Channel Allocation Using Pricing in Satellite Networks

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***Abstract*— We consider allocating a set of parallel channels among multiple users in a satellite network. A novel MAC protocol is proposed based on pricing that allocates network resources efficiently according to users' demand. In this paper, we first characterize the Pareto efficient throughput region in a single satellite network. The existence of a equilibrium price is presented. Furthermore, we show that such equilibrium price is unique. In the multiple satellites case, the Pareto efficient throughput region is also described. We then show that the equilibrium price exists and is unique. The resulting throughput at the equilibrium is shown to be Pareto efficient also.**

I. Introduction

Future satellite communication networks are envisioned to provide diverse quality of service based on user's demand. Hence, it is vital to have a Medium Access Control (MAC) protocol that provides fair and efficient channel access for each user. In this paper, we propose a novel MAC protocol based on pricing that allocates network resources efficiently in response to users' demand.

Specifically, we consider a communication network with multiple satellites, collectively acting as a network manager, who wish to allocate network uplink capacity efficiently among a set of users, each endowed with a utility function depending on their data rate. We assume that each satellite uses a separate channel for communication, such as using different frequency band for receiving. Each user has data that needs to be sent to the satellite network, and there may be multiple satellites that a user on the ground can communicate with, or switched diversity termed in [1]. Therefore, the data rate for each user here is the rate at which each user can access the satellite network by sending its data to any satellite within its view.

Slotted aloha is used here as the multi-access scheme for its simplicity. Other multi-access schemes can be used in conjunction with our pricing scheme to provide QoS as well. Due to different path loss and fading, the channel gain from a user to different satellites within its view can be different. Therefore, during a single time slot, a user has to decide not only whether it should transmit but also to which satellite it will transmit. To control users' transmission rates, each satellite will set a price that may differ from satellite to satellite for each *successfully received* packet. Based on the price set by each satellite, a user determines its target satellite and the transmission probability to maximize its net payoff, which is the utility of its received rate minus the cost.

It is well-known that the throughput of a slotted aloha system is low. Therefore, to efficiently utilize the available resource is a reasonable objective for the network manager. In this paper, we want to explore the use of pricing as a control mechanism to achieve efficiency. To do so, we need to define the meaning of efficiency in the context of a slotted aloha system. With a wire-line, such as optical fiber, of capacity R, an allocation is efficient as along as the sum of the bandwidth allocated to each individual user is equal to R, i.e., no waste of bandwidth. With a collision channel in the aloha system, no simple extension of the wire-line case exists. We therefore use a concept called *Pareto efficiency* for allocating resource in a collision channel. By definition, a feasible allocation $(s_1, s_2, \cdots, s_n)$ is Pareto efficient if there is no other feasible allocation $(s'_1, s'_2, \cdots, s'_n)$ such that $s'_i \geq s_i$ for all $i = 1, \cdots, n$ and $s'_i > s_i$ for some i.

The multiple satellites communication networks considered here differ from the multichannel aloha networks in only one aspect–the channel quality associated with the path from user to the satellite is different in the multiple satellites case. This difference gives us insight on how to best utilize the multiple channels available to users. A multichannel aloha network consists of M parallel, equal capacity channels for transmission to one receiver shared by a set of users. The M channels can be implemented based on either Frequency Division Multiplexing or Time Division Multiplexing approaches. When a user has a packet to send, it will *randomly* select one channel to transmit. This random selection of the channel is largely due to the lack of coordination among competing users. Intuitively, we would expect that the throughput of the system will be higher if the coordination in channel selection among users was available. As we show in this paper, in multiple satellite networks, different prices and channel states are two mechanisms that enable the coordination in channel selection among the competing users.

The multi-channel slotted aloha problem has been studied by numerous researchers. In [4], the authors develop a distributed approach for power allocation and scheduling in a wireless network where users communicate over a set of parallel multi-access fading channels, as in an OFDM or multi-carrier system. In [5], the authors shows how to improve the classic multichannel slotted aloha protocols by judiciously using redundant transmissions. The use of pricing strategy to control the behavior of users who are sharing a single channel using aloha medium access protocol was investigated in [6]. A game theoretical model

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for users competing for the limited resources is provided. Multiple channels and the associated channel states for the users are not considered in their work.

The organization of this paper is as follows. In section II, we characterize the Pareto efficient throughput region in a single satellite network. The existence of a equilibrium price is presented. Furthermore, we show that such equilibrium price is unique. In section III, we describe the Pareto efficient throughput region in a multi-satellites environment given that coordination of satellite selection was allowed among users. We then show that the equilibrium price exists and is unique. The resulting throughput at the equilibrium is shown to be Pareto efficient also. An multiple satellites example, where the competitive equilibrium is explicitly calculated, is given in Section IV. Section IV concludes the paper.

