Improving Age of Information in Wireless Networks With Perfect Channel State Information

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Abstract-Age of information (AoI), defined as the time that elapsed since the last received update was generated, is a newly proposed metric to measure the timeliness of information updates in a network. We consider AoI minimization problem for a network with general interference constraints, and time varying channels. We propose two policies, namely, virtual-queue based policy and age-based policy when the channel state is available to the network scheduler at each time step. We prove that the virtual-queue based policy is nearly optimal, up to a constant additive factor, and the age-based policy is at-most a factor of 4 away from optimality. Comparison with previous work, which derived age optimal policies when channel state information is not available to the scheduler, demonstrates significant improvement in age due to the availability of channel state information. Our analysis relies on the age conservation law and age-square conservation law developed in this paper, which hold more generally and may be of independent interest.

Index Terms—Age of information (AoI), wireless networks, scheduling, information freshness, channel state information.

I. INTRODUCTION

TIMELY delivery of information updates is gaining increasing relevance with the emergence of cyberphysical systems, internet of things, and unmanned aerial vehicular networks. In unmanned aerial vehicular networks, timely delivery of status updates, such as vehicle position and velocity, may be critical to network safety [1], [2]. In internet of things or cyber-physical systems, timely delivery of sensor information can significantly improve the overall system performance [3].

Age of information (AoI) is a recently proposed time evolving measure of information freshness that is defined as the time that elapsed since the last received update was generated by the source [4], [5]. Figure 1 shows the typical evolution of AoI at a destination node, as a function of time. Upon reception of a new update packet AoI drops to the time that elapsed since the generation of the packet, and grows linearly until the next delivery. Therefore, AoI is a destination centric measure, unlike packet delay, and is

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Fig. 1. Time evolution of age, $A_e(t)$, of a link *e*. Times t_i and t'_i are instances of *i*th packet generation and reception, respectively. Upon reception of packet *i*, the age is reset to $t'_i - t_i + 1$.

better suited for applications involving dissemination of time sensitive information. Peak and average age are two metrics of AoI. Peak age is defined as the average of all the peaks in the AoI curve, shown in Figure 1, whereas the average age is time average of the AoI.

In [4], a simulation study considered AoI in a network of vehicles exchanging status updates. Motivated by [4], AoI was analyzed for several queueing models [5]–[22]. The advantage of having parallel servers and setting packet deadlines was studied in [6]–[9] and [11]–[13], respectively. The problem of determining an optimal, or near-optimal, queue scheduling discipline for minimizing AoI was considered in [15].

However, prior to this work, AoI minimization for communication links operating in a wireless network with interference had received very little attention. A problem of scheduling finitely many update packets under physical interference constraints was shown to be NP-hard in [23]. Age for a broadcast network, where only a single link can be activated at any time, was studied in [24], [25]. Distributed ALOHA like random access to minimize AoI was considered in [26], [27]. Age in multi-hop wireless network has been studied in [28].

In [29], we considered the problem of age minimization for a wireless network under general interference constraints, and time varying channel. We considered two types of sources: *active sources*, which generate fresh information in every slot, and *buffered sources*, which cannot generate fresh information in every slot. We showed that for a network with active sources, a stationary scheduling policy, which schedules links according to a stationary probability distribution, is peak age optimal and factor-2 average age optimal. We also showed that the same scheduling policy, with a certain packet generation

1063-6692 © 2020 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See https://www.ieee.org/publications/rights/index.html for more information. rate control, is nearly optimal in the buffered case. In [29], however, the space of policies was limited to not using the channel state information. In this paper, we consider scheduling policies which have perfect channel state information $\mathbf{S}(t)$ at every time slot t.

We consider a wireless network consisting of several communication links E. Each link is a source-destination pair, and need to transmit update packets across. Packets cannot be transmitted over all links simultaneously due to the wireless interference constraints, and a transmission may fail because of channel errors. We consider the space of scheduling policies which have perfect channel state information at every time slot t, and minimize peak and average age. We first show, by considering a simple example, that AoI can be considerably improved, over the proposed optimal policies in [29], by utilizing the current channel state information.

We then show that the peak age minimization problem can be formulated as a network utility maximization problem. Using this, we propose the virtual-queue based scheduling policy which constructs virtual queues $Q_e(t)$ for each link $e \in E$. The virtual queues evolve in time, depending on whether a successful transmission occurred over the link or not. The set of links to be scheduled, at time t, is determined by a max-weight policy:

$$m_t = \arg \max_{m \subset E} \sum_{e \in m} w_e S_e(t) Q_e(t),$$

where w_e are predefined weights on each link, and $S_e(t)$ denotes the channel state of link e. We show that this policy nearly achieves peak age optimality, up to a small additive constant.

Next, we propose age-based policies, which use age $A_e(t)$ of each link $e \in E$, instead of the virtual queues $Q_e(t)$ to determine the schedule at time t:

$$m_t = \arg \max_{m \subset E} \sum_{e \in m} w_e S_e(t) g\left(A_e(t)\right),$$

where $g(\cdot)$ is some non-decreasing function. We show that for a specific choice $g(x) = x^2 + \beta x$, the policy is at most a factor of 4 away from the optimal peak and average age. However, simulation results suggest that this age-based policy performs much better in practice, and can be very close to the optimal average age for certain networks.

Our analysis relies on the *age conservation law* and *age-square conservation law* developed in Section III, which hold more generally and may be of independent interest.

In numerical simulations, we also observe the benefit/utility of using channel state information in scheduling to minimize age, especially when the network has 'high' level of interference or 'bad' channel quality. Even though channel state information may not be perfectly available in certain network settings, this work establishes the utility of acquiring such channel state information for scheduling to minimize age.

A preliminary version of this work was presented at WiOpt 2018 [30]. In [30], however, we made restricting assumptions on the space of scheduling policies. In this work, we have relaxed those restricting assumptions. This work also develops age conservation laws, which are critical in establishing the

results in this paper, and may be of independent interest. In [31], we extended the virtual-queue based and age-based policies described above to the case when the channel state information is not available.

A. Literature Review

The notion of Age-of-information was developed in [4], [5], [32]. In [5], average age was analyzed and optimized for update generation rate, for simple queueing systems such as M/M/1, M/D/1, and D/M/1. This motivated several researchers to study peak and average age for various queueing systems [4]–[8], [11]–[13], [18], [20], [33]–[38]. Many of these works focused on optimizing peak and average age over the update generation and service rate. Queue scheduling disciplines and packet management strategies to minimize age were also considered. The advantage of having parallel servers towards improving age was demonstrated in [6]–[8]. Age improvements by having smaller buffer sizes and introducing packet deadlines, in which a packet deletes itself after the expiration of its deadline, was demonstrated in [4], [11] and [11]–[13], respectively.

Age, for updates traversing a network of queues, was considered in [36]. The last come first serve queue scheduling discipline, with preemptive service, was shown to be age optimal, when the service times are exponentially distributed [36]. Optimal update generation policy to improve age was investigated in [16]. AoI for energy harvesting communication systems was considered in [39]–[44], while AoI for gossip type information dissemination was analyzed in [32], [45].

Little work existed, prior to this, on link scheduling for age minimization. In [23], [46], link scheduling problem for age minimization, in transmitting finitely many updates, was posed. It was shown to be NP-hard under the physical and protocol model for interference. Scheduling for age minimization in a broadcast network, in which at most one link can be activated simultaneously, was considered first in [24]. Index based policies were proposed and were shown to be factor 8 optimal. In [25], age minimization in a broadcast network with no channel uncertainties was considered. The problem was formulated as a Markov Decision Process, and several structural results on optimal policies were derived.

In [29], a simple randomized stationary scheduling policy was shown to be peak age optimal and factor-2 average age optimal for single-hop wireless networks under general interference constraints. This result was presented independently for broadcast networks, and its optimality proved for broadcast networks under throughput constraints in [47] and [48], respectively. [29] also proved an important separation principle for the special case when the sources are not active, i.e. they cannot generate fresh updates for every transmission. It was shown that the peak age optimal randomized stationary policy, designed assuming active sources, could be nearly optimal.

Broadcast networks under ALOHA-like scheduling protocol, in which the link's attempt transmission with some attempt probabilities, was considered in [49]. Exact solution for ALOHA-like distributed scheduling policies under more general interference constraints was obtained in [27]. Age minimization for multi-hop wireless networks under general interference was considered in [28], [50], whereas multi-hop, multi-cast networks was studied in [51]. Scheduling for age minimization, when different sensors view correlated information, has been considered in [52]. Age minimization under power constraints has been recently considered in [53], while a game theoretic view on users competing to minimize age is studied in [54].

