The mobile agent then transmits the head-of-line packet from the ith queue when it reaches the location of the ith ground terminal. The FCFS queue assumption is motivated by uncontrollable MAC layer queues, where the generated updates get queued for transmission [10], [25]. We, now, not only have to design the trajectory of the mobile agent, but also determine the optimal rate at which the central terminal generates information updates for each ground terminal. We show that the average-peak age optimal randomized trajectory for information gathering, along with a simple update generation rate, is at most a factor-$O(\mathcal{H})$ optimal, in both peak and average age. Also derived is an explicit formula for average-peak age of the discrete time Ber/G/1 queue with vacations, which may be of independent interest.

We describe the system model in Section 2. The gathering and dissemination versions are studied in Section 3 and Section 4, respectively. We present simulation results in Section 5, and conclude in Section 6.

2 System Model

We consider a central terminal that needs to communicate with a set of ground terminals V. The ground terminals are equipped with low power, low range radio communication devices, and cannot directly communicate with the central terminal, or with each other. An autonomous mobile agent m, is used as a relay between the central terminal and the ground terminals, while moving across the geographical region where the ground terminals are spread.

The mobility of the agent is constrained by a *mobility graph* $G = (V, E)$, where m can travel from ground terminal i to ground terminal j only if $(i, j) \in E$. The graph G, thus, constraints the set of allowable moves. We consider a time-slotted system, with slot duration normalized to unity. In the duration of a time-slot, the mobile agent stays at a ground terminal to gather or disseminate information, and moves to any of its neighbours in G for the next time-slot. The mobility graph can be constructed from the limitations of a slot duration, distances between ground terminals, and speed of the mobile agent. We consider this abstract graph mobility model because of its analytical tractability. This allows us to gain insight into practical system design and explore the tradeoffs between computational complexity and optimality.

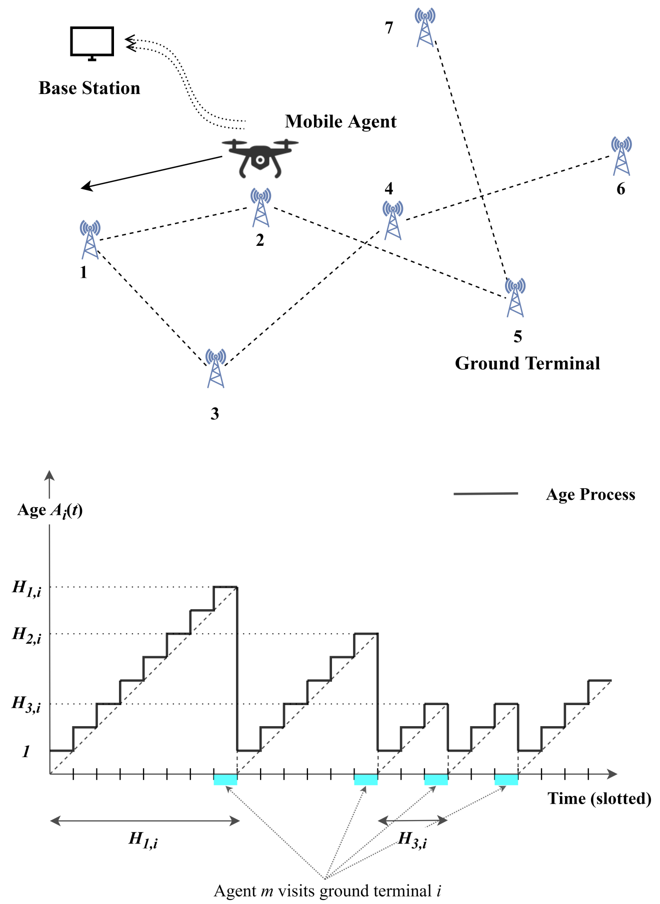


Fig. 1: Information Gathering: time evolution of age $A_i(t)$; $H_{k,i}$ is the k^{th} inter-return time to terminal i.

In the information gathering problem, every time the mobile agent reaches a ground terminal $i \in V$, the ground terminal sends a fresh update to the mobile agent, which is immediately relayed to the central terminal. The age $A_i(t)$, at the central terminal, for the ground terminal i drops to 1. When the mobile agent is not at the ground terminal i, the age $A_i(t)$ increases linearly. See Figure 1. The evolution of $A_i(t)$ in this case can be written as:

$$A_i(t+1) = \begin{cases} A_i(t) + 1, & \text{if } m(t) \neq i \\ 1, & \text{if } m(t) = i \end{cases} \tag{1}$$

where $m(t)$ denotes the location of the mobile agent at time t. Note that the age evolution depends on the trajectory that the mobile agent follows on the mobility graph G.

In the information dissemination setting, the mobile agent receives updates from a central terminal to be disseminated to each ground terminal through queues. The mobile agent enqueues updates generated stochastically in a set of $|V|$ FCFS queues, one for each ground terminal. It transmits the head-of-line update

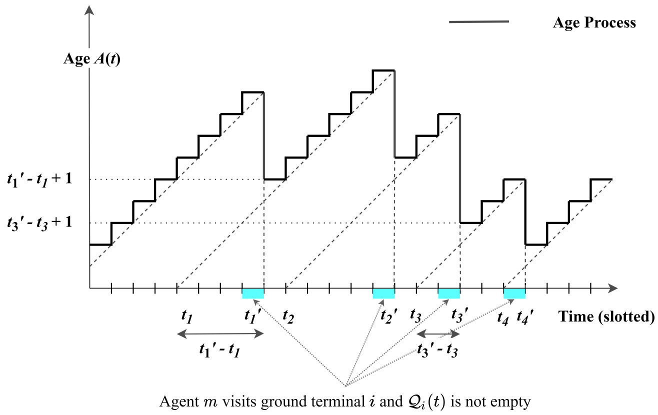


Fig. 2: Information Dissemination: time evolution of age $A_i(t)$; t_k, t_k' are the generation and reception times of the k^{th} status update for terminal i.

in queue i to ground terminal i when it reaches i. The system designer has no direct control over the generation process or the FCFS queues, however, it can control the update generation rate λ_i, for each ground terminal i.

The age $A_i(t)$, at the ground terminal i, increases by 1 every time the mobile agent is not at i, or when it is at i but the queue is empty. Otherwise, a successful delivery of the head-of-line update occurs in time slot t, and the age $A_i(t)$ drops to the age of the head-of-line update in queue i. See Figure 2. This evolution of age $A_i(t)$ can be written as:

$$A_i(t+1) = \begin{cases} A_i(t)+1, & \text{if } m(t) \neq i \\ A_i(t)+1, & \text{if } m(t) = i \text{ and } \mathcal{Q}_i(t) = \emptyset \\ t - G_i(t) + 1, & \text{if } m(t) = i \text{ and } \mathcal{Q}_i(t) \neq \emptyset \end{cases}, \tag{2}$$

where $G_i(t)$ is the time of generation of the head of line packet in queue i, at time t, and $\mathcal{Q}_i(t)$ denotes the set of packets in the mobile agent's queue i at time t.

2.1 Age Metrics

AoI is an evolving function of time. We consider two time average metrics of AoI. Average age, for ground terminal i, is defined as the time averaged area under the age curve:

$$A_i^{\text{ave}} \triangleq \limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A_i(t). \tag{3}$$

In Figures 1 and 2, we see that the age $A_i(t)$ peaks before a new update is delivered. When ground terminals generate updates at will, a fresh update is delivered every time the mobile agent visits i, i.e. $m(t) = i$. Whereas, when updates are stochastic and queued, a new update is delivered whenever $m(t) = i$ and the queue $\mathcal{Q}_i(t) \neq \emptyset$. The average-peak age A_i^{p}, for ground terminal i, is defined as an average of all the peaks in the age evolution curve $A_i(t)$. Peaks in the age process are defined as all time-slots when an update is delivered, i.e. the age does not increase linearly. Thus two consecutive visits to a terminal count as two peaks. The average-peak age can be written as

$$A_i^{\text{p}} \triangleq \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A_i(t) \mathbb{1}_{\{m(t)=i\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i\}}}, \tag{4}$$

in the gathering setting and

$$A_i^{\text{p}} \triangleq \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A_i(t) \mathbb{1}_{\{m(t)=i, \mathcal{Q}_i(t)\neq\emptyset\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i, \mathcal{Q}_i(t)\neq\emptyset\}}}, \tag{5}$$

in the dissemination setting.

Finally, we define the network average-peak age and average age to be

$$A^{\text{p}} = \sum_{i\in V} w_i A_i^{\text{p}} \quad \text{and} \quad A^{\text{ave}} = \sum_{i\in V} w_i A_i^{\text{ave}}, \tag{6}$$

where $w_i > 0$ are weights representing the relative importance of a ground terminal i. Our goal is to minimize network peak and average age. Minimization of peak and average AoI in wireless networks has become a very active topic of research in recent years [10], [26]–[29].

2.2 Trajectory Space

Given a trajectory $\mathcal{T}$, we first define $f_i(\mathcal{T})$ as the fraction of time-slots, the trajectory $\mathcal{T}$, is at ground terminal i:

$$f_i(\mathcal{T}) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{1}_{\{m(t)=i\}}. \tag{7}$$

Using this definition, we define $\mathbb{T}$ to denote the space of trajectories that are of interest for the purpose of age minimization.

$$\mathbb{T} = \{ \text{ Trajectory } \mathcal{T} \mid f_i(\mathcal{T}) \text{ exists and is positive } \forall\, i \in V \ \},$$

For a trajectory $\mathcal{T} \in \mathbb{T}$, the limit (7) exists and is positive for all $i \in V$. This requirement is to ensure that we consider *reasonable* trajectories: those which visit each ground terminal a strictly positive fraction of the time.

Peak and average age depend on the trajectory $\mathcal{T} \in \mathbb{T}$. We use $A^{\text{p}}(\mathcal{T})$ and $A^{\text{ave}}(\mathcal{T})$ to denote network peak and average age, respectively, for $\mathcal{T} \in \mathbb{T}$. Since our goal is minimization of age metrics, we will trivially ignore all trajectories in $\mathbb{T}$ that do not have bounded average or peak age.

3 Information Gathering

In this section, we consider the problem of information gathering when fresh updates are generated at will. We define optimal peak and average age to be

$$A_{\mathcal{G}}^{\text{p}*} = \min_{\mathcal{T}\in\mathbb{T}} A^{\text{p}}(\mathcal{T}), \quad \text{and} \quad A_{\mathcal{G}}^{\text{ave}*} = \min_{\mathcal{T}\in\mathbb{T}} A^{\text{ave}}(\mathcal{T}), \tag{8}$$

where $\mathbb{T}$ denotes the space of trajectories for the mobile agent.

To find optimal trajectories, we first consider randomized ones, where the mobile agent moves according to a random walk on the mobility graph. We shall show that for average-peak age optimality, such randomized trajectories suffice. We then show that the average age optimization is NP-hard, and propose a heuristic randomized trajectory. In Section 3.4, we propose an age-based trajectory for better average age performance.

3.1 Randomized Trajectories

We start by defining the class of randomized trajectories. Note that we discuss randomized trajectories because they are easy to implement and analyze. The performance guarantees we derive, on the other hand, will hold over the space of all trajectories, not just randomized ones.

Definition A trajectory $m(t)$, on mobility graph G, is said to be a *randomized trajectory* if $m(t)$ is an irreducible Markov chain defined by a transition probability matrix $\mathbf{P}$:

$$\mathbb{P}\left[m(t+1)=j|m(t)=i\right]=P_{i,j}, \tag{9}$$

for all t and $i,j\in V$, where $P_{i,j}=0$ for $(i,j)\notin E$.

