Optimizing Information Freshness in Wireless Networks under General Interference Constraints

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***Abstract*—Age of information (AoI) is a recently proposed metric for measuring information freshness. AoI measures the time that elapsed since the last received update was generated. We consider the problem of minimizing average and peak AoI in wireless networks under general interference constraints. When fresh information is always available for transmission, we show that a stationary scheduling policy is peak age optimal. We also prove that this policy achieves average age that is within a factor of two of the optimal average age. In the case where fresh information is not always available, and packet/information generation rate has to be controlled along with scheduling links for transmission, we prove an important *separation principle*: the optimal scheduling policy can be designed assuming fresh information, and independently, the packet generation rate control can be done by ignoring interference. Peak and average AoI for discrete time G/Ber/1 queue is analyzed for the first time, which may be of independent interest.**

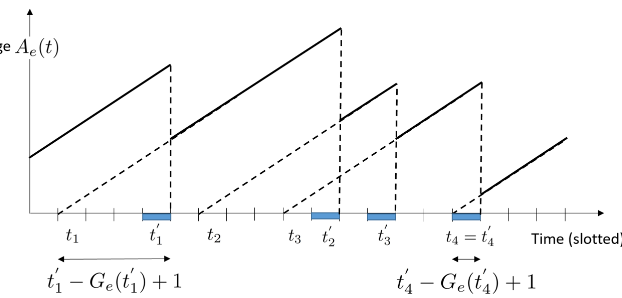

Fig. 1. Time evolution of age, $A_e(t)$, of a link e. Times t_i and t_i' are instances of ith packet generation and reception, respectively. Given the definition $G_e(t_i') \triangleq t_i$, the age is reset to $t_i' - G_e(t_i') + 1$ when the ith packet is received.

I. Introduction

Exchanging status updates, in a timely fashion, is an important functionality in many network settings. In unmanned aerial vehicular (UAV) networks, exchanging position, velocity, and control information in real time is critical to safety and collision avoidance [2], [3]. In internet of things (IoT) and cyber-physical systems, information updates need to be sent to a common ground station in a timely fashion for better system performance [4]. In cellular networks, timely feedback of the link state information to the mobile nodes, by the base station, is necessary to perform opportunistic scheduling and rate adaptation [5], [6].

Traditional performance measures, such as delay or throughput, are inadequate to measure the timeliness of the updates, because delay or throughput are packet centric measures that fail to capture the timeliness of the information from an application perspective. For example, a packet containing stale information is of little value even if it is delivered promptly by the network. In contrast, a packet containing freshly updated information may be of much greater value to the application, even if it is slightly delayed.

A new measure, called Age of Information (AoI), was proposed in [7], [8] that measures the time that elapsed since the last received update was generated. Figure 1 shows evolution of AoI for a destination node as a function of time. The AoI, upon reception of a new update packet, drops to the time elapsed since generation of this packet, and grows linearly otherwise. AoI being a destination-node centric measure, rather than a packet centric measure like throughput or delay, is more appropriate to measure timeliness of updates.

In [7], AoI was first studied for a vehicular network using simulations. Nodes generated fresh update packets periodically at a certain rate, which were queued at the MAC layer first-in-first-out (FIFO) queue for transmission. An optimal packet generation rate was observed that minimized age. It was further observed that the age could be improved by controlling the MAC layer queue, namely, by limiting the buffer size or by changing the queueing discipline to last-in-first-out (LIFO). However, the MAC layer queue may not be controllable in practice. This lead to several works on AoI under differing assumptions on the ability to control the MAC layer queue.

Since [7], age of information has mostly been analyzed for a single link case, by modeling the link as a queue. Age for M/M/1, M/D/1, and D/M/1 queues, under FIFO service, was analyzed in [8] while multiclass M/G/1 and G/G/1 queues, under FIFO service, were studied in [9]. Age for a M/M/∞ was analyzed in [10], which studied the impact of out-of-order delivery of packets on age, while the effect of packet errors or packet drops on age was studied in [11]. Age for LIFO queues was analyzed under various arrival and service time distributions in [12]–[14].

Many of the applications where age is an important metric involve wireless networks, and interference constraints are one of the primary limitations to system performance. However, theoretical understanding of age of information under interference constraints has received little attention thus far. In [15], the problem of scheduling finite number of update packets under physical interference constraint for age minimization was shown to be NP-hard. Age for a broadcast network, where

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only a single link can be activated at any time, was studied in [16], [17], and preliminary analysis of age for a slotted ALOHA-like random access was done in [18].

In this paper, we consider the problem of minimizing age of information in wireless networks under general interference constraints, and time-varying links. We consider average age, which is the time average of the age curve in Figure 1, and peak age, which is the average of all the peaks in the age curve in Figure 1, as the metric of performance. We obtain simple scheduling policies that are optimal, or nearly optimal.

We consider two types of sources: *active sources* and *buffered sources*. Active sources can generate a new update packet for every transmission, i.e., fresh information is always available for transmission. Buffered sources, on the other hand, can only control the rate of packet generation, while the generated packets are buffered in the MAC layer FIFO queue for transmission.

For a network with active sources, we show that a stationary scheduling policy, in which links are activated according to a stationary probability distribution, is peak age optimal. We also show that this policy achieves average age that is within factor of two of the optimal average age. Moreover, we prove that this optimal policy can be obtained as a solution to a convex optimization problem.

For a network with buffered sources, we consider Bernoulli and periodic packet generation, that generate update packets at a certain rate. We design a rate control and scheduling policy to minimizes age. We show that if rate control is performed assuming that there is no other link in the network, and scheduling is done in the same way as in the active source case, then this is close to the optimal age achieved by jointly minimizing over stationary scheduling policies and rate control. This *separation principle* provides an useful insight towards the design of age optimal policies, as scheduling and rate control are typically done at different layers of the protocol stack.

To the best of our knowledge this is the first work to consider the age minimization problem in a general wireless network setting. Moreover, peak and average age for the discrete time FIFO G/Ber/1 queue is analyzed for the first time, which may be of independent interest.

The rest of this paper is organized as follows: We describe the system model in Section II. Age minimization for active sources is considered in Section III, where we also characterize the stationary policy that minimizes peak age under a general interference model. Age minimization for buffered sources is discussed in Section IV. Numerical results are presented in Section V, and we conclude in Section VI.

II. System Model

We consider a wireless communication network as a graph $G = (V, E)$, where V is the set of nodes and E is the set of communication links between the nodes in the network. Time is slotted and the duration of each slot is normalized to unity. Due to wireless interference constraints, not all links can be activated simultaneously [19]. We call a set $m \subset E$ to be a *feasible activation set* if all links in m can be activated simultaneously without interference, and denote by $\mathcal{A}$ the collection of all feasible activation sets. We call this the *general interference model*.