In this work, and it's preliminary versions [30], [31], we propose age-based scheduling and virtual queue based scheduling for single-hop wireless networks under general interference constraints. Such age-based policies have since been proposed for broadcast networks [47], [48]. More recently, age-based policies have been extended to problems when information is no longer freshly available at the source [55], [56].

B. Organization

System model is introduced in Section II and some useful identities on AoI are proved in Section III. We propose and analyze the virtual-queue based scheduling policy and the age-based scheduling policy in Section IV and Section V, respectively. Simulation results are discussed in Section VI. We conclude in Section VII.

II. SYSTEM MODEL

Consider a wireless network G = (V, E), where V denotes the set of nodes and E the set of directed links. Each directed link is a source-destination pair which need to be activated for the source to transmit update packets to the destination. Not all links can be activated simultaneously. Thus, we call a set $m \subset E$ that can be activated simultaneously without interference a *feasible activation set*. We use A to denote the collection of all feasible activation sets. We consider a slotted time system, where the slot duration is normalized to unity.

We use $S_e(t)$ to denote the channel state process, where $S_e(t) = 1$ if the channel is in the ON state at time t and $S_e(t) = 0$ if the channel is in the OFF state at time t. The space of all channel states is given by $\mathbb{S} = \{0, 1\}^{|E|}$. We consider an i.i.d. channel model, in which the channel state process $\{S_e(t)\}_{t\geq 0}$ is independent and identically distributed (i.i.d.) across time t, with $\gamma_e = \mathbb{P}[S_e(t) = 1] > 0$, for all $e \in E$. Further, the channel processes $\{S_e(t)\}_{t\geq 0}$ are independent across links e; but may not be identically distributed.

We use $U_e(t) \in \{0, 1\}$ to denote transmission decision on link e at time t. We set $U_e(t) = 1$ if link e is scheduled to transmit at time t. For a transmission to be successful the channel state must also be ON. Thus, a successful transmission occurs over link e, at time t, if and only if $U_e(t)S_e(t) = 1$.

We consider *active nodes*, which transmit fresh information at every transmission opportunity. We define the age $A_e(t)$, of a link e at time t, to be the time that elapsed since the last successful activation of link e. We consider the discrete time evolution of age. Figure 2 shows the evolution of $A_e(t)$ for a link e over discrete time slots $t \in \{0, 1, 2, \ldots\}$. $A_e(t)$ drops to 1 upon a successful activation of link e, and increases by 1 in every slot in which there is no successful activation of link



Fig. 2. Discrete time evolution of age of link e, namely $A_e(t)$, as a function of time slots t.

e, i.e.,

$$A_e(t+1) = \begin{cases} A_e(t) + 1 & \text{if } U_e(t)S_e(t) = 0\\ 1 & \text{if } U_e(t)S_e(t) = 1. \end{cases}$$
(1)

This age evolution equation can be written compactly as,

$$A_e(t+1) = 1 + A_e(t) - U_e(t)S_e(t)A_e(t),$$
(2)

for all $t \ge 0$ and $e \in E$. For the ease of presentation, we assume that $A_e(0) = 0$ for all links $e \in E$.¹

We consider two age measures, namely, average age and peak age. Average age is the area under the age curve in Figure 2, while peak age is the average of all the peaks of the age curve. Note that the sum of all the peaks, until time t, in the age curve can be expressed as $\sum_{\tau=0}^{t} U_e(\tau) S_e(\tau) A_e(\tau)$. This is because $U_e(\tau) S_e(\tau) = 1$ only at times when age peaks. We, therefore, define the peak age of a link e to be

$$\overline{A}_{e}^{\mathbf{p}} = \limsup_{t \to \infty} \frac{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}(\tau) + A_{e}(t)\right]}{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) + 1\right]}.$$
 (3)

The average age, of link e, is defined as:

$$\overline{A}_{e}^{\text{ave}} = \limsup_{t \to \infty} \frac{1}{t} \mathbb{E} \left[\sum_{\tau=0}^{t} A_{e}(\tau) \right].$$
(4)

The network average and peak age are defined to be the weighted sum of link edges:

$$\overline{A}^{\text{ave}} = \sum_{e \in E} w_e \overline{A}_e^{\text{ave}} \quad \text{and} \quad \overline{A}^{\text{p}} = \sum_{e \in E} w_e \overline{A}_e^{\text{p}}.$$
 (5)

Without loss of generality, we assume the weights w_e to be positive, and normalized to sum to 1: $\sum_{e \in E} w_e = 1$. We are interested in designing link scheduling policies that minimize the network peak and average age.

A. Scheduling Policies

A scheduling policy determines the set of links $m_t \subset E$ that will be activated at each time t, i.e., $m_t = \{e \in E | U_e(t) = 1\}$. The policy can make use of the past history of link activations and observed channel states to make this decision, i.e., at each time t, the policy π will determine m_t as a function of the set

$$\bar{\mathcal{H}}(t) = \{ \mathbf{U}(\tau), \mathbf{S}(\tau'), \mathbf{A}(\tau') \mid 0 \le \tau < t, \ 0 \le \tau' \le t \}, \quad (6)$$

¹All the lemmas and proofs can be extended when $A_e(0)$ is a constant away from 0. The results stated in the theorems, however, remain unchanged if $A_e(0)$ are taken to be some constants.

where $\mathbf{U}(\tau)$, $\mathbf{S}(\tau)$, and $\mathbf{A}(\tau)$ denote the vectors $(U_e(\tau))_{e \in E}$, $(S_e(\tau))_{e \in E}$, and $(A_e(\tau))_{e \in E}$, respectively.

We consider centralized scheduling policies, in which this information is available to a scheduler, which is also able to implement its scheduling decision. Let $\overline{\Pi}$ denote the space of all such scheduling policies, which decide on the current action m_t , using the history $\overline{\mathcal{H}}(t)$, for all t. Our goal is to minimize peak and average age over the space of scheduling policies $\pi \in \overline{\Pi}$. We define the optimal peak and average age to be

$$\overline{A}^{p*} = \min_{\pi \in \overline{\Pi}} \overline{A}^{p}(\pi) \quad \text{and} \quad \overline{A}^{ave*} = \min_{\pi \in \overline{\Pi}} \overline{A}^{ave}(\pi).$$
(7)

Here, and throughout the paper, we use the min notation to denote the technically correct inf.

B. Advantage of Knowing Channel State Information

In this subsection, we illustrate the advantage of using the channel state in scheduling for age minimization. In [29], we considered age minimization with unknown channel state information. Specifically, we considered all policies which scheduled feasible activation set $m_t \in A$, at time t, as a function of the history

$$\mathcal{H}(t) = \{ \mathbf{U}(\tau), \mathbf{S}(\tau), \mathbf{A}(\tau') \big| 0 \le \tau < t \text{ and } 0 \le \tau' \le t \}.$$
(8)

The history $\mathcal{H}(t)$, unlike $\mathcal{H}(t)$ in (6), does not include the current channel state $\mathbf{S}(t)$. We showed in [29] that stationary policies, which schedule links according to a probability distribution, which is independent of $\mathcal{H}(t)$, is in fact peak age optimal and factor-2 average age optimal.

In stationary scheduling policies, every feasible activation set $m \in A$ is assigned a fixed probability x_m with which it is activated in slot t, independent across slots. The probability that a link $e \in E$ is activated in a slot is given by

$$f_e = \sum_{m:e \in m} x_m,\tag{9}$$

for all $e \in E$. This set of equations can be compactly written as $\mathbf{f} = M\mathbf{x}$. Note that an activated link will result in a successful transmission if the corresponding channel is in the ON state. Therefore, the probability of a successful transmission on a link e in any slot is given by $\alpha_e = \gamma_e f_e$.

Further, notice that, if a link e is successfully activated with probability $\alpha_e = \gamma_e f_e$ in each slot, independent across slots, then the time since last transmission, i.e. age $A_e(t)$, is geometrically distributed with rate $\frac{1}{\gamma_e f_e}$. In [29], we showed that the peak age of link e equals this rate $\frac{1}{\gamma_e f_e}$, under any stationary policy. As a result, the peak age for the stationary policy, determined by distribution **x**, is given by $\overline{A}^p = \sum_{e \in E} \frac{w_e}{\gamma_e f_e}$, and thus, the optimal peak age is given by

$$\overline{A}^{\mathbf{p}^*} = \underset{\mathbf{x}, \mathbf{f}}{\text{Minimize}} \sum_{e \in E} \frac{w_e}{\gamma_e f_e},$$

subject to $\mathbf{f} = M\mathbf{x},$
 $\mathbf{1}^T \mathbf{x} \le 1 \text{ and } \mathbf{x} \ge 0.$ (10)

The peak age optimal stationary policy is obtained by solving (10). We will now argue that in the case when the channel



Fig. 3. Plot of achievable successful link activation frequency regions for the two link network, in which only one link can be activated at a time. Shown are regions when channel state is observed (grey) and unobserved (black).

state information is available for scheduling, smaller age than what is given by (10) can be achieved.