For convenience, we shall refer to $m(t)$, defined above, as the randomized trajectory $\mathbf{P}$, where $\mathbf{P}$ to denote the matrix with entries $P_{i,j}$. Note that $P_{i,j}$ is the probability that the mobile agent, when at ground terminal i, moves to ground terminal j for the next time slot. The constraint: $P_{i,j}=0$ for $(i,j)\notin E$, ensures that the randomized trajectory adheres to the mobility constraints defined by G. We further define two matrices associated with every randomized trajectory $\mathbf{P}$ that will come in handy later. Π is an $n\times n$ matrix in which every row is π, the stationary distribution of $\mathbf{P}$. Thus, $\Pi_{i,j}\triangleq\pi_j,\ \forall i,j\in V$. Using this, we define $Z\triangleq(I-\mathbf{P}+\Pi)^{-1}$, called the fundamental matrix of the Markov chain $\mathbf{P}$. The elements of Z are represented by z_{ij}.

We assume in the definition of a randomized trajectory $\mathbf{P}$, that $m(t)$ is an irreducible Markov chain over the state space V. This is desired, since the mobile agent has to traverse through all the nodes, repeatedly, for a positive fraction of time, or otherwise the resulting average-peak and average age would be unbounded.

For any randomized trajectory $\mathbf{P}$, we obtain explicit expressions for network peak and average age. We use the notation $A^{\text{p}}(\mathbf{P})$ and $A^{\text{ave}}(\mathbf{P})$ to show explicit dependence of peak and average age on the randomized trajectory $\mathbf{P}$.

Theorem *1.* The network average-peak and average age for a randomized trajectory $\mathbf{P}$ is given by

$$A^{\text{p}}(\mathbf{P})=\sum_{i\in V}\frac{w_i}{\pi_i},\quad\text{and}\quad A^{\text{ave}}(\mathbf{P})=\sum_{i\in V}\frac{w_i z_{ii}}{\pi_i}, \tag{10}$$

where π is the unique stationary distribution obtained by solving $\pi\mathbf{P}=\pi$ and z_{ii} are diagonal elements of the matrix $Z\triangleq(I-\mathbf{P}+\Pi)^{-1}$, where Π is an $n\times n$ matrix with entries $\Pi_{i,j}\triangleq\pi_j,\ \forall i,j\in V$.

Proof: The key step in proving the result above is to observe that the average-peak age of the ground terminal i, namely A_i^{p}, depends only on the mean of return times to terminal i; see Figure 1. Whereas, the average age A_i^{ave} for i depends on both, the mean and the variance, of return times to terminal i.

Given a randomized trajectory $\mathbf{P}$, the mean of return times to terminal i is given by $\frac{1}{\pi_i}$, while the second moment of the return times is given by $\frac{-1}{\pi_i}+\frac{2z_{ii}}{\pi_i^2}$; see [30, Theorem 4.5.2]. Using this fact, we are able to obtain the explicit expressions for peak and average age. Let A_i^p be the peak age for ground terminal i. We define $H_{k,i}$ to be the k^{th} inter-return time to ground terminal i. Then, the k^{th} age peak for $A_i(t)$ has a value of $H_{k,i}$. Let K be the total number of returns to i over a time-horizon T. Then, the expected peak age of ground terminal i is given by

$$A_i^p=\lim_{T\to\infty}\mathbb{E}\left[\frac{\sum\limits_{t=1}^{t=T}A_i(t)\mathbb{1}_{\{m(t)=i\}}}{\sum\limits_{t=1}^{t=T}\mathbb{1}_{\{m(t)=i\}}}\right]=\lim_{K\to\infty}\mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{t=K}H_{k,i}\right]. \tag{11}$$

Note that return times to a ground terminal i are i.i.d. random variables given a randomized trajectory $\mathbf{P}$. So, we can use the law of large numbers to get

$$A_i^p=\mathbb{E}[H_{1,i}]=\frac{1}{\pi_i}, \tag{12}$$

where π_i is the stationary distribution for Markov chain $\mathbf{P}$. The last equality follows from the fact that the expected return time to a state i for an irreducible Markov chain is given by the inverse of its stationary probability. Thus, the network age is given by

$$A^{\text{p}}=\sum_{i\in V}w_iA_i^p=\sum_{i\in V}\frac{w_i}{\pi_i}. \tag{13}$$

For average age, we define a renewal-reward process using $H_{k,i}$ as our i.i.d. renewal intervals and sum of age $A_i(t)$ during each interval as our reward. Let $T_{k,i}=\sum_{l=1}^{k-1}H_{l,i}$ be the starting time of the kth renewal. The total reward in between two visits to ground terminal i is the sum of the i^{th} age process $A_i(t)$ across all time-slots during that interval.

Note that, for the k^{th} renewal interval, $A_i(t)$ grows from 1 to $H_{k,i}$ over the $H_{k,i}$ time-slots. Thus, the total reward for the k^{th} renewal interval is given by -

$$\sum_{t=T_{k,i}}^{t=T_{k,i}+H_{k,i}}A_i(t)=\sum_{a=1}^{H_{k,i}}a=\frac{H_{k,i}^2+H_{k,i}}{2}. \tag{14}$$

Note that this reward is also i.i.d. across renewals as it depends only on $H_{k,i}$. Thus, by application of the elementary renewal theorem for renewal-reward processes we get

$$A_i^{\text{ave}}=\lim_{T\to\infty}\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T}A_i(t)\right]=\frac{\mathbb{E}[H_{1,i}^2+H_{1,i}]}{2\mathbb{E}[H_{1,i}]}. \tag{15}$$

For irreducible Markov chains, we know the following results hold [30, Ch.4]:

$$\mathbb{E}[H_{1,i}]=\frac{1}{\pi_i},\forall i\in V\text{ and} \tag{16}$$

$$\mathbb{E}[H_{1,i}^2]=\frac{-1}{\pi_i}+\frac{2z_{ii}}{\pi_i^2}, \tag{17}$$

for all $i\in V$, where z_{ii} is the i^{th} diagonal element of the matrix $Z=(I-P+\Pi)^{-1}$, with Π being a matrix in which all rows are the stationary distribution vector π: $\Pi_{i,j}=\pi_j$ for all $i,j\in V$.

Substituting (16) and (17) in (15), we get

$$A_i^{\text{ave}}=\frac{z_{ii}}{\pi_i}, \tag{18}$$

for all $i\in V$, and therefore,

$$A^{\text{ave}}=\sum_{i\in V}w_iA_i^{\text{ave}}=\sum_{i\in V}\frac{w_iz_{ii}}{\pi_i}. \tag{19}$$

□

3.2 Average-Peak Age Minimization

We first formulate the average-peak age minimization problem over the space of randomized trajectories. We shall see that a peak age optimal randomized trajectory suffices for optimality over the space of all trajectories.

We note that when updates can be generated at will, the average-peak age of terminal i is simply the reciprocal of its throughput. This is because the throughput for terminal i is simply π_i while its peak age is $1/\pi_i$.

Now, using the results in Theorem 1, we can write the average-peak age minimization problem over the space of randomized trajectories as:

$$\begin{aligned} \underset{\mathbf{P},\pi}{\text{Minimize}} \quad & \sum_{i\in V} \frac{w_i}{\pi_i}, \\ \text{subject to} \quad & P_{i,j} \geq 0,\ \forall (i,j), \text{ and } \mathbf{P1} = \mathbf{1}, \\ & \pi\mathbf{P} = \pi,\ \mathbf{1}^T\pi = 1, \text{ and } \pi_i \geq 0\ \forall i \\ & P_{i,j} = 0,\ \forall (i,j) \notin E, \\ & \mathbf{P} \text{ is irreducible.} \end{aligned} \tag{20}$$

Note that $\mathbf{P}$ characterizes a randomized trajectory, while π is the unique stationary distribution associated with it.

This problem is difficult to solve because the irreducibility constraint cannot be expressed in a simple, solvable manner. Further, relaxing the irreducibility constraint can yield a trivial solution like $\mathbf{P} = I$, which are neither irreducible nor anywhere close to optimal. It is also important to note that this problem is distinct from throughput optimization under the same setup. Throughput optimization over the space of randomized trajectories can be formulated with similar constraints as in (20) but with a different objective function, namely: $\underset{\mathbf{P},\pi}{\text{Maximize}} \sum_{i\in V} w_i\pi_i$. While both problems depend only on the average rate of visiting each ground terminal, the different objective functions lead to different optimal trajectories. The peak age problem can in fact be interpreted as proportional fair throughput optimization but with different weights. We discuss this in detail in the Appendix A.

The problem (20) can be transformed to finding an irreducible $\mathbf{P}$, with a given stationary distribution. This is a simpler problem and can be solved using the Metropolis-Hastings algorithm.

Lemma *1.* Let $\pi_i^* \triangleq \frac{\sqrt{w_i}}{\sum_{j\in V}\sqrt{w_j}}$, for all $i \in V$, to be a distribution on V, and a randomized trajectory $\mathbf{P}$ satisfy $\pi^*\mathbf{P} = \pi^*$. Then, $(\pi^*, \mathbf{P})$ solves (20).

Proof: Suppose we could choose any stationary distribution π on V. Then to minimize the network peak age, we would need to solve the following optimization problem

$$\begin{aligned} \underset{\pi}{\text{Minimize}} \quad & \sum_{i\in V} \frac{w_i}{\pi_i}, \\ \text{subject to} \quad & \sum_i \pi_i = 1, \pi_i \geq 0, \forall i \in V. \end{aligned} \tag{21}$$

Using KKT conditions for the optimization problem (21), it is straightforward to see that

$$\pi_i^* = \frac{\sqrt{w_i}}{\sum_i \sqrt{w_i}}, \forall i \in V. \tag{22}$$

Clearly, if we could find a randomized trajectory $\mathbf{P}$ that achieves this stationary distribution π^*, then it would be average-peak age optimal. Thus, any randomized trajectory $\mathbf{P}$ that satisfies $\pi^* = \pi^*\mathbf{P}$ is peak age optimal. □

Observe that the expression above implies that the fraction of time spent at a node is proportional to the square root of its weight. This is similar to the "square root principle" first derived in peer-to-peer settings in [31]. Similar square root based scheduling results have been derived for minimizing age in single hop networks [28], [32]

Lemma 1 implies that a randomized trajectory $\mathbf{P}$, that satisfies $\pi^*\mathbf{P} = \pi^*$, is peak age optimal, over the space of all randomized trajectories. We now construct one such randomized trajectory: for π^* given in Lemma 1, define a Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$:

$$P_{i,j}^{\text{mh}} = \begin{cases} P_{i,j}^{\text{rw}} \min(1, \frac{\pi_j^* P_{j,i}^{\text{rw}}}{\pi_i^* P_{i,j}^{\text{rw}}}), & \text{if } i \neq j \text{ and } (i,j) \in E \\ 1 - \sum_{j:j\neq i} P_{i,j}^{\text{mh}}, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}, \tag{23}$$

where

$$P_{i,j}^{\text{rw}} = \begin{cases} \frac{1}{d_i}, & \text{if } i \neq j \text{ and } (i,j) \in E \\ 0, & \text{otherwise} \end{cases}, \forall i,j \in V, \tag{24}$$

and d_i equals the out degree of terminal i in the mobility graph G. It is known that such a randomized trajectory $\mathbf{P}^{\text{mh}}$ satisfies $\pi^*\mathbf{P} = \pi^*$ [33, Ch.11]. We use this to show that this trajectory is average-peak age optimal. Further, $\mathbf{P}^{\text{mh}}$ is in fact a *reversible* Markov chain because of the way it is constructed. It satisfies detailed balance equations: $\pi_i\mathbf{P}_{ij}^{\text{mh}} = \pi_j\mathbf{P}_{ji}^{\text{mh}}, \forall i,j$. Using the definition of $\mathbf{P}^{\text{mh}}$ and Lemma 1, it is easy to see that the Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$ solves (20), i.e. it is average-peak age optimal over the space of *all randomized trajectories.*

Till now, we considered randomized trajectories, where the mobile agent moves from terminal i to j with probability $P_{i,j}$. We now show that for peak age optimality, such a randomization suffices, i.e. the trajectory we found is optimal over *all trajectories* (include possibly complicated history dependent ones).