A non-interfering transmission over link e does not always succeed due to channel errors. We let $R_e(t) \in \{1, 0\}$ denote the channel error process for link e, where $R_e(t) = 1$ if a non-interfering transmission over link e succeeds and $R_e(t) = 0$ otherwise. We assume $R_e(t)$s to be independent across links, and i.i.d. across time with $\gamma_e = \mathbf{P}\left[R_e(t) = 1\right] > 0$, for all $e \in E$. We assume that the channel error process $R_e(t)$ is not observable by the source nodes, but the channel success probabilities γ_e are known, or can be measured separately.

We consider two types of sources, namely, *active source* and *buffered source*. An active source can generate a new update packet at the beginning of each slot for transmission, while discarding old update packets that were not transmitted. Thus, for an active source, a transmitted packet always contains fresh information. Packets generated by a buffered source, on the other hand, get queued before transmission, and may contain 'stale' information. The source cannot control this FIFO queue, and thus, the update packets have to incur queueing delay. A buffered source, however, can control the packet generation rate.

The age $A_e(t)$ of a link e evolves as shown in Figure 1. When the link e is activated successfully in a slot, the age of link e is reduced to the time elapsed since the generation of the delivered packet. $A_e(t)$ grows linearly in absence of any communication over e. This evolution can be simply described as

$$A_e(t+1) = \begin{cases} t - G_e(t) + 1 & \text{if } e \text{ is activated at } t \\ A_e(t) + 1 & \text{if } e \text{ is not activated at } t \end{cases}, \quad (1)$$

where $G_e(t)$ is the generation time of the packet delivered over link e at time t. In the active source case, for example, $G_e(t) = t$ since a new update packet is made available at the beginning of each slot. Thus, in this case, the age of link e is equal to the time elapsed since an update packet was transmitted over it, i.e., the last *activation* of link e.

We define two metrics to measure long term age performance over a network of interfering links. The *weighted average age*, given by,

$$A^{\text{ave}} = \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{e \in E} w_e A_e(t), \quad (2)$$

where w_e are positive weights denoting the relative importance of each link $e \in E$, and the *weighted peak age*, given by,

$$A^{\text{p}} = \limsup_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{e \in E} w_e A_e\left(T_e(i)\right), \quad (3)$$

where $T_e(i)$ denotes the time at which link e was successfully activated for the ith time. Peak age is the average of age peaks, which happen just before link activations. Without loss of generality we assume that $w_e > 0$ for all e.

A. Scheduling Policies

A scheduling policy is needed in order to decide which links to activate at any time slot. It determines the set of links $m(t) \subset E$ that will be activated at each time t. The policy can make use of the past history of link activations and age to make this decision, i.e., at each time t the policy π will determine $m(t)$ as a function of the set

$$\mathcal{H}(t) = \{m(\tau), \mathbf{A}(\tau')|0 \le \tau \le t-1 \text{ and } 0 \le \tau' \le t\}, \quad (4)$$

where $\mathbf{A}(t) = (A_e(t))_{e \in E}$. Note that $\mathbf{R}(t) \notin \mathcal{H}(t)$, i.e., the current channel state $\mathbf{R}(t)$ is not observed before making decision at time t. Furthermore, the information regarding past channel states is available only through age evolution. We consider centralized scheduling policies, in which this information is centrally available to a scheduler, which is able to implement its scheduling decisions.

Given such a policy π, define the *link activation frequency* $f_e(\pi)$, for a link e, to be the fraction of times link e is successfully activated, i.e.,

$$f_e(\pi) = \lim_{T \to \infty} \frac{\sum_{t=1}^{T} \mathbb{I}_{\{e \in m(t), m(t) \in \mathcal{A}\}}}{T}, \quad (5)$$

where $m(t)$ is the set of links activated at time t. Note that $f_e(\pi)$ is not the frequency of successful activations, as channel errors can render an activation of a link unsuccessful. If $f_e(\pi) = 0$ for a certain link e then the average and peak age will be unbounded. We, therefore, limit our attention to the set of policies Π for which $f_e(\pi)$ is well defined and strictly positive for all $e \in E$:

$$\Pi = \left\{\pi \middle| f_e(\pi) \text{ exists and } f_e(\pi) > 0 \; \forall e \in E\right\}. \quad (6)$$

We define the set of all feasible link activation frequencies, for policies described above:

$$\mathcal{F} = \left\{\mathbf{f} \in \mathbb{R}^{|E|} \mid f_e = f_e(\pi) \; \forall \; e \in E \text{ and some } \pi \in \Pi\right\}.$$

This set can be characterized by linear constraints as

$$\mathcal{F} = \left\{\mathbf{f} \in \mathbb{R}^{|E|} \mid \mathbf{f} = M\mathbf{x}, \; \mathbf{1}^T\mathbf{x} \le 1 \text{ and } \mathbf{x} \ge 0\right\}, \quad (7)$$

where $\mathbf{x}$ is a vector in $\mathbb{R}^{|\mathcal{A}|}$ and M is a $|E| \times |\mathcal{A}|$ matrix with elements

$$M_{e,m} = \begin{cases} 1 & \text{if } e \in m \\ 0 & \text{otherwise} \end{cases}, \quad (8)$$

for all links e and feasible activation sets $m \in \mathcal{A}$.

A simple sub-class of policies, which do not use any past history, is the class of stationary policies. In it, links are activated independently across time according to a stationary distribution. We define a stationary policy as follows:

Definition *1 (Stationary Policies):* Let $B_e(t) = \{e \in m(t), m(t) \in \mathcal{A}\}$ be the event that link e was activated at time t. Then, the policy π is stationary if

1) $B_e(t)$ is independent across t, and
2) $\Pr(B_e(t_1)) = \Pr(B_e(t_2))$ for all $t_1, t_2 \in \{1, 2, \ldots\}$,

for all $e \in E$.

The following are two examples of stationary policies.

Example 1: Set $p_e \in (0,1)$ for all $e \in E$, and let a policy attempt transmission over link e with probability p_e, independent of other link's attempts.

Example 2: Assign a probability distribution $\mathbf{x} \in \mathbb{R}^{|\mathcal{A}|}$ over the collection of feasible activation sets, $\mathcal{A}$. Then, in each slot, activate the set $m \in \mathcal{A}$ with probability x_m, independent across time. We call this the *stationary centralized policy*. For this policy,

$$\Pr(B_e(t)) = \sum_{m: e \in m} p(m) = (M\mathbf{p})_e, \quad (9)$$

for all $e \in E$ and slots t.

We will see in the next section that in the active source case, a stationary centralized policy is peak age optimal, and is within a factor of 2 from the optimal average age, over the space Π. Motivated by this, in Section IV-B, we will specifically consider only stationary centralized policies for the buffered sources.

III. Minimizing Age with Active Sources

We consider a network where all the sources are active. Since the age metrics depend on the policy $\pi \in \Pi$ used, we make this dependence explicit by the notation $A^{\text{ave}}(\pi)$ and $A^{\text{p}}(\pi)$. We use $A^{\text{ave}*}$ and $A^{\text{p}*}$ to denote the minimum average and peak age, respectively, over all policies in Π.