II. A Single Satellite Network

We consider an uplink communication scenario in a single satellite network with n users. Each user has unlimited number of packets in its buffer need to be transmitted. As in standard slotted ALOHA model, if two or more packets are transmitted during the same time slot, we assume no packets will be received at the satellite. Now, let z_i denote the transmission probability of user i. The probability that user i's packet is successfully received by the satellite is then denoted as $s_i = z_i \prod_{j \neq i}(1 - z_j)$. We further denote the constant channel state coefficient from user i to the satellite as c_i. Assume all users transmitting at a constant power P. Given user i's transmission during a particular time slot was successful, the throughput of that time slot for user i can be written as $q_i = g_i(c_i, P)$, where g_i is a concave function (e.g.,Shannon capacity equation). Thus, the data rate that user i received can be written as $q_i \cdot s_i$. User i, therefore, receives a *utility* equal to $U_i(q_i \cdot s_i)$, where the utility is measured in monetary units. The utility function $U_i(\cdot)$ is assumed to be concave, strictly increasing, and continuously differentiable. As mentioned in most literature, concavity corresponds to the assumption of elastic traffic.

We therefore use a concept called *Pareto efficient* for allocating resource in a collision channel. By definition, a feasible allocation $(s_1, s_2, \cdots, s_n)$ is Pareto efficient if there is no other feasible allocation $(s'_1, s'_2, \cdots, s'_n)$ such that $s'_i \geq s_i$ for all $i = 1, \cdots, n$ and $s'_i > s_i$ for some i. Here, the allocation is in terms of the success probability of each packet instead of the actual data rate $q_i \cdot s_i$. As we will mention later, it is sufficient for us to consider s_i's only. The following theorem gives the capacity region (i.e., the pareto efficient allocation) of the aloha system considered here.

Theorem 1: Given a set of transmission probabilities $(z_1, z_2, \cdots, z_n)$, the resulting allocation $(s_1, s_2, \cdots, s_n)$ is Pareto efficient if and only if $z_1 + z_2 + \cdots + z_n = 1$.

Proof: First, we will find the capacity region or the Pareto efficient $(s_1, s_2, \cdots, s_n)$. We begin by considering the following optimization problem:

$$\begin{aligned} \max_{z_1, z_2, \cdots, z_n} \quad & s_1 + s_2 + \cdots + s_n \\ \text{subj.} \quad & \frac{s_2}{s_1} = \alpha_2, \cdots, \frac{s_n}{s_1} = \alpha_n \end{aligned} \tag{1}$$

The Lagrangian is given by:

$$\begin{aligned} & L(s_1, \cdots, s_n, \lambda_2, \cdots, \lambda_n) \\ & = (1 - \sum_{i=2}^{n} \lambda_i \alpha_i) s_1 + \sum_{i=2}^{n} (1 + \lambda_i) s_i \\ & = \beta_1 s_1 + \beta_2 s_2 + \cdots + \beta_n s_n \end{aligned} \tag{2}$$

where $\beta_1 = (1 - \sum_{i=2}^{n} \lambda_i \alpha_i)$ and $\beta_i = 1 + \lambda_i$ for $i = 2, \cdots, n$. Substituting $s_i = z_i \prod_{j \neq i}(1 - z_j)$ and differentiating $L(s_1, \cdots, s_n, \lambda_2, \cdots, \lambda_n)$ with respect to z_i's, we have

$$\frac{\partial L}{\partial z_i} = \beta_i \prod_{j \neq i}(1 - z_j) - \sum_{k \neq i} \beta_k z_k \prod_{j \neq k, j \neq i} (1 - z_j) \tag{3}$$

Next, we claim that the solution to the system of equations ($\frac{\partial L}{\partial z_1} = 0, \cdots, \frac{\partial L}{\partial z_n} = 0$) has the following form:

$$z_i = \frac{\sum_{j \neq i} \beta_j - (n-2)\beta_i}{\sum_{i=1}^{n} \beta_i} \tag{4}$$

We will now show that the above solution form indeed solves the system of equations. Substituting Eq.(4) into Eq.(3), the first term of Eq.(3) is given by:

$$\begin{aligned} \beta_i \prod_{j \neq i}(1 - z_j) &= \beta_i \prod_{j \neq i} \frac{(n-1)\beta_j}{\sum_{k=1}^{n} \beta_k} \\ &= \frac{(n-1)^{n-1}}{(\sum_{k=1}^{n} \beta_k)^{n-1}} \beta_1 \cdot \beta_2 \cdots \beta_n \end{aligned} \tag{5}$$