Theorem *2.* The Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$ is average-peak age optimal over the space of *all trajectories* $\mathbb{T}$, namely $A^{\text{p}*}(\mathbf{P}^{\text{mh}}) = A_{\mathcal{G}}^{\text{p}*}$.

Proof: We establish a more general result. Namely, any randomized trajectory which satisfies $\pi^*\mathbf{P} = \pi^*$, where $\pi_i^* = \frac{\sqrt{w_i}}{\sum_{j\in V}\sqrt{w_j}}$, is peak age optimal over the space of *all trajectories* in $\mathbb{T}$:

$$A^{\text{p}*}(\mathbf{P}) = A_{\mathcal{G}}^{\text{p}*}.$$

To prove this, it suffices to argue that the average-peak age for any trajectory in $\mathbb{T}$ is lower bounded by $\sum_{i\in V} \frac{w_i}{\pi_i^*}$, where π^* is as given in Lemma 1.

Let $H_{k,i}$ to be the k^{th} inter-return time to node i. If K is the total number of returns to ground terminal i over a time horizon

T, then the average-peak age A_i^{p} is given by

$$A_i^{\text{p}} = \limsup_{T\to\infty} \frac{\sum_{t=1}^{T} A_i(t) \mathbb{1}_{\{m(t)=i\}}}{\sum_{t=1}^{T} \mathbb{1}_{\{m(t)=i\}}} = \limsup_{K\to\infty} \frac{1}{K} \sum_{k=1}^{k=K} H_{k,i}. \tag{25}$$

Now, the fraction of time-slots in which the mobile agent is at ground terminal i, is given by

$$f_i = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} \mathbb{1}_{\{m(t)=i\}}}{T} = \lim_{K\to\infty} \frac{K}{\sum_{k=1}^{k=K} H_{k,i}} = \frac{1}{A_i^{\text{p}}}, \tag{26}$$

and therefore, $A^{\text{p}} = \sum_{i\in V} w_i A_i^{\text{p}} = \sum_{i\in V} \frac{w_i}{f_i}$. The limits in the above equation exist with f_i being strictly positive for all trajectories $\mathcal{T} \in \mathbb{T}$ because of the way the trajectory space $\mathbb{T}$ is defined (See Section 2.2). Note that f_i, being the fraction of time-slots the mobile agent is at terminal i, define a distribution over V. Thus, A^{p} can be lower bounded by

$$A^{\text{p}} = \sum_{i\in V} w_i A_i^{p} \geq \min_{\{f_i\geq 0,\ \sum_i f_i = 1\}} \sum_{i\in V} \frac{w_i}{f_i} = \sum_{i\in V} \frac{w_i}{\pi_i^*}, \tag{27}$$

where the last equality is obtained by solving the optimization problem, just as in the proof of Lemma 1. Thus, the minimum peak age over all trajectories in which every ground terminal is visited a strictly positive fraction of time is lower bounded by the peak age achieved by any Markov chain with a stationary distribution of π^*. □

We are able to obtain a peak age optimal trajectory, namely $\mathbf{P}^{\text{mh}}$. Further, the matrix $\mathbf{P}^{\text{mh}}$ can be computed in polynomial time; in $O(|V|^2)$ time. Therefore, the average-peak age minimization problem is solved in polynomial time. For details on how to derive the Metropolis-Hastings Markov chain and a nice geometric interpretation, see [34] and [35].

3.3 Average Age Minimization

We now consider the average age minimization problem. We will show later that solving the average age minimization problem is hard. So, we start by deriving a lower bound on average age. Intuitively, if the mobility graph is better connected then it should yield a lower age. This is because a better connected mobility graph imposes fewer restrictions on mobility. The following result obtains a lower bound on network average age by comparing it with the network average age of a complete graph.

Theorem 3. For any trajectory $\mathcal{T} \in \mathbb{T}$, the network average age is lower bounded by

$$A^{\text{ave}}(\mathcal{T}) \geq \frac{1}{2} \sum_{i\in V} \left(\frac{w_i}{\pi_i^*} + w_i \right), \tag{28}$$

where $\pi_i^* = \frac{\sqrt{w_i}}{\sum_{j\in V} \sqrt{w_j}}$ for all $i \in V$.

Proof: Let $H_{k,i}$ be the k^{th} inter-return time to ground terminal i, and K be the total number of returns to i over a time-horizon T. Then the average age A_i^{ave} is given by (see proof of Theorem 1):

$$A_i^{\text{ave}} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A_i(t) = \lim_{K\to\infty} \frac{\sum_{k=1}^{K} (H_{k,i}^2 + H_{k,i})}{2 \sum_{k=1}^{K} H_{k,i}}. \tag{29}$$

Define the empirical first and second moment of return times be $\hat{H}_i \triangleq \frac{1}{K} \sum_{k=1}^{K} H_{k,i}$ and $\hat{H}_i^{(2)} \triangleq \frac{1}{K} \sum_{k=1}^{K} H_{k,i}^2$, respectively. Further, define $\hat{\text{Var}}_i \triangleq \hat{H}_i^{(2)} - \hat{H}_i^2$ to be the empirical variance of return times. From (29), we have

$$A_i^{\text{ave}} = \frac{1}{2} + \lim_{K\to\infty} \frac{\hat{H}_i^{(2)}}{2\hat{H}_i} = \frac{1}{2} + \lim_{K\to\infty} \frac{\left(\hat{H}_i\right)^2 + \hat{\text{Var}}_i}{2\hat{H}_i}. \tag{30}$$

Using Cauchy-Schwarz inequality, we can obtain $\hat{\text{Var}}_i \geq 0$. Applying this to (30), we get

$$A_i^{\text{ave}} \geq \frac{1}{2} + \lim_{K\to\infty} \frac{\hat{H}_i}{2}, \tag{31}$$

Let f_i be the fraction of time-slots in which the mobile agent is at ground terminal i. Then,

$$f_i = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} \mathbb{1}_{\{m(t)=i\}}}{T} = \lim_{K\to\infty} \frac{K}{\sum_{k=1}^{K} H_{k,i}} = \frac{1}{\lim_{K\to\infty} \hat{H}_i}, \tag{32}$$

since f_i is well defined and positive for all trajectories in $\mathbb{T}$. Substituting (32) in (31) we get $A_i^{\text{ave}} \geq \frac{1}{2} + \frac{1}{2f_i}$, for all i, and

$$A^{\text{ave}} = \sum_{i\in V} w_i A_i^{\text{ave}} \geq \frac{1}{2} \sum_{i\in V} w_i + \frac{1}{2} \sum_{i\in V} \frac{w_i}{f_i}. \tag{33}$$

Note that f_i, being the fraction of time-slots the mobile agent is at terminal i, is a distribution over V. Thus, the average age in (33) can be lower bounded by

$$\begin{aligned} A^{\text{ave}} &\geq \frac{1}{2} \sum_{i\in V} w_i + \frac{1}{2} \min_{\{f_i\geq 0,\ \sum_i f_i = 1\}} \sum_{i\in V} \frac{w_i}{f_i}, \\ &= \frac{1}{2} \sum_{i\in V} w_i + \frac{1}{2} \sum_{i\in V} \frac{w_i}{\pi_i^*}, \end{aligned}$$

which proves the result. □

Note that the term $\sum_{i\in V} \frac{w_i}{\pi_i^*}$ is nothing but the optimal peak age $A_{\mathcal{G}}^{\text{p}*}$; see Theorem 2. Furthermore, the lower bound in Theorem 3 is independent of the trajectory $\mathcal{T}$. Therefore, we get

$$A_{\mathcal{G}}^{\text{ave}*} = \min_{\mathcal{T}\in\mathbb{T}} A^{\text{ave}}(\mathcal{T}) \geq A_{\text{LB}}^{\text{ave}} = \frac{1}{2} A_{\mathcal{G}}^{\text{p}*} + \frac{1}{2} \sum_{i\in V} w_i, \tag{34}$$

where $\mathbb{T}$ is the space of all trajectories. It must be noted that a similar result was derived in the case of link scheduling for age minimization in [10]. The similarity of the result is rooted in the fact that the information gathering problem in the complete graph case is equivalent to the link scheduling problem in [10], in which at most one link can be activated simultaneously.

We now argue that in the symmetric setting, namely $w_i = 1/|V|\ \forall\ i \in V$,[1] the average age minimization problem is NP-hard

1. The weights w_i only measure relative significance of ground terminals. Thus, setting $w_i = 1/|V|\ \forall\ i \in V$ is equivalent to setting $w_i = w_j\ \forall\ i, j \in V$.

Theorem 4. The problem of finding an average age optimal trajectory is NP-hard in the symmetric setting of $w_i = 1/|V|\ \forall\, i \in V$.

Proof: To prove NP-hardness, we establish equivalence between the average age minimization problem and the Hamiltonian cycle problem, in the symmetric setting. We know that more connected the graph, lower is its network average age. Therefore, the average age for $G = (V, E)$ is lower bounded by the average age for the complete graph $K(V)$, given by $\frac{(|V|+1)}{2}$. This lower bound can be obtained by using Theorem 3 and setting $w_i = 1/|V|,\ \forall i$.

If the graph is Hamiltonian, we can achieve this average age lower bound by setting the trajectory equal to a Hamiltonian cycle. This is because in a cyclical trajectory, the agent visits every terminal exactly once in every $|V|$ time-slots. Further, if the graph is not Hamiltonian, the optimal average age is strictly greater than $\frac{(|V|+1)}{2}$. This is because in the absence of a cycle on graph G, the agent cannot visit every terminal exactly once every $|V|$ time-slots. Therefore, if an algorithm were to solve the average age problem then the same algorithm could be used to determine whether the graph G is Hamiltonian or not; which is the Hamiltonian cycle problem. Since the Hamiltonian cycle problem is NP-complete, the average age minimization problem must be NP-hard. □

3.3.1 A Heuristic Randomized Trajectory

Motivated by the peak age optimality results of the previous section, we restrict ourselves to the space of randomized trajectories, and propose a heuristic, called the *fastest-mixing reversible randomized trajectory*, and prove an average age performance bound for it.

Using the results in Theorem 1, the average age minimization problem over the space of randomized trajectories can be written as

$$
\begin{aligned}
\underset{\mathbf{P},\pi,\mathbf{Z}}{\text{Minimize}} \quad & \sum_{i\in V} \frac{w_i z_{ii}}{\pi_i}, \\
\text{subject to} \quad & P_{i,j} \geq 0,\ \forall\,(i,j),\ \text{ and }\ \mathbf{P}\mathbf{1} = \mathbf{1}, \\
& \pi\mathbf{P} = \pi,\ \mathbf{1}^T\pi = 1,\ \text{and } \pi_i \geq 0\ \forall i \\
& P_{i,j} = 0,\ \forall (i,j) \notin E, \\
& \mathbf{P} \text{ is irreducible}, \\
& \Pi_{i,j} = \pi_j\ \forall\,(i,j), \\
& Z = (I - \mathbf{P} + \Pi)^{-1}.
\end{aligned}
\tag{35}
$$

Here, $\mathbf{P}$ is the randomized trajectory and π the unique stationary distribution corresponding to $\mathbf{P}$. Solving (35) can be computationally complex. Not only do we have the irreducibility constraint, but also a non-linear constraint in $Z = (I - \mathbf{P} + \Pi)^{-1}$.

We next upper bound the network average age, for any randomized trajectory $\mathbf{P}$ of the mobile agent. We first define mixing time for a randomized trajectory.

To do this, we first discuss the notion of stopping rules and stopping times in a Markov chain. A stopping rule is a rule that observes the walk on a Markov chain and, at each step, decides whether or not to stop the walk based on the walk so far. The time at which the walk stops, called the stopping time, is a random variable. Note that in our discussion, Markov chains define trajectories over ground terminals and state distributions refer to probability distributions over the set V of ground terminals.