We first characterize the peak age for any policy $\pi \in \Pi$, and show that a stationary centralized policy is peak age optimal.

Theorem *1:* For any policy $\pi \in \Pi$, the peak age is given by

$$A^{\text{p}}(\pi) = \sum_{e \in E} \frac{w_e}{\gamma_e f_e(\pi)}. \quad (10)$$

As a consequence, for every $\pi \in \Pi$ there exists a stationary policy $\pi_{\text{st}} \in \Pi$ such that $A^{\text{p}}(\pi) = A^{\text{p}}(\pi_{\text{st}})$. Thus, a stationary policy is peak age optimal.

Proof: Consider a stationary policy with link activation frequency $f_e = \mathbf{P}[B_e(t)]$. Since the channel process $\{R_e(t)\}_t$ is i.i.d. it follows that the link e is successfully activated in a slot with probability $f_e\gamma_e$. This implies that the inter-(successful) activation time of link e is geometrically distributed with mean $\frac{1}{\gamma_e f_e}$, and therefore, the peak age is $\sum_{e \in E} \frac{w_e}{\gamma_e f_e}$. The same result extends to general non-stationary policies in Π, because the existence of the limit (5) for any $\pi \in \Pi$ ensures that the link activation process $U_e(t) = \mathbb{I}_{\{e \in m(t), m(t) \in \mathcal{A}\}}$ is ergodic. The detailed arguments are presented in Appendix A. ■

Theorem 1 implies that the peak age minimization problem can be written as

$$\begin{aligned} \underset{\mathbf{f}}{\text{Minimize}} \quad & \sum_{e \in E} \frac{w_e}{\gamma_e f_e} \\ \text{subject to} \quad & \mathbf{f} \in \mathcal{F}, \end{aligned} \quad (11)$$

where $\mathcal{F}$ - given in (7) - is the space of all link activation frequencies for policies in Π. We discuss solutions to (11) under general, and more specific, interference constraints in Section III-A.

Theorem 1 implies that a stationary centralized policy is peak age optimal. We will next show that a peak age optimal stationary policy is also within a factor of 2 from the optimal average age. We first show an important relation between peak and average age for any policy $\pi \in \Pi$.

Theorem *2:* For all $\pi \in \Pi$ we have

$$A^{\text{p}}(\pi) \leq 2A^{\text{ave}}(\pi) - \sum_{e \in E} w_e. \tag{12}$$

Proof: The result is a direct implication of Cauchy-Schwartz inequality. See Appendix B. ■

Let $A^{\text{p}*} = \min_{\pi \in \Pi} A^{\text{p}}(\pi)$ and $A^{\text{ave}*} = \min_{\pi \in \Pi} A^{\text{ave}}(\pi)$ be the optimal peak and average age, respectively, over the space of all policies in Π. Since the relation (12) holds for every policy $\pi \in \Pi$, it is natural to expect it to hold at the optimality. This is indeed true.

Corollary *1:* The optimal peak age is bounded by

$$A^{\text{p}*} \leq 2A^{\text{ave}*} - \sum_{e \in E} w_e. \tag{13}$$

Proof: Since $A^{\text{p}*}$ is the optimal peak age we have $A^{\text{p}*} \leq A^{\text{p}}(\pi)$ for any policy π. Substituting this in (12) we get $A^{\text{p}*} \leq 2A^{\text{ave}}(\pi) - \sum_{e \in E} w_e$, for all $\pi \in \Pi$. Minimizing the right hand side over all $\pi \in \Pi$ we obtain the result. ■

We next show that for any stationary policy the average and peak age are equal.

Lemma *1:* We have $A^{\text{ave}}(\pi) = A^{\text{p}}(\pi)$ for any stationary policy $\pi \in \Pi$.

Proof: See Appendix C. ■

An immediate implication of Corollary 1 and Lemma 1 is that a stationary peak age optimal policy is also within a factor of 2 from the optimal average age.

Theorem *3:* If π_C is a stationary policy that minimizes peak age over the policy space Π then the average age for π_C is within factor 2 of the optimal average age. Specifically,

$$A^{\text{ave}*} \leq A^{\text{ave}}(\pi_C) \leq 2A^{\text{ave}*} - \sum_{e \in E} w_e. \tag{14}$$

Proof: See Appendix D. ■

Theorem 3 tells us that the stationary peak age optimal policy obtained by solving (11) is within a factor of 2 of optimal average age. Motivated by this, we next characterize solutions to the problem (11).

A. Optimal Stationary Policy π_C

The peak age minimization problem (11) over $\mathcal{F}$ can be written as

$$\begin{array}{ll} \underset{\mathbf{x} \in \mathbb{R}^{|\mathcal{A}|}, \mathbf{f} \in \mathbb{R}^{|E|}}{\text{Minimize}} & \sum_{e \in E} \frac{w_e}{\gamma_e f_e} \\ \text{subject to} & \mathbf{f} = M\mathbf{x} \\ & \mathbf{1}^T \mathbf{x} \leq 1, \; \mathbf{x} \geq 0 \end{array} \tag{15}$$

Note that this is a convex optimization problem in standard form [20]. The solution to it is a vector $\mathbf{x} \in \mathbb{R}^{|\mathcal{A}|}$ that defines a probability distribution over link activation sets $\mathcal{A}$, and determines a stationary centralized policy that minimizes peak age. Average age for this policy, by Theorem 3, is also within a factor of 2 from the optimal average age. We denote this stationary centralized policy by π_C.

We first characterize the optimal solution to (15) for any $\mathcal{A}$. Given $\mathbf{x} \in \mathbb{R}^{|\mathcal{A}|}$, a probability distribution over the link activation sets $\mathcal{A}$, $\mathbf{f} = M\mathbf{x} \in \mathbb{R}^{|E|}$ is the vector of induced link activation frequencies. Now, define $\mu_m(\mathbf{x})$-weight for every feasible link activation set $m \in \mathcal{A}$ as

$$\mu_m(\mathbf{x}) = \sum_{e \in m} \frac{w_e}{\gamma_e \left(M\mathbf{x}\right)_e^2} = \sum_{e \in m} \frac{w_e}{\gamma_e f_e^2}. \tag{16}$$

Clearly, $\mu_m(\mathbf{x}) > 0$ for every m. We now characterize the optimal solution to (15) in terms of $\mu_m(\mathbf{x})$-weights.

Theorem *4:* $\mathbf{x} \in \mathbb{R}^{|\mathcal{A}|}$ solves (15) if and only if there exist a $\mu > 0$ such that

1) For all $m \in \mathcal{A}$ such that $x_m > 0$ we have $\mu_m(\mathbf{x}) = \mu$
2) $x_m = 0$ implies $\mu_m(\mathbf{x}) \leq \mu$
3) $\sum_{m \in \mathcal{A}} x_m = 1$ and $x_m \geq 0$

Further, μ is the optimal peak age $A^{\text{p}*}$.