Similarly, the second term of Eq.(3) is given by:

$$\begin{aligned} & \sum_{k \neq i} \beta_k z_k \prod_{j \neq k, j \neq i} (1 - z_j) = \sum_{k \neq i} z_k \frac{(n-1)^{(n-2)} \prod_{j \neq i} \beta_j}{(\sum_{j=1}^{n} \beta_j)^{(n-2)}} \\ & = \frac{(n-1)^{(n-2)}}{(\sum_{j=1}^{n} \beta_j)^{(n-1)}} (\prod_{j \neq i} \beta_j) \left[\sum_{k \neq i} (\sum_{j \neq k} \beta_j - (n-2)\beta_k) \right] \\ & = \frac{(n-1)^{(n-2)}}{(\sum_{j=1}^{n} \beta_j)^{(n-1)}} (\prod_{j \neq i} \beta_j)(n-1)\beta_i \end{aligned} \tag{6}$$

Comparing the two terms, we see that Eq.(4) is indeed the solution to Eq.(3). $\sum_{i=1}^{n} z_i = 1$ follows trivially. Also, note that the set z_i's given in Eq. (4) is a stationary point for the function $L(\cdot)$. It is straight forward to see that the set z_i's given in Eq.(4) cannot be a minimum point of the function $L(\cdot)$. Hence, the set of z_i's given in Eq.(4) must maximize $L(\cdot)$.

We have shown that for an Pareto efficient allocation, the sum of individual transmission probability has to be one. Conversely, if the sum of individual transmission probability is one, we know it is a solution to the optimization problem defined above for appropriately chosen α_i's. Therefore, the resulted s_i's must be Pareto efficient. ■

The utility function of each user, $U_i(\cdot)$, is not available to the satellite in general. Therefore, we consider a pricing scheme for controlling the transmission probability. We assume that the satellite, or network manager, treats all users the same (i.e., the satellite does not *price discriminate*). In our case, the price per successfully received packet charged by the satellite is the same for all users.

Given a price p per successfully received packet and other users' transmission probability z_j for $j \neq i$, user i acts to maximize the following payoff function over $0 \leq z_i \leq 1$:

$$\begin{aligned} & U_i(z_i \prod_{j\neq i}(1-z_j)\cdot q_i) - z_i \prod_{j\neq i}(1-z_j)\cdot p \\ & = U_i(s_i \cdot q_i) - s_i \cdot p \end{aligned} \tag{7}$$

The first term represents the utility to user i if it receives a data rate of $s_i \cdot q_i$, and the second term represents the price that user i pays to the network manager. We say a set $(s_1, \cdots, s_n, p)$ with $s_i = z_i \prod_{j\neq i}(1-z_j)$ for $i = 1, \cdots, n$ and $p \geq 0$ is a *competitive equilibrium* if users maximize their payoff as defined in (7), and the satellite sets a price p so as to make $\sum_{i=1}^{n} z_i = 1$ (i.e., network is efficiently utilized).

The following theorem shows the existence of a unique competitive equilibrium for the pricing scheme considered here.

Theorem 2: Assume for each user i, the utility function U_i is concave, strictly increasing, and continuously differentiable. Then there exists a unique competitive equilibrium.

Proof: We first provide the condition for users to be in the equilibrium. At an equilibrium point, user i chooses a transmission probability z_i to maximize its payoff, $U_i(s_i \cdot q_i) - s_i \cdot p$ which is equivalent to the following conditions:

$$U_i'(q_i \cdot z_i \prod_{j\neq i}(1-z_j)) = \frac{p}{q_i}, \quad \text{if } 0 < z_i < 1 \tag{8}$$

$$U_i'(0) \leq \frac{p}{q_i}, \quad \text{if } z_i = 0 \tag{9}$$

$$U_i'(q_i \prod_{j\neq i}(1-z_j)) \geq \frac{p}{q_i}, \quad \text{if } z_i = 1 \tag{10}$$

