Scheduling Policies for Age Minimization in Wireless Networks with Unknown Channel State

Rajat Talak, Igor Kadota, Sertac Karaman, and Eytan Modiano

Abstract-Age of information (AoI) is a recently proposed metric that measures the time elapsed since the generation of the last received information update. We consider the problem of AoI minimization for a network under general interference constraints, and time varying channel. We study the case where the channel statistics are known, but the current channel state is unknown. We propose two scheduling policies, namely, the virtual queue based policy and age-based policy. In the virtual queue based policy, the scheduler schedules links with maximum weighted sum of the virtual queue lengths, while in the age-based policy, the scheduler schedules links with maximum weighted sum of a function of link AoI. We prove that the virtual queue based policy is peak age optimal, up to an additive constant, while the age-based policy is at most factor 4 away from the optimal age. Numerical results suggest that both the proposed policies are, in fact, very close to the optimal.

I. Introduction

Age of information (AoI), at the destination node, is the time elapsed since the last received information update was generated at the source node. AoI, upon reception of a new update packet drops to the time elapsed since generation of the packet, and grows linearly otherwise. Unlike packet delay, AoI measures the lag in obtaining information at the destination node, and is therefore more suited for applications involving dissemination of time sensitive information.

AoI was recently proposed in [1], [2]. In [1], AoI was studied for a network of vehicles exchanging status updates packets, via simulations, where it was shown that the AoI is minimized at a certain optimal packet generation rate. It was further shown that AoI can be improved by changing the queue discipline of the MAC layer FIFO queue to last-in-first-out (LIFO). This observation was proved under a general network setting in [3]. Motivated by [1], AoI was analyzed for several queueing models [2], [4]–[8].

However, age minimization for a network under general interference constraints and time varying channels has received very little attention. A problem of scheduling finitely many update packets under physical interference constraints was shown to be NP-hard in [9]. Age for a broadcast network, where only a single link can be activated at any time, was studied in [10], [11]. Some preliminary analysis of age for a slotted ALOHA like random access was done in [12], while age minimization under throughput constraints for a broadcast

The authors are with the Laboratory for Information and Decision Systems (LIDS) at the Massachusetts Institute of Technology (MIT), Cambridge, MA. {talak, kadota, sertac, modiano}@mit.edu

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network, in which only a single link can be activated at a time, was only recently studied in [13].

In this paper, we considered the problem of age minimization for a wireless network under general interference constraints, and time varying channels. We consider active sources, which generate fresh information in every slot, and single-hop flows for which all source and destination nodes share a link. We propose two policies, namely, a virtual queue based policy π_Q and an age-based policy π_A that takes into account the AoI in making decisions. In the queue based policy π_Q , each link maintains a virtual queue, and a set of non-interfering links with the highest weighted sum of virtual queue lengths is scheduled, in every time slot. In the age-based policy, however, the set of non-interfering links with the highest weighted sum of link-AoIs are scheduled.

We show that the virtual-queue based policy is peak age optimal, up to an additive factor, and that the age-based policy is at most a factor 4 away from the optimal peak and average age. A similar result was obtained for broadcast network, in which at most one link can be activated simultaneously, recently in [13]. Numerical simulations indicate that both the policies are very close to the optimal peak and average age, and outperform the stationary policy proposed in [14], especially when the network interference is high.

This is an extension of our recent work in [14], [15], where we proposed similar policies when the channel states are perfectly known for each slot.

II. SYSTEM MODEL

The wireless network is modeled as a graph G=(V,E), where V denotes the set of nodes and E the set of directed links. We consider a slotted time system, where the slot duration is normalized to unity. Due to wireless interference, not all links can be activated simultaneously. We call a set $m \subset E$ that can be activated simultaneously without interference as a feasible activation set, and use $\mathcal A$ to denote the collection of all feasible activation sets.

We use $U_e(t)$, which equals either 0 or 1, to denote whether the link e is activated or not, respectively, at time t. Not every attempted transmission on a link is successful due to channel errors, and we use $S_e(t) \in \{0,1\}$ to denote the state of link e at time t. If $S_e(t) = 1$ then an attempted transmission over e at time t succeeds, and fails otherwise. A successful transmission occurs over link e, at time t, if and only if $U_e(t)S_e(t) = 1$.

