Capacity and Delay Scaling for Broadcast Transmission in Highly Mobile Wireless Networks

Rajat Talak, Sertac Karaman, and Eytan Modiano, *Fellow, IEEE*

Abstract—Futuristic communication network formed by autonomously operated, unmanned aerial vehicles, has piqued researchers interests in highly mobile wireless networks. Exchanging safety critical information, with low latency and high throughput, in such systems is of paramount importance. We study the broadcast capacity and minimum delay scaling laws for such highly mobile wireless networks, in which each node has to disseminate packets to all other nodes in the network. In particular, we consider a cell partitioned network under an IID mobility model, in which each node chooses a new position at random, every time slot. We derive scaling laws for broadcast capacity and minimum delay as a function of the network size. We propose a simple first-come-first-serve flooding scheme, which nearly achieve both capacity and minimum delay scaling. Thus, in contrast to what has been speculated in the literature, we show that there is nearly no tradeoff between capacity and delay. Our results also show that high mobility does not improve broadcast capacity. Our analysis makes use of the theory of Markov Evolving Graphs (MEGs), and develops two new bounds on flooding time in MEGs by relaxing the previously required expander property assumption. Simulation results verify our analysis, and throw up interesting open problems.

Index Terms—Wireless networks, broadcast, throughput-delay tradeoff, flooding time, scaling laws, Markov evolving graphs

1 Introduction

Interest in mobile wireless networks has increased in recent years due to the emergence of unmanned aerial vehicle (UAV) networks. Dense networks of small UAVs are being used in a wide range of applications including product delivery, disaster management, environmental monitoring, surveillance, and more [1], [2], [3], [4], [5], [6], [7]. We envision a futuristic scenario, in which UAVs belonging to multiple vendors and users will share a common airspace. In which case, the need for exchanging safety critical information, with low latency and high throughput, becomes even more prominent.

For such non-cooperative network of UAVs, an important communication operation that needs to be performed is that of all-to-all broadcast, where in, each vehicle or node broadcasts its current state or location information to all other vehicles in its vicinity. We aim to analyze capacity and delay, and propose policies, for all-to-all broadcast operation in a network consisting of mobile UAVs.

Network of UAVs is different from other mobile networks such as the traditional mobile ad-hoc network (MANET) and vehicular network (VANET) [5]. Speed of vehicles in UAV networks, for example, can be much larger: speeds in MANET and VANETs are expected to be around 2 and 20-30 m/s, respectively, the speeds in UAV networks can be as high as 100 m/s [5]. So, if several UAVs are conducting separate operations in a 500 m $\times$ 500 m region, then the UAVs can get from one end of the region to the other in a matter of a few seconds. As a result, the network topology changes rapidly, and the communication network formed by UAVs is much more likely to get partitioned frequently. The communication and traffic sessions, in this case the broadcast sessions, must run over such intermittently connected, rapidly changing, network topology.

Various models have been considered in the literature to model such mobile nodes. IID mobility model, is the simplest one, in which, the node choose a new position in the region, independently of other nodes, in each time-slot. In Markov mobility model, the region is partitioned into cells, and nodes choose to move to neighboring cells, according to a given transition probability. In random waypoint model (RWP), a node selects a new location, randomly in the region, and moves towards it at a given speed. Mobility of a node has also been modeled as a Brownian motion and Levy processes [8].

Among all these models, the IID mobility model results in fastest mobility. This mobility model was used in [9], [10] to capture the impact of high mobility, and the resulting intermittent network connectivity, on throughput and delay. Moreover, this model serves as a good, first approximation model for UAV networks where rapid mobility and intermittent connectivity are common. It also resembles the RWP and Brownian Motion models at high speeds. We therefore consider the IID mobility model in this work to study a highly mobile wireless network.

We consider a cell partitioned network with N nodes, shown in Fig. 1, in which a unit square is partitioned into C cells. Due to interference, only a single packet transmission can take place in the cell at a given time, and all other nodes in the cell can correctly receive the packet. Different cells can have simultaneous packet transmissions. This simple model captures the essential features of interference and helps obtain key insights into its impact on throughput and

• *The authors are with the Laboratory for Information and Decision Systems (LIDS), Massachusetts Institute of Technology (MIT), Cambridge, MA 02139. E-mail: {talak, sertac, modiano}@mit.edu.*

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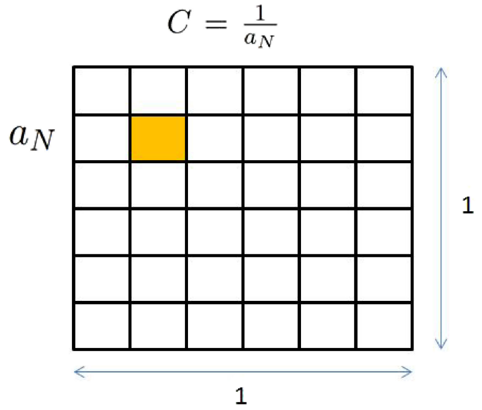

Fig. 1. Network partitioned into $C = \frac{1}{a_N}$ cells. Each cell of area a_N.

delay [9], [11], [12]. Nodes move according to the IID mobility model. At the end of every time slot, each node chooses a new cell uniformly at random.

Analyzing capacity and delay for finite network is known to be a hard problem, and in order to gain insights, we study the network as N scales to ∞ [8]. We study all-to-all broadcast capacity and delay scaling as a function of node density. Here, capacity is defined as the maximum rate at which each node can transmit packets to all other nodes in the system and delay as the average time taken by a packet to reach every node in the system. We say that a network is dense if the number of vehicles or nodes per cell is increasing with N, and sparse otherwise. Thus, if the cell size grows as $cN^{-\alpha}$, for some $c > 0$, then the network is *dense* for $0 < \alpha < 1$ and *sparse* for $\alpha \geq 1$.

We show that as the network gets more dense the all-to-all broadcast capacity increases to reach a maximum scaling of $1/N$. Interestingly, delay decreases as the network gets denser. In fact, both, capacity and delay attain their best scaling in N when the cell size is just smaller than order $1/N$, i.e., when $\alpha = 1 - \epsilon$ for a small positive ϵ. We further note that the best per-node capacity scaling of $1/N$ is the same as that can be achieved in a static wireless network, thus, mobility does not improve network capacity. This is in contrast to the unicast case where it was shown in [13] that mobility improves capacity. Our scaling results are summarized in Table 1.

We propose a simple first-come-first-serve (FCFS) flooding scheme, and its variant the threshold-based flooding scheme, which achieve capacity scaling, up to a $\log N$ factor from the optimal when the network is sparse, and up to a $\log \log N$ factor from the optimal when the network is dense. Both the schemes also achieve the minimum delay scaling when the network is sparse, and up to a factor of $\log \log N$ from minimum delay when the network is dense. Thus, nearly optimal throughput and delay scaling is achieved simultaneously.

The IID mobility model was analyzed for unicast and multicast operations in [9] and [10], respectively, using standard probabilistic arguments. In contrast, we use the abstraction of Markov evolving graphs (MEG), and flooding time bounds for MEGs [14]. An MEG is a discrete time Markov chain with state space being a collection of graphs with N nodes. An MEG of the IID mobility model can be constructed by drawing an edge between two nodes in the same cell and viewing the network as a graph at each time step. Flooding time, is then, the time it takes for a single packet to reach all nodes from a single source node.

TABLE 1
Capacity and Average Delay

	Capacity	
	Upper bound (Theorem 1)	FCFS flooding (Equation (62))
Sparse: $\alpha \geq 1$	$\frac{1}{N^\alpha}$	$\frac{1}{N^\alpha}\frac{1}{\log N}$
Dense: $0 < \alpha < 1$	$\frac{1}{N}$	$\frac{1}{N}\frac{1}{\log \log N}$
	Average Delay	
	Lower bound (Theorem 2)	FCFS flooding (Equations (63) and (61))
Sparse: $\alpha \geq 1$	$N^{\alpha-1}\log N$	$N^{\alpha-1}\log N$
Dense: $0 < \alpha < 1$	1	$\log \log N$

A flooding time bound for MEGs was derived in [14]. It relied on an expander property which states that whenever m nodes have the packet then in the next slot at least km new nodes will receive the packet with high probability, for some $k > 0$. However, this strong requirement does not always hold. For example, when the IID mobility model is sparse, this expander property cannot be guaranteed. We derive two new bounds on flooding time in MEGs by relaxing the strong expander property requirements imposed in [14]. These new bounds are of independent theoretical interest. A part of this work first appeared in MobiHoc 2017 [15].

