# Impact of Interference and Channel Assignment on Blocking Probability in Wireless Networks

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***Abstract*— In a wireless network, blocking probability of connection-oriented traffic depends on the transmission radius of the nodes and the channel assignment scheme. In this work we study these two aspects for simple wireless networks. Specifically, we present blocking probability analysis for a wireless line network with random channel allocation to the incoming calls. Based on the expressions derived from our model, we then study the effect of transmission radius of the nodes on blocking probability. We show that for a line network using a larger transmission radius substantially reduces the blocking probability of the calls; while for a more dense grid network using a smaller transmission radius is better. Finally, we present a novel non-rearranging channel assignment algorithm for multi-hop calls in a general network. Our algorithm significantly reduces call blocking probability when compared to other algorithms.**

## I. Introduction

A multi-hop wireless network is a cooperative network where data streams may be transmitted over multiple wireless links to reach the destination. The network structure depends on the transmission radius of the nodes and can be adjusted by varying the transmission power. Recently, there has been substantial research work on Quality of Service (QoS) guarantees for ad-hoc networks with various performance metrics [1], [2], [3]. In this work, we consider blocking probability as a performance measure and explore its dependence on the transmission radius of the nodes. In the rest of the paper a channel refers to a time slot in a TDMA system and a distinct frequency band for a FDMA system.

This paper addresses the following issues. We first present an exact blocking probability analysis for a single channel wireless line network. We then construct a model to compute the blocking probability in the multiple channel case for the random channel assignment policy. Using our model we study the following tradeoff: A smaller transmission radius of the nodes incurs less interference on each hop but the calls have to hop through many nodes to reach the destination. As the nodes not only serve the external call requests but also the internal requests from other nodes, multi-hopping increases the internal load in the network. On the other hand a larger transmission radius reduces the number of hops of a call but increases the interference constraints at each hop. We examine this tradeoff between increased interference and increased internal load in relation to its effect on blocking probability. For analytical simplicity we focus on two network topologies: the line and the grid network. We show that in a line network larger transmission radius can in fact substantially reduce the blocking probability of the calls. However, for a grid network with an underlying more dense node topology we show that it is more desirable to use smaller transmission radius. This suggests that for networks with low node density the effect of increased interference due to a larger radius is offset by a reduction in the total effective load experienced by the nodes. Finally, we address the issue of dynamic channel assignment to the incoming calls given the network link structure (i.e. given the transmission radii of the nodes). Note that for multi-hop calls a channel must be allocated on each hop such that the wireless constraints are satisfied. Here, we develop a novel non-rearranging algorithm (Section V) and compare its performance with other algorithms namely the rearrangement, random and first fit algorithms.

The problem of dynamic channel assignment has been extensively considered in the context of cellular networks ([11], [12], [13] and references therein). However there are significant differences between the two networks. For example, in a cellular network the communication is with the nearest base-station over a single wireless link; whereas in a multi-hop wireless network calls hop through various links to reach the destination. This imposes additional complexity as non-conflicting channels must be allocated on the wireless links along the source-destination path. Another difference between the two networks is that a cellular network has a regular structure which makes the set of interfering cells fixed; whereas in a multi-hop wireless network the set of interfering nodes depends on the node topology and their transmission radii.

Steady state blocking probability as a performance metric for connection-oriented traffic has been widely used by researchers in studying various other networks. Some of this work includes [16], [17] in all-optical networks; [7], [8], [9], in circuit-switched networks; and [10], [13], in cellular networks. Blocking probability relates to the system throughput in the following sense. Let the system throughput be defined as the set of arrival rates such that the steady state average blocking probability is less than some threshold $\beta$. Consider two transmission and channel assignment schemes such that for all arrival rates Scheme 1 has a lower steady state average blocking probability than Scheme 2. This implies that Scheme 1 has a larger throughput than Scheme 2. Thus in this sense reducing blocking probability leads to an increase in the supportable load for a given threshold $\beta$.

The rest of the paper is organized as follows. In Section II, we describe the wireless interference model. Section III presents blocking probability analysis for a line network. In Section IV, we consider the effect of transmission radius on blocking probability in a line and a grid network. In Section V, we present channel assignment algorithms for a wireless network and simulation results that compare their performance. Finally, Section VI concludes the paper.

## II. Wireless Model

We assume a disk model of interference and define the transmission radius of a node, say $T$, as the radius of a circle centered at $T$ outside which the signal due to node $T's$ transmission is negligible. Within the transmission radius of node $T$ we assume complete interference of the signal transmitted by $T$ with other ongoing transmissions and no interference outside the circle. We say that a direct wireless link exists between any two nodes if they lie within transmission radius of each other. All nodes in the network have an omnidirectional antenna and transmit with constant power thereby having equal transmission radius. We investigate a wireless network whose topology does not change over time.