We assume the channel process $\{S_e(t)\}_{t\geq 0}$ to be independent and identically distributed (i.i.d.) across time, with $\gamma_e = \mathbb{P}\left[S_e(t) = 1\right] > 0$ for all $e \in E$. Note that the channel

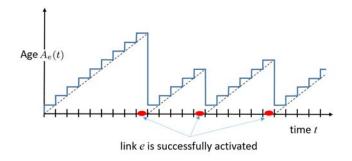


Fig. 1. Evolution of age of link e, namely $A_e(t)$, as a function of time t.

is not identical across links e, and we allow for the channel success probability γ_e to be different across links.

We consider *active nodes*, which transmit fresh information at every transmission opportunity. We define age $A_e(t)$, of a link e at time t, to be the time elapsed since the last successful activation of link e. Figure 1 shows evolution of age $A_e(t)$ for a link e. Age $A_e(t)$ reduces to 1 upon a successful activation of link e, while it increases by 1 in every slot in which there is no successful activation of link e. This age evolution equation can be written as

$$A_e(t+1) = 1 + A_e(t) - U_e(t)S_e(t)A_e(t), \tag{1}$$

for all $e \in E$, and t > 0.

For a link e, we define average age to be the area under the age curve in Figure 1. This can be written as

$$\overline{A}_e^{\text{ave}} = \limsup_{t \to \infty} \mathbb{E} \left[\frac{1}{t} \sum_{\tau=0}^{t-1} A_e(\tau) \right]. \tag{2}$$

We see that the age curve in Figure 1 peaks whenever the link e is successfully activated. We define peak age, for a link e, to be the average of all the peaks. This is given by

$$\overline{A}_{e}^{p} = \limsup_{t \to \infty} \frac{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau) A_{e}(\tau)\right]}{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_{e}(\tau) S_{e}(\tau)\right]}, \quad (3)$$

because the numerator is the sum of age peaks until time t, while the denominator counts the number of age peaks until time t. Note that, this follows because the age peaks occur whenever link e is successfully activated, which is exactly when $S_e(\tau)U_e(\tau)=1$.

Average and peak age of the network ${\cal G}$ is then defined as the weighted sum of link ages:

$$\overline{A}^{\text{ave}} = \sum_{e \in E} w_e \overline{A}_e^{\text{ave}} \quad \text{and} \quad \overline{A}^{\text{p}} = \sum_{e \in E} w_e \overline{A}_e^{\text{p}},$$
 (4)

where $w_e > 0$ are positive weights. We are interested in designing policies that minimize peak and average age of the network.

A. Scheduling Policies

A scheduling policy determines the set of links $m_t \subset E$ that will be activated at each time t. We consider policies that can make use of current and past age, and previous decisions,

when making the decision at time t, but not the current channel state. That is, the scheduler at each time t determines m_t as a function of the set

$$\mathcal{H}(t) = \{ \mathbf{U}(\tau), \mathbf{A}(\tau') \mid 0 \le \tau < t, \ 0 \le \tau' \le t \}. \tag{5}$$

Note that knowledge of age until time t, and decisions until time t-1, implies complete knowledge of the channel state $\mathbf{S}(\tau)$ until time $\tau \leq t-1$. This is because age $A_e(\tau)$ drops whenever $S_e(\tau)U_e(\tau)=1$, and continues to grow if $S_e(\tau)=0$. We consider centralized scheduling policies, in which this information $\mathcal{H}(t)$ is centrally available to a scheduler. This assumption is consistent with that in network scheduling literature [16].