Eq.(9) represents the case that the price set by the satellite is too high; therefore, user i will not transmit anything. Similarly, Eq.(10) indicates that the price per successfully received packet is too low; hence, user i will always transmit. We consider the case that each user's utility function is strictly concave. Since the utility U_i is strictly concave, strictly increasing, and continuously differentiable, U_i' is a continuous, strictly decreasing function with its domain $[0, q_i]$ and range over the interval $[a, b]$ where b could be infinity. Consequently, the inverse U_i', say V_i, is also well defined over the interval $[a, b]$, and it is continuous and strictly decreasing. We can write Eq.(8) as the following:

$$s_i = \frac{1}{q_i} V_i(\frac{p}{q_i}) \tag{11}$$

We can think of the s_i's as the *desired* throughput for user i given the price p, even though the set $(s_1, \cdots, s_n)$ may not be feasible (i.e., there does not exist a set $(z_1, \cdots, z_n)$ and $0 \leq z_i \leq 1$ such that $s_i = z_i \prod_{j\neq i}(1-z_j)$). The set $(s_1, s_2, \cdots, s_n)$ forms a strictly decreasing continuous trajectory in $\mathcal{R}^n$ from $(1, 1, \cdots, 1)$ to $(0, 0, \cdots, 0)$ as p increases. The continuity property of the trajectory is due to the continuity of V_i.

For any Pareto optimal allocation $(s_1, s_2, \cdots, s_n)$, we must have $\sum_{i=1}^{n} z_i = 1$. For convenience, we write $\vec{z} = (z_1, z_2, \cdots, z_n)$ and $\vec{s} = (s_1, s_2, \cdots, s_n)$. Then, let $A = \{\ \vec{z} \mid z_i \geq 0$ for $i = 1, \cdots, n$ and $\sum_{i=1}^{n} z_i = 1\}$. Moreover, let $f_i(\vec{z}) = z_i \prod_{j\neq i}(1-z_j)$. Thus, the mapping $\mathbf{f}$ of A into $\mathcal{R}^n$ is defined by:

$$\mathbf{f}(\vec{z}) = (f_1(\vec{z}), \cdots, f_n(\vec{z}))$$

Since each of the functions $f_1, \cdots, f_n$ is continuous, $\mathbf{f}$ is continuous as well. We then have the set $B = \{\mathbf{f}(\vec{z}) \mid \vec{z} \in A\}$ is compact because A is compact. Thus, the set B forms a surface in $\mathcal{R}^n$ that separates the point $(1, 1, \cdots, 1)$ from the origin. To see this, we use induction. In the two dimensional case, this is obviously true. Now, suppose this statement is true for the n-dimensional case. For the $n+1$ dimensional case, let's look at the boundary points of the simplex $\sum_{i=1}^{n+1} z_i = 1$. The boundary points has dimension n. Thus, the resulting mapping is closed surface from induction hypothesis. The following figure illustrate the idea by going from two dimension to three dimension.

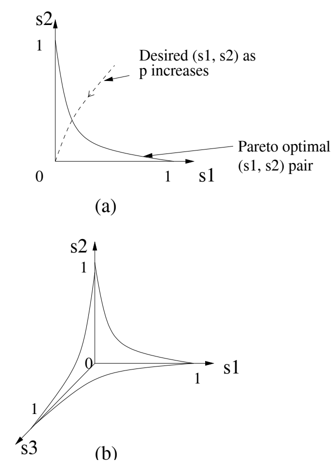

Fig. 1. (a) The relationship between a Pareto optimal (s_1, s_2). (b) The relationship of a Pareto optimal (s_1, s_2, s_3).

Therefore, for the continuous trajectory $(s_1, s_2, \cdots, s_n)$ parameterized by the price p to go from $(1, 1, \cdots, 1)$ to the origin, it must intersect with the set B at a unique point. That point is the unique competitive equilibrium in our pricing scheme. ■

III. Multi-satellite System

A. The Pareto Optimal Throughput Region of A Multi-satellite System

In a network with multiple satellites, we assume that users can simultaneously transmit to different satellites using different frequencies during the same time slot. The case that a user can transmit to only one single satellite during a time slot is a special case of the model where simultaneous transmissions are allowed. We let $z_{(i,j)}$ denote the transmission probability from user i to satellite j. Similarly, let $q_{(i,j)}$ denote the quality of the channel from user i to satellite j. The set of users that transmits to satellite j is denoted as A_j. The set of satellites that user i

transmits to is denoted as B_i. The probability of a success transmission from user i to satellite j is denoted as $s_{(i,j)}$. We also assume that users are backlogged. A graph $G = (V, E)$ can be used to represent the connections between users and satellites, where V is a set of nodes representing the users and the satellites; the edge (i, j) is in E if $z_{(i,j)}$ is positive. We first consider the case where the channels from the users to the satellites are all identical. The Pareto optimal throughput region of this multiple satellites system with identical channel quality is given by the following theorem:

***Theorem** 3:* Given a multi-satellite system represented by a connected graph $G = (V, E)$, the resulting throughput is Pareto optimal if and only if the following two conditions are satisfied:

1) there is no cycle in the graph G
2) $\sum_i z_{(i,j)} = 1 \; \forall \; j$

For a multi-satellite system that cannot be represented by a *connected* graph, we can consider each disconnected part of that graph separately. The following figure is a graphical representation of a possible communication scenario.