Mixing Time [36] The hitting time from a state distribution σ_1 to σ_2 on a Markov chain is the minimum expected stopping time over all stopping rules that, beginning at σ_1, stop in the exact distribution of σ_2. In other words, it is the expected number of steps that the optimal stopping rule takes to move from σ_1 to σ_2. This is denoted by $\mathcal{H}(\sigma_1, \sigma_2)$. The mixing time $\mathcal{H}$ of a Markov chain $\mathbf{P}$ is then defined as

$$
\mathcal{H} \triangleq \sup_{\sigma\in\mathbf{\Delta}(V)} \mathcal{H}(\sigma, \pi), \tag{36}
$$

where $\mathbf{\Delta}(V)$ is the collection of all distributions on V and π is the stationary distribution of $\mathbf{P}$. In other words, it is the expected time taken to reach stationarity using the optimal stopping rule and starting at the worst initial distribution. We provide further discussion on hitting times, stopping rules and mixing times in Appendix B. For more details, see [36].

Lemma 2. The network average age for a randomized trajectory $\mathbf{P}$ is upper bounded by

$$
A^{\text{ave}}(\mathbf{P}) = \sum_{i\in V} \frac{w_i z_{ii}}{\pi_i} \leq 4\mathcal{H}A^{\text{p}}(\mathbf{P}) + \sum_{i\in V} w_i, \tag{37}
$$

where $\mathcal{H}$ denotes the mixing time of the randomized trajectory $\mathbf{P}$.

Proof: First, we define the quantity $\mathcal{Z} \triangleq \max_i \sum_j |z_{ij} - \pi_j|$, called the discrepancy of the randomized trajectory $\mathbf{P}$. This definition implies that $z_{ii} \leq \mathcal{Z} + \pi_i,\ \forall i \in V$. Thus, we get the following upper bound:

$$
\sum_{i\in V} \frac{w_i z_{ii}}{\pi_i} \leq \sum_{i\in V} \left(\frac{w_i \mathcal{Z}}{\pi_i} + w_i\right). \tag{38}
$$

However, from [36, Theorem 5.1] we know that $\mathcal{Z} \leq 4\mathcal{H}$, where $\mathcal{H}$ is the mixing time of the randomized trajectory $\mathbf{P}$. Thus, we have the required result

$$
\sum_{i\in V} \frac{w_i z_{ii}}{\pi_i} \leq \sum_{i\in V} \left(\frac{4 w_i \mathcal{H}}{\pi_i} + w_i\right) = 4\mathcal{H}A^{\text{p}}(\mathbf{P}) + \sum_{i\in V} w_i,
$$

where the last equality follows from Theorem 1. □

We use this relation and suggest the following heuristic for minimizing age: *Find the fastest mixing randomized trajectory* $\mathbf{P}$ *on the mobility graph* G *that minimizes peak age.*

From the proof of Theorem 2, we know that for a randomized trajectory $\mathbf{P}$ to be peak age optimal all we need is its stationary distribution to satisfy $\pi = \pi^*$, where π^* is as defined in Lemma 1. It, therefore, suffices to find $\mathbf{P}$ that satisfies $\pi_i = \pi_i^*, \forall i$, and simultaneously minimizes the mixing time $\mathcal{H}$. Note that while it is computationally feasible to find the fastest mixing *reversible* Markov chain on a graph, this is not the case if we also consider non reversible chains. So we limit ourselves to reversible Markov chains and call this the *fastest-mixing reversible randomized trajectory*. We use $\mathbf{P}^*$ to denote it. The following result provides a way to obtain $\mathbf{P}^*$ by solving a convex program.

Theorem 5. The fastest mixing *reversible* randomized trajectory can be found by solving the following convex optimization problem:

$$\begin{aligned} \underset{\mathbf{P}}{\text{Minimize}} \quad & \mu(\mathbf{P}) = ||\mathbf{P} - \Pi^*||_2, \\ \text{subject to} \quad & P_{i,j} \geq 0, \ \forall (i,j), \\ & \mathbf{P}\mathbf{1} = \mathbf{1}, \\ & \pi_i^* P_{i,j} = \pi_j^* P_{j,i}, \ \ \Pi_{i,j}^* = \pi_i^* \ \forall \ i,j \in V, \\ & P_{i,j} = 0, \forall (i,j) \notin E. \end{aligned} \tag{39}$$

Here $||A||_2$ denotes the spectral norm of matrix A and $\pi_i^* = \frac{\sqrt{w_i}}{\sum_{j \in V} \sqrt{w_j}}$, $\forall i \in V$.

Proof: From [34, Section 6] , we know that the fastest mixing reversible Markov chain on a graph $G(V, E)$ having an *arbitrary* stationary distribution π can be found by formulating the following convex program:

$$\begin{aligned} \underset{\mathbf{P}}{\text{Minimize}} \quad & ||D^{1/2}\mathbf{P}D^{-1/2} - qq^T||_2, \\ \text{subject to} \quad & P_{i,j} \geq 0, \ \forall (i,j) \\ & \mathbf{P}\mathbf{1} = \mathbf{1}, \\ & \pi_i^* P_{i,j} = \pi_j^* P_{j,i}, \forall i,j \in V \\ & P_{i,j} = 0, \forall (i,j) \notin E. \end{aligned} \tag{40}$$

Here $D = \text{diag}(\pi^*)$ and $q = (\sqrt{\pi_1^*}, \sqrt{\pi_2^*}, ..., \sqrt{\pi_n^*})$. Left and right multiplying $(D^{1/2}\mathbf{P}D^{-1/2} - qq^T)$ by matrices $D^{-1/2}$ and $D^{1/2}$, respectively, does not change the spectral norm; since $\mathbf{P}$ has the same eigen-values as $D^{1/2}\mathbf{P}D^{-1/2}$ and qq^T has the same eigen-values as $D^{-1/2}qq^TD^{1/2}$ [34]. Further, observe that $D^{-1/2}qq^TD^{1/2} = qq^T = \Pi^*$, where $\Pi_{i,j}^* = \pi_i^* \ \forall \ i,j \in V$. Thus, the optimization problem reduces to (39). This proves the required result. See Appendix C for a more detailed discussion. □

This convex program (39) finds a *reversible* randomized trajectory $\mathbf{P}^*$ on G that is closest to the stationary randomized walk Π^*, in the spectral norm sense. Detailed balance equations $\pi_i^* P_{i,j} = \pi_j^* P_{j,i}, \forall i, j$ are constraints that we impose for finding $\mathbf{P}^*$ that has provably minimum mixing time over a sufficiently large class of trajectories, namely reversible Markov chains. In practice, however, we can relax this constraint to global balance $\pi^*\mathbf{P} = \pi^*$ and get non reversible trajectories whose performance is better. We discuss this in Appendix C.

We note that $\mathbf{P}^*$ is peak age optimal on graph G, since its stationary distribution is π^*. Further, the problem (39) and its relaxation for non reversible chains can both be solved in polynomial time by converting it to a semi-definite program [34].

We now bound the average age performance of the fastest-mixing randomized trajectory.

Theorem 6. The network average age of the fastest-mixing randomized trajectory is at most $8\mathcal{H}$-factor away from the optimal average age:

$$\frac{A^{\text{ave}}(\mathbf{P}^*)}{A_{\mathcal{G}}^{\text{ave}*}} \leq 8\mathcal{H}, \tag{41}$$

where $\mathcal{H}$ is the mixing time of $\mathbf{P}^*$.

Proof: Note that the peak age for the fastest-mixing randomized trajectory $\mathbf{P}^*$ is given by $A^{\text{p}}(\mathbf{P}^*) = \sum_{i \in V} \frac{w_i}{\pi_i^*}$, since $\pi^*\mathbf{P}^* = \pi^*$. From Theorem 3, a lower bound on average age is given by

$$A_{\text{LB}}^{\text{ave}} = \sum_{i \in V} \frac{1}{2} \left(\frac{w_i}{\pi_i^*} + w_i \right) = \frac{1}{2} A^{\text{p}}(\mathbf{P}^*) + \frac{1}{2} \sum_{i \in V} w_i. \tag{42}$$

To prove the result, it suffices to argue that $A^{\text{ave}}(\mathbf{P}^*)/A_{\text{LB}} \leq 8\mathcal{H}$. From (42) and Lemma 2, we get

$$\begin{aligned} \frac{A^{\text{ave}}(\mathbf{P}^*)}{A_{\text{LB}}^{\text{ave}}} &\leq \frac{4\mathcal{H}A^{\text{p}}(\mathbf{P}^*) + \sum_{i \in V} w_i}{\frac{1}{2}A^{\text{p}}(\mathbf{P}^*) + \frac{1}{2}\sum_{i \in V} w_i}, & (43) \\ &\leq 8\mathcal{H}, & (44) \end{aligned}$$

since $\mathcal{H}$ is always greater than or equal to 1. □

We note that we could have derived a similar mixing time bound for the Metropolis-Hastings chain $\mathbf{P}^{\text{mh}}$ introduced earlier. However, that bound would be worse than the bound for $\mathbf{P}^*$ since the mixing time of $\mathbf{P}^{\text{mh}}$ is necessarily larger than that of the fastest-mixing reversible chain. This is because $\mathbf{P}^{\text{mh}}$ is also a reversible Markov chain.

To further see the usefulness of the fastest-mixing randomized trajectory, and Theorem 6, consider a random geometric graph $\mathcal{G}(n, r)$. The graph consists of n nodes spread over a unit square with a link between every two nodes that are within a distance r. If v is the physical speed of the mobile agent, then r must equal $v\tau$, where τ is the slot duration.We know that mixing time of reversible chains on $\mathcal{G}(n, r)$ is upper bounded by $O\left(\frac{\log n}{r^2}\right)$ [37], and therefore, the fastest-mixing randomized trajectory would be at most $O\left(\frac{\log n}{v_{max}^2 \tau^2}\right)$ factor optimal. For highly connected graphs, such as Dirac graphs in which the degree of each node is at least $|V|/2$, we have constant factor of optimality; since the mixing times are $O(1)$. [38] establishes a connection between the existence of long paths in graphs and their mixing times and that it is hard to find even constant factor approximations to the problem of finding the longest path on a general graph.

3.4 Age-based Trajectories

In the last two sub-sections, we proposed two randomized trajectories, namely $\mathbf{P}^{\text{mh}}$ and $\mathbf{P}^*$. Both were peak age optimal, while the latter was also factor-$\mathcal{H}$ average age optimal. We also noted that solving the average age problem is generally hard. We now propose an age-based trajectory which can be constant factor age optimal.

Age-based trajectory In every time slot, agent m moves to the location that has the highest weighted function of $A_i(t)$. Specifically, if $m(t) = i$ then

$$m(t+1) = \arg \max_{j:(i,j) \in E} w_j g\left(A_j(t)\right), \tag{45}$$

for all $i, j \in V$ and time t, where $g(\cdot)$ is an increasing function. We assume that ties are broken in order of vertex indices.

Examples of functions include $g(a) = a$ and $g(a) = a + a^2$. The idea for an age-based trajectory comes from results on age optimal scheduling [27], [28] that develop index based methods which are constant factor optimal. In the symmetric setting, where

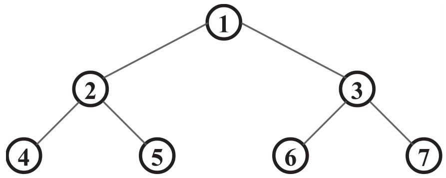


Fig. 3: Mobility graph restricted to a binary tree.

$w_i = 1 \; \forall \; i \in V$, the function $g(\cdot)$ does not matter and the agent moves greedily to the neighbouring node with the highest age.