For such a policy π we define link activation frequency to be

$$f_e(\pi) = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{I}_{\{e \in m_t, m_t \in \mathcal{A}\}},$$
 (6)

for all $e \in E$. Note that, $U_e(t) = \mathbb{I}_{\{e \in m_t, m_t \in \mathcal{A}\}}$, and thus, the link activation frequency doesn't count the channel uncertainties. However, it is clear that if $f_e(\pi) = 0$ or not well defined, for some e, then the peak and average age would be infinity or not-well defined. We consider Π to be the class of all policies for which the link activation frequencies $\mathbf{f}(\pi)$ are well defined and positive:

$$\Pi = \{ \pi | f_e(\pi) \text{ exists and is positive } \}.$$
 (7)

We define optimal peak and average age to be

$$\overline{A}^{p*} = \min_{\pi \in \Pi} \overline{A}^p(\pi)$$
 and $\overline{A}^{ave*} = \min_{\pi \in \Pi} \overline{A}^{ave}(\pi)$. (8)

An important fact about the policy space Π is that the space of all feasible link activation frequencies $\mathcal{F} = \{\mathbf{f}(\pi) \mid \pi \in \Pi\}$, is given by (see [16]):

$$\mathcal{F} = \left\{ \mathbf{f} \in \mathbb{R}^{|E|} \middle| \mathbf{f} = M\mathbf{x}, \ \mathbf{1}^T\mathbf{x} \le 1, \text{ and } \mathbf{x} \ge 0 \right\},$$
 (9)

where M is a $|E| \times |\mathcal{A}|$ matrix such that $M_{e,m} = 1$ if and only if $e \in m$, and 0 otherwise, for all $e \in E$ and $m \in \mathcal{A}$.

B. Stationary Policies

An important sub-space of Π , which do not use any past history, is the space of stationary policies. In it, a feasible activation set $m \in \mathcal{A}$ is activated with probability x_m , in every slot; we have $\sum_{m \in \mathcal{A}} x_m = 1$. The link activation frequencies, for this policy, are then given by $f_e = \sum_{m:e \in m} x_m$, which can be written as $\mathbf{f} = M\mathbf{x}$, where M is the same $|E| \times |\mathcal{A}|$ matrix in (9). Therefore, any link activation frequency in the set \mathcal{F} , in (9), can be achieved by a stationary policy.

The following result proves that there exists a stationary policy that is peak age optimal. This was proved in [14], and we will use it to prove bounds on our proposed policies.

Theorem 1: The optimal peak age \overline{A}^{p*} is given by

$$\overline{A}^{p*} = \underset{\mathbf{f}, \mathbf{x} \in [0, 1]^{|A|}}{\text{Minimize}} \sum_{e \in E} \frac{w_e}{\gamma_e f_e},$$
subject to $\mathbf{f} = M\mathbf{x},$

$$\sum_{m \in A} x_m \le 1,$$
(10)

and the solution \mathbf{x}^* to (10) yields a stationary policy that is peak age optimal, call it π_C . Furthermore, the peak age and average age for π_C are equal and bounded by

$$\overline{A}^{p*} = \overline{A}^{p}(\pi_C) = \overline{A}^{ave}(\pi_C) \le 2\overline{A}^{ave*} - \sum_{e \in E} w_e.$$
 (11)

Proof: This result is proved in our recent work [14]. To intuitively see the result, note that for a stationary policy with distribution \mathbf{x} and link activation frequencies $\mathbf{f} = M\mathbf{x}$, every link e is successfully activated with probability $\gamma_e f_e$ in every slot. As a result, the age $A_e(t)$, which is the time since last activation is geometrically distributed with mean $\frac{1}{\gamma_e f_e}$. It turns out that the peak age of the link, is indeed, given by $\overline{A}_e^p = \frac{1}{\gamma_e f_e}$. As a consequence, the peak age of the network is given by $\overline{A}_e^p = \sum_{e \in E} \frac{w_e}{\gamma_e f_e}$, and the optimal peak age is given by (10).

Furthermore, (11) primarily follows because the peak and average age are equal for any stationary policy [14].

A consequence of Theorem 1 is that the stationary peak age optimal policy π_C is also factor 2 average age optimal. This can be seen from (11). Further, this bound is tight in the sense that for certain networks, average age of the stationary policy π_C is indeed factor 2 away from optimality.

To see this, consider E links only one of which can be activated at any given time t. Also, assume there to be no channel uncertainties, i.e. $S_e(t)=1$ for all t and e. If weights are all equal, i.e. $w_e=1$ for all e, then the optimal stationary policy π_C activates link e with probability 1/|E|. The average and peak age then is given by

$$A^{\text{ave}}(\pi_C) = A^{\text{p}}(\pi_C) = |E|^2.$$
 (12)

However, if we schedule links in E in round robbin manner, the peak age would still be $|E|^2$ but the average age would improve to |E|(|E|-1)/2.