1.1 Previous Work

In [7], we considered the impact of wireless interference constraints on the ability to exchange timely control information in UAV networks. We showed that, in guaranteeing location awareness of other vehicles in the networks, wireless interference constraints can limit mobility of aerial vehicles in such networks. This result motivates us to study the delay and capacity scalings of all-to-all broadcast in mobile wireless networks.

Broadcast has been studied before in the contexts of disseminating data packets in wireless ad-hoc networks [16], [17], sensor information in sensor networks, and in exchanging intermediate variables in distributed computing [18]. Scaling laws for capacity and delay in wireless networks have received significant attention in the literature. Capacity scaling for unicast traffic, in which each node sends packets to only one other destination node, was analyzed in [19], [20]. It was shown that the capacity scales as $1/\sqrt{N \log N}$ with increasing N. Minimum delay scaling for the static unicast network was analyzed in [11], where it was also shown that it is not possible to simultaneously achieve minimum delay and capacity. This implied a tradeoff between capacity and delay. In [13], it was shown that if the nodes were mobile, then a constant per node capacity that does not diminish with N can be achieved. The seminal works of [19] and [13] led to the analysis of capacity and delay scaling under various mobility models including IID [9], Markov [11], Brownian motion [21], and Random Waypoint [22]. Capacity-delay tradeoffs were observed in each of these settings.

Broadcast has been studied in static wireless networks in [16], [17], [23], [24]. It was shown that the per-node broadcast capacity scales as $1/N$ in static wireless networks [17]. However, to the best of our knowledge, optimal delay scalings for static broadcast has not been analyzed. In [10], the authors conjectured a capacity-delay tradeoff for multicast, and by implication for broadcast as a special case, under IID mobility. However, in this paper, we show that there is nearly no capacity-delay tradeoff for broadcast. In particular, we propose a scheme that (nearly) achieves both capacity and minimum delay, which is up to a $\log\log N$ factor when the network is dense and up a $\log N$ factor when the network is sparse. Moreover, we show that the capacity scaling does not improve with mobility, unlike in the unicast case [13].

Although, throughput and delay scalings have been investigated under various communication operations and mobility models for the past 15 years, the same problem under broadcast has not been thoroughly analyzed even for the simplest IID mobility model. In [10], delay bounds were obtained for multicast, however, these bounds are very weak when applied to the all-to-all broadcast operation. By using and extending the theory of MEGs developed in [14] we are able to obtain tight bounds on delay.

Flooding time bounds on MEG have been used for various network models in [14], [25], [26]. To the best of our knowledge, this is the first time that these techniques are being used in the mobility setting. Moreover, the new bounds derived in Section 3 could be of independent interests and can also be applied to models considered in [14], [25], [26].

1.2 Organization and Notations

The paper is organized as follows. In Section 2 we describe the system model, and derive bounds on capacity and minimum delay. In Section 3, we summarize the flooding time upper bound result of [14], and derive two new upper bounds on flooding time for MEGs. In Section 4, we apply these results to our setting and, in Section 5, we use it to propose and analyse the FCFS flooding scheme. We propose a single-hop scheme in Section 6 that achieves capacity for a sparse network. We conclude and discuss open problems in Section 8.

In this paper we make extensive use of order notation. For infinite sequences $\{a_N\}$ and $\{b_N\}$, $a_N = O(b_N)$ implies $\lim_{N\to\infty} \frac{a_N}{b_N} \leq c_1$ for some $c_1 > 0$ and $a_N = \Theta(b_N)$ implies $a_N = O(b_N)$ and $b_N = O(a_N)$. We write $a_N \leq_N b_N$ if there exists a $N_0 \geq 1$ such that for all $N \geq N_0$ we have $a_N \leq b_N$. Positive constants are denoted by $c_1, c_2, \ldots$.

2 System Model and Fundamental Limits

Consider the network of Fig. 1 with N nodes that are uniformly distributed over a unit square. The size of each cell is $a_N = \frac{1}{C} = cN^{-\alpha}$, for some $\alpha > 0$ and $c > 0$. We consider a slotted time system, with the duration of each slot normalized to unity. The duration of each slot is sufficient to complete the transmission of a single packet. We use the IID mobility model of [9] in which each node, at the end of every slot, chooses a new cell/location uniformly at random, and independent of other node's locations.

Packets arrive at each node according to a Poisson process, at rate λ. Note that the arrivals happen over continuous time, and therefore, two or more packets can arrive during a slot. We use the Poisson arrival assumption as it simplifies analysis, and can be relaxed in most of the paper.

We now obtain two fundamental limits on the system performance, namely, an upper-bound on rate λ and a lower-bound on achievable delay.

2.1 Capacity

Each node receives an inflow of packets at rate λ, and each of these packets have to be broadcast to all other nodes in the network. A communication scheme is said to achieve a rate of λ if at this arrival rate the average number of backlogged packets in the network does not increase to infinity. The capacity of the network is the maximum achievable rate. We start with a simple upper-bound on the capacity.

Theorem 1. *The achievable rate λ is bounded by*

$$\lambda \leq \frac{1}{2(N-1)}\left(1-(1-a_N)^{N-1}\right) \tag{1}$$

$$= \begin{cases} \Theta\left(\frac{1}{N^{\alpha}}\right) & \text{if } \alpha \geq 1 \text{ (sparse)} \\ \Theta\left(\frac{1}{N}\right) & \text{if } 0 < \alpha < 1 \text{ (dense)} \end{cases}. \tag{2}$$

Proof: Let λ be the rate achieved by a scheme. If $X_h(T)$ is the number of packets delivered to the destination in exactly h hops by time T then for an $\epsilon > 0$ we have

$$\frac{1}{T}\sum_{h\geq 1} X_h(T) > N(N-1)\lambda - \epsilon, \tag{3}$$

for all $T > T_\epsilon$, for some $T_\epsilon > 0$.

If $Z_i^k(t)$ is a binary random variable which equals 1 if there are k nodes in cell i in slot t then the total number of packet receptions by time T is at most $\sum_{i=1}^{C}\sum_{k=2}^{N}\sum_{t=1}^{T}(k-1)Z_i^k(t)$. Hence

$$\sum_{h\geq 1} hX_h(T) \leq \sum_{i=1}^{C}\sum_{k=2}^{N}\sum_{t=1}^{T}(k-1)Z_i^k(t). \tag{4}$$

Combining Equations (3) and (4) we obtain

$$\begin{aligned}\sum_{i=1}^{C}\sum_{k=2}^{N}\frac{1}{T}\sum_{t=1}^{T}(k-1)Z_i^k(t) &\geq \frac{1}{T}\sum_{h\geq 1} hX_h(T),\\ &= \frac{1}{T}X_1(T) + \frac{1}{T}\sum_{h\geq 2} hX_h(T),\\ &\geq \frac{1}{T}X_1(T) + \frac{2}{T}\sum_{h\geq 2} X_h(T).\end{aligned}$$

Using Equation (3) we obtain

$$\begin{aligned}\sum_{i=1}^{C}\sum_{k=2}^{N}\frac{1}{T}\sum_{t=1}^{T}(k-1)Z_i^k(t) &\geq \frac{1}{T}X_1(T)\\ &+ 2\left(N(N-1)\lambda - \epsilon - \frac{1}{T}X_1(T)\right).\end{aligned}$$

Taking $T \to +\infty$ we have

$$\begin{aligned}\sum_{i=1}^{C}\sum_{k=2}^{N}(k-1)p(k) &\geq Cp + 2(N(N-1)\lambda - \epsilon - Cp),\\ &= 2N(N-1) - 2\epsilon - Cp,\end{aligned} \tag{5}$$

where $p(k)$ is the probability that there are k nodes in a cell and p is the probability that there are at least two nodes in a cell; we use the fact that $\limsup_{T\to+\infty}\frac{X_1(T)}{T} \leq Cp$. Taking $\epsilon \to 0$, we obtain

$$2N(N-1)\lambda \leq Cp + C\sum_{k=2}^{N}(k-1)p(k). \tag{6}$$

Substituting $p(k) = \binom{n}{k} a_N^k (1-a_N)^{N-k}$ and computing the binomial sum we obtain

$$2N(N-1)\lambda \leq N\Big(1-(1-a_N)^{N-1}\Big). \tag{7}$$

Therefore

$$(N-1)\lambda \leq \frac{1}{2}\Big(1-(1-a_N)^{N-1}\Big), \tag{8}$$

$$= \frac{1}{2}\left(1-\left(1-\frac{c}{N^\alpha}\right)^{N-1}\right). \tag{9}$$

When $0 < \alpha < 1$, we have $N/N^\alpha \to \infty$. In which case

$$(N-1)\lambda \leq \frac{1}{2}\left(1-\left(1-\frac{c}{N^\alpha}\right)^{N-1}\right) = \Theta(1). \tag{10}$$