For any two nodes $T$ and $R$, we say that node $R$ is a neighbor of node $T$ if $R$ lies within the transmission circle of $T$. Since the nodes have equal transmission radius $T$ is also a neighbor of $R$. Let the set consisting of neighbors of $T$ and $R$ be denoted as $\mathcal{N}_T$ and $\mathcal{N}_R$ respectively. Consider the data transfer on a single link, $T \rightarrow R$, in channel $\gamma$. For this call to be successfully serviced, the following criteria needs to be satisfied.

1) Nodes $T$ and $R$ must not be involved in any other call transmission/reception in channel $\gamma$. This criterion ensures that a node cannot simultaneously serve two different calls in the same channel.
2) Neighbors of $T$ ($P \in \mathcal{N}_T$, excluding $R$) must not *receive* from any other node in channel $\gamma$. Otherwise the transmission from $T$ will interfere at $P$.
3) Neighbors of $R$ ($Q \in \mathcal{N}_R$, excluding $T$) must not *transmit* to any other node in channel $\gamma$. Otherwise the transmission from $Q$ will interfere at $R$.

The above "idealized" model approximates realistic interference assumptions and is commonly used in the study of network layer issues in wireless networks [1], [14], [15].

We focus on connection-oriented traffic which models the QoS calls in the network and also permits an understanding of the trade-offs in a shared resource environment. A connection-oriented call requires a dedicated channel on each hop along the path. These channels are held up while the call is in progress and simultaneously released at the end of the call. We assume that all calls require a *single* channel for service on each hop. For simplicity of mathematical arguments and length considerations, we consider only bi-directional calls in this paper. A call between any two nodes $T, R$ is defined as bi-directional if there is data transfer in both directions $T \rightarrow R$ and $R \rightarrow T$. The reader is referred to [4] for an analysis of uni-directional calls that involve data transfer only in one direction. For a bi-directional call a node can both transmit and receive data in the reserved channel and thus all the three conditions stated earlier must be satisfied. A node is labelled inactive in channel $\gamma$ if it is not involved in transmission/reception in that channel and active otherwise. With this notation, we get the following spatial channel re-use constraint. For a bi-directional call on link $T \leftrightarrow R$ to be successful in channel $\gamma$, neighbors of node $T$ excluding $R$ and neighbors of node $R$ excluding $T$ must be inactive.

Figure 1 illustrates a single hop bi-directional data transfer between nodes $T$ and $R$ in channel $\gamma$. Nodes $T$ and $R$ cannot service any other call in channel $\gamma$. Neighbors of node $T$ ($T_1$, $T_2$) and neighbors of node $R$ ($R_1, R_2$) must be inactive while call $T \leftrightarrow R$ is active. In the figure all data transfers marked 'X' must not take place for call $T \leftrightarrow R$ to be successful.

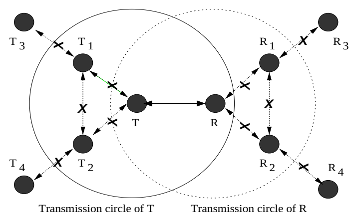


Fig. 1. Interference model for a bi-directional transmission ($T \leftrightarrow R$).

## III. Blocking Probability Analysis in a Wireless Line Network

In this section we develop an analysis for the blocking probability of a call in a line network. The expressions derived here form the basis for the study in Section IV where we consider the effect of transmission radius on blocking probability. In addition to facilitating elegant solutions, a line network is an important network in practice and serves as a good starting point in understanding network tradeoffs. We begin with the analysis of a single channel network for which an exact solution is obtained and then extend it to multiple channels in the next subsection by making simplifying approximations.

### A. Single Channel

Consider a wireless line network with nodes located unit distance apart at positions $x = -m, -m+1, ..., m$. We label these nodes as $X_{-m}, X_{-m+1}, ..., X_m$. Let there be a single channel with each node having a transmission radius of $r$ ($r \geq 1$, $r \in Z^+$, a positive integer) and let all calls in the network be $r$ units long (i.e. between nodes $X_k$ and $X_{k+r}$, $k \in Z$). The calls are single hop as each node can communicate directly with a node $r$ units apart. Calls $X_k \leftrightarrow X_{k+r}, \forall k$ arrive according to an independent Poisson process of rate $\lambda$. The call holding period of each call is independent and identically distributed according to an Exponential distribution with mean $1/\mu$ [1]. If a call cannot be accepted then it is dropped otherwise it occupies the channel while in progress. We refer to this network as a wireless line network with radius $r$ or WLN-r for short.

As an example illustrating the constraints on the successful service of a call consider the WLN-1 ($r = 1$) network. If $C_k$ denotes a call between nodes $X_k$ and $X_{k+1}$ then following

[1]The result holds even for general service distributions with mean $1/\mu$ as the product form solution for the steady state distribution still holds.

Section II call $C_k$ can be successfully serviced if node $X_{k-1}$ (neighbor of $X_k$) and node $X_{k+2}$ (neighbor of $X_{k+1}$) are inactive. This implies that calls $C_{k-2}$, $C_{k-1}$, $C_{k+1}$, $C_{k+2}$ must be inactive (Figure 2). The constraints for the WLN-r network are more involved and follow along similar lines.