The above example shows that beyond stationary policies, average age could be improved by resorting to periodic policies. One way to resort to periodic policies is to schedule links based on age $A_e(t)$. For example, the above round robbin policy can be induced by having the link with the largest age $A_e(t)$, or the largest $g\left(A_e(t)\right)$ for an increasing function $g(\cdot)$, transmit in every slot t. In this paper, we propose policies which do just that.

In Sections III, we propose a virtual queue based policy, which schedules a feasible activation set m with largest weighted virtual queue lengths, rather than largest age. In Section IV, we propose an age-based policy that schedules feasible activation sets m with maximum $\sum_{e \in m} w_e \gamma_e g\left(A_e(t)\right)$, for $g(x) = x^2 + \beta x$.

III. VIRTUAL-QUEUE BASED POLICY

We first present a lemma that states a conservation law for age. Intuitively, it states that for any policy $\pi \in \Pi$, the sum of all age peaks is equal to the total time elapsed plus a small insignificant term that goes to 0 as $t \to \infty$.

Lemma 1: For any policy $\pi \in \Pi$ we have

$$\lim_{t \to \infty} \mathbb{E}\left[\frac{1}{t} \sum_{\tau=0}^{t-1} U_e(\tau) S_e(t) A_e(\tau)\right] = 1, \tag{13}$$

for all $e \in E$.

Proof: See Appendix A in our technical report [17].
A direct consequence of Lemma 1 is that the peak age minimization problem $\min_{\pi \in \Pi} \overline{A}^p(\pi)$ reduces to

$$\begin{array}{ll} \underset{\alpha \geq 0, \pi \in \Pi}{\operatorname{Minimize}} & \sum_{e \in E} \frac{w_e}{\alpha_e}, \\ \text{subject to} & \liminf_{t \to \infty} \mathbb{E} \left[\frac{1}{t} \sum_{\tau=0}^{t-1} U_e(\tau) \right] \geq \frac{\alpha_e}{\gamma_e} \quad \forall \ e \in E. \end{array} \tag{14}$$

This follows primarily because the numerator in (3), after dividing by t, is equal to 1, due to Lemma 1. We prove this equivalence in Appendix B, in our technical report [17]. This result is significant because it shows that the peak age minimization problem is independent of the age evolution equation. This is the reason why peak age minimization problem is much simpler than minimizing average age.

We now propose a policy that solves the peak age minimization problem (14). We construct a virtual queue $Q_e(t)$, which is reduced by at most 1 upon a successful transmission over link e and increased otherwise. These queue lengths determine the 'value' of scheduling link e in time slot t. Therefore, a set $m_t \in \mathcal{A}$ that maximizes $\sum_{e \in m} w_e \gamma_e Q_e(t)$ is activated in slot t. This virtual-queue based policy, π_Q , is described below. Here, V > 0 is any chosen constant.

Virtual Queue based policy π_Q Start with $Q_e(0)=1$ for all $e\in E$. At time t,

1) Update $Q_e(t)$ as

$$Q_e(t) = \left[Q_e(t-1) + \sqrt{\frac{V}{Q_e(t-1)}} - S_e(t-1)U_e(t-1)\right]_{+1},$$

for all $e \in E$, where $[x]_{+1} = \max\{x, 1\}$.

2) Schedule activation set m_t given by

$$m_t = \arg\max_{m \in \mathcal{A}} \sum_{e \in m} w_e \gamma_e Q_e(t).$$
 (15)

Note that, we require the product $U_e(t-1)S_e(t-1)$ at time t in order to update the virtual queue lengths. This is possible with $\mathcal{H}(t)$, as the products $U_e(t-1)S_e(t-1)$ can be inferred from current age vector and past actions. We now prove that the policy π_Q is nearly peak age optimal.

Theorem 2: The peak age under for policy π_Q is bounded by

$$\overline{A}^{p}(\pi_{Q}) \le \overline{A}^{p*} + \frac{1}{2} \sum_{e \in E} w_{e} + \frac{1}{2V} \sum_{e \in E} w_{e},$$
 (16)

where \overline{A}^{p*} is the optimal value of (10).