Hence, $\lambda = O(1/N)$. When $\alpha \geq 1$, either $N/N^\alpha \to 0$ or $N/N^\alpha \to c_1$ for some $c_1 > 0$. This implies

$$(N-1)\lambda \leq \frac{1}{2}\left(1-\left(1-\frac{c}{N^\alpha}\right)^{N-1}\right) = \Theta(N/N^\alpha). \tag{11}$$

Hence, $\lambda = O(1/N^\alpha)$. □

We first note that the proof does not use the assumption that the packet arrival processes are Poisson. Thus, the result holds for general arrival process of rate λ. Second, the obtained capacity bound, is in fact, achievable. As we shall see, a single-hop scheme proposed in Section 6 achieves this capacity when the network is sparse, and a FCFS flooding scheme in Section 5 achieves capacity, up to a $\log\log N$ factor, when the network is dense.

Typically, one expects to have larger broadcast capacity with increasing cell sizes, i.e., with decreasing α. A larger cell size implies more nodes in a given cell, and hence, more receptions per slot can occur by exploiting the broadcast nature of the wireless medium. Theorem 1, however, shows that the capacity remains constant at $\Theta(\frac{1}{N})$ for $0 < \alpha < 1$. This is because, larger cell sizes also result in fewer transmission opportunities in every slot due to interference. As a result, capacity remains constant when $0 < \alpha < 1$.

2.2 Minimum Delay

Another important performance measure is the delay. The delay of a packet is defined as the time from the arrival of the packet to the time the packet reaches all its $N-1$ destination nodes. The delay of a communication scheme is the average delay, averaged over all packets in the network. To obtain a lower-bound on the network's delay performance we define a *single packet flooding* scheme that transmits a single packet to all other nodes in the network. As we show later, this lower-bound provides a fundamental limit on delay.

Single packet flooding scheme: At the beginning of the first slot, only a single node has the packet.

1) In every cell, randomly select one packet carrying node to be the transmitter in that slot. If no such node exists in a cell no transmission occurs in that particular cell.
2) In each cell, the transmitter node (if present) transmits the packet to all other nodes in the cell.
3) If all nodes have the packet then terminate the process, otherwise repeat from step 1.

The single packet flooding scheme is clearly the fastest way to disseminate a packet to all nodes in the network. Hence, a lower bound is given by the time it takes for a single packet to reach all other nodes under the single packet flooding scheme.

We now provide a lower-bound the the average delay, by merely computing a lower-bound on the time it takes the single packet flooding scheme to run its course.

Theorem 2. *Any achievable average delay $\overline{D}$ is lower-bounded by*

$$\overline{D} \geq \begin{cases} \Theta(N^{\alpha-1}\log N) & \text{if } \alpha \geq 1 \text{ (sparse)} \\ \Theta(1) & \text{if } 0 < \alpha < 1 \text{ (dense)} \end{cases}. \tag{12}$$

Proof. As a lower-bound we compute the time it takes for the single packet flooding scheme to terminate. Let K_t denote the number of nodes that have the packet after t slots; where $K_1 = 1$. Let T_N be the flooding time, i.e., the first time when $K_t = N$. Let A_i, for $1 \leq i \leq K_t$, be the number of new nodes to which node i transmits the packet in slot $t+1$. We then have

$$K_{t+1} = K_t + \sum_{i=1}^{K_t} A_i. \tag{13}$$

Since $\mathbb{E}[A_i|K_t] \leq (N-1)a_N$, we have

$$\mathbb{E}[K_{t+1}|K_t] = \mathbb{E}\left[K_t + \sum_{i=1}^{K_t} A_i | K_t\right], \tag{14}$$

$$\leq K_t(1+(N-1)a_N), \tag{15}$$

for all $t \geq 1$. Applying this recursively, we obtain

$$\mathbb{E}[K_t] \leq (1+(N-1)a_N)^t. \tag{16}$$

Now, using Markov inequality we have

$$\mathbb{E}[T_N] \geq t\mathbb{P}[T_N > t]. \tag{17}$$

The event $\{T_N > t\}$ is same as $\{K_t < N\}$. Hence, we have

$$\mathbb{E}[T_N] \geq t\mathbb{P}[K_t < N], \tag{18}$$

$$= t(1-\mathbb{P}[K_t \geq N]), \tag{19}$$

$$\geq t\left(1-\frac{\mathbb{E}[K_t]}{N}\right), \tag{20}$$

where the last inequality follows from Markov inequality. Using Equation (16), we obtain

$$\mathbb{E}[T_N] \geq t\left(1-\frac{1}{N}(1+(N-1)a_N)^t\right), \tag{21}$$

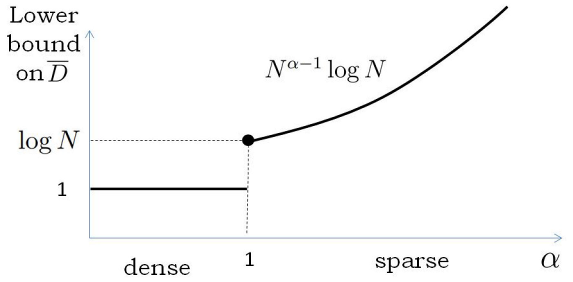

Fig. 2. Lower bound on achievable average delay $\overline{D}$ as a function of α.

for all $t \geq 1$. Since Equation (21) is a valid lower-bound for all values of $t \geq 1$, setting $t = \frac{1/2 \log N}{\log(1+(N-1)a_N)}$ for $\alpha \geq 1$ and $t = \frac{1/2 \log N^{\alpha}}{\log(1+(N-1)a_N)}$ for $0 < \alpha < 1$ yields the result. □

We first note that the proof of Theorem 2 did not use any property of the arrival process. It merely obtained a lower-bound on the single packet flooding time. Thus, the result in Theorem 2 does not depend on the assumption that the arrival process is Poisson.

In Fig. 2, we plot the lower-bound on average delay $\overline{D}$ as a function of α. We observe that as the network gets sparser the number of nodes receiving the flooded packet per cell decreases, thereby, increasing the broadcast delay. Thus, the lower-bound is a non-decreasing function of α. However, for $0 < \alpha < 1$ the delay bound is a constant $O(1)$, and remains unchanged. Clearly, if $C = 1$, i.e., if the entire network is a single cell, then the broadcast delay will be 1 as the packet can reach all other nodes in a single transmission.

In the next two sections we show that this lower-bound on average delay is in fact achievable, up to $\log\log N$ factor.

3 Flooding Time in Markov Evolving Graphs

In order to gain further insights into the flooding time of the packet flooding scheme we use the theory of Markov evolving graphs (MEG), to help us derive the necessary upper bound on the flooding time. We start with a brief introduction to MEG and a review of pertinent results.

Let $\mathcal{G}$ be a family of graphs with node set $[N] = \{1, 2, \ldots N\}$. The Markov chain $\mathcal{M} = (G_t)_{t\in\mathbb{N}}$, where $G_t \in \mathcal{G}$, with state space $\mathcal{G}$ is called a MEG. Note that $\mathcal{G}$ is a finite set. For our network model of Fig. 1, if we draw edge between i and j whenever both nodes i and j lie in the same cell, the resulting time evolving graph is an MEG. When the MEG has a unique stationary distribution we call it a stationary MEG.[1] In this work, we assume that a stationary MEG starts from its stationary distribution. The IID mobility model results in one such stationary MEG, as every graph formation can follow any other in $\mathcal{G}$. We now describe the single packet flooding scheme in MEG.

Single packet flooding for a MEG: In the first slot only a single node s has the packet, i.e. $I_1 = \{s\}$. Here, $I_t \subset [N]$ denotes the set of nodes that have the packet at time t. In every slot $t \geq 1$:

1) Identify the neighbors of I_t that are not in I_t

$$N(I_t) = \{\text{neighbours of } I_t \text{ in } G_t \backslash I_t\}. \tag{22}$$

2) Transmit the packet to each node in $N(I_t)$. We, thus, have

$$I_{t+1} = I_t \bigcup N(I_t). \tag{23}$$

3) If $I_t = [N]$ then stop, else start again from Step 1.