In fact, we observe that the age-based trajectory is a repeated depth-first traversal of the mobility graph G. This can be verified easily when the mobility graph is a tree. Consider the tree in Figure 3, and assume that we start at the root node 1 with age for all nodes being zero. The trajectory of the agent following the rule described above would be $1 \rightarrow 2 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 6 \rightarrow 3 \rightarrow 7 \rightarrow 3 \rightarrow 1 ...$ This is precisely the depth-first traversal of the tree graph.

In the symmetric setting, where $w_i = 1 \; \forall \; i \in V$, we now prove that the age-based trajectory is factor-2 optimal.

Theorem *7.* In the symmetric setting $w_i = 1 \; \forall \; i \in V$, the network average age A^{ave} for the age-based trajectory is bounded by

$$\frac{A^{\text{ave}}}{A^{\text{ave}*}_{\mathcal{G}}} \leq \frac{2|V|+1}{|V|+1} \leq 2, \tag{46}$$

for any increasing function $g(\cdot)$.

Proof: The number of steps taken to cover every vertex of a graph by performing a depth first search (DFS) traversal is upper bounded by $2|V|$, since every vertex is visited at least once and the sum total of visits after the first visit to all nodes is at most $|V|$. This is because every repeated visit to a vertex means that at least one new vertex was visited. Thus, every location gets visited at least once in every $2|V|$ time-slots. This implies that the average age of every terminal can be upper bounded by $\frac{(2|V|+1)}{2}$.

However, from our earlier discussion, we know that the average age of any terminal is lower bounded by $\frac{(|V|+1)}{2}$ if all the weights are 1. Combining the upper and lower bounds, we have the required result. $\square$

This age-based policy can be implemented in an online fashion if the mobile agent has access to age $A_i(t)$ of the neighboring terminals. The complexity of implementing this trajectory is then at most linear in the time-horizon and $|V|$. However, it also suggests a polynomial time "offline" algorithm that does not need knowledge of ages or computation in every time-slot to achieve the same result -

1) Let the starting node be any $v \in V$. Compute a depth-first traversal on the graph $G(V, E)$ starting at node v.
2) Compute the shortest path from the last visited node in the dfs traversal to v.
3) Append the path from step 2 to the dfs traversal in step 1. Follow this trajectory plan iteratively.

Note that both the dfs traversal and the shortest path can be computed beforehand in polynomial time. Using exactly the same arguments as for the greedy algorithm, this trajectory plan also achieves factor 2 optimality in the equal weight setting.

The utility of index-based policies is in situations where we include unreliable packet deliveries or time-varying weights and mobility graphs in our model. While Markov chain based analysis works only for fixed graphs known beforehand, the age-based trajectory can be easily modified for use in dynamic settings. Showing performance guarantees in such settings is also an interesting line of future work.

4 Information Dissemination

We now consider information dissemination. The central terminal generates updates independently at rates λ_i, according to Bernoulli processes. The generated updates get queued in the ith FCFS queue for the ith ground terminal. The mobile agent follows a trajectory $\mathcal{T}$, and transmits the head-of-line update in queue i, when it reaches the ith location. The FCFS queue assumption is motivated by uncontrollable MAC layer queues implemented in practice, where the generated updates get queued for transmission [10], [25].

We assume that the UAV can deliver only one packet from the queue whenever it visits a node. However, the key ideas that we present in this section would still apply if the UAV is able to send at most a fixed number of packets to a node in every visit. This is because even if the UAV can empty a large number of packets from the queue in every visit, there might be packets remaining in the queue if the arrival rate is high. This queuing phenomenon and optimal choice of arrival rate is what we study in the information dissemination problem.

Our objective is to minimize the network peak age and average age over the space of update generation rates $\boldsymbol{\lambda}$ and all trajectories $\mathbb{T}$:

$$A^{\text{p}*}_{\mathcal{D}} = \min_{\mathcal{T} \in \mathbb{T}, \boldsymbol{\lambda}} \sum_{i \in V} w_i A^{\text{p}}_i, \quad \text{and} \quad A^{\text{ave}*}_{\mathcal{D}} = \min_{\mathcal{T} \in \mathbb{T}, \boldsymbol{\lambda}} \sum_{i \in V} w_i A^{\text{ave}}_i, \tag{47}$$

where A^{p}_i denotes average-peak age and A^{ave}_i denotes the average age of terminal i. Their evolution is given by (2). For notational convenience, we have omitted their explicit dependence on $\mathcal{T} \in \mathbb{T}$ and $\boldsymbol{\lambda}$.

Motivated by results for the gathering setting, we begin by considering randomized trajectories. We assume that the time spent by an update waiting at the head-of-line of the queue is its *effective service time*. Note that an update arriving in queue i when the queue is non empty sees updates ahead of it leaving the system once every visit to i. Thus, it sees the *effective service times* for updates ahead of it to be equal to the inter-visit times to the terminal i. The same holds for the arriving update as well. However, when an update arrives to an empty queue i, its effective service time is dependent on where the mobile agent was when the upate arrived and how long it takes to reach i again.

Since the analysis of age for such a queueing system with non i.i.d. service may be difficult, we provide an upper bound, by comparing the ith queue with a discrete time Ber/G/1 queue with vacations. Whenever the ith queue is empty we pretend that it goes on a vacation, with vacation times having the same distribution as inter-visit times; otherwise the service times for the queue are just inter-visit times. In this upper bound system, when an update arrives into the ith queue when it is empty, we do not allow it to be sent in the next visit by the mobile agent, but the visit after that.

This ensures i.i.d. service times and vacation times and allows us to analyze the system.

The age process of such an FCFS queue is clearly an upper bound for the age process $A_i(t)$. This is because the total time in the system for packets arriving into a non-empty queue are identically distributed to the original FCFS queue while packets arriving into an empty queue in the upper bound system spent extra time waiting. Thus, we upper bound the peak age A_i^{p} and average age A_i^{ave}, by the peak and average age of this Ber/G/1 queue with vacations. We first analyze peak and average age of a Ber/G/1 queue with i.i.d. vacations and service times.

4.1 Age for Ber/G/1 Queue with Vacations

Consider a FCFS Ber/G/1 queue with vacations, where an arrival occurs with probability λ, the service times S are generally distributed with mean $\mathbb{E}[S] = 1/\mu$, and the vacation times V are also distributed the same as S.

We obtain an expression for the average-peak age of a discrete time Ber/G/1 queue with vacations. Further, we derive an upper bound on average age under a negative correlation assumption.

Lemma *3.* The average-peak age for a discrete time FCFS Ber/G/1 queue with i.i.d. vacations and service is given by

$$A^{\text{p}} = \frac{1}{\lambda} + \frac{1}{\mu} + \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{\mathbb{E}\left[V^2\right]}{2\mathbb{E}\left[V\right]} - \frac{1}{2}, \tag{48}$$

where $\rho = \frac{\lambda}{\mu}$, while the average age is upper-bounded by peak age, namely $A^{\text{ave}} \leq A^{\text{p}}$.

Proof: The peak age for a FCFS queue is given by

$$A^{\text{p}} = \mathbb{E}\left[T + X\right], \tag{49}$$

where T denotes the time an update spends in the queue and X is the inter-arrival time between two updates. Given that vacation times are distributed i.i.d according to random variable V, we have

$$\mathbb{E}[T] = \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{1}{\mu} + \frac{\mathbb{E}\left[V^2\right]}{2\mathbb{E}\left[V\right]} - \frac{1}{2}, \tag{50}$$

where S denotes the service time distribution. Substituting this and $\mathbb{E}\left[X\right] = \frac{1}{\lambda}$ in (49), we obtain the expression for peak age. We now derive the expression for average system time $\mathbb{E}\left[T\right]$ seen in (50).

4.1.1 *Derivation of System Time*

The proof is a discretized version of the proof for M/G/1 queues with vacations using residual service times as discussed in [39].

Let us define the residual service time for an update at time t, given by $R(t)$, as the amount of time remaining until the update currently at the head of the queue is complete, excluding the current time-slot. If the queue is empty, $R(t)$ equals zero.

From [39] we know that the expected waiting time in the queue can be found using the residual service times as follows

$$\mathbb{E}\left[T_Q\right] = \frac{\mathbb{E}\left[R\right]}{1-\rho}, \tag{51}$$

where $\rho = \frac{\lambda}{\mu}$, $\mathbb{E}\left[S\right] = \frac{1}{\mu}$ and $\mathbb{E}\left[R\right] = \lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{t=T} R(t)\right]$. As in [39], $\mathbb{E}\left[R\right]$ can be computed using a graphical argument. Let service times for the mth packet be X_m, and let the kth vacation time be V_k. Let the total number of packets served be $M(T)$ and the total number of vacations be $L(T)$, over the entire time-horizon T. Then, we have

$$\frac{1}{T}\sum_{t=0}^{t=T} R(t) = \frac{1}{2}\frac{M(T)}{T}\frac{\sum_{m=1}^{M(T)}(X_m^2 - X_m)}{M(T)} + \frac{1}{2}\frac{L(T)}{T}\frac{\sum_{k=1}^{L(T)}(V_k^2 - V_k)}{L(T)}. \tag{52}$$

Using the strong law of large numbers and the fact that $\frac{M(T)}{T} \to \lambda$ and $\frac{L(T)}{T} \to \frac{(1-\rho)}{\mathbb{E}[V]}$, we get

$$\mathbb{E}\left[R\right] = \frac{\lambda(\mathbb{E}\left[S^2\right] - \mathbb{E}\left[S\right])}{2} + \frac{(1-\rho)(\mathbb{E}\left[V^2\right] - \mathbb{E}\left[V\right])}{2\mathbb{E}\left[V\right]}. \tag{53}$$

Combining (51), (53), and the fact that total time spent in the system by a packet is given by the sum of its waiting time in the queue and its processing time, we get

$$\mathbb{E}\left[T\right] = \mathbb{E}\left[S + T_Q\right] = \frac{1}{\mu} + \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{\mathbb{E}\left[V^2\right]}{2\mathbb{E}\left[V\right]} - \frac{1}{2}, \tag{54}$$

since $\mathbb{E}\left[S\right] = \frac{1}{\mu}$. We now show the second part of the Lemma, that the average age is upper bounded by the peak age.

4.1.2 *Average Age*

Consider a FCFS Ber/G/1 queue with i.i.d. vacations and service times. Let the packet inter-arrival times be $X_1, X_2, ...$ Let T_n be the total time spent in the system by the n^{th} packet. Then, the average age is given by [6]:

$$A^{\text{ave}} = \frac{1}{\lambda} + \lambda\mathbb{E}[X_n T_n], \tag{55}$$

where $\frac{1}{\lambda} = \mathbb{E}[X_n]$. To evaluate the term $\mathbb{E}[X_n T_n]$, we observe that larger inter-arrival times X_n between packets mean lesser wait times in the system T_n for individual packets. This suggests X_n and T_n are negatively correlated and that $\mathbb{E}\left[X_n T_n\right] \leq \mathbb{E}\left[X_n\right]\mathbb{E}\left[T_n\right]$. This is a commonly stated observation in AoI literature [6], [40] but a general proof hasn't appeared before. In Appendix D we provide a proof of this result for FCFS queues with no vacations. Proving this for our vacation system becomes challienging, but we provide simulation results which strongly indicate that the result holds in both the original system and Ber/G/1 queues with i.i.d. vacations and service times. Therefore, we present this result as the assumption below:

Assumption *1.* Consider a Ber/G/1 queue with i.i.d. vacations and service times, where the distribution of a vacation is the same as that of a service time. Then, packet inter-arrival times X_n are negatively correlated with packet system times T_n, i.e.

$$\mathbb{E}\left[X_n T_n\right] \leq \mathbb{E}\left[X_n\right]\mathbb{E}\left[T_n\right]. \tag{56}$$

If Assumption 1 is true, then we have the required result:

$$A^{\text{ave}} \leq \frac{1}{\lambda} + \lambda\mathbb{E}[X_n]\mathbb{E}\left[T_n\right] = \mathbb{E}\left[X_n\right] + \mathbb{E}\left[T_n\right] = A^{\text{p}}.$$

For the remainder of this work, we assume that Assumption 1 holds and the average age A^{ave} is upper bounded by peak age A^{p}. $\square$

4.2 Age Minimization Problem

Using Lemma 3, we now obtain an upper-bound on both network peak and average age for a given randomized trajectory $\mathbf{P}$ and update generation rates $\boldsymbol{\lambda}$.