Proof: The problem (15) is convex and Slater's conditions are trivially satisfied as all constraints are affine [20]. Thus, the KKT conditions are both necessary and sufficient. We use the KKT conditions to derive the result. See Appendix E for a detailed proof. ■

Theorem 4 implies that at the optimal distribution $\mathbf{x}$, all $m \in \mathcal{A}$ with positive probability, $x_m > 0$, have equal $\mu_m(\mathbf{x})$-weights, while all other $m \in \mathcal{A}$ have smaller $\mu_m(\mathbf{x})$-weights.

Although the set $\mathcal{A}$ is very large, it is mostly the case that only a small subset of it is assigned positive probability. In the following we show that only the maximal sets in $\mathcal{A}$ are assigned positive probability, thereby reducing the number of constraints in (15).

Corollary *2:* If $\mathbf{x}$ is the optimal solution to (15) then $x_m = 0$ for all non-maximal sets $m \in \mathcal{A}$.

Proof: Let $m \in \mathcal{A}$ be a non-maximal set. Thus, there exists a $\overline{m} \in \mathcal{A}$ such that $m' \subsetneq \overline{m}$. By definition of $\mu_m(\mathbf{x})$ we have $\mu_{m'}(\mathbf{x}) < \mu_{\overline{m}}(\mathbf{x})$. Thus, if $x_{m'} > 0$ then we would have $\mu = \mu_{m'}(\mathbf{x}) < \mu_{\overline{m}}(\mathbf{x})$ which is a contradiction. ■

The optimization problem (15), although convex, has a variable space that is $|\mathcal{A}|$-dimensional, and thus, its computational complexity increases exponentially in $|V|$ and $|E|$. It is, however, possible to obtain the solution efficiently in certain specific cases.

1) Single-Hop Interference Network: Consider a network $G = (V, E)$ where links interfere with one another if they share a node, i.e., if they are adjacent. For this network, every feasible activation set is a matching on G, and therefore, $\mathcal{A}$ is a collection of all matchings in G. As a result, the constraint set in (15) is equal to the matching polytope [21]. The problem of finding an optimal schedule reduces to solving a convex optimization problem (15) over a matching polytope. This can be efficiently solved (i.e., in polynomial time) by using the Frank-Wolfe algorithm [22], and the separation oracle for matching polytope developed in [23].

2) K-Link Activation Network: Consider a network $G = (V, E)$ in which at most K links can be activated at any given time; we label links $E = \{0, 2, \ldots |E| - 1\}$. Such interference constraints arise in cellular systems where the K represent the number of OFDM sub-channels or number of sub-frames available for transmission in a cell [6].

The set $\mathcal{A}$ is a collection of all subsets of E of size at most K. This forms a uniform matroid over E [21]. As a result, the constraint set in (15) is the uniform matroid polytope. It is known that the inequalities $\sum_{e \in E} f_e \leq K$ and $0 \leq f_e \leq 1$, for all $e \in E$, are necessary and sufficient to describe this polytope [21]. Thus, the peak age minimization problem (15) reduces to

$$\begin{aligned} \underset{\mathbf{f} \in [0,1]^{|E|}}{\text{Minimize}} \quad & \sum_{e \in E} \frac{w_e}{\gamma_e f_e} \\ \text{subject to} \quad & \sum_{e \in E} f_e \leq K \end{aligned} \tag{17}$$

Since the number of constraints is now linear in $|E|$, this problem can be solved using standard convex optimization algorithms [20].

IV. Minimizing Age with Buffered Sources

We now consider a network with buffered sources, where each source generates update packets according to a Bernoulli process. The generated packets get queued at the MAC layer FIFO queue for transmission. We restrict our scope to stationary policies.

Let π be a stationary policy with link activation frequency f_e for link e. Then the service of the link e's MAC layer FIFO queue is Bernoulli at rate $\gamma_e f_e$. The buffered source (link e), in effect, behaves as a discrete time FIFO G/Ber/1 queue. In the following subsection, we derive peak and average age for a discrete time G/Ber/1 queue. We use these results for the network case in sub-sections IV-B and IV-C.

A. Discrete Time G/Ber/1 Queue

Consider a discrete time FIFO queue with Bernoulli service at rate μ. Let the source generate update packets at epoches of a renewal process. Let X denote the inter-arrival time random variable with general distribution F_X. Note that X takes values in $\{1, 2, \ldots\}$. We assume $\lambda = \mathbb{E}\left[X\right]^{-1} < \mu$.

We derive peak and average age for this G/Ber/1 queue. Age for continuous time FIFO M/M/1 and D/M/1 queues was analyzed in [8]. We will see that the results for the discrete time FIFO Ber/Ber/1 and D/Ber/1, which can be obtained as special cases of our G/Ber/1 results, differ from their continuous time counterparts, namely M/M/1 and D/M/1.

To help derive peak and average age, we first analyze the system time T for a packet in the G/Ber/1 queue.

Lemma *2:* The system time, T, in a FIFO G/Ber/1 queue is geometrically distributed with rate α^*, where α^* is the solution to the equation

$$\alpha = \mu - \mu M_X\left(\log(1 - \alpha)\right), \tag{18}$$

where $M_X(\alpha) = \mathbb{E}\left[e^{\alpha X}\right]$ denotes the moment generating function of the inter-arrival time X.

Proof: See Appendix F ■

Note that the α^* depends on the distribution F_X. Using Lemma 2, we can now compute peak and average age for the G/Ber/1 queue.

Theorem *5:* For G/Ber/1 queue with update packet generation rate λ and service rate μ the peak age is given by

$$A^{\text{p}} = \frac{1}{\alpha^*} + \frac{1}{\lambda}, \tag{19}$$

while the average age is given by

$$A^{\text{ave}} = \lambda \left[\frac{M_X''(0)}{2} + \frac{1}{\alpha^*} M_X'\left(\log(1 - \alpha^*)\right)\right] + \frac{1}{\mu} + \frac{1}{2}, \tag{20}$$

where α^* is given by (18), and $M_X = \mathbb{E}\left[e^{\alpha X}\right]$ is the moment generating function of the inter-generation time X.

Proof: The peak age is given by [9]

$$A^{\text{p}} = \mathbb{E}\left[T + X\right], \tag{21}$$

where T is the steady state system time, and X is the inter-arrival time of update packets. Since $\mathbb{E}\left[X\right] = \frac{1}{\lambda}$ and $\mathbb{E}\left[T\right] = \frac{1}{\alpha^*}$, form Lemma 2, the result follows. The proof for A^{ave} is given in Appendix G. ■

We now use this result to obtain optimal rate control and scheduling policy for the buffered case.