Proof: The proof is in our technical report [17]. Theorem 2 shows that even when the channel statistics are not known the optimal peak age $A^{\mathrm{p}*} = \sum_{e \in E} \frac{w_e}{\gamma_e f_e^*}$ can be achieved, barring an additive factor of $\frac{1}{2} \sum_{e \in E} w_e$, with arbitrary precision by selecting an appropriate V.

IV. AGE-BASED POLICY

In this section, we propose an age-based policy to minimize age of the network. To gain an intuitive understanding of the proposed policy, we first provide for an equivalent characterization of average age. Note that the average age for a link e is given by $\overline{A}_e^{\text{ave}} = \limsup_{t \to \infty} \mathbb{E}\left[\frac{1}{t}\sum_{t=0}^{t-1} A_e(\tau)\right]$. The following result provides a different characterization of the average age in terms of $A_e^2(t)$.

Lemma 2: For any $\pi \in \Pi$, we have

$$\overline{A}_e^{\text{ave}} = \frac{1}{2} \limsup_{t \to \infty} \mathbb{E} \left[\frac{1}{t} \sum_{\tau=0}^{t-1} \gamma_e U_e(\tau) B_e(\tau) \right] + \frac{1-\beta}{2},$$

for all $e \in E$, where $B_e(t) = A_e^2(t) + \beta A_e(t)$ and $\beta \in \mathbb{R}$.

Proof: See Appendix D in our technical report [17]. For an intuitive understanding of Lemma 2, note that average age is essentially time averaged area of the triangles formed by the age curve in Figure 1. Note that $S_e(t)U_e(t)A_e^2(t)$ are square of age peaks in the age curve Figure 1, and $\frac{1}{2}S_e(t)U_e(t)A_e^2(t)$ is the area of the triangle, because $S_e(t)U_e(t) = 1$ only at the instances when there is a successful transmission on link e. We can replace $S_e(t)$ with γ_e because $S_e(t)$ is independent of $U_e(t)$ and $A_e(t)$. An additional term of $\beta A_e(t)$ is possible due to Lemma 1.

Lemma 2 implies that average age minimization problem over $\pi \in \Pi$ can be equivalently posed to minimize

$$\limsup_{t \to \infty} \mathbb{E}\left[\frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{e \in E} w_e \gamma_e U_e(\tau) B_e(\tau)\right]. \tag{17}$$

Since, age reduces to 1 after a link activation, it makes intuitive sense to choose $\mathbf{U}(t)$ such that as

$$\mathbf{U}(t) = \arg\max_{\mathbf{U}'(t)} \sum_{e \in E} w_e \gamma_e U_e'(t) \left[A_e^2(t) + \beta A_e(t) \right], \quad (18)$$

in time slot t. This, in the least, should minimize age in the next slot. We now propose this age-based policy:

Age-based Policy π_A The policy activates links $m_t \in$ A in slot t given by:

$$m_t = \arg\max_{m \in \mathcal{A}} \sum_{e \in m} w_e \gamma_e \left[A_e^2(t) + \beta A_e(t) \right],$$
 (19)

for all $t \geq 1$.

The following result shows that the average and peak age of policy π_A is within a factor of 4 from the respective optimal.

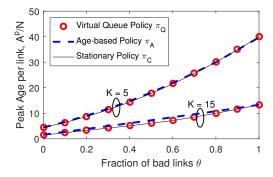


Fig. 2. Peak age per link, namely A^p/N , as a function of the fraction of bad nodes θ , for various policies.

Theorem 3: The policy π_A is at most factor-4 peak and average age optimal, i.e.,

$$\overline{A}^{\text{ave}}(\pi_A) \le 4\overline{A}^{\text{ave*}} - c_1(\beta) \sum_{e \in E} w_e, \tag{20}$$

and

$$\overline{A}^{p}(\pi_{A}) \le 4\overline{A}^{p*} - c_{2}(\beta) \sum_{e \in E} w_{e}, \tag{21}$$

where $c_1(\beta) = \frac{10+2\beta-\beta^2}{4}$ and $c_2(\beta) = \frac{4+2\beta-\beta^2}{2}$. *Proof:* We prove this by minimizing an appropriately defined Lyapunov drift plus a penalty function. The detailed proof is given in Appendix E, in our technical report [17]. We note that β can be chosen to improve the additive factor of optimality. The best bounds, for both peak and average age, occur when $\beta = 1$, for which both $c_1(\beta)$ and $c_2(\beta)$ are maximized.