Lemma *4.* For a randomized trajectory $\mathbf{P}$ and packet generation rates $\boldsymbol{\lambda}$, the peak and average age for a ground terminal i is upper-bounded by

$$A_i^{\text{UB}} = \frac{1}{\pi_i}\left[1 + z_{ii} + \frac{1}{\rho_i} + \frac{z_{ii}\rho_i}{1-\rho_i}\right] - \frac{\rho_i}{1-\rho_i} - 1, \tag{57}$$

for all $i \in V$, where π is the unique stationary distribution of $\mathbf{P}$, $Z = (I - \mathbf{P} + \Pi)^{-1}$, Π is a matrix with all rows equal to the stationary distribution vector π, and $\rho_i \triangleq \frac{\lambda_i}{\pi_i}$.

Proof: See Appendix E. $\square$

We propose a policy, i.e. a randomized trajectory $\mathbf{P}$ and update generation rate $\boldsymbol{\lambda}$, that minimizes the age upper-bound $A^{\text{UB}} = \sum_{i \in V} w_i A_i^{\text{UB}}$:

Definition Separation Principle Policy

1) Mobile agent follows the randomized trajectory $\mathbf{P}^*$ obtained by solving (39).
2) Generate updates for the ground terminal i at rate

$$\lambda_i^* = \frac{\pi_i^*}{1 + \sqrt{z_{ii}^* - \pi_i^*}}, \tag{58}$$

where $\pi_i^* = \frac{\sqrt{w_i}}{\sum_{j\in V} w_j}$ and z_{ii} are diagonal elements of the matrix $Z = (I - \mathbf{P}^* + \Pi^*)^{-1}$.

We call it the separation principle policy for two reasons. Firstly, $\mathbf{P}^*$ is the fastest-mixing randomized trajectory, which we proposed for minimizing average age earlier. Secondly, the update generation rate for the ground terminal i, depends only on z_{ii} and π_i, which are functions of the first and second moments of the return times to terminal i under trajectory $\mathbf{P}^*$ (see (16) and (17)). We now bound the performance of this separation principle policy.

Theorem *8.* The peak and average age of the separation principle policy is bounded by

$$\frac{A^{\text{p}}}{A_{\mathcal{D}}^{\text{p}*}} \leq 4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2 \text{ and } \frac{A^{\text{ave}}}{A_{\mathcal{D}}^{\text{ave}*}} \leq 8\mathcal{H} + 8\sqrt{\mathcal{H}} + 4,$$

where $\mathcal{H}$ is the mixing time of the randomized trajectory $\mathbf{P}^*$.

Proof: We formulate the upper bound age minimization problem and use an approach similar to Lemma 2 and Theorem 7. We want to solve the upper bound age minimization problem, which can be stated as:

$$\begin{aligned} \underset{\mathbf{P},\boldsymbol{\rho}}{\text{Minimize}} \quad & \sum_{i\in V} w_i A_i^{\text{UB}}, \\ \text{subject to} \quad & P_{i,j} \geq 0,\ \forall (i,j), \\ & \mathbf{P}\mathbf{1} = \mathbf{1}, \\ & P_{i,j} = 0,\ \forall (i,j) \notin E, \\ & \mathbf{P} \text{ is irreducible.} \end{aligned} \tag{59}$$

We first find the optimal packet generation rates given a random walk $\mathbf{P}$. Observe that the optimal queue utilization factors ρ_i can be solved for given any fixed irreducible random walk $\mathbf{P}$, i.e.

$$\rho_i^*(\mathbf{P}) = \arg \min_{\rho_i \in [0,1]} A_i^{\text{UB}}(\mathbf{P}, \rho_i) = \frac{1}{1 + \sqrt{z_{ii} - \pi_i}} \tag{60}$$

Note that $z_{ii} \geq \pi_i$ so the equation above is well defined. This is because var$(H_i) \geq 0$ and the first and second moments of H_i are given by (16) and (17). Now, the age under $\rho_i^*(\mathbf{P})$ is given by:

$$\min_{\rho_i \in [0,1]} A_i^{\text{UB}}(\mathbf{P}, \rho_i) = A_i^{\text{UB}}(\mathbf{P}, \rho_i^*) = \frac{z_{ii} - \pi_i + 2\sqrt{z_{ii} - \pi_i} + 2}{\pi_i}. \tag{61}$$

Thus, the upper bound age minimization problem reduces to

$$\begin{aligned} \underset{\mathbf{P}}{\text{Minimize}} \quad & \sum_{i\in V} w_i \left(\frac{z_{ii} - \pi_i + 2\sqrt{z_{ii} - \pi_i} + 2}{\pi_i}\right), \\ \text{subject to} \quad & P_{i,j} \geq 0,\ \forall (i,j), \mathbf{P}\mathbf{1} = \mathbf{1}, \\ & P_{i,j} = 0,\ \forall (i,j) \notin E, \\ & \mathbf{P} \text{ is irreducible.} \end{aligned} \tag{62}$$

Now, we can relate the network age upper bound, given a random walk $\mathbf{P}$, to its mixing time $\mathcal{H}$. We assume optimal packet generation rates $\rho_i^*(\mathbf{P})$.

$$\begin{aligned} \sum_{i\in V} w_i A_i^{\text{UB}}(P, \rho_i^*(P)) &= \sum_{i\in V} w_i \left(\frac{z_{ii} - \pi_i + 2\sqrt{z_{ii} - \pi_i} + 2}{\pi_i}\right), \\ &\leq \sum_{i\in V} w_i \left(\frac{\mathcal{Z} + 2\sqrt{\mathcal{Z}} + 2}{\pi_i}\right), \\ &\leq \sum_{i\in V} w_i \left(\frac{4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2}{\pi_i}\right), \end{aligned}$$

where inequalities follow from the same argument as in the proof of Lemma 2. Setting $\mathbf{P} = \mathbf{P}^*$, we obtain

$$\sum_{i\in V} w_i A_i^{\text{UB}}(\mathbf{P}^*, \rho_i^*(\mathbf{P}^*)) \leq \sum_{i\in V} w_i \left(\frac{4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2}{\pi_i^*}\right), \tag{63}$$

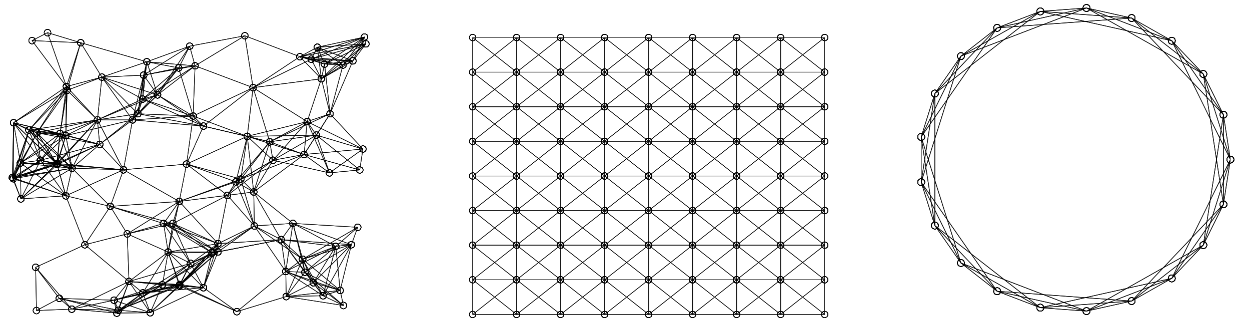
where $\mathcal{H}$ is the mixing time of P^*. Note that $\sum_{i\in V} \frac{w_i}{\pi_i^*}$ is the optimal peak age for information gathering, i.e. $A_{\mathcal{G}}^{\text{p}*} = \sum_{i\in V} \frac{w_i}{\pi_i^*}$. This gives,

$$\frac{A^{\text{UB}}(\mathbf{P}^*, \boldsymbol{\rho}^*)}{A_{\mathcal{G}}^{\text{p}*}} \leq 4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2. \tag{64}$$

Due to the presence of queues we have $A_{\mathcal{G}}^{\text{p}*} \leq A_{\mathcal{D}}^{\text{p}*}$. This, (64), and the fact that $A^{\text{p}}(\mathbf{P}^*, \boldsymbol{\rho}^*) \leq A^{\text{UB}}(\mathbf{P}^*, \boldsymbol{\rho}^*)$, yields the peak age bound on the separation principle policy:

$$\frac{A^{\text{p}}(\mathbf{P}^*, \lambda^*)}{A_{\mathcal{D}}^{\text{p}*}} \leq 4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2,$$

since $\boldsymbol{\rho}^* = \boldsymbol{\lambda}^*$.



(a) (b) (c)

Fig. 4: (a) A random geometric graph with 100 nodes, (b) A grid graph with 81 nodes and diagonal edges, and (c) A 3-connected ring or cycle graph with 21 nodes.

From the discussion following Theorem 3, we know that $2A_{\mathcal{G}}^{\text{ave}*} \geq A_{\mathcal{D}}^{\text{p}*}$. Also, $A_{\mathcal{G}}^{\text{ave}*} \leq A_{\mathcal{D}}^{\text{ave}*}$ and $A^{\text{ave}}(\mathbf{P}^*, \boldsymbol{\rho}^*) \leq A^{\text{UB}}(\mathbf{P}^*, \boldsymbol{\rho}^*)$. Combining these with (64) gives us

$$\frac{A^{\text{ave}}(\mathbf{P}^*, \lambda^*)}{A_{\mathcal{D}}^{\text{ave}*}} \leq 8\mathcal{H} + 8\sqrt{\mathcal{H}} + 4, \tag{65}$$

since $\boldsymbol{\rho}^* = \boldsymbol{\lambda}^*$.

□

The separation principle policy is factor $O(\mathcal{H})$ peak age and average age optimal. It is worthwhile to note that a similar separation principle policy was established in a completely different setting of scheduling links for age minimization in [10]. Theorem 8 generalizes that result to a graph.

5 Simulation Results

We test the performance of our proposed trajectories on three different kinds of mobility graphs: random geometric graphs $\mathcal{G}(n, \frac{2}{\sqrt{n}})$,[2] grid graphs with diagonal edges, and 3-connected ring or cycle graphs; see Figure 4. We use n to denote the number of ground terminals, namely $n = |V|$. For the age-based policy, we set the function $g(a) = a^2 + a$, inspired by the index based policies in [10]. Link weights are picked uniformly at random from the interval $(1, 2]$ in an independent manner. We run our simulations for a total of 50000 time-slots, to get a good estimate of the peak and average age.

We first consider information gathering, and plot average-peak and average age for all the proposed trajectories of the mobile agent: the Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$, fastest mixing randomized trajectory $\mathbf{P}^*$, and age-based trajectory. Figure 5 plots peak age as a function of network size n for the random geometric graph $\mathcal{G}(n, 2/\sqrt{n})$. We observe that the peak age for all the three proposed trajectories achieves the optimal value. This is in line with Theorems 2 and 5 where we showed that the two randomized trajectories - the Metropolis-Hastings chain $\mathbf{P}^{\text{mh}}$ and the fastest mixing chain $\mathbf{P}^*$, achieve the optimal stationary distribution π^* and hence the optimal average-peak age. Figure 5 suggests, in addition, that even the age-based trajectory for the mobile agent is peak age optimal.