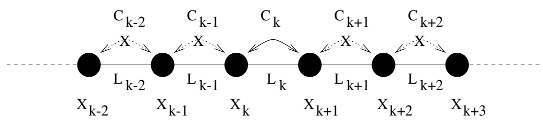


Fig. 2. Constraints representing the simultaneous service of calls.

*Theorem 1:* The blocking probability of a call in a WLN-r, $r \geq 1, r \in Z^+$, network with the length of the line network tending to infinity and $\nu = \lambda/\mu$ is,

$$P_B = 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}} \tag{1}$$

where $x$ is the unique root in (0,1] of $\nu x^{2r+1} + x = 1$

*Proof:* The proof is omitted for length considerations but can be found in [4]. ■

The limiting argument in the derivation of Theorem 1 eliminates edge effects and yields the above simple expression that very closely approximates blocking probability of finite length line networks (See [4] for more details).

It is worthwhile comparing Equation 1 with the standard M/M/1/1 blocking probability expression [6]. This gives useful insights into the blocking probability behavior of a link $X_k \leftrightarrow X_{k+r}$ in a shared linear environment as compared to an identical link when it is isolated. The blocking probability of a M/M/1/1 system with load $\tilde{\nu}$ is given by,

$$P_B = \frac{\tilde{\nu}}{1 + \tilde{\nu}} \tag{2}$$

Let $\tilde{\nu}$ be such that the $P_B$ in Equation 2 equals that obtained from Equation 1 for load $\nu$. Thus $\tilde{\nu}$ captures the effects of a shared channel and is the load on an isolated link of WLN-r that would have the same blocking probability as experienced within the line network with symmetrical load $\nu$. As we see later, $\tilde{\nu}$ plays an important role in the analysis for the multiple channel case.

$$\frac{\tilde{\nu}}{1+\tilde{\nu}} = 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}} \tag{3}$$

$$\tilde{\nu} = \frac{1 + (2r\nu - 1)x^{2r+1}}{x^{2r+1}}, \nu x^{2r+1} + x = 1 \tag{4}$$

Define the *effective load factor* $g$ as, $g = \tilde{\nu}/\nu$; then $g$ can be expressed as,

$$g = \frac{1 + (2r\nu - 1)x^{2r+1}}{\nu x^{2r+1}} \tag{5}$$

The low load and the high load regimes can be studied by taking the limit $\nu \to 0$ and $\nu \to \infty$ respectively in Eqn 5. This yields $\lim_{\nu \to 0} g = 4r + 1$ and $\lim_{\nu \to \infty} g = 2r + 1$. Thus at low loads $\tilde{\nu} \approx (4r+1)\nu$ and at high loads $\tilde{\nu} \approx (2r+1)\nu$.

### B. Multiple Channels

In this section we extend the analysis of WLN-r to the case of multiple channels. We consider the random policy for assigning channels to the incoming calls. In this policy the new channel request on a link is assigned a channel randomly from among the free channels on that link. Free channels refer to those channels such that the acceptance of a call in those channels does not violate the wireless constraints. The random policy is non-rearranging (see Section V) and easy to implement practically. However its exact analysis is complicated by the fact that to make a channel allocation decision we must have knowledge of the channels already occupied by the ongoing calls. This makes the state space for this system very large and an analysis of the steady state probability distribution intractable. Interestingly, since the random policy does not differentiate between the channels we can construct an approximate model of the system based on the effective load concept. We show later that the model closely predicts the blocking probability values obtained from simulation results.

Consider the link $L_k$ ($X_k \leftrightarrow X_{k+r}$) of the line network. For now, assume that there is only a single channel $\gamma$ in the network and denote its state on link $L_k$ as $S_k$. We model $S_k$ as a three state process, the free state ($\mathcal{F}$), the busy state ($\mathcal{B}u$) and the blocked state ($\mathcal{B}l$) as shown in Figure 3. The link $L_k$ is said to be in the blocked state if the channel is occupied by a call on an interfering link. It is in the busy state if there is a call in progress. Let $Y_{\mathcal{F}\to\mathcal{B}l}$ be the random variable denoting the amount of time $L_k$ spends in state $\mathcal{F}$ before making a transition to state $\mathcal{B}l$.