Let T_N be the *flooding time*, i.e., the time it takes for this process to terminate. Note that, this scheme reduces to the single packet flooding scheme of Section 2 for our network model. An upper bound on flooding time was derived in [14]. This bound depended on the MEG satisfying certain expander properties. We summarize this result in Theorem 3, and provide two new bounds on flooding time in Theorems 4 and 5.

The expander property of MEG is defined in terms of the expander property of a static graph [14].

Definition 1. *A graph $G = ([N], E)$ is said to be $([h_0, h_1], k)$-expander if for every $I \subset [N]$ such that $h_0 < |I| \leq h_1$ we have*

$$|N(I)| \geq k|I|, \tag{24}$$

where $N(I)$ is the set of all neighbours of nodes in I that are not already in I.

We now use this to define the expander property of MEG.

Definition 2. *Stationary MEG $\mathcal{M} = (G_t)_{t\in\mathbb{N}}$ is $([h_0, h_1], k)$-expander with probability p if*

$$\mathbb{P}[G_0 \text{ is } ([h_0, h_1], k)\text{−expander}] \geq p. \tag{25}$$

If the graph is $([h-1, h], k)$-expander then for notational simplicity we say that it is (h, k)-expander. To show that a stationary MEG is (h, k)-expander we have to evaluate the probability

$$\mathbb{P}\left[\bigcap_{|I|=h} \{|N(I)| \geq k|I|\}\right]. \tag{26}$$

Recall that for two sequences $\{a_N\}_{N\geq 1}$ and $\{b_N\}_{N\geq 1}$, we write $a_N \leq_N b_N$ if there exists a $N_0 \geq 1$ such that for all $N \geq N_0$ we have $a_N \leq b_N$. The following upper bound on flooding time was derived in [14].

Theorem 3. *[14] For a stationary MEG, if*

$$\mathbb{P}\left[\bigcap_{i=1}^{s}\{G_0 \text{ isan } ([h_{i-1}, h_i], k_i)\text{−expander}\}\right] \geq_N 1 - \frac{c_1}{N^2}, \tag{27}$$

for some $c_1 > 0$, $1 = h_0 \leq h_1 < h_2 < \cdots < h_s = \frac{N}{2}$, a non-increasing sequence $k_1 \geq k_2 \geq \cdots \geq k_s > 0$, and $s \in \{2, 3, \ldots \frac{N}{2}\}$ then the flooding time

$$T_N = O\left(\sum_{i=1}^{s} \frac{\log(h_i/h_{i-1})}{\log(1+k_i)}\right), \tag{28}$$

with probability at least $1 - \frac{c_2}{N}$ for some $c_2 > 0$.

1. Since the state space $\mathcal{G}$ is finite, it always has at least one stationary distribution.

A stationary MEG may not always satisfy the expander property required by Equation (27). In such a case, we provide the following two bounds for flooding time for a stationary MEG.

Theorem 4. *If for every $h \in 1, 2, \dots N-1$ and for all $I \subset [N]$ with $|I| = h$, there exists a function $p(h)$ such that $\mathbb{P}[N(I) \geq 1] \geq_N p(h) > 0$ then the flooding time*

$$T_N = O\left(\sum_{h=1}^{N-1} \frac{1}{p(h)}\right), \tag{29}$$

with probability at least $1 - e^{-c_1 N}$ for some $c_1 > 0$.

Proof. We denote $X \sim \mathrm{Geo}(p)$ when X is a geometrically distributed random variable with parameter p, that is, $\mathbb{P}[X = k] = p(1-p)^{k-1}$ for all $k \geq 1$. □

Note that if only a single packet transmission was to take place at the occurrences of the events $\{N(h) \geq 1\}$, the flooding time would be much larger. We now write this mathematically. For all $h \in \{1, 2, \dots N-1\}$, define X_h as follows: Let X_h be the time it takes for the event $\{N(h) \geq 1\}$ to occur. If for no time t, did we have $|I_t| = h$, let $X_h \sim \mathrm{Geo}(\mathbb{P}[N(h) \geq 1])$. We have

$$T_N \leq \sum_{h=1}^{N-1} X_h. \tag{30}$$

Since $\mathbb{P}[N(h) \geq 1] \geq_N p(h)$ we can construct random variables Z_h, on the same probability space, such that $Z_h \sim \mathrm{Geo}(p(h))$ and $X_h \leq_N Z_h$ a.s., for all h. This implies

$$T_N \leq \sum_{h=1}^{N-1} X_h \leq_N \sum_{h=1}^{N-1} Z_h. \tag{31}$$

Now, we use the following concentration bound:

Lemma 1. *Let $X_1, X_2, \dots X_n$ be independent geometrically distributed random variables with parameters $0 < p_1 \leq p_2 \leq \cdots \leq p_n$, i.e., $\mathbb{P}[X_i = t] = p_i(1-p_i)^{t-1}$ for all $t \geq 1$. Let $S_n = \sum_{i=1}^{n} X_i$ and*

$$\mu = \mathbb{E}[S_n] = \frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n}. \tag{32}$$

Then, for some $c \geq 2$,

$$\mathbb{P}[S_n > c(\mu + t)] \leq (1-p_1)^t \exp\{-(2c-3)n/4\}. \tag{33}$$

Proof. The proof is given in [27]. □

Using this concentration bound on $\{Z_1, \dots Z_{N-1}\}$ and substituting $t = \mu = \sum_{h=1}^{N-1} \frac{1}{p(h)}$ we obtain

$$\mathbb{P}\left[\sum_{h=1}^{N-1} Z_h > 2c_1\mu\right] \leq (1-p^*)^{\mu} \exp\left\{-\frac{2c_1 - 3}{4}(N-1)\right\}, \tag{34}$$

for some $c_1 \geq 2$, where $p^* = \min_{h \in \{1,2,\dots N-1\}} p(h)$. Note that $(1-p^*)^{\mu} \leq 1$. We, thus, have

$$\mathbb{P}\left[\sum_{h=1}^{N-1} Z_h > 2c_1\mu\right] \leq \exp\left\{-\frac{2c_1 - 3}{4}(N-1)\right\} \tag{35}$$

$$= \Theta(\exp\{-c_2 N\}), \tag{36}$$

for some positive constant c_2. From Equations (31) and (36) we have

$$\mathbb{P}\left[T_N \leq 2c_1 \sum_{h=1}^{N-1} \frac{1}{p(h)}\right] \geq_N 1 - \exp\{-c_2 N\}. \tag{37}$$

Notice that instead of $\mathbb{P}[N(I) \geq 1] \geq_N p(h) > 0$ if we have the condition $\mathbb{P}[N(I) = 1] \geq_N p(h) > 0$ the same result holds, as a mere corollary, since $\mathbb{P}[N(I) \geq 1] \geq \mathbb{P}[N(I) = 1]$.

Theorem 4, does not use any expander properties of the MEG. It can happen that a stationary MEG satisfies the expander property for some subsets $I \subset [N]$ but not all. In this case Theorem 4 may not give a very tight bound. We can combine the ideas of Theorems 3 and 4 to establish the following result.

Theorem 5. *For a stationary MEG if*

1) *there exists a $s \in \{2, 3, \dots \frac{N}{2}\}$, strictly increasing sequence $1 < h_1 < h_2 < \cdots < h_s = \frac{N}{2}$, and a non-increasing sequence $k_2 \geq k_3 \geq \cdots \geq k_s > 0$ such that*

$$\mathbb{P}\left[\bigcap_{i=2}^{s} \{G_0 \text{ is } ([h_{i-1}, h_i], k_i)\text{−expander}\}\right] \geq_N 1 - \frac{c_1}{N^2}, \tag{38}$$

for some $c_1 > 0$,

2) *for $1 \leq h \leq h_1$, for all $I \subset [N]$ such that $|I| = h$ we have*

$$\mathbb{P}[N(I) = 1] \geq_N p(h) > 0, \tag{39}$$

and

3) *$h_1 \geq c_2 \log N$ is such that*

$$\lim_{N \to \infty} \frac{h_1}{\log N} = \infty, \tag{40}$$

then

$$T_N = O\left(\sum_{h=1}^{h_1} \frac{1}{p(h)} + \sum_{i=2}^{s} \frac{\log(h_i/h_{i-1})}{\log(1+k_i)}\right), \tag{41}$$

with probability at least $1 - c_2/N$ for some $c_2 > 0$.