B. Bernoulli Generation of Update Packets

Using Theorem 5, we now derive peak and average age for Bernoulli packet generation. Let λ_e be the packet generation rate for link e. If link e is getting served at link activation frequency f_e under a stationary policy π, then its peak age is given by

$$A_e^{\text{p}}(f_e, \rho_e) = \begin{cases} \frac{1}{\gamma_e f_e}\left[\frac{1}{\rho_e} + \frac{1}{1-\rho_e}\right] - \frac{\rho_e}{1-\rho_e}, & \text{if } \gamma_e f_e < 1 \\ \frac{1}{\rho_e}, & \text{if } \gamma_e = 1, f_e = 1 \end{cases}, \tag{22}$$

while its average age is given by

$$A_e^{\text{ave}}(f_e, \rho_e) = \begin{cases} \frac{1}{\gamma_e f_e}\left[1 + \frac{1}{\rho_e} + \frac{\rho_e^2}{1-\rho_e}\right] - \frac{\rho_e^2}{1-\rho_e}, & \text{if } \gamma_e f_e < 1 \\ 1 + \frac{1}{\rho_e}, & \text{if } \gamma_e f_e = 1 \end{cases}, \tag{23}$$

where $\rho_e = \frac{\lambda_e}{\gamma_e f_e}$. See Appendix H for a detailed derivation.

In order to minimize AoI, unlike in the active source case, we need to jointly optimize over packet generation rates λ_e, or ρ_e, and scheduling policy π. Using (22), the peak age minimization problem is given by

$$\begin{aligned} A_{\mathcal{B}}^{\text{p}*} = &\underset{\mathbf{f}, \boldsymbol{\rho} \in [0,1]^{|E|}}{\text{Minimize}} \quad \sum_{e \in E} w_e A_e^{\text{p}}(f_e, \rho_e) \\ &\text{subject to} \quad \mathbf{f} \in \mathcal{F} \end{aligned} \tag{24}$$

where $\mathcal{F}$ is the set of feasible link activation frequencies; see (7). Similarly, the average age minimization problem is given by

$$\begin{aligned} A_{\mathcal{B}}^{\text{ave}*} = &\underset{\mathbf{f}, \rho \in [0,1]^{|E|}}{\text{Minimize}} \quad \sum_{e \in E} w_e A_e^{\text{ave}}(f_e, \rho_e) \\ &\text{subject to} \quad \mathbf{f} \in \mathcal{F} \end{aligned} \tag{25}$$

We now derive an important separation principle which leads to a simple and practical solution to these problems.

For $\gamma_e f_e < 1$, the peak and average age for link e can be upper bounded by as follows:

$$A_e^{\text{p}}(f_e, \rho_e) \leq \frac{1}{\gamma_e f_e}\left[\frac{1}{\rho_e} + \frac{1}{1-\rho_e}\right], \tag{26}$$

and

$$A_e^{\text{ave}}(f_e, \rho_e) \leq \frac{1}{\gamma_e f_e}\left[1 + \frac{1}{\rho_e} + \frac{\rho_e^2}{1-\rho_e}\right]. \tag{27}$$

The upper bounds in (26) and (27) are, in fact, the peak age and average age for the M/M/1 queue [8], [9]. It is easy to see that the peak age upper bound is minimized with $\rho = \frac{1}{2}$ and the average age upper bound is minimized with $\rho \in [0, 1]$ that solves $\rho^4 - 2\rho^3 + \rho^2 - 2\rho + 1 = 0$. This is approximately given by $\rho \approx 0.53$ [8].

We make the following observation.

Result *1:* Minimizing the upper-bound in (26) over ρ, which occurs at $\rho = \frac{1}{2}$, results in peak age that is within an additive factor of 1 of the optimal peak age, i.e.,

$$A_e^{\text{p}}\left(f_e, \frac{1}{2}\right) - \min_{\rho \in [0,1]} A_e^{\text{p}}(f_e, \rho) \leq 1, \tag{28}$$

for all $f_e \in (0, 1)$. Similarly, if $\overline{\rho}$ minimizes the upper-bound in (27), then

$$A_e^{\text{ave}}(f_e, \overline{\rho}) - \min_{\rho \in [0,1]} A_e^{\text{ave}}(f_e, \rho) \leq 1, \tag{29}$$

for all $f_e \in (0, 1)$.

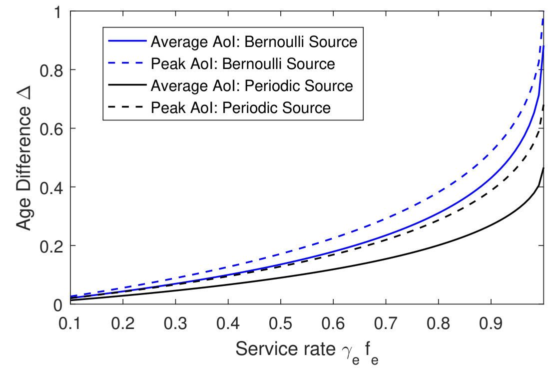

Fig. 2. Plot of age difference $\Delta = A_e(f_e, \overline{\rho}) - \min_{\rho_e} A_e(f_e, \rho_e)$ as a function of service rate $\gamma_e f_e$ for Bernoulli and periodic packet generation.

To see this, consider the age difference function

$$\Delta = A_e(f_e, \overline{\rho}) - \min_{\rho_e} A_e(f_e, \rho_e) \tag{30}$$

for both peak and average age. Since it depends on f_e and γ_e only through the product $\gamma_e f_e$, we can verify the result by plotting Δ as a function of $\gamma_e f_e$, which is the service rate for link e. Figure 2 plots Δ, for both peak and average age, as a function of $\gamma_e f_e$. We see that the age difference Δ is always below 1, thereby, validating Result 1. The proof for (28) is in Appendix I, while we conjecture (29).

Motivated by this we propose the following separation principle:

1) Schedule links according to the stationary policy π_C that minimizes peak age in the active source case. Here, π_C is obtained as a solution to problem (15).
2) Choose $\rho_e = \overline{\rho}$, for all $e \in E$, that minimizes the upper bound in (26) for peak age and upper bound in (27) for average age.
3) Generate update packets according to a Bernoulli process of rate $\lambda_e = \overline{\rho}\gamma_e f_e$.

We call this the separation principle policy (SPP). Note that the rate control $\rho_e = \overline{\rho}$ is the same for all links. We now prove that the SPP is close to the optimal peak and average age, namely, $A_{\mathcal{B}}^{\text{p}*}$ and $A_{\mathcal{B}}^{\text{ave}*}$, respectively.

Theorem *6:* Let $\mathbf{f}^*$ be the link activation frequency vector of the stationary policy π_C.