To see the difference between age minimization under known and unknown channel process consider the two link example shown in Figure 3. In this example, only one link can be activated at a time. Let the weights $w_1 = w_2 = 1$ for the two links, and the channel success probabilities be $\gamma_1 = \gamma_2 = 0.5$. When the channel state $\mathbf{S}(t) = (S_1(t), S_2(t))$ is unavailable the peak age minimization problem is given by (from (10)):

$$\overline{A}^{p*} = \underset{f_1, f_2}{\text{Minimize}} \frac{1}{\gamma_1 f_1} + \frac{1}{\gamma_2 f_2},$$

subject to $f_1 + f_2 \leq 1,$
 $f_1 \geq 0 \text{ and } f_2 \geq 0.$ (11)

Here, f_1 denotes the fraction of times link 1 is scheduled and f_2 denotes the fraction of times link 2 is scheduled. Since $\gamma_1 = \gamma_2 = 0.5$, the optimal solution to (11) is given by $f_1^* = f_2^* = 0.5$, i.e. with probability 0.5 each link gets scheduled in each slot, and as a result the optimal peak age is $\overline{A}^{p*} = 8$.

However, if we can observe the channel state $\mathbf{S}(t)$ in every slot before making scheduling decision, we can achieve even smaller age than $\overline{A}^{p*} = 8$. Consider the following policy: schedule link 1 whenever $S_1(t) = 1$, and otherwise link 2. The successful link activation frequency on link 1 is then $\alpha_1 = \gamma_1 = 0.5$, while on link 2 it is $\alpha_2 = \gamma_2(1 - \gamma_1) = 0.25$. The peak age is given by $\overline{A}^p = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} = 6 < \overline{A}^{p*} = 8$. This happens primarily because the set of achievable successful link activation frequencies, namely α_e , is larger in the case when the channel can be observed before deciding on the schedule in each slot. In Figure 3, we show these regions in the observed and unobserved channel state case for the two link example.

This shows that when the channel state information is available for making scheduling decisions, the network age performance can be improved. In Sections IV and V, we will propose two scheduling policies which make use of the current channel state. In the next section, we present some age identities, which bring out the inter-relation between the channel state process, scheduling decisions, and the age process. These identities hold under greater generality, and will be used in proving performance bounds on the proposed policies in Sections IV and V.

III. AGE IDENTITIES

The age evolution in (2) shows that the channel state process, the scheduling decisions, and the age processes are interrelated. We first present two identities, namely, age-conservation law and the age-square conservation law. We then prove that for any scheduling policy, the peak age is upper-bounded by twice the average age, under the same policy. The results presented here hold in greater generality, in the sense, that we do not make any assumptions on the channel state process or the scheduling decisions.

Let us define finite time horizon versions of peak and average age. The t-slot average age, of link e, is defined as:

$$A_{t,e}^{\text{ave}} = \frac{1}{t} \sum_{\tau=0}^{t} A_e(\tau), \qquad (12)$$

and the t-slot peak age, of link e, is defined as:

$$A_{t,e}^{p} = \frac{\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}(\tau) + A_{e}(t)}{\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) + 1}.$$
 (13)

The finite time horizon, network average and peak age are defined to be the weighted sum of link edges:

$$A_t^{\text{ave}} = \sum_{e \in E} w_e A_{t,e}^{\text{ave}} \quad \text{and} \quad A_t^{\text{p}} = \sum_{e \in E} w_e A_{t,e}^{\text{p}}, \qquad (14)$$

where the weights w_e are positive, and normalized to sum to 1: $\sum_{e \in E} w_e = 1.$

We first present the age conservation law. It states that for any scheduling policy, the sum of all age peaks is equal to the total time elapsed.

Lemma 1: For any scheduling policy, and for all $e \in$ E, we have t-1

$$\sum_{\tau=0}^{1} U_e(\tau) S_e(\tau) A_e(\tau) + A_e(t) = t.$$
(15)

Proof: See Appendix A.

The lemma states that the sum of all age peaks, and the residual age at time t, equals the total time that elapsed, namely, t. Note that the result holds even if link e is never scheduled. The result doesn't depend on the scheduling policy, and is an invariant principle that holds for all triplets of age $\{A_e(t)\}_t$, channel process $\{S_e(t)\}_t$, and the decision variables $\{U_e(t)\}_t$.

We now present the *age-square conservation law*, which states that the sum of squares of the age peaks is equal to the average age.

formed by the age curve in Figure 2. Since $S_e(t)U_e(t)A_e(t)$ either equals the age peaks in Figure 2, or is 0, the term $\sum_{\tau=0}^{t-1} \frac{1}{2} S_e(\tau) U_e(\tau) A_e^2(\tau)$ is the sum of the areas of all the

Proof: See Appendix B.

E, we have

 $\frac{1}{2}A_e^2(t)$ captures the area formed by the residual age. Just like the age conservation law, the age-square conservation law holds for any all triplets of age $\{A_e(t)\}_t$, channel process $\{S_e(t)\}_t$, and the decision variables $\{U_e(t)\}_t$. Notice that Lemma 2 would hold even when the link e is never scheduled.

triangles formed by age peaks in Figure 2. The last term

Lemma 2: For any scheduling policy, and for all $e \in$

 $A_{t,e}^{\text{ave}} = \frac{1}{t} \left\{ \frac{1}{2} \sum_{i=1}^{t-1} U_e(\tau) S_e(\tau) A_e^2(\tau) + \frac{1}{2} A_e^2(t) \right\} + \frac{1}{2}.$

For an intuitive understanding of Lemma 2, note that aver-

age age is essentially the time averaged area of the triangles

We will typically be interested in peak and average age performance of the scheduling policies. In the following lemma, we prove a useful relation between the peak and average age, that holds for any scheduling policy.

Lemma 3: For any policy
$$\pi$$
, we have
 $A_t^p(\pi) \le 2A_t^{ave}(\pi) - 1,$ (16)

and

$$\overline{A}^{p}(\pi) \le 2\overline{A}^{avc}(\pi) - 1, \qquad (17)$$

for all $e \in E$, and any t > 1.

Proof: This result follows by a direct application of the Cauchy-Schwartz inequality. See Appendix D.

An important implication of this result is as follows. If we want to minimize peak and average age over some policy space $\overline{\Pi}$, then the same relation in Lemma 3 will hold for the optimal peak and average age, namely, $\overline{A}^{p*} \leq 2\overline{A}^{ave*} - 1$, where \overline{A}^{p*} and \overline{A}^{ave*} denote the peak and average age, respectively. This result will provide with a natural lowerbound on the optimal average age \overline{A}^{ave*} in terms of the optimal peak age.

In the next two sections, we propose scheduling policies, that use the channel state information, to minimize peak and average age.

IV. VIRTUAL QUEUE-BASED POLICY

In II-A, we defined the policy space $\overline{\Pi}$ to be the policies which make use of the entire history $\mathcal{H}(t)$, and not just the current channel state S(t). In Section II-B, we saw that scheduling using the channel states can improve age. In this section, we propose a virtual-queue based policy, which uses only the current channel state information to make the scheduling decisions. We will prove that these policies can be nearly peak age optimal.

(16)

We first note that a direct consequence of the age conservation law, namely Lemma 1, is that the peak age minimization problem $\min_{\pi \in \overline{\Pi}} \overline{A}^p(\pi)$ reduces to

$$\begin{split} \underset{\alpha \ge 0, \pi \in \overline{\Pi}}{\text{Minimize}} & \sum_{e \in E} \frac{w_e}{\alpha_e}, \\ \text{subject to } \liminf_{t \to \infty} \mathbb{E} \left[\frac{1}{t} \sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau) \right] \ge \alpha_e \quad \forall \ e \in E. \end{split}$$

$$\end{split}$$

$$\end{split}$$

$$\end{split}$$

We prove this equivalence in Appendix C. This result is significant because it shows that the peak age minimization problem is independent of the age evolution equation. For this reason peak age minimization is much simpler than average age minimization.