Suppose that the present state $S_k = \mathcal{F}$. If we knew the state of channel $\gamma$ on other links then $Y_{\mathcal{F}\to\mathcal{B}l}$ is an exponential random variable with rate equal to the sum of the rates of the competing arrival processes on interfering links. The number of such competing processes will vary depending on the present state of other links. Thus, conditioned on the state of the network $Y_{\mathcal{F}\to\mathcal{B}l}$ is exponentially distributed. However, unconditionally $Y_{\mathcal{F}\to\mathcal{B}l}$ has a general distribution. Based on this observation, we model the unconditioned $Y_{\mathcal{F}\to\mathcal{B}l}$ by approximating it as an exponential random variable with rate $\lambda'$. The random variable $Y_{\mathcal{B}l\to\mathcal{F}}$ has a general distribution with mean $1/\mu'$. Figure 3 shows the transition rates of $S_k$. The single

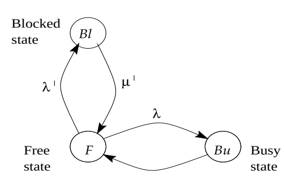


Fig. 3. Three state Markov process model of a channel on a link.

channel blocking probability $P_B$ can be computed by solving the detailed balance equation (see [5]) of the three state Markov process. Let $\nu' = \lambda'/\mu'$ and $\nu = \lambda/\mu$; then,

$$\tilde{\nu} = \nu' + \nu = \frac{P_B}{1 - P_B} \tag{6}$$

Thus, we can interpret the effective load $\tilde{\nu}$ as consisting of two components; the external load $\nu$ and the load seen by the link $\nu'$ that makes the channel blocked. Using the expression for $P_B$ from Equation 1 we get,

$$\tilde{\nu} = \nu' + \nu = \frac{1 + (2r\nu - 1)x^{2r+1}}{x^{2r+1}} \quad (7)$$

Generalizing to the multiple channel case, define the state of a link as $X(t) \equiv (X_{bu}(t), X_{bl}(t))$ where $X_{bu}$ is the number of busy channels and $X_{bl}$ the number of blocked channels on that link at time $t$. Let the total number of channels available in the network be $p$. At any time $t$, the state $X(t) \equiv (X_{bu}(t), X_{bl}(t))$ must satisfy $X_{bu}(t) + X_{bl}(t) \leq p$. Following the single channel process and the fact that the random policy does not differentiate among the channels we approximate the transition rates among the states of the process $X(t)$ as shown in Figure 4.

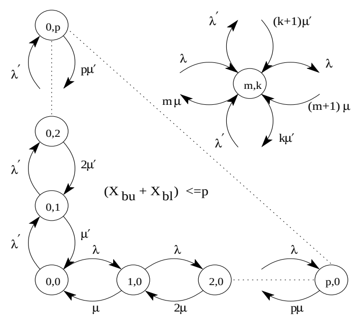


Fig. 4. State transition diagram for the random assignment policy.

Let $\pi(i,j)$ denote the steady state probability that $X$ takes value $(i,j)$. Then the steady state probability of blocking $P_B^{rand}$ is equal to $\sum_{i+j=p} \pi(i,j)$. Solving the detailed balance equations (see [5]) we get,

$$P_B^{rand} = \frac{\frac{\tilde{\nu}^p}{p!}}{1 + \tilde{\nu} + \frac{\tilde{\nu}^2}{2!} + ... + \frac{\tilde{\nu}^p}{p!}} \quad (8)$$

Denoting $E(\nu, p)$ as the Erlang B formula ([6]) for load $\nu$ and $p$ servers, we have,

$$P_B^{rand} = E(\tilde{\nu}, p) \quad (9)$$

Plots comparing the predicted and the actual simulation results for 20 channels with $r = 2$ and $r = 10$ are shown in Figure 5. The length of the line network simulated is 50 nodes and blocking probability is computed for the center call to minimize edge effects. As seen from the figure the theoretical values closely predict the simulation results even for large $r$.

## IV. Effect of Transmission Radius on Blocking Probability

This section explores the dependence of blocking probability on transmission radius. The primary motivation is the following tradeoff: If the nodes have a smaller transmission radius then the interference constraints on each hop are fewer but the calls hop through many links to reach the destination. As the nodes not only serve the external call requests but also the internal requests from other nodes, multi-hopping increases the internal load in the system. On the other hand a larger transmission radius reduces the number of hops of a call but increases the interference constraints at each hop. The effect of this tradeoff between increased interference and increased internal load is considered next. We will restrict our attention to the line and the grid network.

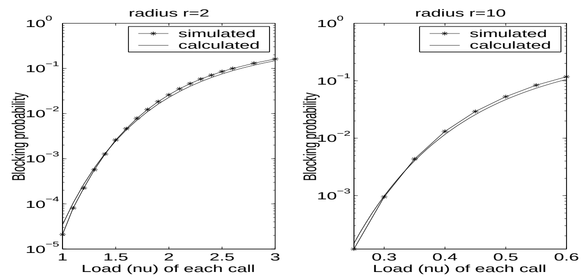


Fig. 5. Comparison of theoretical and simulated values for $r = 2$ and $r = 10$ with 20 channels.

### A. Line Network

We begin by considering the following simple but non-trivial example that lends itself to an exact analysis and also clearly highlights the problem. Consider a line network with *two channels* and all calls of *length two*; i.e. between nodes that are two units apart. The arrival process of each call is an independent Poisson process of rate $\lambda$ and the holding time is i.i.d with mean $1/\mu$. Consider two schemes, the first in which nodes have a transmission radius of unity ($r = 1$) and thus the calls are two hops long. Each call requires a distinct channel on each hop as adjacent links are interfering. The channels are assigned using the rearrangement channel assignment policy (Section V) as it uses the channel resources optimally.