In Figure 6 we plot the average age performance of the proposed trajectories, as a function of network size n. Also plotted

2. Setting $r = \frac{2}{\sqrt{n}}$ for random geometric graphs ensures connectivity w.h.p.

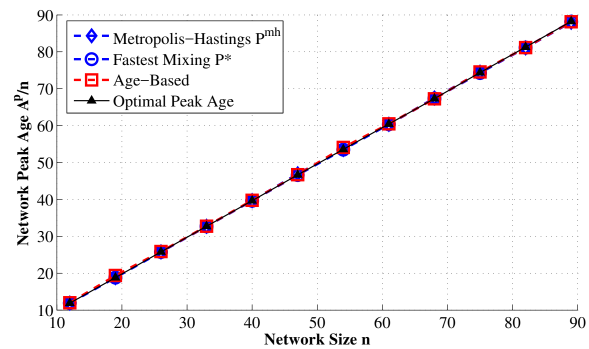


Fig. 5: Information Gathering on $\mathcal{G}(n, 2/\sqrt{n})$: network peak age as a function of network size n for several proposed trajectories of the mobile agent.

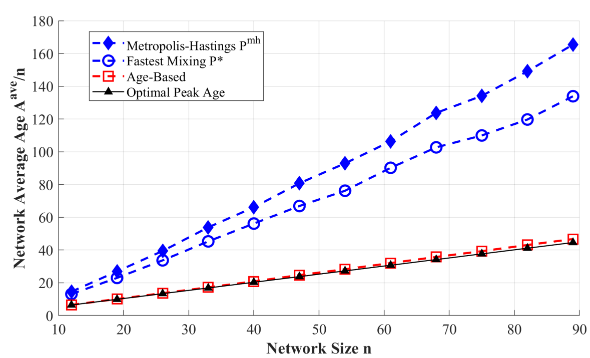


Fig. 6: Information Gathering on $\mathcal{G}(n, 2/\sqrt{n})$: network average age as a function of network size n for several proposed trajectories of the mobile agent (averaged over 20 runs).

is the lower bound for average age derived in Theorem 3. We see that the age-based policy is nearly average age optimal, while the fastest mixing randomized trajectory $\mathbf{P}^*$ has a lower average age, and hence better performance, than the Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$. This is to be expected given the mixing time upper-bound on average age derived in Lemma 2 and the fact that $\mathbf{P}^*$ has a smaller mixing time than $\mathbf{P}^{\text{mh}}$ by construction.

Theorem 6 proved that the fastest mixing randomized trajectory $\mathbf{P}^*$ is at least factor-$8\mathcal{H}$ optimal. Figure 6 validates this

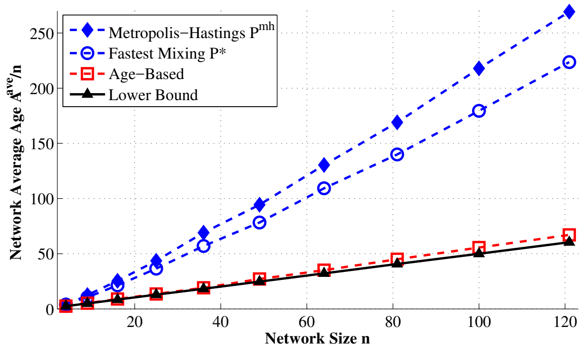


Fig. 7: Information Gathering on the Grid graph: network average age as a function of network size n for several proposed trajectories of the mobile agent.

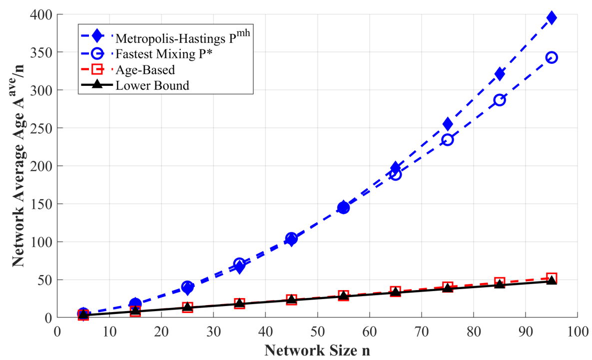


Fig. 8: Information Gathering and the Ring graph: network average age as a function of network size n for several proposed trajectories of the mobile agent (averaged over 20 runs).

conclusion: for example, for $n = 90$ ground terminals, the average age for the fastest mixing randomized trajectory $\mathbf{P}^*$ is approximately a factor 3 away from the lower bound.

In Figures 7 and 8 we plot the average age performance for several proposed trajectories, as a function of the network size. The age-based policy, again outperforms the two randomized trajectories, and is nearly optimal. We observe that the average age for the fastest mixing randomized trajectory $\mathbf{P}^*$, namely $A^{\text{ave}}(\mathbf{P}^*)$, is much worse in the ring graph than in the grid graph. This is because the mixing time for the ring graph is much larger than for the grid graph. Similar observation holds in comparing $\mathcal{G}(n, 2/\sqrt{n})$ and the grid graph. Note that the mixing times for both the 2D grid graph and $\mathcal{G}(n, 2/\sqrt{n})$ grow as $O(n \log n)$ [41] while for a ring graph, it grows as $O(n^2 \log n)$ [42].

In Figure 9, we simulate the performance of the separation principle policy in the dissemination setting, for graph $\mathcal{G}(n, 2/\sqrt{n})$, and compare its age performance with the gathering setting. We observe a significant deterioration of age, as a function of network size n, under stochastic updates and FCFS queues in comparison to fresh updates. This, we note, is the cost of uncontrollable queues in the system on age performance.

6 Conclusion

We considered the trajectory planning problem for a mobile agent, that traverses through a mobility graph G, to help timely exchange of information updates between a central terminal and a set of ground terminals V. For information gathering, we showed that

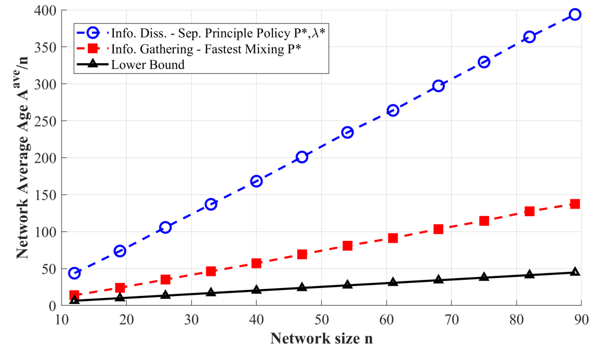


Fig. 9: Network average age as a function of network size (averaged over 20 runs).

a randomized trajectory, namely the fastest-mixing randomized trajectory, is peak age optimal and factor-$\mathcal{H}$ average age optimal. We showed that obtaining an average age optimal trajectory can be NP-hard, while we constricted the peak age optimal trajectory in polynomial time. To improve the average age, we proposed an age-based policy, and showed it to be factor-2 average age optimal, in a symmetric setting. For information dissemination, we proposed a separation principle policy, in which the mobile agent follows the fastest mixing randomized trajectory with a simple rate control. We proved that the separation principle policy is factor-$O(\mathcal{H})$ optimal, in both peak and average age.

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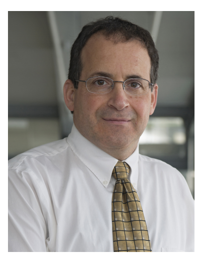

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Appendix A
Peak Age and Throughput Optimality

First, we note that **when fresh updates are generated at will** both peak age and throughput of a ground terminal depend only on the fraction of time the mobile agent spends on it. Further, we can construct a randomized trajectory $\mathbf{P}$ that achieves any possible stationary distribution on the set of terminals V using the Metropolis-Hastings algorithm and the fact that $G(V, E)$ is connected. Since peak age and throughput depend only on the stationary distribution, it is sufficient to look at randomized trajectories for both peak age and throughput optimality. In addition, it is sufficient to find the stationary distributions that achieve optimality for the respective metrics, since it is easy to find the trajectory once we have a target distribution.

So, we now discuss the relationship between throughput optimal and peak age optimal trajectories. From Lemma 1, we know that to minimize the network peak age, we need to solve the following optimization problem

$$\begin{aligned} \underset{\pi}{\text{Minimize}} \quad & \sum_{i \in V} \frac{w_i}{\pi_i}, \\ \text{subject to} \quad & \sum_i \pi_i = 1, \\ & \pi_i \geq 0, \forall i \in V. \end{aligned} \tag{1}$$

Using KKT conditions we get that

$$\pi_i^* = \frac{\sqrt{w_i}}{\sum_i \sqrt{w_i}}, \forall i \in V. \tag{2}$$

. Similarly, if we considered weighted sum throughput as the metric of interest, then our optimization problem would become

$$\begin{aligned} \underset{\pi}{\text{Maximize}} \quad & \sum_{i \in V} w_i \pi_i, \\ \text{subject to} \quad & \sum_i \pi_i = 1, \\ & \pi_i \geq 0, \forall i \in V. \end{aligned} \tag{3}$$

It is easy to see that the distribution that solves this problem is simply to put all the probability mass on the terminal with the highest weight. In other words, the throughput optimal trajectory, under weighted-sum throughput, is to just stay at the terminal with the highest weight. Note that this is clearly not peak age optimal. In fact, the peak age for all other ground terminals goes to infinity. Thus, throughput optimization does not directly correspond with peak age optimization.

However, let's now we consider proportional fair throughput optimization with *square-root weights*. The optimization problem is given by:

$$\begin{aligned} \underset{\pi}{\text{Maximize}} \quad & \sum_{i \in V} \sqrt{w_i} \log(\pi_i), \\ \text{subject to} \quad & \sum_i \pi_i = 1, \\ & \pi_i \geq 0, \forall i \in V. \end{aligned} \tag{4}$$

Using KKT conditions we can show that

$$\hat{\pi}_i = \frac{\sqrt{w_i}}{\sum_i \sqrt{w_i}}, \forall i \in V \tag{5}$$

maximizes this weighted-sum proportional fair throughput metric. Note that $\hat{\pi} = \pi^*$, thus peak age optimality is in fact equivalent to proportional fair throughput with square root weights.

Appendix B
Stopping Rules, Hitting Times and Mixing Times

A detailed discussion on stopping rules, hitting times and mixing times is outside the scope of this work, but we provide a brief discussion to make the definitions clear. We refer the reader to [36] for details.

A *stopping rule* is a rule that observes the walk on a Markov chain and, at each step, decides whether or not to stop the walk based on the walk so far. This decision can be probabilistic. Thus, a stopping rule maps walks to the probability of continuing the walk.

Hitting times $H(i, j)$ are usually defined as the expected time taken for a random walk to hit a state j starting from a given state i. Note that for any irreducible Markov chain, the quantities $H(i, j)$ are bounded $\forall i, j$.

Hitting times also have a more generalized definition in Markov chain literature [36] based on distributions and stopping rules. The *hitting time* $\mathcal{H}(\sigma, \tau)$ from a state distribution σ to τ on a Markov chain is the minimum expected stopping time over all stopping rules that, beginning at σ, stop in the exact distribution of τ.

We first show that there exists a naive stopping rule T that takes any irreducible Markov chain from an arbitrary distribution σ to another arbitrary distribution τ in a finite number of steps *in expectation*. The naive rule is as follows: start the chain with distribution σ, sample a state j according to the distribution τ and run the chain until it reaches j. The distribution of the Markov chain at the end of this stopping rule is simply τ since it stops at state j with probability τ_j. Further, the expected time to stop if it started at state i and stopped at state j is simply $H(i, j)$, since it is the expected time taken to go from state i to state j. Observe that we start at state i with probability σ_i and end at state j with probabiltiy τ_j. Thus, the expected length of this stopping rule is given by:

$$\mathbb{E}[\mathrm{T}] = \sum_{i,j} \sigma_i \tau_j H(i, j).$$

where $H(i, j)$ are the *statewise hitting times* defined earlier. Clearly, we can reach any distribution τ starting from any distribution σ in a finite number of steps, *on average*.