In [29], we showed that when the current channel states were not available, a randomized scheduling policy is peak age optimal. We now define a sub-class of policies, which always contain a peak age optimal policy. These policies do not use any past history, but only the current channel state S(t), and are defined as follows [57]:

S-Only Policy: For each observed channel state $S \in S$ we assign a probability distribution p(S, m) over the set of feasible activation sets $m \in A$. If channel state $\mathbf{S}(t)$ is observed then the activation set $m \in A$ is activated for slot t with probability $p(\mathbf{S}(t), m)$.

For an S-only policy, the rate at which a successful transmission occurs over link e is given by

$$\alpha_e = \mathbb{E}\left[U_e(t)S_e(t)\right] = \mathbb{P}\left[U_e(t)S_e(t) = 1\right]$$
$$= \gamma_e \mathbb{P}\left[U_e(t) = 1|S_e(t) = 1\right], (19)$$

for all $e \in E$. The space of all such rates α will depend on channel statistics γ_e , and thus, we use $\Lambda_S(\gamma)$ to denote this space of all feasible α using S-only policies. For the two link example in Figure 3, $\Lambda_S(\gamma)$ is exactly the grey region of successful link activation frequencies (α_1, α_2) . Let $\Lambda(\gamma)$ be the space of rates α achievable under all policies in $\overline{\Pi}$. Then, it is known that $\Lambda(\gamma)$ equals $\Lambda_S(\gamma)$ [57], i.e., S-only policies are sufficient to achieve the full rate region. This will help us show that an S-only policy can be nearly peak age optimal.

We first characterize the optimal peak age by showing that there exists a S-only policy that is nearly peak age optimal.

Theorem 1: The optimal peak age
$$\overline{A}^{p*}$$
 is given by
 $\overline{A}^{p*} = \underset{\alpha}{\text{Minimize}} \sum_{e \in E} \frac{w_e}{\alpha_e},$
subject to $\alpha \in \Lambda_S(\gamma)$, (20)

and as a consequence, for any $\epsilon > 0$, there exists a S-only policy π^{ϵ} that attains a peak age $\overline{A}^{p}(\pi^{\epsilon}) \leq \overline{A}^{p*} + \epsilon$.

Proof: In Appendix E, we show that the peak age minimization problem over the space of S-only policies can be written as (20).

Theorem 1 can be used to obtain a S-only policy that is nearly peak age optimal. However, the search space $\Lambda_S(\gamma)$ is usually difficult to characterize for general interference constraints. Another issue is that, solving (20), requires exact knowledge of the channel statistics γ_e . We propose a scheduling policy that attains near optimal peak age, even when the channel statistics γ_e is not known apriori.

We now propose a policy that solves the peak age minimization problem (18). Note that a policy π can decide on the activation set m_t , at time t, based on the entire history $\mathcal{H}(t)$. However, we do not need the entire history to make a choice at time t but only a representation of it. To do so, we construct virtual queue $Q_e(t)$ for each link e, which decreases by (at most) 1 upon a successful transmission over link e and increases otherwise. These queue lengths determine the 'value' of scheduling link e in time slot t. Therefore, a set $m_t \in \mathcal{A}$ that maximizes $\sum_{e \in m_t} w_e Q_e(t) S_e(t)$ is activated in slot t. This virtual-queue based policy, π_Q , is described below. Here, V > 0 is any chosen constant.

Virtual-queue based policy π_Q Start with $Q_e(0) = 1$ for all $e \in E$. At time t,

1) Schedule activation set m_t given by

$$m_t = \arg\max_{m \in \mathcal{A}} \sum_{e \in m} w_e Q_e(t) S_e(t), \qquad (21)$$

2) Update $Q_e(t)$ as

$$\begin{aligned} Q_e(t+1) &= \left[Q_e(t) + \sqrt{\frac{V}{Q_e(t)}} - U_e(t)S_e(t)\right]_{+1},\\ \text{for all } e \in E, \text{ where } [x]_{+1} &= \max\{x,1\}. \end{aligned}$$

We now show that the virtual-queue based policy approximately solves the peak age minimization problem (18). The policy π_Q is nearly peak age optimal up to an additive factor.

Theorem 2: The peak age for policy π_Q is bounded by

$$\overline{A}^{\mathrm{p}}(\pi_Q) \le \overline{A}^{\mathrm{p}*} + \left(\frac{1}{2} + \frac{1}{2V}\right), \qquad (22)$$

where \overline{A}^{p*} is the optimal value of (20).

Proof: Let $\alpha_e(t) = \sqrt{\frac{V}{Q_e(t)}}$ and $\overline{\alpha}_e(t) = \frac{1}{t} \sum_{\tau=0}^{t-1} \alpha_e(\tau)$ for all $t \ge 0$ and $e \in E$. Also, let $g(\alpha) = \sum_{e \in E} \frac{w_e}{\alpha_e}$ (i.e., the objective function in our optimization problem (18)). The proof is divided into three parts:

Part A: For all time t, we have

$$\limsup_{t \to \infty} \mathbb{E}\left[g\left(\overline{\boldsymbol{\alpha}}(t)\right)\right] \le \overline{A}^{\mathsf{p}*} + \frac{1}{2} \sum_{e \in E} w_e + \frac{1}{2V} \sum_{e \in E} w_e.$$
(23)

Part B: The virtual queue $\mathbf{Q}(t)$ is mean rate stable, i.e., for all $e \in E$ we have

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E}\left[Q_e(t)\right] = 0.$$
(24)

Part C: If $\mathbf{Q}(t)$ is mean rate stable then

$$\liminf_{t \to \infty} \mathbb{E}\left[\overline{\alpha}_e(t)\right] \le \liminf_{t \to \infty} \frac{1}{t} \mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau)\right], \quad (25)$$

and

$$\overline{A}^{p}(\pi_{Q}) \leq \limsup_{t \to \infty} \mathbb{E}\left[g\left(\overline{\alpha}(t)\right)\right].$$
(26)

The proofs of Part A, B, and C are given in Appendix F. Since the virtual queues are mean rate stable, by Part B, (25) and (26) are true. From (23) and (26) we get the result in (22).

Theorem 2 shows that even when the channel statistics are not known the optimal peak age \overline{A}^{p*} can be achieved, up to an additive factor of $\frac{1}{2} \sum_{e \in E} w_e$, with arbitrary precision. The precision can be chosen by selecting V. For example, we may obtain peak age of at most $\overline{A}^{p*} + \frac{1}{2} \sum_{e \in E} w_e + \delta$ by setting $V = \frac{1}{2\delta} \sum_{e \in E} w_e$.

V. AGE-BASED POLICY

The virtual-queue based policy, in the previous section, relied on constructing virtual queues for each link, which would accurately weigh the importance of scheduling the link at time t. Instead of constructing artificial link weights to measure the importance of scheduling a link, we could use the actual age. We define age-based policies, which schedule links as a function of links' age $A_e(t)$. It is intuitive that scheduling a set of links with high age is better, at least in a myopic sense. We define an age-based policy to be one that schedules a set of links $m_t \in \mathcal{A}$ with maximum weight $\sum_{e \in m} w_e S_e(t) h(A_e(t))$, where $h(\cdot)$ is an increasing function.

Age-based Policy π_A The policy activates links $m_t \in \mathcal{A}$ in slot t given by: $m_t = \arg \max_{m \in \mathcal{A}} \sum_{e \in m} w_e S_e(t) h(A_e(t)),$ (27) for all $t \ge 1$.

The following Lemma gives an alternate characterization of average age, which helps us deduce a function h() to use in the age-based policy.

Lemma 4: Define $B_e(t) = A_e^2(t) + \beta A_e(t)$ for all t and $e \in E$, and any given $\beta \in \mathbb{R}$. Then, for any scheduling policy, and for all $e \in E$, we have

$$A_{t,e}^{\text{ave}} = \frac{1}{t} \left\{ \frac{1}{2} \sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau) B_e(\tau) + \frac{1}{2} B_e(t) \right\} + \frac{1-\beta}{2}.$$

Proof: The proof follows from Lemma 1 and Lemma 2.