The second scheme considers nodes with a transmission radius of two units ($r = 2$) and hence all calls are single hop. Here, we consider a sub-optimal channel assignment policy that selects a channel randomly from the two channels for each new arriving call. If the channel is free (non-blocked and non-busy) then it is allocated otherwise the incoming call is dropped. The policy clearly under utilizes the channels as it rejects a call if the randomly selected channel is not free without considering the state of the other channel. It performs a simple random splitting of the incoming arrival stream into two independent Poisson processes of rate $\lambda/2$ applied to each channel.

The following theorem shows that even with this very inefficient random policy the second scheme ($r = 2$) has a lower blocking probability as compared to the unit radius case for all finite load $\nu$. Thus, for any fixed blocking probability threshold $\beta$ the supportable load $\nu$ is higher for the second scheme than the first one. This increased throughput performance comes at the expense of higher transmission energy. Thus, the result highlights that in networks with low node density increased transmission power can lead to better network performance.

*Theorem 2:* The blocking probability for case $r = 2$ is lower than the blocking probability for case $r = 1$ for all load $\nu = \lambda/\mu$ satisfying $0 < \nu < \infty$.

*Proof:* The proof is omitted for brevity and can be found in [4]. ∎

Next, we draw a similar conclusion in a more general setup.

Consider a line network with $p$ channels and all calls of length $k > 1$ ($k = 1$ is the trivial case). The traffic model is identical to that considered earlier. We consider the random channel allocation policy and use the blocking probability expression derived in Section III-B. We compare the following transmission schemes.

*Scheme 1:* The transmission radius of the nodes is $s(< k)$ and hence each call is $n = \frac{k}{s}$ hops long. For technical reasons we take only those $s$ for which $n$ is an integer. We consider the low blocking regime as it is practically significant and also justifies our simplifying approximations. In the low blocking probability regime almost all calls get served and the average load on each link ($X_m \leftrightarrow X_{m+s}, \forall m$) is $\approx n\nu$ which is the sum of the loads of all calls hopping through that link. Assuming this load to be Poisson the probability $P_L$ that none of the channels on a link are free can be computed by considering this as an equivalent system with load $n\nu$ on each link and $r = s$. Using Equations 7 and 9 we get,

$$\tilde{\nu_1} = \frac{1 + (2k\nu - 1)x^{2s+1}}{x^{2s+1}}\ , \ (n\nu x^{2s+1} + x = 1) \qquad (10)$$

$$P_L = E(\tilde{\nu_1}, p) \qquad (11)$$

For a $n$ hop call to be served it must not be blocked on any hop along the entire length of the path. Therefore, the blocking probability of the call is greater than its blocking probability on the first hop. As the latter value is $P_L$ we get,

$$P_B^1 > P_L = E(\tilde{\nu_1}, p) \qquad (12)$$

*Scheme 2 :* The transmission radius of the nodes is $k$ and hence each call is single hop. For this system, Theorem 1 gives the exact blocking probability in the single channel case. The blocking probability for multiple channels follows from Section III-B. Thus we have,

$$\tilde{\nu_2} = \frac{1 + (2k\nu - 1)x^{2k+1}}{x^{2k+1}}\ , \ (\nu x^{2k+1} + x = 1) \qquad (13)$$

$$P_B^2 = E(\tilde{\nu_2}, p) \qquad (14)$$

Observe that the tradeoff can be understood by examining the polynomial equations in $x$ for the two schemes (Eqns 10 and 13). In Scheme 1 we have a higher load $n\nu$ but a smaller exponent $2s+1$ of $x$ due to less interference at each hop; while in Scheme 2 the load is $\nu$ but a higher exponent $2k + 1$ of $x$ due to more interference. It can be shown that $P_B^2 < P_L < P_B^1, \forall \nu > 0, k > 1$, $k$ integer. The proof is omitted for brevity.

The intuition behind this result is that for a line network with a sparse node topology the blocking probability increase due to a larger set of interfering nodes (larger radius) is smaller as compared to an increase due to larger effective link load caused by multi-hopping.

Figure 6 presents simulation plots verifying this claim. The blocking probability of the center call is computed in each simulation to reduce edge effects. The first plot has all calls of length 3 with two schemes of radius 1 and 3. The second plot has all calls of length 6 with radius 1, 2, 3 and 6. Note that the reduction in blocking probability by using a larger transmission radius is a few orders of magnitude and this difference increases with the length of the calls.

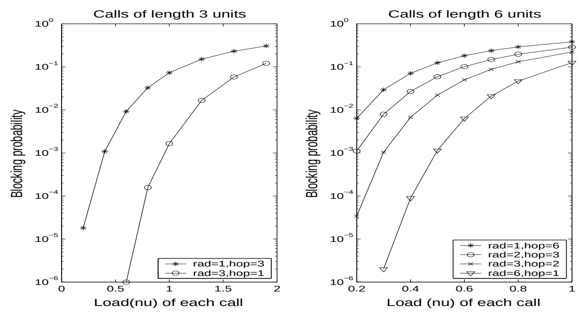


Fig. 6. Line network with calls of length 3 and 6 and 20 channels.