V. NUMERICAL RESULTS

We now evaluate the performance of the proposed policies. We consider a N=20 link network, with interference constraints such that at most K links can be activated at any given time. We set link weights to unity, i.e., $w_e = 1$ for all $e \in E$. We let the links to be either 'good', with channel success probability $\gamma_e=\gamma_{\rm good}=0.9,$ or 'bad' with channel success probability $\gamma_e = \gamma_{\rm bad} = 0.1$. We use θ to denote the fraction of bad links in the network. We simulate the policies π_{O} , π_{A} , and the peak age optimal stationary policy π_{C} of [14], over a horizon of 10^5 time slots.

We first set V=1 for policy π_Q and $\beta=1$ for policy π_A , and evaluate the policies. In figures 2 and 3 we plot the per-link peak and average age, $A^{p}(\pi)/N$ and $A^{ave}(\pi)/N$, respectively, for all policies $\pi \in \{\pi_Q, \pi_A, \pi_C\}$. As to be expected, we see that, increasing the fraction of 'bad' channels θ or increasing interference, i.e. reducing K, increases age.

In Figure 2, we further see that in all the cases, of θ and K, the peak age of the proposed policies π_Q and π_A coincide with the peak age optimal stationary policy π_C . Thus, the proposed policies are nearly, if not, peak age optimal under the current network setting. We observe similar behavior for several other networks, not presented here.

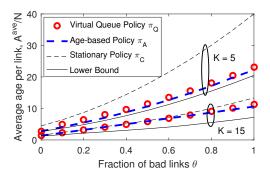


Fig. 3. Average age per link, namely $A^{\rm ave}/N$, as a function of the fraction of bad nodes θ , for various policies.

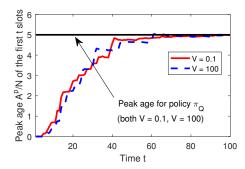


Fig. 4. Peak age per link $A^p(\pi_Q)/N$, computed for the first t slots, as a function of time t, for V=0.1 and V=100. Also plotted is the per-link peak age $A^p(\pi_Q)/N$ achieved over a much larger time horizon.

In Figure 3, however, we observe a larger gap between average age of policy π_C and proposed policies π_Q and π_A , especially when K=5 than when K=15. This shows that the proposed policies perform much better than the stationary peak age optimal policy π_C under high interference (smaller K). In Figure 3, we also plot the average age lower bound, obtained from (11). We observe that the proposed policies are much closer to the average age lower bound in the high interference case (small K). This also shows that the age-based policy π_A performs better than the bound derived in Theorem 3. We believe that better bounds on $A^p(\pi_A)$ and $A^{ave}(\pi_A)$ are possible.

The virtual queue based policy π_Q and the age-based policy π_A have free parameters V and β , respectively, which need to be chosen apriori. Figure 4 plots the per-link peak age A^p/N , computed over the first t slots, as a function of time t. We observe that the choice of V has nearly no effect on the convergence time of the algorithm. In Figure 5, we plot the per-link peak and average age for the age-based policy π_A , when K=5,15 and $\theta=0.25$. We observe that $\beta\approx 0$ is the safest bet for the age-based policy π_A .

VI. CONCLUSION

We considered the problem of age minimization for a wireless network under general interference constraints, and time varying channel. We proposed two policies: virtual queue based policy and age-based policy. We proved that the virtual

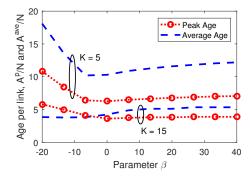


Fig. 5. Per link peak and average age for policy π_A , namely $A^p(\pi_A)/N$ and $A^{ave}(\pi_A)/N$, as a function of parameter β .

queue based policy is peak age optimal, barring an additive factor, while the age-based policy is at most factor 4 away from the optimal peak and average age. Using numerical simulations, we demonstrated that both the proposed policies are, in fact, very close to optimal.

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