1) Peak age of the stationary policy π_C with rate control $\rho_e = 1/2$ is bounded by

$$A^{\text{p}}(\mathbf{f}^*, \mathbf{1}/2) \leq A_{\mathcal{B}}^{\text{p}*} + \sum_{e \in E} w_e. \tag{31}$$

2) Average age of the stationary policy π_C with rate control $\rho_e = \overline{\rho}$ is bounded by

$$A^{\text{ave}}(\mathbf{f}^*, \overline{\rho}\mathbf{1}) \leq A_{\mathcal{B}}^{\text{ave}*} + \sum_{e \in E} w_e, \tag{32}$$

where $\overline{\rho} \in [0,1]$ is the unique solution to

$$\rho^4 - 2\rho^3 + \rho^2 - 2\rho + 1 = 0. \tag{33}$$

Proof: See Appendix J. ■

Theorem 6, therefore, says that when we restrict to stationary policies, separation between rate control and scheduling is nearly optimal. That is, if the rate control (choosing ρ_e) is performed assuming that there are no other contending links, and link scheduling is done by assuming active sources then the resulting solution is close to optimal. This is a significant observation for the design of age optimal policies because the rate control and scheduling policies are implemented at different layers of the protocol stack.

1) Performance Bounds under Non-Stationary Policies: It is conceivable that a non-stationary policy may perform significantly better then stationary policies. Such a policy may attempt transmissions or schedule links depending on queue backlogs. We now show that the SSP is a constant factor away from optimality.

Let $A_{\mathcal{QB}}^{\text{p}*}$ and $A_{\mathcal{QB}}^{\text{ave}*}$ denote the minimum peak and average age that can be achieved under all scheduling policies and rate controlled Bernoulli arrivals.

Corollary *3:* Let $\mathbf{f}^*$ be a vector of link activation frequencies under the stationary policy π_C.

1) Peak age of the stationary policy π_C with rate control $\rho_e = 1/2$ is bounded by

$$A_{\mathcal{QB}}^{\text{p}*} \leq A^{\text{p}}\left(\mathbf{f}^*, \mathbf{1}/2\right) \leq 4A_{\mathcal{QB}}^{\text{p}*}. \tag{34}$$

2) Average age of the stationary policy π_C with rate control $\rho_e = \overline{\rho}$ is bounded by

$$A_{\mathcal{QB}}^{\text{p}*} \leq A^{\text{p}}\left(\mathbf{f}^*, \overline{\rho}\mathbf{1}\right) \leq 2\alpha A_{\mathcal{QB}}^{\text{p}*}, \tag{35}$$

where $\alpha = 1 + \frac{1}{\overline{\rho}} + \frac{\overline{\rho}^2}{1-\overline{\rho}}$ and $\overline{\rho} \in [0,1]$ is the unique solution to

$$\rho^4 - 2\rho^3 + \rho^2 - 2\rho + 1 = 0. \tag{36}$$

Proof: See Appendix K. ■

We know that the $\rho \in [0,1]$ that solves (33) is ≈ 0.53. This yields $\alpha = 1 + \frac{1}{\rho} + \frac{\rho^2}{1-\rho} \approx 3.48$. Thus, Corollary 3 implies that the average age of the stationary policy π_C with rate control $\lambda_e = \rho\gamma_e f_e$ is at most a factor of $2\alpha \approx 7$ away from optimality. It is important to note that the constant factors of optimality in Corollary 3 are independent of the network size. These factors, namely 4 and α, are equal to the optimal peak and average age upper-bounds of a single link network, where we have $\gamma_e f_e = 1$.

C. Periodic Generation of Update Packets

Since the packet generation is entirely a design parameter, we now consider the case where the sources generate update packets periodically. We will derive optimal packet generation period D_e, or equivalently rate $\lambda_e = 1/D_e$, for each link e along with a scheduling policy in order to minimize age.

Using Theorem 5 we can obtain peak and average age for a link e under any stationary policy π and periodic arrivals. Let f_e be the link activation frequency for link e under policy π, and D_e be the period of packet generation for link e. Then the peak age is given by

$$A^{\text{p}}\left(f_e, \rho_e\right) = \frac{1}{\gamma_e f_e}\left[\frac{1}{\rho_e} + \frac{1}{\sigma_e^*}\right], \tag{37}$$

where $\rho_e = \left(D_e \gamma_e f_e\right)^{-1}$ and σ_e^* is the solution to the equation $\sigma = 1 - \left(1 - \sigma\gamma_e f_e\right)^{D_e}$. Average age for the same link e is given by

$$A_e^{\text{ave}}\left(f_e, \rho_e\right) = \frac{1}{\gamma_e f_e}\left[\frac{1}{2\rho_e} + \frac{1}{\sigma_e^*}\right] + \frac{1}{2}. \tag{38}$$

See Appendix L for a detailed derivation.

Our objective is to minimize $\sum_{e \in E} w_e A_e^{\text{p}}(f_e, \rho_e)$ for peak age and $\sum_{e \in E} w_e A_e^{\text{ave}}(f_e, \rho_e)$ for average age, where $\mathbf{f}$ and $\boldsymbol{\rho}$ take values in $\mathcal{F}$ and $[0,1]^{|E|}$, respectively. Let $A_{\mathcal{D}}^{\text{p}*}$ and $A_{\mathcal{D}}^{\text{ave}*}$ denote the minimum peak and average age, jointly optimized over scheduling policy π and rate control ρ. We first upper-bound peak and average age just as we did for the Bernoulli packet generation case.

Lemma *3:* The peak and average age for link e is upper bounded by

$$A_e^{\text{p}}\left(f_e, \rho_e\right) \leq \frac{1}{\gamma_e f_e}\left[\frac{1}{\hat{\sigma}_e} + \frac{1}{\rho_e}\right], \tag{39}$$

and

$$A_e^{\text{ave}}\left(f_e, \rho_e\right) \leq \frac{1}{\gamma_e f_e}\left[\frac{1}{2\hat{\sigma}_e} + \frac{1}{\rho_e}\right] + \frac{1}{2}, \tag{40}$$

where $\hat{\sigma}_e$ solves $\hat{\sigma}_e = 1 - e^{-\frac{\hat{\sigma}_e}{\rho_e}}$.

Proof: Using the fact that $1 - x \leq e^{-x}$, we have

$$1 - \left(1 - f_e\gamma_e\sigma\right)^{D_e} \geq 1 - e^{-\sigma/\rho_e}, \tag{41}$$

where $\rho_e = \frac{1}{D_e\gamma_e f_e}$. This implies that if σ^* solves $\sigma = 1 - (1 - \gamma_e f_e\sigma)^{D_e}$ and $\hat{\sigma}$ solves $\sigma = 1 - e^{-\sigma/\rho_e}$ then $\hat{\sigma} \leq \sigma^*$. The result follows from this. ■

It is important to note that σ^* in (37) and (38) is different from $\hat{\sigma}$ in Lemma 3. In particular, σ^* depends on f_e and D_e while $\hat{\sigma}$ is a function only of $\frac{1}{D_e\gamma_e f_e}$, which is queue occupancy ρ_e. As a result the $\rho_e = \overline{\rho}$ that minimizes the upper-bound(s) in Lemma 3 is independent of f_e. We now make the following important observation on choosing $\rho_e = \overline{\rho}_e$, which minimizes the upper-bounds, and in choosing the best ρ.