Proof. $I_t \subset [N]$ denotes the number of nodes that have the packet at time $t \geq 1$. Let T_1 be the first time at which at least h_1 nodes get the packet, i.e.,

$$T_1 = \min\{t \geq 1 | |I_t| \geq h_1 \text{ and } |I_1| = 1\}, \tag{42}$$

and $T_{2:N} = T_N - T_1$. Clearly, $T_{2:N}$ will be less than the time it takes for the packet to reach all nodes if the system were to start with exactly h_1 nodes carrying the packet, i.e.,

$$T_{2:N} \leq T'_{2:N} = \min\{t \geq 1 | |I_t| = N \text{ and } |I_1| = h_1\}. \tag{43}$$

Following the same arguments listed in [14] for the proof of Theorem 3, while using the expander property Equation(38), we have

$$T^{'}_{2:N} = O\left(\sum_{i=2}^{s} \frac{\log(h_i/h_{i-1})}{\log(1+k_i)}\right), \tag{44}$$

with probability at least $1 - c_1/N$ for some $c_1 > 0$.

Following the same arguments in the proof of Theorem 4, while using Equation (39), yields

$$T_1 = O\left(\sum_{h=1}^{h_1} \frac{1}{p(h)}\right), \tag{45}$$

with probability at least $1 - \exp\{-c_2 h_1\}$ for some $c_2 > 0$. From Equation (40), it is clear that $h_1 > \gamma \log N$ for any $\gamma > 0$. This implies

$$1 - \exp\{-c_2 h_1\} \geq 1 - \exp\{-c_2 \gamma \log N\}, \tag{46}$$

$$\geq 1 - \frac{1}{N^{c_2\gamma}}, \tag{47}$$

for any $\gamma > 0$. Choosing any $\gamma \geq 1/c_2$ yields

$$T_1 = O\left(\sum_{h=1}^{h_1} \frac{1}{p(h)}\right), \tag{48}$$

with probability at least $1 - c_3/N$ for some $c_3 > 0$. We know that $T_N \leq T_1 + T^{'}_{2:N}$. Using Equations (44) and (48) we obtain the desired result. □

The results also hold if we replace the condition $\mathbb{P}[N(I) = 1] \geq_N p(h) > 0$ with

$$\mathbb{P}[N(I) \geq 1] \geq_N p(h) > 0. \tag{49}$$

Theorems 3, 4, and 5 give a high probability upper bound on flooding time, and not an upper bound on average flooding time. In the next section we apply these results to obtain a high probability upper bound on flooding time for our network model, and show that it nearly scales as the lower bound on average flooding time obtained in Theorem 2 of Section 2. In Section 5, we use this fact to propose a FCFS flooding scheme that achieves the high probability upper bound as its average delay.

4 Flooding Time for the IID Mobility Model

We now apply the high probability upper bounds on flooding time from Theorems 3, 4, and 5 of Section 3 to our network model. As to which of the three results we use depends on whether the network is sparse or dense. Let $\mathcal{M}$ denote the stationary MEG for our network model of Fig. 1, and let G_0 be its stationary distribution.

To apply these theorems, we first make the following observation about the single packet flooding scheme: if h nodes have the packet at a given time slot then the number of nodes that will receive the packet in the next slot, $N(h)$, is a binomial random variable $\mathrm{Bin}(N-h, 1-(1-a_N)^h)$.

To see this, let $H = \{1, 2, \ldots h\}$ and $\overline{H} = \{h+1, h+2, \ldots N\}$ denote the set of nodes that have and do not have the packet at a given time slot, respectively. For the node i that has not received the packet, i.e., $i \in \overline{H}$, let X_i be a binary valued random variable that is 1 if node i receives the packet in the next slot and 0 otherwise. The probability that the node i does not receive the packet in the next slot is the probability that no node of H lies in the same cell as node i. This happens with probability $(1-a_N)^h$ as locations of node's are independent and identically distributed (i.i.d.). Hence, $\mathbb{P}[X_i = 0] = (1-a_N)^h$. Also, the X_is are independent across $i \in \overline{H}$ as, again, the node locations are i.i.d. and uniform. Since $N(h) = \sum_{i \in H} X_i$ the result follows.

Using this, we prove a scaling bound on the flooding time.

Theorem 6. *The flooding time is*

$$T_N = \begin{cases} O(N^{\alpha-1}\log N) & \text{if } \alpha \geq 1 \text{ (sparse)} \\ O(\log\log N) & \text{if } 0 < \alpha < 1 \text{ (dense)} \end{cases}, \tag{50}$$

with probability at least $1 - \frac{c_1}{N}$ *for some* $c_1 > 0$.

Proof. We derive this by showing the expander properties of the network $\mathcal{M}$. We use the following lemma. □

Lemma 2. *If* $X_1, X_2, \ldots X_{g(n)}$ *are binomial random variables such that*

$$c_1 f(n) \leq_N \mathbb{E}[X_h] \leq_N c_2 f(n), \tag{51}$$

for some positive constants c_1 *and* c_2, *where* $g(n)$ *and* $f(n)$ *are increasing functions of* n. *Then there exists an* $\eta \in (0,1)$ *and a positive constant* c_3 *such that*

$$\mathbb{P}[X_h < \eta c_1 f(n)] \leq_N e^{-c_3 f(n)}, \tag{52}$$

for all $h \in \{1, 2, \ldots g(n)\}$.

Proof. See Appendix D, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2019.2923733. □

We split the proof into three cases: $0 < \alpha < 1$, $1 \leq \alpha < 2$, and $\alpha \geq 2$.

1) $0 < \alpha < 1$: In this case, the expander properties of Theorem 3 hold. Note that

$$\mathbb{E}[N(h)] = (N-h)\Big[1 - (1 - c/N^{\alpha})^h\Big]. \tag{53}$$

It is also easy to see that $1 - (1-c/N^{\alpha})^h = \Theta(h/N^{\alpha})$ if $h/N^{\alpha} \to 0$, and $1 - (1-c/N^{\alpha})^h = \Theta(1)$ if $h/N^{\alpha} \to \infty$. When $h/N^{\alpha} = \Theta(1)$, both are true. We, therefore, have

$$\mathbb{E}[N(h)] = \begin{cases} \Theta(Nh/N^{\alpha}) & \text{for } 1 \leq h \leq N^{\alpha} \\ \Theta(N) & \text{for } N^{\alpha}+1 \leq h \leq N/2 \end{cases}. \tag{54}$$

Since, in both cases we have $\mathbb{E}[N(h)] \to \infty$, we can use Lemma 2, the concentration bound on the binomial distribution, to show that the event $\{N(h) \geq c_1 \mathbb{E}[N(h)]\}$ occurs with high probability for some $0 < c_1 < 1$. This proves that the graph is $(h, k(h))$-expander where $k(h) = c_1 \frac{\mathbb{E}[N(h)]}{h}$, i.e.,

$$\mathbb{P}\left[\bigcap_{h=2}^{N/2} \{G_0 \text{ is } (h, k(h))\text{−expander}\}\right] \geq_N 1 - \frac{c_2}{N^2}, \tag{55}$$

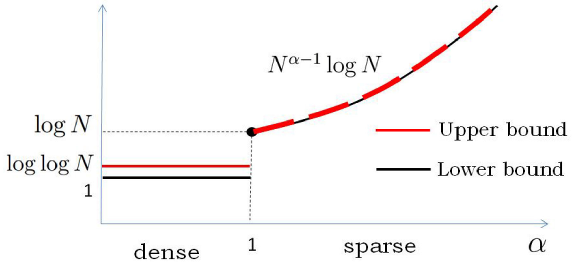

Fig. 3. High probability upper bound and the average lower-bound on flooding time T_N as a function of α.

for some $c_2 > 0$ where

$$k(h) = \begin{cases} c_3 N^{1-\alpha} & \text{for } 1 \leq h \leq N^\alpha \\ c_4 \frac{N}{h} & \text{for } N^\alpha + 1 \leq h \leq N/2 \end{cases}, \tag{56}$$

for some $c_3, c_4 > 0$. See Appendix A, available in the online supplemental material, for a detailed proof. This satisfies the expander property requirements of Theorem 3. Applying Theorem 3, we obtain

$$T_N = O(\log\log N), \tag{57}$$

with probability at least $1 - \frac{c_5}{N}$ for some $c_5 > 0$. We prove this in Appendix A, available in the online supplemental material.