Lemma 4 implies that average age minimization problem over $\pi \in \overline{\Pi}$ can be equivalently posed as minimizing

$$\limsup_{t \to \infty} \mathbb{E}\left[\frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{e \in E} w_e U_e(\tau) S_e(\tau) B_e(\tau) + \frac{1}{2} B_e(t)\right],$$
(28)

where $B_e(\tau) = A_e^2(\tau) + \beta A_e(\tau)$, for all $\tau \ge 0$, $e \in E$, and any chosen $\beta \in \mathbb{R}$. Since, age reduces to 1 after a link activation it makes intuitive sense to choose $\mathbf{U}(t)$ such that as

$$\mathbf{U}(t) = \arg\max_{\mathbf{U}'(t)} \sum_{e \in E} w_e U'_e(t) S_e(t) \left[A_e^2(t) + \beta A_e(t) \right], \quad (29)$$

in time slot t. This, in the least, should minimize age in the next slot. Motivated by this, we choose the function $h(\cdot)$ to be $h(x) = x^2 + \beta x$.

We next show that the average and peak age of policy π_A , with $h(x) = x^2 + \beta x$, is within a factor of 4 from the respective optimal values.

Theorem 3: The age-based policy π_A , with $h(x) = x^2 + \beta x$, is at most factor-4 peak and average age optimal, i.e.,

$$\overline{A}^{\text{ave}}(\pi_A) \le 4\overline{A}^{\text{ave}*} - c_1(\beta) \sum_{e \in E} w_e, \tag{30}$$

and

$$\overline{A}^{\mathbf{p}}(\pi_A) \le 4\overline{A}^{\mathbf{p}*} - c_2(\beta) \sum_{e \in E} w_e, \tag{31}$$

where
$$c_1(\beta) = \frac{20+8\beta-\beta^2}{8}$$
 and $c_2(\beta) = \frac{8+8\beta-\beta^2}{4}$.

Proof: To obtain the bound we define two functions f(t) and $\Delta(t)$, where f(t) is a representation of our objective function, which is age at time t, while $\Delta(t)$ is the drift of a certain Lyapunov function L(t). We obtain a bound on $\mathbb{E}[f(t) + \Delta(t)|\mathbf{A}(t)]$ where $\mathbf{A}(t)$ denote the vector of all $A_e(t)$. Telescoping $f(t) + \Delta(t)$ over T slots then yields the result. The detailed proof is given in Appendix G.

We note that $\beta \in \mathbb{R}$ can be chosen to improve the additive factor of optimality. The best bounds, for both peak and average age, occur when $\beta = 4$, for which both $c_1(\beta)$ and $c_2(\beta)$ are maximized. In the next section, we evaluate the age-based policy for different choices of β . We also compare it with the virtual-queue based policy π_Q from Section IV.

VI. SIMULATION RESULTS

Consider a network of N = 20 links, in which at most K links can be activated at any given time. We study the performance of our proposed scheduling policies for this network via simulations. We set $w_e = 1/N$ for all links e. We assume links to be either 'good', in which case $\gamma_e = \gamma_{good} = 0.9$, or 'bad' in which case $\gamma_e = \gamma_{bad} = 0.1$. We use n_{bad} to denote the number of bad links in the network. We simulate the policies π_Q , π_A , and the optimal policy for the unknown channel case, proposed in [29], over 10^5 time slots.



Fig. 4. Network peak age \overline{A}^{p} for various policies as a function of K.



Fig. 5. Network average age \overline{A}^{ave} for various policies as a function of K.

In Figure 4 and 5, we plot the network peak and average age, namely \overline{A}^{p} and \overline{A}^{ave} , as a function of K. Here, we have chosen the parameters V = 1 for the virtual-queue policy π_Q , and $\beta = 1$ for the age-based policy π_A . Also, $n_{bad} = 5$. Also plotted in Figures 4 and 5, is the case when the channel state is not observed, i.e., scheduling decisions are made only using history $\mathcal{H}(t)$. We plot the peak age optimal policy π_C of [29], while in Figure 5, we also plot a lower-bound on average age that can be achieved by any such policy [29], since π_C is not average age optimal.

We observe that the gap between the optimal policy π_C in the unknown channel case and policies π_Q , π_A of the known channel case is large when K is small, and diminishes as K increases. Smaller K implies more network interference, as fewer links can be activated simultaneously. This shows that there is a significant utility, in terms of age reduction, in knowing the channel state, especially when the network suffers from a lot of interference.

In Figure 6 and 7 we plot the network peak and average age as a function of the fraction of nodes with bad channel, namely $\theta = \frac{n_{\text{bad}}}{N}$. We observe that the gap between the optimal policy π_C in the unknown channel state case, and our policies π_Q and π_A of the known channel case, increases as the fraction θ increases. This indicates that if the channel statistics of the network are poor then there is a significant utility, in terms of age reduction, in knowing the channel state information. For example, when all channels are 'bad', i.e. $\theta = 1$, the gap is as large as 4 fold.

We also observe that the peak and average age of the virtualqueue based policy π_Q and the age-based policy π_A nearly coincide. The two scheduling policies are quite distinct in their construction, and there is no reason to expect a similar performance, unless they are both near optimal.



Fig. 6. Network peak age \overline{A}^{p} for various policies as a function of θ .



Fig. 7. Network average age \overline{A}^{ave} for various policies as a function of θ .



Fig. 8. Network peak age $A_t^p(\pi_Q)$ computed till time t for the virtual-queue policy π_Q for V = 0.1 and V = 100. Also plotted is the peak age $\overline{A}^p(\pi_Q)$ achieved over a much larger time horizon.

From Theorem 2 we know that the virtual-queue based policy is nearly peak age optimal, barring an additive factor of $\frac{1}{2} + \frac{1}{2V}$. This factor equals 1 in the simulation results, as we have chosen V = 1. Figures 4 and 6 show that the age-based scheduling policy π_A also attains approximately the same peak age. This implies that the age-based policy π_A is also near peak age optimal. We cannot make a similar claim about the average age performance of the two policies. We can at best claim, using Theorem 3, that the average age performance of the virtual-queue based policy is also about factor-4 away from the optimal.

A. Choice of Parameters V and β

We now analyze performance of our proposed policies π_Q and π_A over the choice of parameters V and β , respectively. Here, we set K = 5 and the number of 'bad' channels also to be $n_{\text{bad}} = 5$. For the virtual-queue based policy π_Q , we observe that the parameter V has nearly no effect on convergence time of the algorithm. To illustrate this,



Fig. 9. Network average and peak age, namely $\overline{A}^{ave}(\pi_A)$ and $\overline{A}^p(\pi_A)$, as a function of parameter β .

in Figure 8, we plot the network peak age $\overline{A}^{p}(\pi_{Q})$ computed over the first t time slots, for two different values of V = 0.1and V = 100. We observe that the peak age measured over the first t slots converged to the peak age $\overline{A}^{p}(\pi_{Q})$ at nearly the same time.

For the age-based policy π_A , we again observe no difference in convergence time with respect to β . Theorem 3 guarantees bounds for any $\beta \in \mathbb{R}$. However, in Figure 9, we observe that the peak and average age achieved by π_A gets worse as β becomes negative. This is because $c_1(\beta)$ and $c_2(\beta)$ in Theorem 3 are large and negative when $\beta < 0$.

VII. CONCLUSION

We considered the problem of age minimization for a singlehop wireless network under general interference constraints and time varying channels. We first argued that the knowledge of channel state information can greatly improve the network's age. We proposed a virtual-queue based policy and an agebased policy, in which the scheduler uses the current channel state information, to minimize age. We proved that the virtualqueue based policy is nearly peak age optimal, up to a constant additive factor, and that the age-based policy is at most a factor 4 away from age optimality. In comparison to the optimal scheduling policies, which do not use the channel state information, we demonstrate several fold improvement in age performance. This, therefore, establishes the utility in obtaining or using the channel state information in scheduling to minimize age. Our proofs relied on the use of age conservation law and age-square conservation law, which were derived under greater generality here, and may be of independent interest.

APPENDIX

A. Proof of Lemma 1

Age evolution for link e can be written as

$$A_e(t+1) = 1 + A_e(t) - U_e(t)S_e(t)A_e(t),$$
 (32)
r all t. As a result, we have

for all t. As a result, we have $A_{t}(t) = A_{t}(0) = \sum_{t=1}^{t-1} (A_{t}(t))$

$$A_{e}(t) - A_{e}(0) = \sum_{\tau=0}^{\tau=0} \left(A_{e}(\tau+1) - A_{e}(\tau) \right),$$

$$= \sum_{\tau=0}^{t-1} \left(1 - U_{e}(\tau) S_{e}(\tau) A_{e}(\tau) \right),$$

$$= t - \sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}(\tau).$$
(33)

Substituting $A_e(0) = 0$, we obtain the result.