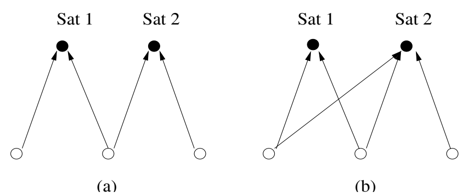

Fig. 2. (a) A graphical representation containing no cycle. (b) A graphical representation containing cycle.

Proof: Condition (2) is straightforward from Theorem 1. We will prove condition (1) here. Suppose that we have m satellites and n users. The probability of success for user i can be written as follows:

$$s_i = \sum_{k \in B_i} z_{(i,k)} \cdot \prod_{j \in A_k} (1 - z_{(j,k)}) \tag{12}$$

A set of transmission probabilities $z_{(i,j)}$ achieving pareto optimality implies that we cannot find a set of small variation $\delta_{z_{(i,j)}}$ on $z_{(i,j)}$ such that the throughput can be improved for all users. Hence, given a set of transmission probability $z_{(i,j)}$, to see whether such transmission probabilities achieves pareto optimality, we need to check whether we can find a set of $\delta_{z_{(i,j)}}$ to improve the throughput performance for some users without decreasing the throughput for other users. For satellite j, if there are k users transmitting to this satellite, we can freely vary the transmission probability by a small amount to only $k - 1$ users in order to satisfy the condition $\sum_i z_{(i,j)} = 1$ (If we change the transmission probability of all k users by a small amount, the condition $\sum_i z_{(i,j)} = 1$ may be violated). In this case, we say that we have $k - 1$ degree of freedom in varying the transmission probabilities. Therefore, for a system with m satellites, the degree of freedom in varying the transmission probabilities is $\sum_{i=1}^{n} |B_i| - m$. For a connected graph, we must have

$$\sum_{i=1}^{n} |B_i| \geq n + m - 1.$$

Similarly, for the connected graph to contain a cycle, we must have

$$\sum_{i=1}^{n} |B_i| \geq n + m.$$

Therefore, for a connected graph contains no cycle, we have

$$\sum_{i=1}^{n} |B_i| = n + m - 1.$$

To satisfy the pareto optimality, from the first order condition, we need to check whether we can find a set of $\delta_{z_{(i,j)}}$ such that

$$\sum_{i,j} \frac{\partial s_i}{\partial z_{(i,j)}} \cdot \delta_{z_{(i,j)}} \geq 0 \quad \forall \; i \tag{13}$$

and

$$\sum_{i,j} \frac{\partial s_i}{\partial z_{(i,j)}} \cdot \delta_{z_{(i,j)}} > 0 \quad \text{for some } i. \tag{14}$$

If we can find a set of $\delta_{z_{(i,j)}}$ satisfying the above equation, the set of transmission probability $z_{(i,j)}$ cannot be pareto optimal transmission probabilities. Since there is a total of n users, we will have n linear equations. The variables in these linear equations are the small variation $\delta_{z_{(i,j)}}$. The number of variable is the degree of freedom in varying the transmission probabilities, which is $\sum_{i=1}^{n} |B_i| - m$. For a graph with cycle, we have $\sum_{i=1}^{n} |B_i| - m \geq n$. In this case, since we have n positive linear equations and $k \geq n$ variables, we can certainly find a set of $\delta_{z_{(i,j)}}$ of dimension k that satisfies Eq.(13) and Eq.(14).

Now suppose that a connected graph G satisfies both conditions of this theorem. If we increase the transmission probability of one link, we must also decrease the transmission probability of some other link due to the constraint that $\sum_i z_{(i,j)} = 1 \forall j$ and the fact that there is no cycle in the graph. Hence, the resulting throughput is pareto optimal. ■

In the case that there is a channel state $q_{(i,j)}$ associated with each channel, the above theorem provides a necessary condition for obtaining the Pareto optimal throughput region.