2) $1 \leq \alpha < 2$: In this case, the expander properties of Theorem 5 hold. Note that $\frac{h}{N^\alpha} \to 0$ for all $1 \leq h \leq N/2$. We, thus, have $(1 - (1 - c/N^\alpha)^h) = \Theta(h/N^\alpha)$. Using the expression for $\mathbb{E}[N(h)]$ in Equation (53) we have $N(h) = \Theta(Nh/N^\alpha) = \Theta(h/N^{\alpha-1})$.

Here, $\mathbb{E}[N(h)]$ does not always go infinity in N. However, we observe that, for all $\beta N^{\alpha-1}\log N + 1 \leq h \leq N/2$ and for any $\beta > 0$, $\mathbb{E}[N(h)] \to \infty$ as $N \to \infty$. We can then use Lemma 2, the concentration bounds for binomial distribution, to derive the following expander property for $\beta N^{\alpha-1}\log N + 1 \leq h \leq N/2$:

$$\mathbb{P}\left[\bigcap_{h > \beta N^{\alpha-1}\log N}^{N/2} \left\{G_0 \text{ is } \left(h, \frac{c_1}{N^{\alpha-1}}\right)\text{−expander}\right\}\right] \geq_N 1 - \frac{c_2}{N^2}, \tag{58}$$

for some $c_1, c_2 > 0$ and provided $\beta > c_3$ for some $c_3 > 0$.

For $1 \leq h \leq \beta N^{\alpha-1}\log N$, $\mathbb{E}[N(h)]$ need not always go to infinity, and can in fact go to zero. Due to this, the network $\mathcal{M}$ does not satisfy any expander property for all $1 \leq h \leq \beta N^{\alpha-1}\log N$. Therefore, we derive a lower-bound on the probability $\mathbb{P}[N(h) \geq 1]$. In particular, there exists $c_3 > 0$ such that

$$\mathbb{P}[N(h) \geq 1] \geq_N c_3\left(1 - \exp\{-h/N^{\alpha-1}\}\right), \tag{59}$$

for all $h \in \{1, 2, \ldots \beta N^{\alpha-1}\log N\}$. See Appendix B, available in the online supplemental material, for a detailed proof. This satisfies the conditions of Theorem 5. From this, one can obtain

$$T_N = O\left(N^{\alpha-1}\log N\right),$$

with probability at least $1 - \frac{c_4}{N}$ for some $c_4 > 0$. We prove this in Appendix B, available in the online supplemental material.

3) $\alpha \geq 2$: In this case, the conditions of Theorem 4 hold. Since $\alpha \geq 2$, we have $h/N^\alpha \to 0$ for all $1 \leq h \leq N/2$. This implies $1 - (1 - c/N^\alpha)^h = \Theta(h/N^\alpha)$. Thus, using Equation (53), we have $\mathbb{E}[N(h)] = \Theta(Nh/N^\alpha) \to 0$ for all $1 \leq h \leq N/2$. This shows that the network $\mathcal{M}$ does not satisfy any expander property. We, therefore, derive a lower-bound on $\mathbb{P}[N(h) = 1]$. There exists a $c_1 > 0$ such that

$$\mathbb{P}[N(h) = 1] \geq_N c_1 \frac{(N-h)h}{N^\alpha}, \tag{60}$$

for all $1 \leq h \leq N - 1$. See Appendix C, available in the online supplemental material, for a detailed proof. This satisfies the condition of Theorem 4, using which one can obtain

$$T_N = O\left(N^{\alpha-1}\log N\right),$$

with probability at least $1 - \frac{c_2}{N}$ for some $c_2 > 0$. We prove this in Appendix C, available in the online supplemental material.

Fig. 3 compares the high probability upper bound with the average lower-bound on flooding time T_N from Theorem 2. We observe a gap of at most $O(\log\log N)$ when $0 < \alpha < 1$. For all other values of α the upper and lower-bounds are of the same order. The lower-bound on flooding time was derived in Theorem 2, which was also the lower-bound on the achievable average delay. In the next section, we show that a simple FCFS flooding scheme achieves the high probability upper bound on flooding time as its achievable average delay.

5 FCFS Flooding Scheme

We propose a scheme that is based on the idea of single packet flooding described in Section 2. In this scheme, only a single packet is transmitted over the entire network at any given time. Packets are served sequentially by the network on a FCFS basis. Each packet gets served for a fixed duration of τ. The packet is dropped if within this duration it is not received by all the other $(N-1)$ nodes. We call this the *FCFS packet flooding* scheme.

FCFS Packet Flooding: Packets arrive at each of the N nodes at rate λ.

1) Among all the packets that have arrived, select the one that had arrived the earliest. At this time only one node, i.e. the source node, has this packet.
2) In every cell, randomly select one packet carrying node (if it exists) as a transmitter.
3) Selected nodes transmit in each cell during the slot while all other nodes in the corresponding cells receive the packet.
4) Repeat Steps 2 and 3 for τ time slots.
5) After τ slots, remove the current packet from the transmission queue and go to Step 1.

Note that in the FCFS flooding scheme, as described, if a cell does not contain a node carrying the earliest packet, no transmission occurs in that cell, during that time slot. This is an inefficiency in the algorithm, which we will fix in Section 5.1.

We refer to τ in the FCFS flooding scheme as the *flooding-time threshold*. The choice of the flooding-time threshold τ is critical as a small value τ will imply that an arriving packet is dropped before it reaches all the nodes much too often, whereas a large value of τ will imply that the packet remains in the system for too long a duration, thereby causing congestion, and affecting network throughput. To ensure a better tradeoff between this packet drop probability and network throughput, we set $\tau = U_N$, which is given by

$$U_N = \begin{cases} c_1 N^{\alpha-1} \log N & \text{if } \alpha \geq 1 \text{ (sparse)} \\ c_2 \log\log N & \text{if } 0 < \alpha < 1 \text{ (dense)} \end{cases}, \tag{61}$$

for some positive constants c_1 and c_2 such that $T_N < U_N$ with probability $1 - \frac{1}{N}$. Such constants exists by Theorem 6. This leads to a vanishingly small packet drop probability. We now obtain the capacity and delay performance of this FCFS packet flooding scheme.

Theorem 7. *The FCFS packet flooding scheme achieves a capacity of*

$$\lambda = \begin{cases} \Theta\left(\frac{1}{N^{\alpha} \log N}\right) & \text{if } \alpha \geq 1 \text{ (sparse)} \\ \Theta\left(\frac{1}{N \log\log N}\right) & \text{if } 0 < \alpha < 1 \text{ (dense)} \end{cases}. \tag{62}$$

Furthermore, the delay achieved at this rate is $\overline{D} = \Theta(U_N)$.

Proof. The packets arrive at each node according to a Poisson process, at rate λ. Thus, the sum packets arrivals in the networks is also a Poisson process of rate $N\lambda$. The service time for each packet under the FCFS packet flooding scheme is nothing but U_N. Thus, the system can be thought of as a M/D/1 queue, with an arrival rate of $N\lambda$ and service time of U_N. The waiting time for such a system is given by [28]

$$\tilde{W} = U_N + U_N \frac{\rho}{2(1-\rho)}, \tag{63}$$

for any arrival rate $N\lambda < \frac{1}{U_N}$, where $\rho = N U_N \lambda < 1$ is the queue utilization. Selecting any $\rho < 1$, we obtain $\tilde{W} = \Theta(U_N)$ and $\lambda = \Theta(\frac{1}{N U_N})$. Substituting U_N from Equation (61), we obtain the result. □

The obtained delay bound in Theorem 7 uses the fact that the arrival process at each node is Poisson. In the proof, we make use of the property that superposition of N Poisson arrival processes is also Poisson. In relaxing the Poisson arrival assumption, we will need to analyze a queueing system in which the arrivals is a sum of independent renewal processes. This is an interesting problem in itself as, to the best of our knowledge, tight bounds for such systems are not known.

Theorem 7 implies that the delay lower-bound of Theorem 2 is achieved, up to a gap of $O(\log\log N)$, when the network is dense, i.e. $0 < \alpha < 1$. We also see that the achieved throughput λ is less than the capacity upper bound of Theorem 1 by a factor of $\log\log N$ when $0 < \alpha < 1$, and by a factor of $\log N$, when $\alpha \geq 1$. The $\log\log N$ gap appears due to the exact same gap between the flooding time upper and lower bounds when $0 < \alpha < 1$. The $\log N$ factor gap for $\alpha \geq 1$ occurs even though the flooding time upper and lower bounds are asymptotically tight. This, we conjuncture, is because the FCFS flooding scheme does not allow simultaneous transmissions of different packets, which leads to inefficient utilization of available transmission opportunities.