Since $\mathcal{H}(\sigma, \tau)$ is defined as the minimum expected stopping time over all stopping rules, it is also finite for any pair of distributions. Finally, we define the mixing time of a chain $\mathbf{P}$ as follows:

$$\mathcal{H} \triangleq \sup_{\sigma \in \mathbf{\Delta}(V)} \mathcal{H}(\sigma, \pi), \tag{6}$$

where $\mathbf{\Delta}(V)$ is the collection of all distributions on V and π is the stationary distribution of $\mathbf{P}$. Clearly, the supremum over a bounded set is bounded. Thus the mixing time $\mathcal{H}$ of an irreducible, aperiodic Markov chain is also bounded.

Appendix C
Mixing Time and Spectral Gap

In Theorem 5, we find the fastest mixing reversible Markov chain on a graph by solving the following convex program:

$$\begin{aligned}
\underset{\mathbf{P}}{\text{Minimize}} \quad & \mu(\mathbf{P}) = ||\mathbf{P} - \Pi^*||_2, \\
\text{subject to} \quad & P_{i,j} \geq 0,\ \forall(i,j), \\
& \mathbf{P1} = \mathbf{1}, \\
& \pi^*\mathbf{P} = \mathbf{P}^T\pi^*,\ \ \Pi^*_{i,j} = \pi^*_i\ \forall\, i,j \in V, \\
& P_{i,j} = 0, \forall(i,j) \notin E.
\end{aligned} \tag{7}$$

This formulation of finding the fastest mixing chain is based on a slightly different notion of mixing time (using spectral gap) rather than the stopping time definition introduced in Section 3.3. In [34], the authors define mixing time using the asymptotic rate of convergence to the stationary distribution. As shown in that work, for reversible chains, this is simply a function of the second largest eigenvalue. Hence, we can use spectral norm in the objective function of the program above.

However, in earlier discussions on bounding the performance of randomized trajectories, we used a different definition of mixing times, based on stopping rules. While it is not obvious, these two definitions are in fact closely related and only differ by at most a constant factor [36].

What is more tricky is the relationship between eigenvalues and mixing times. As shown in [34] for reversible chains on a graph with a fixed stationary distribution, the mixing time is indeed minimized by optimizing the spectral norm as done above. However, if chains can be irreversible, the relationship between mixing times and spectral gaps is not straightforward [43]. In fact, we do not know of a way to formulate a simple program to find the fastest mixing (possibly irreversible) Markov chain on a graph.

A simple way to get around this is to find the fastest mixing reversible Markov chain using (39) and also a relaxation of (39) over irreversible chains by replacing local balance constraints with global balance. Then, we can use whichever trajectory that has the lower mixing time. This ensures that we do not restrict ourselves to reversible chains in practice. However, we cannot guarantee that the chain found is the fastest mixing over all trajectories, including irreversible ones, but just the reversible ones.

Appendix D
Negative Correlation of X_n and T_n

We first prove the result for Ber/G/1 FCFS queues with no vacations. Let the service time of the nth packet be S_n and let the inter-arrival time between the $n-1$th packet and the nth packet be X_n. Let T_n be the time packet n spends in the system. Then,

$$T_n = \max\{T_{n-1} - X_n, 0\} + S_n. \tag{8}$$

Since the arrival of the nth packet cannot affect how long packet $n-1$ stays in the system, we know that T_{n-1} is independent of X_n. Similarly, since packet service times are independent of arrivals, we know that S_n is independent of X_n. We will compute $\mathbb{E}[T_n|X_n]$ using the fact that T_{n-1} and S_n are independent of X_n.

$$\mathbb{E}[T_n|X_n] = \sum_{t=1}^{\infty} \mathbb{P}(T_{n-1} = t)\max\{t - X_n, 0\} + \mathbb{E}[S]. \tag{9}$$

Note that $\mathbb{P}(T_{n-1} = t)\max\{t - X_n, 0\}$ is a monotone decreasing function of X_n for any value of t. Since the sum of monotone functions is monotone and $\mathbb{E}[S]$ is a constant independent of X_n, we get that $\mathbb{E}[T_n|X_n]$ is a monotone decreasing function of X_n. We will denote this function as $f(X_n)$. We analyze the covariance between X_n and T_n.

$$\begin{aligned}
&\text{cov}(X_n, T_n) = \mathbb{E}[X_nT_n] - \mathbb{E}[X_n]\mathbb{E}[T_n] \\
&= \mathbb{E}[\mathbb{E}[X_nT_n|X_n]] - \mathbb{E}[X_n]\mathbb{E}[T_n] \\
&= \mathbb{E}[X_n\mathbb{E}[T_n|X_n]] - \mathbb{E}[X_n]\mathbb{E}[\mathbb{E}[T_n|X_n]] \\
&= \mathbb{E}[X_nf(X_n)] - \mathbb{E}[X_n]\mathbb{E}[f(X_n)] \\
&= \mathbb{E}[(X_n - \mathbb{E}[X_n])(f(X_n) - \mathbb{E}[f(X_n)])] \\
&= \mathbb{E}[(X_n - \mathbb{E}[X_n])(f(X_n) - \mathbb{E}[f(X_n)] + f(\mathbb{E}[X_n]) - f(\mathbb{E}[X_n]))] \\
&\overset{(75A)}{=} \mathbb{E}[(X_n - \mathbb{E}[X_n])(f(X_n) - f(\mathbb{E}[X_n]))] \\
&\quad + \mathbb{E}[(X_n - \mathbb{E}[X_n])(f(\mathbb{E}[X_n]) - \mathbb{E}[f(X_n)])] \\
&\overset{(75B)}{=} \mathbb{E}[(X_n - \mathbb{E}[X_n])(f(X_n) - f(\mathbb{E}[X_n]))] \\
&\overset{(75C)}{\leq} 0,
\end{aligned} \tag{10}$$

To get (75B), we show that the second term in (75A) is zero.

$$\begin{aligned}
&\mathbb{E}\Big[(X_n - \mathbb{E}[X_n])\big(f(\mathbb{E}[X_n]) - \mathbb{E}[f(X_n)]\big)\Big] \\
&= \big(f(\mathbb{E}[X_n]) - \mathbb{E}[f(X_n)]\big)\mathbb{E}\big[(X_n - \mathbb{E}[X_n])\big] \\
&= \big(f(\mathbb{E}[X_n]) - \mathbb{E}[f(X_n)]\big)\big(\mathbb{E}[X_n] - \mathbb{E}[X_n]\big) = 0.
\end{aligned}$$

The first equality follows since $(f(\mathbb{E}[X_n]) - \mathbb{E}[f(X_n)])$ is a constant and it can be pulled out of the expectation. The second equality similarly follows since $\mathbb{E}[X_n]$ is a constant and can be pulled out of the expectation. Now, it is easy to see that the term is zero.

(75C) follows since $f(\cdot)$ is a monotone decreasing function and $(a-b)(f(a)-f(b)) \leq 0, \forall a, b$. This concludes the proof since we have show that X_n and T_n are negatively correlated.

This approach is not easy to extend for the case with vacations. This is primarily because the recursion for system time involves the residual time of the current vacation interval when the packet enters an empty queue. So, we provide numerical evidence of negative correlation in both the original system (which has non i.i.d. service) and for Ber/G/1 queues with i.i.d vacations and service times.

In Figure 1, we plot the network peak and average age under our proposed trajectory and transmission rates for the information dissemination setting. We observe that the average age is always upper bounded by the peak age irrespective of the network size or topology. Figures 2, 3, and 4 plot peak and average age for Ber/G/1 queues with i.i.d. vacations and service intervals. We observe that irrespective of the arrival rates and vacation/service time distributions peak age is always an upper bound for the average age. This strongly suggests that Assumption 1 is true.

Appendix E
Proof of Lemma 4

Consider a randomized trajectory $\mathbf{P}$ and Bernoulli arrival rates $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots)$. From the arguments made in Section 4, we know that the peak age for the ground terminal i is upper-bounded by the peak age of a discrete time FCFS Ber/G/1 queue with

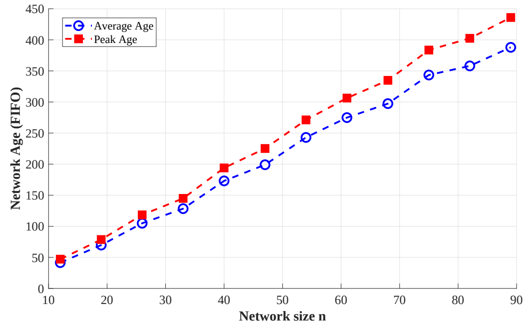

Fig. 1: Network peak and average age as a function of network size n for the dissemination setting

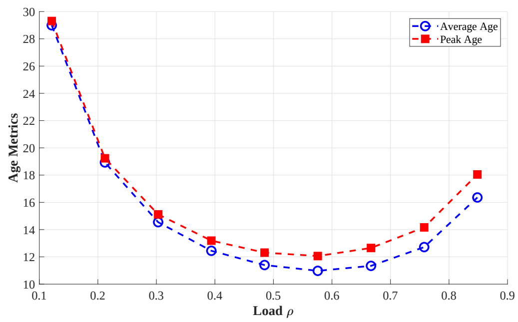

Fig. 2: Peak and average age v/s load for a Ber/G/1 queue with geometric vacations and service.

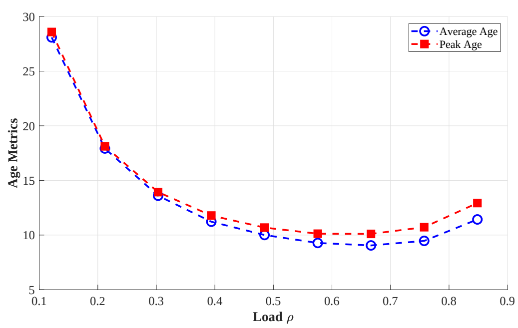

Fig. 3: Peak and average age v/s load for a Ber/G/1 queue with bounded uniform random vacations and service.

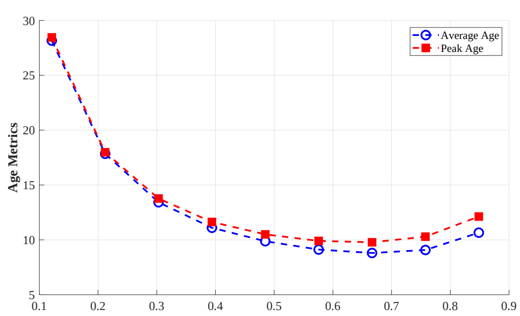

Fig. 4: Peak and average age v/s load for a Ber/G/1 queue with deterministic vacations and service.

vacations, for which the service times and vacation times have the same distribution as the inter-visit times $H_{1,i}$. Applying Lemma 3, we obtain

$$A_i^{\mathrm{p}} \leq \frac{1}{\pi_i}\left[1 + z_{ii} + \frac{1}{\rho_i} + \frac{z_{ii}\rho_i}{1-\rho_i}\right] - \frac{\rho_i}{1-\rho_i} - 1 \triangleq A_i^{\mathrm{UB}}, \quad (11)$$

where we have used the first and second moment of inter-visit times $H_{1,i}$ [30, Ch.4]:

$$\mathbb{E}[H_{1,i}] = \frac{1}{\pi_i},\ \mathbb{E}[H_{1,i}^2] = \frac{-1}{\pi_i} + \frac{2z_{ii}}{\pi_i^2}, \forall i \in V. \quad (12)$$

Similarly, we know that the average age for the ground terminal i is also upper-bounded by the average age for the FCFS Ber/G/1 queue with vacations. Using the fact that $A^{\mathrm{ave}} \leq A^{\mathrm{p}}$ for the Ber/G/1 queue with vacations (see Lemma 3, we get $A_i^{\mathrm{ave}} \leq A_i^{\mathrm{UB}}$.