Now, let's consider a network consisting of only two satellites for simplicity. We investigate how these two satellites can each set their own prices, p_1 and p_2 respectively, to achieve Pareto optimal throughput region. The objective for user i is to maximize the following function:

$$U_i(s_{(i,1)} \cdot q_{(i,1)} + s_{(i,2)} \cdot q_{(i,2)}) - s_{(i,1)} p_1 - s_{(i,2)} p_2 \tag{15}$$

where

$$s_{(i,k)} = z_{(i,k)} \cdot \prod_{j \in A_k} (1 - z_{(j,k)}) \tag{16}$$

The term $s_{(i,1)} \cdot q_{(i,1)} + s_{(i,2)} \cdot q_{(i,2)}$ denotes the throughput of user i. We first assume that the transmission probability $z_{(i,1)}$ is independent of the transmission probability $z_{(i,2)}$ for user i. That is, user i can transmit to both satellites during the same time slot. The case that user i can send to only one satellite during a time slot is the same as the case which allows simultaneous transmission when $z_{(i,1)} + z_{(i,2)} \leq 1$. To increase the utility function in Eq.(15) by a small amount, user i can increase either $s_{(i,1)}$ or $s_{(i,2)}$. The marginal costs are $p_1/q_{(i,1)}$

and $p_2/q_{(i,2)}$ respectively. Thus, if $p_1/q_{(i,1)}$ is strictly less than $p_2/q_{(i,2)}$, user i will transmit to satellite 1 only. To maximize Eq.(15), the following equation must be satisfied:

$$\frac{\partial}{\partial s_{(i,1)}} U_i(s_{(i,1)} \cdot q_{(i,1)}) = \frac{p_1}{q_{(i,1)}} \tag{17}$$

Likewise, if $p_2/q_{(i,2)}$ is strictly less than $P_1/q_{(i,1)}$, user i will transmit to satellite 2 only. The following equations must be satisfied to maximize Eq.(15):

$$\frac{\partial}{\partial s_{(i,2)}} U_i(s_{(i,2)} \cdot q_{(i,2)}) = \frac{p_2}{q_{(i,2)}} \tag{18}$$

In the case that $p_1/q_{(i,1)} = p_2/q_{(i,2)}$, user i can transmit to either satellite, and the following equation holds:

$$U_i'(s_{(i,1)} \cdot q_{(i,1)} + s_{(i,2)} \cdot q_{(i,2)}) = \frac{p_1}{q_{(i,1)}} = \frac{p_2}{q_{(i,2)}} \tag{19}$$

Following the single satellite case, in the m satellites case we say a set $(s_{(1,1)}, s_{(1,2)}, \cdots, s_{(n,1)}, s_{(n,2)}, p_1, \cdots, p_m)$ with $s_{(i,k)}$ defined in Eq.(16) and $p_j \geq 0$ for $j = 1, \cdots, m$ is a *competitive equilibrium* if users maximize their payoff, and satellites set a price vector $(p_1, \cdots, p_m)$ so as to make $\sum_{i=1}^{n} z_{(i,j)} = 1$ for $j \in \{1, \cdots, m\}$. To test for equilibrium, given the price set by the satellite, we ask whether a particular user has the desire to change its transmission strategy. That is, a user will take the price as fixed, and decide the optimal action based on this price. We also make the following channel diversity assumptions:

1) There does not exist i and j such that $q_{(i,k)} = q_{(j,k)}$ for all k.
2) $q_{(i,k_1)} \neq q_{(i,k_2)}$ for all k_1 and k_2.

Assumption 1 implies that no two users have identical channel to both satellites. Assumption 2 implies that, for each user, the channel states to different satellites are different. The following theorem shows the existence of a competitive equilibrium in a multi-satellites environment.

Theorem 4: With the channel diversity assumption, given that each user's utility function U_i is concave, strictly increasing, and continuously differentiable, there exists a unique competitive equilibrium in this network with n users and m satellites.

Proof: We first consider the case that $m = 2$ and $n = 3$ for illustration. Because $V_i = U_i'$ is strictly decreasing, as the price p_j increases, the desired throughput for each user also decreases, moving closer to a feasible point (i.e., $\sum_{i \in A_j} z_{(i,j)} = 1$). Eventually, the desired throughput meet a feasible point. This part is the same as the single satellite part. However, as one satellite decreases or increases the price p_1, it may cause a user, say user 1, to start transmitting to the other satellite. This happens when $p_1/q_{(1,1)} = p_2/q_{(1,2)}$. If the user's desired throughput is r, it can choose $s_{(1,1)}$ and $s_{(1,2)}$ such that $s_{(1,1)} \cdot q_{(1,1)} + s_{(1,2)} \cdot q_{(1,2)} = r$. For fixed r, $s_{(1,1)}$ is a continuous function of $s_{(1,2)}$. If p_1 is too high, user 1 could start transmitting to satellite 2, thus forcing satellite 2 to change its price to meet the Pareto operating point. In case that two prices are decoupled, we have two desired operating points, one for each satellite, with two control parameters. In case that prices are coupled, we can control one price and one transmitting probability to get the two desired operating points. In both cases, we have two control parameters, thus are able to get to the equilibrium point.