We summarize these results in Table 1. Unlike the unicast case, where a capacity-delay tradeoff has been observed [9], [11], [22], nearly no such tradeoff exists for the broadcast problem, and both capacity and minimum delay can be nearly achieved simultaneously.

1) Implementing FCFS Flooding Scheme. The FCFS flooding scheme, as stated, is hard to implement in a real system, as it requires the earliest packet arrived, among all packets in the network, to be serviced at any given time. We now present a slightly modified version of the FCFS flooding scheme, which overcomes this difficulty by a slight modification. Instead of choosing the earliest arrived packet among all packets in the network, it picks the earliest arrived packet in each cell. This selection of the earliest packet among nodes in a single cell can be ensured in short time span by using contention based conflict resolution mechanisms [29], [30]. We describe the resulting threshold-based flooding scheme.

Threshold-Based Flooding Scheme. In each time slot and in each cell, we select a node, which has the earliest arrived packet, across all packets in that cell. The selected nodes broadcast the earliest arrived packet to all other nodes in the cell. We fix a flooding time threshold τ, and like in the FCFS flooding scheme, a packet is dropped after it spends τ time slots in the system.

Comparing this scheme with the FCFS flooding scheme, it is clear that the threshold-based flooding scheme is easier to implement. Furthermore, unlike the FCFS flooding scheme, the threshold-based flooding scheme does not let a cell with more than two nodes remain idle with no transmissions. Using stochastic dominance arguments, it can be shown that the expected queue length (total number of packets waiting to be served in the network) for the threshold-based flooding scheme is upper-bounded by the queue length for the FCFS flooding scheme. This implies that Theorem 7 also holds for the threshold-based flooding scheme.

6 SINGLE HOP SCHEME

We now propose a single-hop scheme that achieves the capacity upper-bound of Theorem 1 when the network is sparse, i.e. $\alpha \geq 1$. In this scheme, a packet reaches its destination from a source in a single hop, i.e. by direct source to destination transmission. This scheme only allows for a single receiver in each cell, thus, ignores the broadcast nature of the wireless medium. The scheme still achieves the upper-bound capacity as the number of nodes in a cell tends to be very small in the sparse case.

Single-Hop Scheme: Each node makes $(N-1)$ copies of an arrival packet, one for each receiving node. Fig. 4 illustrates this for node 1, where a copy of an arriving packet at node 1 is transferred to each of the queues $Q_{1,j}$ for all $2 \leq j \leq N$.

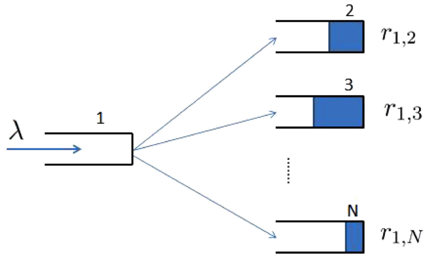

Fig. 4. Node 1 makes $(N-1)$ copies of every arriving packet, one for each queue $Q_{1,j}$ for $2 \leq j \leq N$. Service rate of $Q_{1,j}$ is denoted by $r_{1,j}$.

1) In each cell, select a pair of nodes at random. If a cell contains fewer than 2 nodes no transmissions occur in that cell.
2) For the selected pair in every cell, assign, uniformly and randomly, one node as a transmitter and the other as receiver.
3) For each transmitter-receiver pair, if the transmitter node has a packet for the receiver node, transmit it, else remain idle.
4) Wait for the next slot to begin, and restart the process from Step 1.

The scheme is opaque to which node pairs are chosen as the source-destination pairs. Thus, every queue $Q_{i,j}$ is activated at the same rate. This implies that all the queues $Q_{i,j}$ have identical service rates. Hence

$$\sum_{i \neq j} r_{i,j} = N(N-1)r_{1,2}. \tag{64}$$

The left hand side of Equation (64) corresponds to the total rate of service opportunities across the network, which is given by Cp, where p is the probability that there are at least two nodes in a cell: $p = 1 - (1-a_N)^N - Na_N(1-a_N)^{N-1}$. Thus, $N(N-1)r_{1,2} = Cp$, which gives

$$r_{1,2} = \frac{Cp}{N(N-1)}. \tag{65}$$

Hence, any arrival rate $\lambda < r_{1,2}$ will yield a stable network under the single-hop scheme. The delay achieved by this scheme is lower-bounded by the delay in the single queue. Since each queue is Poisson arrival and geometric service times, the waiting time in each queue can be computed using the Pollaczek-Khinchine formula (see [28])

$$\bar{W} = \frac{\lambda \mathbb{E}[S^2]}{2(1-\lambda/r_{1,2})}, \tag{66}$$

where S is geometrically distributed random variable, with rate $r_{1,2}$. Substituting $\mathbb{E}[S^2] = \frac{2-r_{1,2}}{r_{1,2}^2}$ and setting $\lambda = \frac{1}{2}r_{1,2}$, we obtain $\bar{W} = \Theta(1/r_{1,2})$. We summarize this in the following result.

Theorem 8. *The single hop scheme achieves a capacity of*

$$\lambda_{\mathrm{SH}} = \begin{cases} \Theta\left(\frac{1}{N^{\alpha}}\right) & \text{if } \alpha \geq 1 \text{ (sparse)} \\ \Theta\left(\frac{1}{N^{2-\alpha}}\right) & \text{if } 0 < \alpha < 1 \text{ (dense)} \end{cases}. \tag{67}$$

Furthermore, the delay achieved at this rate is

$$\overline{D}_{\mathrm{SH}} \geq \begin{cases} \Theta(N^{\alpha}) & \text{if } \alpha \geq 1 \text{ (sparse)} \\ \Theta(N^{2-\alpha}) & \text{if } 0 < \alpha < 1 \text{ (dense)} \end{cases}. \tag{68}$$

Hence, the single hop scheme achieves the capacity upper-bound for $\alpha \geq 1$. Thus, the capacity upper bound in Theorem 1 is indeed achievable. In the derivation of the result we have used the Poisson arrival assumption only in Equation (66), which applies the Pollaczek-Khinchine formula. The Poisson arrival assumption here can be relaxed by using the waiting time upper-bound formula for G/G/1 queues (see [28]). Doing so will add a constant term, namely the inter-arrival time variance, to the waiting time in Equation (66), and not change the scaling obtained in the theorem.

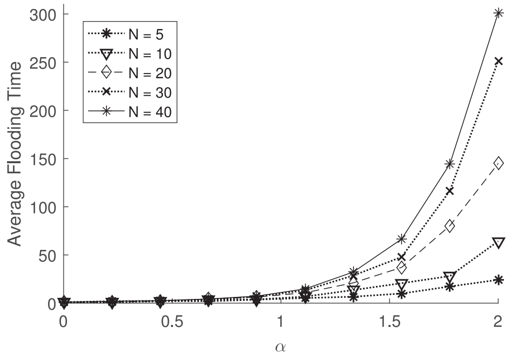

Fig. 5. Plot of average flooding time $\mathbb{E}[T_N]$ as a function of α.

7 Simulation Results

In this section, we first validate our flooding time results using Monte Carlo simulations. We consider a network with N nodes and $C = \lfloor N^{\alpha} \rfloor$ cells. In Fig. 5, we plot the average flooding time $\mathbb{E}[T_N]$, for the single packet flooding scheme, as a function of $\alpha \in (0,2)$ for various values of N. We observe that the flooding time increases as α increases, i.e., as the network gets sparser. Also note that Fig. 5 matches with our asymptotic illustration of flooding-time bounds in Fig. 3, which plots the average single-packet flooding time lower-bound derived in Theorem 2, and the high probability upper-bound on T_N derived in Theorem 6.

In Fig. 5, for $\alpha < 1$, we observe a very small increase in the average flooding time $\mathbb{E}[T_N]$, as a function of N, akin to either a $\log N$ or $\log\log N$. However, for $\alpha > 1$, we see a polynomial increase of the average flooding time $\mathbb{E}[T_N]$ in N. A comparison with Fig. 3 implies the correctness of the derived results. In Fig. 6, we plot the average flooding time $\mathbb{E}[T_N]$, in log-scale, as a function of N, for various values of α. From Theorems 6 and 2, loosely speaking, we have $T_N \sim N^{\alpha-1}\log N$ for $\alpha > 1$ and $T_N \sim \log\log N$ for $\alpha < 1$. Plotting on log-scale, this implies $\log T_N \sim (\alpha-1)\log N + \log\log N$ for $\alpha > 1$ and $T_N \sim \log\log\log N \approx$ constant, for $\alpha < 1$. We observe exactly this scaling behavior in Fig. 6. For higher values of α, namely $\alpha = 2$ and $\alpha = 3$, we see that the log-scale plot of $\mathbb{E}[T_N]$ has the form of $\log N$ with scaling determined by α, whereas for $\alpha < 1$, the log-scale plot of $\mathbb{E}[T_N]$ is pretty much a constant.