B. Proof of Lemma 2

We know that the age of link *e* evolves as

$$A_e(t+1) = 1 + A_e(t) - U_e(t)S_e(t)A_e(t), \qquad (34)$$

for all t. Squaring this we obtain

$$A_e^2(t+1) = 1 + A_e^2(t) + U_e^2(t)S_e^2(t)A_e^2(t) + 2A_e(t) - 2U_e(t)S_e(t)A_e^2(t) - 2U_e(t)S_e(t)A_e(t).$$
(35)

Since $U_e(t)S_e(t) \in \{0,1\}$, we have $U_e^2(t)S_e^2(t) = U_e(t)S_e(t)$. Substituting this in (35) we get

$$A_e^2(t+1) - A_e^2(t) = 1 + 2A_e(t) - U_e(t)S_e(t)A_e^2(t) - 2U_e(t)S_e(t)A_e(t), \quad (36)$$

for all t. Telescoping this over t time slots we get

$$A_{e}^{2}(t) - A_{e}^{2}(0) = \sum_{\tau=0}^{t-1} \left(A_{e}^{2}(\tau+1) - A_{e}^{2}(\tau) \right),$$

$$= t + 2 \sum_{\tau=0}^{t-1} A_{e}(\tau) - \sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}^{2}(\tau)$$

$$- 2 \sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}(\tau).$$
(37)

Applying Lemma 1, we obtain

$$\sum_{\tau=0}^{t} A_e(\tau) = \frac{t}{2} + \frac{1}{2} \sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau) A_e^2(\tau) + \frac{1}{2} A_e^2(t).$$
(38)

Dividing both sides by t yields the result.

C. Derivation of the Peak Age Minimization Problem Using Lemma 1, we first show that

$$\overline{A}_{e}^{\mathbf{p}} = \frac{1}{\liminf_{t \to \infty} \mathbb{E}\left[\frac{1}{t} \sum_{\tau=0}^{t} \sum_{e \in E} U_{e}(\tau) S_{e}(\tau)\right]}, \quad (39)$$

for every $e \in E$. To see this, note that the peak age of link e is given by

$$\overline{A}_{e}^{\mathbf{p}} = \limsup_{t \to \infty} \frac{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}(\tau) + A_{e}(t)\right]}{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) + 1\right]},$$
$$= \limsup_{t \to \infty} \frac{t}{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) + 1\right]},$$
$$= \frac{1}{\liminf_{t \to \infty} \mathbb{E}\left[\frac{1}{t} \sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau)\right]}, \qquad (40)$$

where the second equality follows from Lemma 1. Since $\overline{A}^{p}(\pi) = \sum_{e \in E} w_e \overline{A}_e^{p}(\pi)$, the peak age minimization problem $\min_{\pi \in \overline{\Pi}} \overline{A}^{p}(\pi)$ can be written as

$$\underset{\pi \in \overline{\Pi}}{\text{Minimize}} \sum_{e \in E} \frac{w_e}{\liminf_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau)}.$$
 (41)

Using auxiliary variables α_e , this can be written as (18).

D. Proof of Lemma 3

Using Cauchy-Schwartz inequality we obtain

$$\begin{pmatrix} \sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau) A_e(\tau) + A_e(t) \\ \leq \left(\sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau) + 1 \right) \\ \times \left(\sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau) A_e^2(\tau) + A_e^2(t) \right),$$

since $U_e^2(\tau)S_e^2(\tau) = U_e(\tau)S_e(\tau)$ as $U_e(\tau)S_e(\tau) \in \{0,1\}$. Applying Lemma 1, we obtain

$$A_{t,e}^{p} = \frac{\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}(\tau) + A_{e}(t)}{\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) + 1} \\ \leq \frac{1}{t} \left(\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}^{2}(\tau) + A_{e}^{2}(t) \right), \quad (42)$$

which is equal to $2A_{e,t}^{ave} - 1$ by Lemma 2. Summing this over $e \in E$, and making use of the normalizing assumption $\sum_{e \in E} w_e = 1$, yields (16). Applying $\limsup x t \to +\infty$ on both sides of (16), yields (17).

E. Proof of Theorem 1

Let π be a S-only policy such that $\pi \in \overline{\Pi}$. Since, the channel process $\{\mathbf{S}(t)\}_{t\geq 0}$ is i.i.d. across time t, the process $\{U_e(t)S_e(t)\}_{t\geq 0}$ is also i.i.d. across t for policy π , as $\mathbf{U}(t)$ is entirely determined by $\mathbf{S}(t)$. Therefore, we have $\alpha_e = \mathbb{E}\left[U_e(t)S_e(t)\right]$ for all $t \geq 0$ and $e \in E$. Using Lemma 1 and definition of peak age we have

$$\overline{A}_{e}^{p} = \frac{1}{\liminf_{t \to \infty} \frac{1}{t} \mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau)\right]},$$
$$= \begin{cases} \frac{1}{\alpha_{e}} & \text{if } \alpha_{e} > 0\\ +\infty & \text{if } \alpha_{e} = 0, \end{cases}$$
(43)

for all $e \in E$. Thus, the problem of peak age minimization over the space of all S-only policies is equivalent to

$$\begin{array}{l} \underset{\alpha}{\text{Minimize }} \sum_{e \in E} \frac{w_e}{\alpha_e}, \\ \text{subject to } \alpha \in \Lambda_{\mathcal{S}}\left(\gamma\right). \end{array}$$
(44)

The optimality of S-only policies in solving (18) follows from Theorem 4.5 in [57]. This proves the result.

F. Proof of Theorem 2

Proof of Part A: Let $L(t) = \frac{1}{2} \sum_{e \in E} w_e Q_e^2(t)$ and $\Delta(t) = L(t+1) - L(t)$. Note that

$$Q_{e}^{2}(t+1) = \left[\max\{Q_{e}(t) + \alpha_{e}(t) - U_{e}(t)S_{e}(t), 1\}\right]^{2}, \\ \leq 1 + \left(Q_{e}(t) + \alpha_{e}(t) - U_{e}(t)S_{e}(t)\right)^{2}, \\ = 1 + \left(\alpha_{e}(t) - U_{e}(t)S_{e}(t)\right)^{2} + Q_{e}^{2}(t) \\ + 2Q_{e}(t)\left(\alpha_{e}(t) - U_{e}(t)S_{e}(t)\right), \\ \leq 1 + V + Q_{e}^{2}(t) + 2Q_{e}(t)\left(\alpha_{e}(t) - U_{e}(t)S_{e}(t)\right),$$

$$(45)$$

where the last inequality follows from the fact that $\alpha_e(t) = \sqrt{\frac{V}{Q_e(t)}} \leq \sqrt{V}$ because $Q_e(t) \geq 1$ for all t. Using (45) we obtain

$$\Delta(t) \le \frac{1+V}{2} \sum_{e \in E} w_e + \sum_{e \in E} w_e Q_e(t) \left(\alpha_e(t) - U_e(t)S_e(t)\right),$$
(46)

for all t. We, therefore, have

$$\begin{split} Vg(\boldsymbol{\alpha}(t)) + \Delta(t) \\ &\leq V \sum_{e \in E} \frac{w_e}{\alpha_e(t)} + \frac{1+V}{2} \sum_{e \in E} w_e \\ &+ \sum_{e \in E} w_e Q_e(t) \left[\alpha_e(t) - U_e(t) S_e(t) \right]. \end{split}$$

Substituting $\alpha_e(t) = \sqrt{V/Q_e(t)}$, which minimizes the right hand side, gives

$$Vg(\boldsymbol{\alpha}(t)) + \Delta(t)$$

$$\leq \sum_{e \in E} 2w_e \sqrt{VQ_e(t)}$$

$$+ \frac{1+V}{2} \sum_{e \in E} w_e - \sum_{e \in E} w_e U_e(t) S_e(t) Q_e(t). \quad (47)$$

Policy π_Q minimizes the right hand side of (47) as it activates set m_t at t which maximizes $\sum_{e \in m_t} w_e S_e(t) Q_e(t)$. Therefore, we can upper bound the right-hand side of (47) by the S-only policy π^{ϵ} in Theorem 1:

$$Vg(\boldsymbol{\alpha}(t)) + \Delta(t)$$

$$\leq \sum_{e \in E} 2w_e \sqrt{VQ_e(t)}$$

$$+ \frac{1+V}{2} \sum_{e \in E} w_e - \sum_{e \in E} w_e U_e^{\pi^{\epsilon}}(t) S_e(t) Q_e(t).$$