### B. Grid Network

We observed in Section III-A that as $\nu \to 0$, $\tilde{\nu} \approx (4r + 1)\nu$. A similar observation can be shown to hold for a single channel general network as well by making the low load approximation i.e. as $\nu \to 0$, $\tilde{\nu} \approx \alpha\nu$ where $\alpha$ = total number of interfering calls + 1 (see [4]). Following Section III-B, the blocking probability at low loads for multiple channels and random channel allocation policy is $P_B = E(\tilde{\nu}, p)$.

Consider an infinite grid network (to avoid edge effects) with all calls of length 3 and load $\nu$. Calls are between nodes $\{x, y\} \to \{x + 3, y\}$ and $\{x, y\} \to \{x, y + 3\}$. The approach can be easily generalized to longer length calls. In the first scenario, transmission radius of each node is 3 and hence all calls are single hop. Here, each link has 134 interfering links all of which carry load $\nu$ [4]. Thus, $\tilde{\nu} = 134\nu + \nu$ and the blocking probability of a call is $P_B \approx E(135\nu, p)$.

In the second scenario, the transmission radius of each node is 1. Here, each link has 23 interfering links including itself. In the low blocking regime almost all calls get served and the average load on each link is $\approx 3\nu$. Treating the system as an equivalent network with load $3\nu$ on each link, the effective load equals $\tilde{\nu} = 23 * 3\nu$. The probability that no channel is free at a link is $E(69\nu, p)$. Making a further simplification that the links block independently the probability that a 3-hop call is blocked $\approx 1 - (1 - E(69\nu, p))^3 \approx 3E(69\nu, p)$.

Clearly, for low $\nu$ and moderate number of channels we have $E(135\nu, p) > 3E(69\nu, p)$ which suggests that it is preferable to use a smaller transmission radius. The intuitive reason is that a grid network has a denser node topology than a line network. As a result the number of interfering links increase much rapidly with an increase in the transmission radius of the nodes leading to higher blocking than using a smaller transmission radius. This suggests a relationship between blocking probability and the density of the nodes in the network and is an interesting future research direction.

Figure 7 presents simulation results that justify this conclusion. The plot shows the blocking probability of the center call in a 20X20 grid with 30 channels. All calls are of length of 3 and two cases of radius 1 and 3 are considered.

## V. CHANNEL ASSIGNMENT ALGORITHMS

Given the transmission radii of the nodes, blocking probability in a wireless network also depends on how we assign the

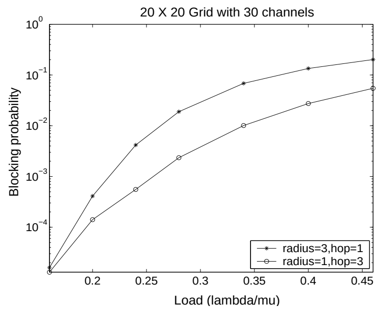


Fig. 7. Grid network with calls of length 3.

channels to the incoming calls. In this section, we propose a dynamic channel assignment algorithm called the Local Channel Reuse Algorithm (LCRA) and present simulation results that compare its performance with other algorithms - random, first fit and rearrangement algorithm.

**Rearrangement Algorithm :** The rearrangement algorithm was first presented in [11] for cellular networks. This policy admits an incoming call even if this requires rearrangement of the allocated channels to the calls in progress. Thus at a new call request the ongoing calls may be re-assigned channels to accommodate the new call. It is clear that this policy cannot be easily implemented in practice. The difficulty lies in the computational burden of searching for the feasible assignments for all the calls. However, as shown in [4] there is a simple characterization of the existence of an assignment for a line network.

**Non-rearranging Algorithms :** Here we consider algorithms that are not allowed to rearrange the channels allocated to the existing calls. Such algorithms are clearly more practical. The algorithms that we study are the random, first fit and the local channel re-use algorithm (LCRA). These algorithms base their decision on the set of free channels available at a node. Free channels refer to those channels such that the acceptance of a call in those channels does not violate the wireless constraints.

Let $\mathcal{F}_N$ be the set of free channels at node $N$. $\mathcal{F}_N$ contains all those channels in which node $N$ and its neighbors are inactive. Similarly, the set of free channels for a link $N \leftrightarrow M$ is the set of all those channels that are free at both nodes $N$ and $M$. We have, $\mathcal{F}_{N\leftrightarrow M} = \mathcal{F}_N \cap \mathcal{F}_M$.

*Single Hop Calls:* Consider a single hop call between nodes $S$ and $D$. The set of available channels on link $S \leftrightarrow D$ is $\mathcal{F}_S \cap \mathcal{F}_D$. If $\tilde{g}()$ denotes the decision function which selects a channel from the set $\mathcal{F}_S \cap \mathcal{F}_D$ then the chosen channel $\gamma_c = \tilde{g}(\mathcal{F}_S \cap \mathcal{F}_D)$. The channels are arbitrarily assigned an index number for the implementation of the algorithms.