For the general n-users case, we know that user i should send to satellite 2 if the following holds:

$$\frac{q_{(i,1)}}{q_{(i,2)}} < \frac{p_1}{p_2}.$$

Also, from our channel diversity assumption, there can be only one user such that

$$\frac{q_{(i,1)}}{q_{(i,2)}} = \frac{p_1}{p_2}.$$

This implies that at most one user can transmit to both satellites.

Now, we show that the equilibrium is indeed unique. Assuming there exists two equilibrium points: $(s_{(1,1)}, \cdots, s_{(n,2)}, p_1, p_2)$ and $(s'_{(1,1)}, \cdots, s'_{(n,2)}, p'_1, p'_2)$, we will show that there is a contradiction. At a equilibrium point, we know that two scenarios are possible: 1) no user transmits to both satellites; 2) there is exactly one user transmitting to both satellites. First, we consider the case that no user transmits to both satellites at both equilibriums. Without loss of generality, we number users such that the following order holds:

$$\frac{q_{(1,1)}}{q_{(1,2)}} > \frac{q_{(2,1)}}{q_{(2,2)}} > \cdots > \frac{q_{(n,1)}}{q_{(n,2)}}.$$

If both equilibrium points have the same graphical representation (i.e., user transmits to the same satellite in both equilibrium), the two equilibrium points have to be identical from the derivation in the single satellite case. Let's now consider the case that two equilibriums points have different graphical representations. Specifically, users 1 to k transmit to satellite one, and users $k+1$ to n transmit to satellite two for the equilibrium $(s_{(1,1)}, \cdots, s_{(k,1)}, s_{(k+1,2)}, \cdots, s_{(n,2)}, p_1, p_2)$. For the equilibrium $(s'_{(1,1)}, \cdots, s'_{(l,1)}, s'_{(l+1,2)}, \cdots, s'_{(n,2)}, p'_1, p'_2)$, users 1 to l transmit to satellite one and users $l+1$ to n transmit to the second satellite, where $l > k$. Since $l > k$, we have

$$\frac{p_1}{p_2} > \frac{p'_1}{p'_2}.$$

If $p_1 < p'_1$, we have $p_2 < p'_2$ from the above equation. With price p_2, the desired throughput at satellite two is $(s_{(k+1,2)}, \cdots, s_{(n,2)})$. Similarly, with price p'_2, the desired throughput at satellite two is $(s'_{(l+1,2)}, \cdots, s'_{(n,2)})$. Since $p_2 < p'_2$, we have the desired throughput $s_{(i,2)} > s'_{(i,2)}$ for all $i \in \{l+1, \cdots, n\}$. We know that $(s'_{(l+1,2)}, \cdots, s'_{(n,2)}, p'_2)$ is at equilibrium in satellite two. Therefore, $(s_{(k+1,2)}, \cdots, s_{(l,2)}, \cdots, s_{(n,2)}, p_2)$ cannot be in equilibrium. That is, there does not exist $(z_{(k+1,2)}, \cdots, z_{(n,2)})$ such that

$$s_{(i,2)} = z_{(i,2)} \prod_{j \neq i, j \in A_2} (1 - z_{(j,2)}) \quad \forall i = \{k+1, \cdots, n\}$$

and $\sum_{i=k+1}^{n} z_{(i,2)} = 1$. Hence, we have a contradiction here. If $p_1 > p'_1$, we get a similar contradiction.

Thus far, we have discussed the case that no user transmits to both satellites for both equilibrium points. If there is exactly one user transmits to both satellites for the two equilibrium points, a similar contradiction can be derived. For the other cases (i.e., one user transmits to both satellites in one equilibrium while no user transmits to both satellites in the other equilibrium), we can get similar contradiction. Therefore, the equilibrium is unique.

■

Corollary 1: The equilibrium throughput obtained using the pricing scheme is Pareto optimal.