Since $\alpha_e^{\epsilon} = \mathbb{E}[U_e^{\epsilon}(t)S_e(t)]$, taking conditional expectation in the above equation we get

$$\mathbb{E}\left[Vg(\boldsymbol{\alpha}(t)) + \Delta(t)|\mathbf{Q}(t)\right] \\
\leq \sum_{e \in E} 2w_e \sqrt{VQ_e(t)} \\
+ \frac{1+V}{2} \sum_{e \in E} w_e - \sum_{e \in E} w_e \alpha_e^{\epsilon} Q_e(t).$$
(48)

From Theorem 1, note that $\overline{A}^{p}(\pi^{\epsilon}) = \sum_{e \in E} \frac{w_{e}}{\alpha_{e}^{\epsilon}} \leq \overline{A}^{p*} + \epsilon$. Substituting this in (48), we can write it as

$$\mathbb{E}\left[Vg(\boldsymbol{\alpha}(t)) + \Delta(t) | \mathbf{Q}(t)\right] \\ \leq V\overline{A}^{\mathsf{p}*} + \frac{1+V}{2} \sum_{e \in E} w_e + V\epsilon \\ -\sum_{e \in E} w_e \alpha_e^{\epsilon} \left[\sqrt{Q_e(t)} - \frac{\sqrt{V}}{\alpha_e^{\epsilon}}\right]^2.$$
(49)

Ignoring the last term in (49), we get

$$\mathbb{E}\left[Vg(\boldsymbol{\alpha}(t)) + \Delta(t) | \mathbf{Q}(t)\right] \le V\overline{A}^{\mathsf{p}*} + \frac{1+V}{2} \sum_{e \in E} w_e + V\epsilon.$$
(50)

Note that no other term depends on π^{ϵ} , except the last term $V\epsilon$. Since we could have chosen any $\epsilon > 0$ to arrive at (50), we must have

$$\mathbb{E}\left[Vg(\boldsymbol{\alpha}(t)) + \Delta(t)|\mathbf{Q}(t)\right] \le V\overline{A}^{\mathsf{p}*} + \frac{1+V}{2}\sum_{e \in E} w_e.$$
 (51)

Now, summing both sides of (51) over the first t time slots we obtain

$$\mathbb{E}\left[V\sum_{\tau=0}^{t-1}g(\boldsymbol{\alpha}(t))\right] + \mathbb{E}\left[L(t) - L(0)\right]$$
$$\leq t\left[V\overline{A}^{\mathsf{p}*} + \frac{1+V}{2}\sum_{e\in E}w_e\right].$$

Since $L(t) \ge 0$, we have

$$\mathbb{E}\left[V\sum_{\tau=0}^{t-1}g(\boldsymbol{\alpha}(t))\right] \leq \mathbb{E}\left[V\sum_{\tau=0}^{t-1}g(\boldsymbol{\alpha}(t))\right] + \mathbb{E}\left[L(t)\right],\\ \leq t\left[V\overline{A}^{\mathsf{p}*} + \frac{1+V}{2}\sum_{e\in E}w_e\right] + \mathbb{E}\left[L(0)\right]$$

Diving by t and taking the limit we get

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E} \left[\sum_{\tau=0}^{t-1} g(\boldsymbol{\alpha}(t)) \right] \leq \overline{A}^{\mathsf{p}*} + \frac{1}{2} \sum_{e \in E} w_e + \frac{1}{2V} \sum_{e \in E} w_e.$$
(52)

Since g is convex, we have $g(\overline{\alpha}(t)) \leq \frac{1}{t} \sum_{\tau=0}^{t-1} g(\alpha(t))$ from Jensen's inequality [58]. Substituting this in (52) yields the result.

Proof of Part B: Since $Vg(\alpha(t)) \ge 0$, from (51) we obtain

$$\mathbb{E}\left[\Delta(t)\right] \le V\left[\overline{A}^{\mathsf{p}*} + \frac{1}{2}\sum_{e \in E} w_e\right] + \frac{1}{2}\sum_{e \in E} w_e.$$
(53)

Summing this over t time slots we get

$$\frac{1}{t}\mathbb{E}\left[L(t)\right] \le \frac{1}{t}\mathbb{E}\left[L(0)\right] + V\left[\overline{A}^{\mathsf{p}*} + \frac{1}{2}\sum_{e\in E}w_e\right] + \frac{1}{2}\sum_{e\in E}w_e.$$
(54)

This implies,

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E}\left[L(t)\right] \le B,\tag{55}$$

where $B = V \left[\overline{A}^{p*} + \frac{1}{2} \sum_{e \in E} w_e\right] + \frac{1}{2} \sum_{e \in E} w_e$. Now, since $L(t) = \frac{1}{2} \sum_{e \in E} w_e Q_e^2(t)$, (55) implies

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E} \left[Q_e^2(t) \right] \le B, \tag{56}$$

and as a consequence $\limsup_{t\to\infty} \frac{1}{\sqrt{t}} \mathbb{E}[Q_e(t)] \leq B$, for all $e \in E$, since $\mathbb{E}[Q_e(t)]^2 \leq \mathbb{E}[Q_e^2(t)]$. This implies

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E}\left[Q_e(t)\right] = 0,\tag{57}$$

for all $e \in E$.

Proof of Part C: The queue evolution equation implies

$$Q_e(\tau+1) \ge Q_e(\tau) + \alpha_e(\tau) - U_e(\tau)S_e(\tau), \tag{58}$$

for any $\tau \ge 0$. Summing this over t times slots yields

$$\overline{\alpha}_{e}(t) + \frac{1}{t}Q_{e}(0) \le \frac{1}{t}\sum_{\tau=0}^{t-1}U_{e}(\tau)S_{e}(\tau) + \frac{1}{t}Q_{e}(t), \quad (59)$$

for all $t \ge 0$. Since $Q_e(t)$ is mean rate stable, taking expected value of (59) and limiting as $t \to \infty$ we obtain

$$\liminf_{t \to \infty} \mathbb{E}\left[\overline{\alpha}_e(t)\right] \le \liminf_{t \to \infty} \frac{1}{t} \mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau) S_e(\tau)\right].$$
 (60)

Since g is a continuous, decreasing function in each α_e we have

$$\overline{A}^{p}(\pi_{Q}) = \sum_{e \in E} \frac{w_{e}}{\liminf_{t \to \infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} U_{e}(t)S_{e}(t)\right]},$$

$$\leq \sum_{e \in E} \frac{w_{e}}{\liminf_{t \to \infty} \mathbb{E}\left[\overline{\alpha}_{e}(t)\right]},$$

$$= \limsup_{t \to \infty} \sum_{e \in E} \frac{w_{e}}{\mathbb{E}\left[\overline{\alpha}_{e}(t)\right]},$$

$$\leq \limsup_{t \to \infty} \mathbb{E}\left[\sum_{e \in E} \frac{w_{e}}{\overline{\alpha}_{e}(t)}\right] = \limsup_{t \to \infty} \mathbb{E}\left[g\left(\overline{\alpha}(t)\right)\right],$$
(61)

where the first equality follows from Lemma 1 and (3), the second inequality follows from (60), while the last inequality follows from Jensen's inequality [59] and definition of $g(\alpha)$.

G. Proof of Theorem 3

Define $L(t)=\frac{1}{2}\sum_{e\in E}w_eA_e^2(t),~\Delta(t)=L(t+1)-L(t),$ and

$$f(t) = \left(1 - \beta \frac{(1-V)}{2}\right) \sum_{e \in E} w_e U_e(t) S_e(t) A_e(t) + \frac{V}{2} \sum_{e \in E} w_e U_e(t) S_e(t) A_e^2(t), \quad (62)$$

for 0 < V < 1, $\beta \in \mathbb{R}$, and all $t \ge 0$. Using age evolution equation $A_e(t+1) = 1 + A_e(t) - U_e(t)S_e(t)A_e(t)$, we obtain

$$\Delta(t) = \frac{1}{2} \sum_{e \in E} w_e + \sum_{e \in E} w_e A_e(t) - \sum_{e \in E} w_e U_e(t) S_e(t) A_e(t) - \frac{1}{2} \sum_{e \in E} w_e U_e(t) S_e(t) A_e^2(t).$$
(63)

Summing (62) and (63) we get

$$f(t) + \Delta(t) = \frac{1}{2} \sum_{e \in E} w_e + \sum_{e \in E} w_e A_e(t) - \frac{(1-V)}{2} \sum_{e \in E} w_e U_e(t) S_e(t) \left[A_e^2(t) + \beta A_e(t)\right].$$
 (64)