*Random Algorithm*: The channel decision function $\tilde{g}()$ chooses a channel randomly from the set of free channels.

*First Fit Algorithm*: The channel decision function $\tilde{g}()$ chooses a channel that has the lowest index among the set of free channels. This algorithm has been studied earlier in WDM optical networks [18].

*Local Channel Re-use Algorithm (LCRA)*: Consider a link $S \leftrightarrow D$ on which the channel needs to be allocated. Let $\mathcal{N}_S$ and $\mathcal{N}_D$ be the neighbors of node $S$ and node $D$ respectively. Let the nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ be denoted as $N_1, N_2, .., N_{|\mathcal{N}_S \cup \mathcal{N}_D|}$ and $\Gamma = F_S \cap \mathcal{F}_D$. LCRA chooses a channel $\gamma_c \in \Gamma$ such that $\gamma_c$ minimizes the number of nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ that have $\gamma_c$ as a free channel in the present state. This leads to blocking of that channel for the least number of neighboring nodes. Mathematically,

$$\begin{aligned} I_{N_i}(\gamma_k) &= 1, \text{ if } \gamma_k \text{ is free at node } N_i \\ &= 0, \text{ otherwise.} \\ \Omega(\gamma_k) &= \text{Total nodes in } \mathcal{N}_S \cup \mathcal{N}_D \text{ with } \gamma_k \text{ free} \\ &= \sum_{N \in \mathcal{N}_S \cup \mathcal{N}_D} I_N(\gamma_k) \\ \gamma_c &= \tilde{g}(\Gamma) = \arg\, \min_{\gamma_k \in \Gamma}\ \Omega(\gamma_k) \end{aligned}$$

If there are more than one $\gamma_k$ that minimize $\Omega()$ then the smallest indexed $\gamma_k$ is selected.

To understand how this algorithm uses the channels in an efficient manner suppose channel $\gamma_c$ is chosen. Then, nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ cannot use channel $\gamma_c$ as long as the allocated call is active. Therefore, all those nodes that had $\gamma_c$ as a free channel before the call request was made remove $\gamma_c$ from their set of free channels. *LCRA minimizes this set of nodes*. The fact that some nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ do not have $\gamma_c$ in their set of free channels also implies that there is presently an active call in their neighborhood but that call does not interfere with the new incoming call on $S \leftrightarrow D$. Choosing such a channel will then lead to a local re-use of the channels. Thus, *LCRA tries to locally re-use the channels*.

*Multihop Calls:* A multihop call is regarded as a sequence of single hop calls where the first call arrives on the first link followed by an arrival on the second link and so on until the last link of the multihop path. With this interpretation, we assign channels for the multihop call by repeating the single hop assignment procedure in a sequence over the multihop path. Along the multihop path if at any link there are no free channels available then the call is dropped.

We next present simulation results that compare the performance of the above stated algorithms in a line and a grid network. We compute the blocking probability of the center call as the edge effects for this call are minimal. In both networks, the transmission radius of each node is fixed at unity. The arrival process of all the calls is Poisson and of the same rate $\lambda$ while the departure time is Exponentially distributed with mean $1/\mu = 1$. The load in the plots equals $\lambda/\mu$.

Figure 8 compares the blocking probability in a line network with 30 nodes, unit length calls and 50 channels. LCRA performs better than both the random and the first fit algorithms. Observe that if we fix a particular value of blocking probability then LCRA can support a higher load for each call as compared to random and first fit algorithms. As expected the rearrangement algorithm has the lowest blocking probability.

Simulating the rearrangement policy in a grid network is practically difficult. Therefore, in a grid network we compare the blocking probability for the random, first fit and the LCRA algorithms. Figure 9 shows the comparison plot for a 20X20 grid with 50 channels and unit length calls. LCRA performs better than both the random and the first fit policy. In a grid network a node has more interfering neighbors as compared to

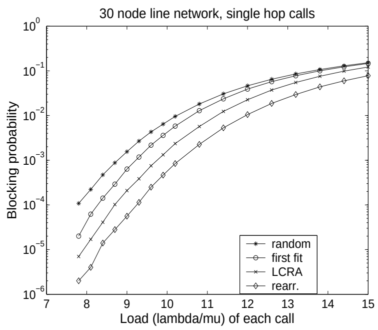


Fig. 8. Line network with unit length calls.

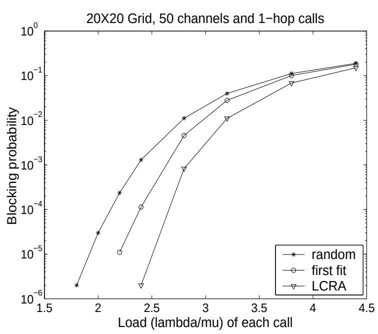


Fig. 9. Grid Network with unit length calls.

a line network. Therefore, spatially re-using the channels will pack the calls onto significantly lesser number of channels and have a greater impact on blocking probability. This is evident from the wider spread between the curves as compared to the line network.