We next implement the FCFS flooding scheme. In implementing the FCFS flooding scheme, one has to compute the flooding time threshold τ. As we saw in Section 5, setting a

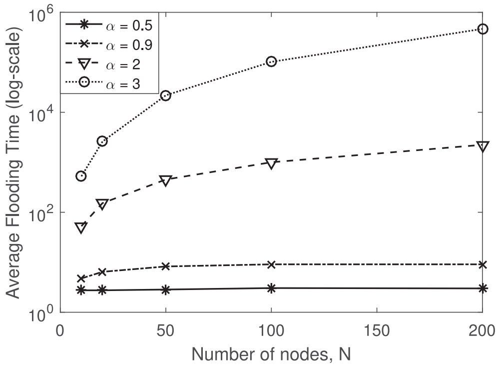

Fig. 6. Plot of average flooding time $\mathbb{E}[T_N]$ as a function of number of nodes N in the network.

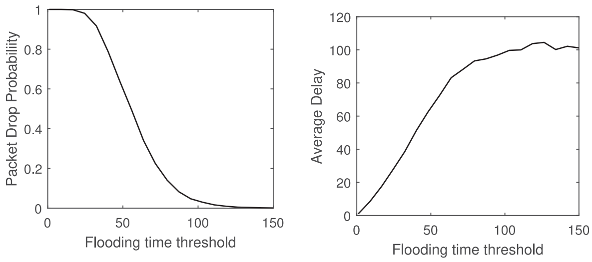

Fig. 7. Pakcet drop probability p_{drop} and average delay $\bar{D}$ as a function of the flooding time threshold, for a network with $N = 10$ nodes, $C = 100$ cells, and an arrival rate $\lambda = 0.001$ packets/s.

threshold τ results in a packet drop probability. In Fig. 7, we plot the packet drop probability and average delay attained by a FCFS flooding scheme, as a function of the flooding time threshold τ. The plot is for a network of $N = 10$ nodes, $C = 100$ cells, and an arrival rate $\lambda = 0.001$ packets/s.

In Section 5, we shows that there exists a threshold that scales as U_N in Equation (61), for some constants c_1 and c_2, such that the capacity and delay given by Theorem 7 is achieved. The proof of this result involved proving that the packet drop probability tends to zero, for this selection of flooding time threshold τ, as $N \to +\infty$. In practice, for finite N, we have to decide on a tradeoff between the packet drop probability and the average delay. In particular, we can set a required packet drop probability p_{drop}, and then compute the minimum flooding time threshold τ that achieves it.

In Fig. 8, we plot the optimal flooding time threshold (in log-scale), obtained numerically for a packet drop probability of $p_{\text{drop}} = 0.01$, as a function of N and two values of α. We assume the arrival rate λ to be half the capacity upper-bound derived in Theorem 1. Also plotted is the average flooding time $\mathbb{E}[T_N]$ and the average delay $\bar{D}$ attained by the corresponding FCFS flooding scheme. We see that the optimal flooding time threshold τ is very close to the expected flooding time. More specifically, we observe that the $\tau = \eta\mathbb{E}[T_N]$, for $\eta \in (1, 1.8)$ is generally the optimal choice. This figure illustrates the importance of studying flooding times for mobile networks, as it helps us design the broadcast algorithms such as FCFS packet flooding and threshold-based flooding scheme, which could be implemented in practice.

8 Conclusion and Open Problems

We considered the problem of all-to-all broadcast transmissions, in a networks of highly mobile nodes. We derived the

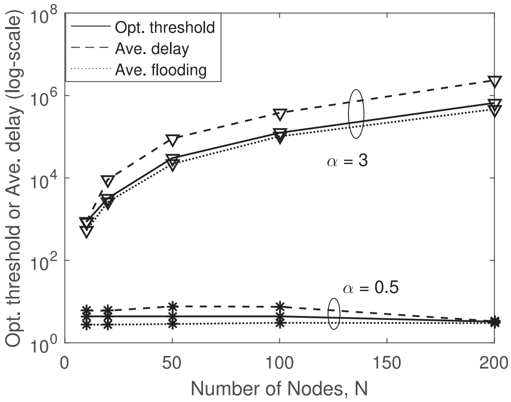

Fig. 8. Plot of optimal threshold for the FCFS flooding scheme, for various values of N and α. Also plotted is the average delay for the corresponding FCFS flooding scheme, and the flooding time.

broadcast capacity and minimum delay scaling, in the number of vehicles N, and showed that the capacity cannot scale better than $1/N$. This, in conjunction with earlier known results for static network [17], proves that the broadcast capacity does not improve with high mobility. This is in contrast with the unicast case for which mobility improves network capacity [13].

We further showed that both, the capacity and minimum delay scalings, can be nearly achieved, simultaneously. We proposed a simple FCFS flooding scheme, and its variant threshold-based flooding scheme, which nearly achieve both the capacity and minimum delay scaling. The flooding time bound for Markov evolving graphs (MEG), proposed in [14], was used to analyze the proposed schemes. We also derived two new bounds on flooding time for MEG, which may be of independent theoretical interest.

In the Simulation Results section, we saw that the optimal flooding time threshold is closely related to the average flooding time for the mobile network model. Therefore, obtaining tight flooding time bounds for more general mobility models such as the random waypoint model, Brownian motion model, and Levy process models, would be of considerable interest. It will also help us analyze the delay of the FCFS packet flooding and the threshold-based flooding scheme, under such general mobility models.

The flooding time bounds derived here, and in [14], would be useful, considering that the Brownian Motion and Levy processes are Markovian in character. It would be interesting to see whether the result obtained here, namely that of (nearly) no capacity-delay tradeoff, holds when the speeds of motion are much slower.

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Rajat Talak received the MSc degree from the Department of Electrical Communication Engineering, Indian Institute of Science (IISc), Bangalore, India, in 2013. He is working toward the PhD degree in the Laboratory of Information and Decision Systems (LIDS), Massachusetts Institute of Technology. He was awarded the Prof. F M Mowdawalla medal for his master's thesis. His research inclination is towards theoretical modeling, analysis, and algorithm design for communication, and networked systems. His current research work is on scheduling algorithms in wireless networks, age of information, large scale networks, and autonomous vehicular networks. He is the recipient of the MobiHoc 2018 best paper award.

Sertac Karaman received the BS degrees in mechanical engineering and in computer engineering from the Istanbul Technical University, Turkey, in 2007, the MS degree in mechanical engineering from the Massachusetts Institute of Technology (MIT), in 2009, and the PhD degree in electrical engineering and computer science from the MIT, in 2012. He is an associate professor of aeronautics and astronautics at the Massachusetts Institute of Technology (since Fall 2012). His research interests lie in the broad areas of robotics and control theory. In particular, he studies the applications of probability theory, stochastic processes, stochastic geometry, formal methods, and optimization for the design and analysis of high-performance cyber-physical systems. He delivered the the Robotics: Science and Systems Early Career Spotlight Talk in 2017. He is the recipient of numerous awards.

Eytan Modiano received the BS degree in electrical engineering and computer science from the University of Connecticut, Storrs, in 1986, and the MS and PhD degrees, both in electrical engineering from the University of Maryland, College Park, Maryland, in 1989 and 1992, respectively. He is a professor with the Department of Aeronautics and Astronautics and associate director of the Laboratory for Information and Decision Systems (LIDS), MIT. Prior to joining the faculty with MIT in 1999, he was a Naval Research Laboratory fellow between 1987 and 1992, a National Research Council post doctoral fellow during 1992-1993, and a member of the technical staff at MIT Lincoln Laboratory between 1993 and 1999. His research is on modeling, analysis and design of communication networks and protocols. He is the co-recipient of numerous best paper awards. He is the editor-in-chief of the *IEEE/ACM Transactions on Networking*. He is a fellow of the IEEE and an associate fellow of the AIAA, and served on the IEEE fellows committee.