Result *2:* If $\overline{\rho}^{\text{p}}$ minimizes the upper-bound on peak age in (39) then

$$A_e^{\text{p}}\left(f_e, \overline{\rho}^{\text{p}}\right) - \min_{\rho \in [0,1]} A_e^{\text{p}}\left(f_e, \rho\right) \leq 1, \quad (42)$$

for all $f_e \in (0, 1)$. Similarly, if $\overline{\rho}^{\text{ave}}$ minimizes the upper-bound on average age in (40) then

$$A_e^{\text{ave}}\left(f_e, \overline{\rho}^{\text{ave}}\right) - \min_{\rho \in [0,1]} A_e^{\text{ave}}\left(f_e, \rho\right) \leq 1, \quad (43)$$

for all $f_e \in (0, 1)$.

Figure 2 plots the age difference

$$\Delta = A_e(f_e, \overline{\rho}) - \min_{\rho_e} A_e(f_e, \rho_e),$$

as a function of $\gamma_e f_e$. We see that the age difference Δ is always below 1, thereby, justifying our result. We do not prove Result 2, but conjecture it to be true. In fact, Δ for peak age is observed to be less than 0.7 and Δ for average age is observed to be less than 0.6.

Motivated by this we again resort to the same SPP as in Section IV-B but with different rate control ρ_e which minimizes upper bounds in (39) and (40). We prove the following bounds for this SPP.

Theorem *7:* Let $\mathbf{f}^*$ be the link activation frequency vector of the stationary policy π_C.

1) Peak age of the stationary policy π_C with rate control $\rho_e = \overline{\rho}^{\text{p}}$ is bounded by

$$A^{\text{p}}(\mathbf{f}^*, \overline{\rho}^{\text{p}}\mathbf{1}) \leq A_{\mathcal{D}}^{\text{p}*} + \sum_{e \in E} w_e, \quad (44)$$

where $\overline{\rho}^{\text{p}}$ minimizes the peak age upper-bound in Lemma 3.

2) Average age of the stationary policy π_C with rate control $\rho_e = \overline{\rho}^{\text{ave}}$ is bounded by

$$A^{\text{ave}}(\mathbf{f}^*, \overline{\rho}^{\text{ave}}\mathbf{1}) \leq A_{\mathcal{D}}^{\text{ave}*} + \sum_{e \in E} w_e, \quad (45)$$

where $\overline{\rho}^{\text{ave}}$ minimizes the average age upper-bound in Lemma 3.

Proof: The proof follows the same line of arguments as that of Theorem 6. ■

Numerically, it can be observed that $\overline{\rho}^{\text{p}} \approx 0.594$ and $\overline{\rho}^{\text{ave}} \approx 0.515$ [8].

1) Performance Bounds over non-Stationary Policies: We now consider the performance of policy π_C with rate control described in Theorem 7. We show that among all policies, which may possibly employ queue based scheduling, the policy and rate control of Theorem 7 is within a constant factor of optimality.

Let $A_{\mathcal{QD}}^{\text{p}*}$ and $A_{\mathcal{QD}}^{\text{ave}*}$ denote the minimum peak and average age that can be achieved under all scheduling policies and rate controlled periodic arrivals. We prove that our SPP is a constant factor away from the optimal age.

TABLE I
RATE CONTROL FOR SEPARATION PRINCIPLE POLICY (SPP) $\overline{\rho} = \frac{\lambda_e}{f_e}$ AND OPTIMALITY OF SPP OVER SPACE OF ALL POLICIES.

Peak Age	$\overline{\rho}$	Factor of optimality
Bernoulli updates	$1/2$	4
Periodic updates	≈ 0.594	≈ 2.15
Average Age	Optimal ρ	Factor of optimality
Bernoulli updates	≈ 0.53	≈ 7
Periodic updates	≈ 0.515	≈ 4.51

Corollary *4:* Let $\mathbf{f}^*$ be the vector of link activation frequencies for policy π_C.

1) Peak age of the stationary policy π_C with rate control $\rho_e = \overline{\rho}^{\text{p}}$ is bounded by

$$A_{\mathcal{QD}}^{\text{p}*} \leq A^{\text{p}}\left(\mathbf{f}^*, \overline{\rho}^{\text{p}}\mathbf{1}\right) \leq \theta A_{\mathcal{QD}}^{\text{p}*}, \quad (46)$$

where $\theta = \frac{1}{\hat{\sigma}} + \frac{1}{\overline{\rho}^{\text{p}}}$, and $\hat{\sigma} \in (0, 1)$ is the unique solution to $\sigma = 1 - e^{-\sigma/\overline{\rho}^{\text{p}}}$.

2) Average age of the stationary policy π_C with rate control $\rho_e = \overline{\rho}^{\text{ave}}$ is bounded by

$$A_{\mathcal{QD}}^{\text{ave}*} \leq A^{\text{ave}}\left(\mathbf{f}^*, \overline{\rho}^{\text{ave}}\mathbf{1}\right) \leq 2\beta A_{\mathcal{QD}}^{\text{ave}*}, \quad (47)$$

where $\beta = \frac{1}{2\hat{\sigma}} + \frac{1}{\overline{\rho}^{\text{ave}}}$, and $\hat{\sigma} \in (0, 1)$ is the unique solution to $\sigma = 1 - e^{-\sigma/\overline{\rho}^{\text{ave}}}$.

Proof: The proof follows the same line of arguments as that of Corollary 3. ■

It is again important to note that θ and β are equal to the peak and average age upper-bounds in Lemma 3 when $\gamma_e f_e = 1$. Numerically, we obtain $\theta \approx 2.151$ and $\beta \approx 2.256$. Therefore, the peak age SPP policy is within a factor of ≈ 2.15 from the optimal peak age, while the average age SPP policy is within a factor of ≈ 4.51 from the optimal average age.

In Table I we summarize the performance of our SPP policies, under both Bernoulli and periodic packet generation, over the space of all policies. If the bounds are tight, then it suggests that periodic packet generation should perform much better than Bernoulli generation.

V. Numerical Results

We consider a K-link activation network with N links. A fraction θ of the links have bad channel with $\gamma_e = \gamma_{\text{bad}}$, while the rest have $\gamma_e = \gamma_{\text{good}} > \gamma_{\text{bad}}$. We let $w_e = 1$ for all the links. Similar results are observed for single-hop interference network.