Finally considering multihop calls, Figure 10 compares the blocking probability for random, first fit, LCRA and the rearrangement algorithms for a line network with 50 channels and all calls 6-hop long. Here again, we obtain similar conclusions with LCRA outperforming both the random and the first fit algorithms.

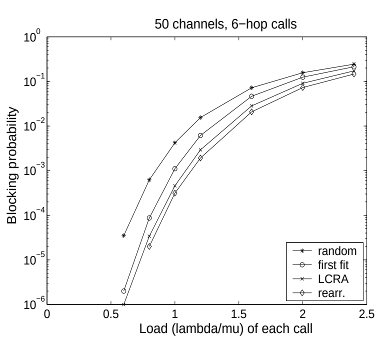


Fig. 10. Line network with 6-hop calls (length 6 units).

## VI. Conclusion

We studied the blocking probability behavior of connection oriented traffic and investigated dynamic channel assignment algorithms for multi-hop wireless networks. We derived both exact and approximate blocking probability formulas for a line network that yielded useful insights into the effect of transmission radius on call blocking. For example, we showed that in the line topology using a large transmission radius though energy costly substantially reduces the blocking probability; while the opposite is true in the more dense grid topology. The relationship between blocking probability and the density of the nodes is an interesting future research direction for efficient network design. We also developed a novel channel assignment algorithm that aims at reducing blocking probability by cleverly packing calls onto channels. We showed through simulations that an efficient channel assignment algorithm can significantly reduce blocking probability; especially for densely connected networks and multi-hop calls. It would also be interesting to investigate channel assignment schemes when the nodes are mobile.

## References

[1] C. Zhu and M. S. Corson, "QoS routing for mobile ad-hoc networks," *INFOCOM 2002*, vol. 2, pp. 958-967, 2002.

[2] S. Chakrabarti and A. Mishra, "QoS Issues in Ad Hoc Wireless Networks," *IEEE Communications Magazine*, February 2001.

[3] C. R. Lin and J.-S. Liu, "QoS Routing in Ad Hoc Wireless Networks," *IEEE JSAC*, vol. 17, no. 8, Aug. 1999, pp. 1426-1438.

[4] Murtaza Zafer, "Channel assignment algorithms and blocking probability analysis for connection-oriented traffic in wireless networks," *M.S. Thesis*, MIT LIDS, 2003. http://web.mit.edu/murtaza/www/

[5] S. Ross, "Stochastic Processes", $2^{nd}$ edn., *Wiley & Sons* 1996.

[6] D. Bertsekas and R. Gallager, "Data Networks", *Prentice Hall*, $2^{nd}$edn. 1992.

[7] F. P. Kelly, "Blocking probabilities in large circuit-switched networks," *Advances in Applied Probability* 18, pp. 473-505 (1986).

[8] D. Mitra, "Asymptotic analysis and computational methods for a class of simple, circuit-switched networks with blocking", *Advances in Applied Probability* vol. 19, pp. 219-239 (1987).

[9] S. Chung and K. W. Ross, "Reduced load approximations for multirate loss networks", *IEEE Trans. on Communications* Vol. 41, no. 8, Aug. 1993.

[10] F. P. Kelly, "Stochastic Models of Computer Communication Systems," *Journal of the Royal Statistical Society, Series B* Volume 47, Issue 3 (1985), 379-395.

[11] D. Everitt and N. Macfadyen, "Analysis of multi-cellular mobile radio telephone systems with loss", *British Telecom Journal*, 1, pp. 37-45, 1983.

[12] A. Kulshreshtha and K. N. Sivarajan, "Maximum packing channel assignment in cellular networks", *IEEE Transactions on Vehicular Technology*, vol. 48, no. 3, May 1999.

[13] G. Boggia and P. Camarda, "Modeling dynamic channel allocation in multicellular communication networks", *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 11, Nov. 2001.

[14] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Trans. on Information Theory*, vol. 46, no. 2, March 2000, pp. 388-404.

[15] K. Kar, S. Sarkar, L. Tassiulas, "Achieving Proportional Fairness using Local Information in Multi-hop Aloha Networks," $41^{st}$ *Allerton Conference On Communication, Control, and Computing*, Oct. 2003.

[16] R. A. Barry and P. A. Humblet, "Models of blocking probability in all-optical networks with and without wavelength changers", *IEEE Journal on Selected Areas in Communications*, vol. 14, no. 5, June 1996.

[17] A. Birman, "Computing approximate blocking probabilities for a class of all-optical networks", *IEEE Journal on Selected Areas in Communications*, vol. 14, no. 5, June 1996.

[18] R. Ramaswami and K.N. Sivarajan, "Routing and wavelength assignment in all-optical networks", *IEEE/ACM Trans. on Networking*, vol. 3,no. 5, pp. 489-500, Oct. 1995.