Policy π_A chooses $\mathbf{U}(t)$ that maximizes

$$\sum_{e \in E} w_e U_e(t) S_e(t) \left[A_e^2(t) + \beta A_e(t) \right], \tag{65}$$

and thus, it minimizes the right-hand side in (64). Therefore, for any other policy π we must have

$$f(t) + \Delta(t) \\\leq \frac{1}{2} \sum_{e \in E} w_e + \sum_{e \in E} w_e A_e(t) \\- \frac{(1-V)}{2} \sum_{e \in E} w_e U_e^{\pi}(t) S_e(t) \left[A_e^2(t) + \beta A_e(t) \right], \quad (66)$$

where $\mathbf{U}^{\pi}(t)$ denotes the action of policy π at time t. Take an $\epsilon > 0$ and substitute the nearly peak age optimal S-only policy π^{ϵ} from Theorem 1 for π in (66). This gives the bound

$$\mathbb{E}\left[f(t) + \Delta(t) \middle| \mathbf{A}(t)\right] \leq \frac{1}{2} \sum_{e \in E} w_e + \sum_{e \in E} w_e A_e(t) - \frac{(1-V)}{2} \sum_{e \in E} w_e \alpha_e^{\epsilon} \left[A_e^2(t) + \beta A_e(t)\right], \quad (67)$$

since $\alpha_e^{\epsilon} = \mathbb{E}\left[U_e^{\pi^{\epsilon}}(t)S_e(t)\right]$ and $U_e^{\pi^{\epsilon}}(t), S_e(t)$ are independent of $A_e(t)$ as π^{ϵ} is a S-only policy. We can further re-write (67) as

$$\mathbb{E}\left[f(t) + \Delta(t) \middle| \mathbf{A}(t)\right] \leq \frac{1}{2} \sum_{e \in E} w_e + \frac{1-V}{2} \sum_{e \in E} w_e \alpha_e^{\epsilon} \left[\frac{\beta^2}{4} + \frac{(1-V)^{-2}}{(\alpha_e^{\epsilon})^2} - \frac{1}{1-V} \frac{\beta}{\alpha_e^{\epsilon}}\right] - \frac{(1-V)}{2} \sum_{e \in E} w_e \alpha_e^{\epsilon} \left[A_e(t) + \frac{\beta}{2} - \frac{(1-V)^{-1}}{\alpha_e^{\epsilon}}\right]^2.$$
(68)

Ignoring the last term, since it is negative, and using the fact that $\alpha_e^{\epsilon} \leq 1$ we have

$$\mathbb{E}\left[f(t) + \Delta(t)\right] \le \frac{(1-V)^{-1}}{2} \sum_{e \in E} \frac{w_e}{\alpha_e^{\epsilon}} + \theta \sum_{e \in E} w_e, \quad (69)$$

where $\theta = \frac{1-\beta}{2} + \frac{(1-V)}{2}\frac{\beta^2}{4}$. Summing this over t time slots we obtain

$$\mathbb{E}\left[\sum_{\tau=0}^{t-1} f(\tau)\right] + \mathbb{E}\left[L(t) - L(0)\right]$$

$$\leq t \left[\frac{(1-V)^{-1}}{2} \sum_{e \in E} \frac{w_e}{\alpha_e^e} + \theta \sum_{e \in E} w_e\right]. \quad (70)$$

Since $L(t) \ge 0$ for all t, we have

$$\mathbb{E}\left[\sum_{\tau=0}^{t-1} f(\tau)\right] \leq \mathbb{E}\left[\sum_{\tau=0}^{t-1} f(\tau)\right] + \mathbb{E}\left[L(t)\right],$$
$$\leq t\left[\frac{(1-V)^{-1}}{2}\sum_{e\in E}\frac{w_e}{\alpha_e^{\epsilon}} + \theta\sum_{e\in E}w_e\right] + \mathbb{E}\left[L(0)\right].$$

Dividing this by t and taking the limit we obtain

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E}\left[\sum_{\tau=0}^{t-1} f(\tau)\right] \le \frac{(1-V)^{-1}}{2} \sum_{e \in E} \frac{w_e}{\alpha_e^{\epsilon}} + \theta \sum_{e \in E} w_e.$$
(71)

Note that $\overline{A}^{p}(\pi^{\epsilon}) = \sum_{e \in E} \frac{w_{e}}{\alpha_{e}^{e}} \leq \overline{A}^{p^{*}} + \epsilon$, by Theorem 1. Substituting this in (71) we get

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E}\left[\sum_{\tau=0}^{t-1} f(\tau)\right] \leq \frac{(1-V)^{-1}}{2} \overline{A}^{\mathsf{p}*} + \theta \sum_{e \in E} w_e + \frac{(1-V)^{-1}}{2} \epsilon.$$
(72)

Note that in (72), only the last term $\frac{(1-V)^{-1}}{2}\epsilon$ depends on π^{ϵ} . Since we could have chosen any $\epsilon > 0$ to arrive at (72), we have

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E}\left[\sum_{\tau=0}^{t-1} f(\tau)\right] \le \frac{(1-V)^{-1}}{2} \overline{A}^{\mathsf{p}*} + \theta \sum_{e \in E} w_e.$$
(73)

Now, from Lemma 3, we know that $\overline{A}^{p*} \leq 2\overline{A}^{ave*} - \sum_{e \in E} w_e$. Substituting this in (73) we get

$$\limsup_{t \to \infty} \mathbb{E}\left[\frac{1}{t} \sum_{\tau=0}^{t-1} f(\tau)\right] \leq \frac{1}{(1-V)} \overline{A}^{\text{ave*}} + \left(\theta - \frac{1}{2(1-V)}\right) \sum_{e \in E} w_e.$$
(74)

Assuming that $\mathbb{E}\left[A_e^2(t)\right]$ is uniformly bounded for all t, we can make use of Lemma 1 and 2 to compute $\limsup_{t\to\infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} f(\tau)\right]$. This gives us

$$\limsup_{t \to \infty} \mathbb{E} \left[\frac{1}{t} \sum_{\tau=0}^{t-1} f(\tau) \right]$$
$$= \sum_{e \in E} w_e + V \overline{A}^{\text{ave}}(\pi_A)$$
$$- \frac{\beta(1-V) + V(1-\beta)}{2} \sum_{e \in E} w_e.$$
(75)

Substituting this in (74) we get

$$\overline{A}^{\text{ave}}(\pi_A) \le \frac{1}{V(1-V)} \overline{A}^{\text{ave*}} - \kappa \sum_{e \in E} w_e, \qquad (76)$$

where κ is given by

$$\kappa = \frac{1}{V} + \frac{1}{2V(1-V)} - \frac{\beta(1-V) + V(1-\beta)}{2V} - \frac{\theta}{V}.$$
 (77)

Substituting V = 1/2 gives (30).

In order to obtain (31), first substitute $\overline{A}^{p}(\pi_{A}) \leq 2\overline{A}^{ave}(\pi_{A}) - \sum_{e \in E} w_{e}$ from Lemma 3 in (75) to get

$$\limsup_{t \to \infty} \frac{1}{t} \mathbb{E} \left[\sum_{\tau=0}^{t-1} f(\tau) \right] \ge \sum_{e \in E} w_e + \frac{V}{2} \overline{A}^{\mathsf{p}}(\pi_A) - \frac{\beta(1-2V)}{2} \sum_{e \in E} w_e.$$
(78)

Combining (78) and (73), and setting V = 1/2, we get the result in (31).

It suffices to argue that the mean $\mathbb{E}\left[A_e^2(t)\right]$ is uniformly bounded for all t. Define a Lyapunov function

 $\tilde{L}(t) = \frac{1}{2} \sum_{e \in E} w_e (A_e(t) + \beta/2 - 1)^2$, and the corresponding drift $\tilde{\Delta}(t) = \tilde{L}(t+1) - \tilde{L}(t)$. Then using the same arguments as in (68) we can obtain

$$\mathbb{E}\left[\tilde{\Delta(t)}|\mathbf{A}(t)\right] \le B_1 - \sum_{e \in E} B_{2,e} \left(A_e(t) + c_e\right)^2, \quad (79)$$

for constants B_1 , $B_{2,e}$, and c_e . Foster-Lyapunov theorem [60, Chap. 6] then implies that the process $\{\mathbf{A}^2(t)\}_t$ is positive recurrent, and that $\mathbb{E}[A_e^2(t)]$ is uniformly bounded.

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