Proof: From the proof of the Theorem 3, we see there cannot be any cycle in the graph even when users having different channel qualities. Let the set of users transmitting to satellite one and satellite two be denoted as A_1 and A_2 respectively. Since $q_{(i,1)}/q_{(i,2)} \geq q_{(j,1)}/q_{(j,2)}$ for all $i \in A_1$ and $j \in A_2$, thus switching the receiving satellite cannot expand the throughput region. Hence, the equilibrium throughput is Pareto optimal. ■

IV. Example

In this example, we consider a communication scenario with two satellites and three users and try to obtain an exact expression of the equilibrium point. The channel conditions are given as follows:

$$\begin{aligned} q_{(1,1)} &= 0.8, \quad q_{(2,1)} = 0.5, \quad q_{(3,1)} = 0.5 \\ q_{(1,2)} &= 0.3, \quad q_{(2,2)} = 0.4, \quad q_{(3,2)} = 0.7 \end{aligned}$$

The utility function for user i is given by the following:

$$U_i(x) = a_i \cdot x^{b_i} \tag{20}$$

where $a_1 = 1, a_2 = 2, a_3 = 1.5$ and $b_1 = b_2 = b_3 = 0.5$. We first make the assumption that user 2 transmits to both satellites; user 1 only transmits to satellite 1 while user 3 transmits to satellite 2 only. If we can find an equilibrium, we know that our assumption is correct. Therefore, the following equations must hold:

$$\begin{aligned} U_1'(s_{(1,1)} \cdot q_{(1,1)}) &= \frac{p_1}{q_{(1,1)}} \\ U_2'(s_{(2,1)} \cdot q_{(2,1)} + s_{(2,2)} \cdot q_{(2,2)}) &= \frac{p_1}{q_{(2,1)}} = \frac{p_2}{q_{(2,2)}} \\ U_3'(s_{(3,2)} \cdot q_{(3,2)}) &= \frac{p_2}{q_{(3,2)}} \end{aligned} \tag{21}$$

We have the following after simplification:

$$\begin{aligned} s_{(1,1)} \cdot 0.8 &= \alpha_1 \cdot p_1^{\frac{1}{b_1-1}} \\ s_{(2,1)} \cdot 0.5 + s_{(2,2)} \cdot 0.4 &= \alpha_2 \cdot p_2^{\frac{1}{b_2-1}} \\ s_{(3,2)} \cdot 0.7 &= \alpha_3 \cdot p_2^{\frac{1}{b_3-1}} \end{aligned} \tag{22}$$

where

$$\begin{aligned} \alpha_1 &= \left(\frac{1}{q_{(1,1)}a_1b_1}\right)^{\frac{1}{b_1-1}} \\ \alpha_2 &= \left(\frac{1}{q_{(2,2)}a_2b_2}\right)^{\frac{1}{b_2-1}} \\ \alpha_3 &= \left(\frac{1}{q_{(3,2)}a_3b_3}\right)^{\frac{1}{b_3-1}} \end{aligned}$$

The set of $s_{(1,1)}$ and $s_{(2,1)}$ such that $z_{(1,1)} + z_{(2,1)} = 1$ are related by the following equation:

$$s_{(1,1)} = (1 - \sqrt{s_{(2,1)}})^2. \tag{23}$$

Similar relation holds for $s_{(2,2)}$ and $s_{(3,2)}$. Hence, we have the following equation:

$$(1 - \sqrt{s_{(1,1)}})^2 \cdot 0.5 + (1 - \sqrt{s_{(3,2)}})^2 \cdot 0.4 = \alpha_2 \cdot p_2^{\frac{1}{b_2-1}} \tag{24}$$

Since user 2 is transmitting to both satellites, the equation $p_1/q_{(2,1)} = p_2/q_{(2,2)}$ holds. We can write $s_{(1,1)}$ and $s_{(3,2)}$ as a function of p_2 only. Substituting $s_{(1,1)}$ and $s_{(3,2)}$ into Eq. (24), we can solve for p_2. From p_2, we can get the unique competitive equilibrium for this example, which is given below:

$$\begin{aligned} p_1 &= 1.097, \quad s_{(1,1)} = 0.166, \quad s_{(2,2)} = 0.081 \\ p_2 &= 0.877, \quad s_{(2,1)} = 0.351, \quad s_{(3,2)} = 0.511 \end{aligned}$$

The transmission probability is given as the following:

$$\begin{aligned} z_{(1,1)} &= 0.407, \quad z_{(2,2)} = 0.285 \\ z_{(2,1)} &= 0.593, \quad z_{(3,2)} = 0.715 \end{aligned}$$

V. Conclusion

In this paper, we investigate how to better utilize the multiple channels available in a satellite network. Specifically, we use pricing as a mechanism to control users' transmission probabilities and exploit different channel qualities to coordinate transmission among users. Hence, the throughput performance of the system is improved. We also characterize the Pareto optimal throughput region for both single satellite network and multiple satellites network. We show that users' throughput is Pareto optimal at the equilibrium price. The characterization of the Pareto optimal throughput region for multiple channels with time varying channel states can be a possible direction for the future research.

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