A. Network with Active Sources

First, consider the case in which all the sources in the network are active sources. We plot and compare the proposed peak age optimal policy π_C (shown in red), a uniform stationary policy that schedules maximal subsets in $\mathcal{A}$ randomly

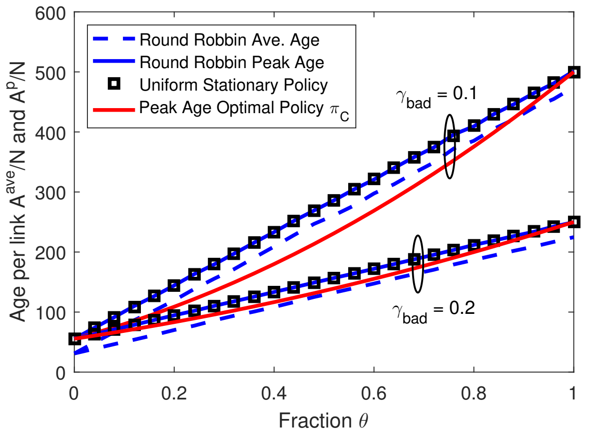

Fig. 3. Plot of AoI as a function of fraction of links, θ, with bad channel. $N = 50$, $K = 1$, $\gamma_{\text{good}} = 0.9$, and $\gamma_{\text{bad}} = 0.1$ and 0.2.

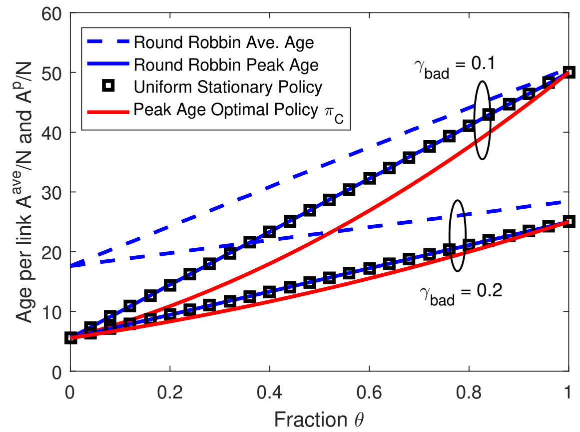

Fig. 4. Plot of AoI as a function of fraction of links, θ, with bad channel. $N = 50$, $K = 10$, $\gamma_{\text{good}} = 0.9$, and $\gamma_{\text{bad}} = 0.1$ and 0.2.

with uniform probability, and a round robbin policy (RR) that schedules K links at a time.

Figure 3 considers the simplest case with $K = 1$, and plots peak and average age per-link, which is A^{p}/N and A^{ave}/N respectively, as a function of θ. Here, the network has $N = 50$ links, $\gamma_{\text{good}} = 0.9$, and two cases of $\gamma_{\text{bad}} = 0.1$ and $\gamma_{\text{bad}} = 0.2$ are plotted. Note that the peak age and average age coincide for stationary policies by Theorem 1. We observe this in simulation. Thus, to reduce clutter, we have plotted only one curve for the peak age optimal policy π_C and the uniform stationary policy.

We observe in Figure 3 that both peak and average age increases as θ, the fraction of links with bad channel, increases. This is to be expected as with more error prone channels, it takes more time for the source to update the destination. For $\gamma_{\text{bad}} = 0.1$, we observe in Figure 3, that the peak age optimal policy π_C achieves the minimum peak age. Furthermore, when the channel statistics are more asymmetric, i.e. θ not near 0 or 1, the average age performance of the peak age optimal policy π_C is better than the round robbin and uniform stationary policy. We also observe that the round robbin policy and uniform stationary policy achieve the same peak age, this validates Theorem 1 which states that any two policies with same link activation frequencies should have the same peak age.

In Figure 3, we see that when the channel statistics across links is more symmetric (i.e., θ closer to 0 or 1), the round robbin policy yields a slightly smaller average age than the peak age optimal policy π_C. In fact, when γ_{bad} is increased to 0.2, the round robbin policy performs better in average age for all θ. However, the average age optimal centralized scheduling policy is yet unknown even for this simple network (with $K = 1$), and hence by Theorem 3, the average age of the peak age optimal policy π_C is at most factor 2 away from the optimal average age, which is consistent with Figure 3.

This problem is exacerbated when we move to $K > 1$, in which case it is difficult to intuit a 'good' policy that minimizes average age. Figure 4 plots average and peak age per link as a function of θ. All the parameters are same as in Figure 3, except that we can activate $K = 10$ links at a time. We observe that, other than ensuring peak age optimality, the proposed policy π_C also far outperforms other policies in terms of its average age. This observation is not limited to the policies presented here, but in general, as it is difficult to come up with average age optimal policies for a network with general interference constraints.

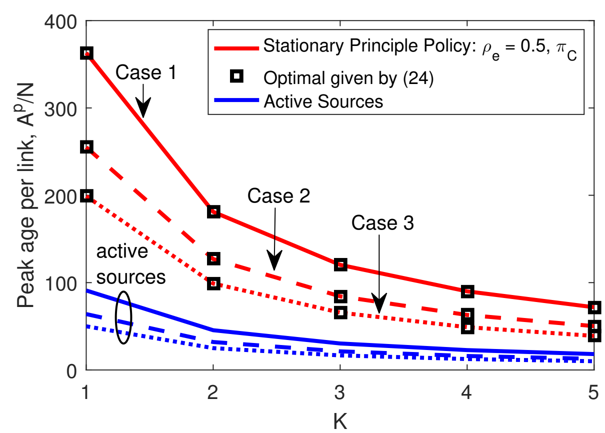

Fig. 5. Plot of peak age, for a network of buffered nodes, as a function of K. Case 1: $N = 50$, $\gamma_{\text{good}} = 0.9$, $\gamma_{\text{bad}} = 0.1$, $n_{\text{bad}} = 7$. Case 2: $N = 10$, $\gamma_{\text{good}} = 0.9$, $\gamma_{\text{bad}} = 0.1$, $n_{\text{bad}} = 7$. Case 3: $N = 50$ and $\gamma_e = 1$ for all links.

B. Network with Buffered Sources

We next consider the sources in the network to be buffered sources. We assume Bernoulli arrival of update packets. We plot three cases to illustrate the near optimality of the separation principle policy (SPP): In Case 1, we have $N = 50$, $n_{\text{bad}} = 7$ links have bad channel, i.e., $\gamma_e = \gamma_{\text{bad}} = 0.1$ while the remaining have good channel $\gamma_e = \gamma_{\text{good}} = 0.9$. In Case 2, we have $N = 10$, $n_{\text{bad}} = 7$, $\gamma_{\text{bad}} = 0.1$, and $\gamma_{\text{good}} = 0.9$. In

Case 3, we consider $N = 50$ and $\gamma_e = 1$ for all links e. We let link weights to be unity, i.e. $w_e = 1$ for all e.

We compare the peak age SPP, which chooses $\rho_e = 1/2$ for every link and the link activation frequency $\mathbf{f}^*$ that solves (17). In Figure 5, we plot the peak age achieved by the peak age SPP and the optimal $A^{\text{p}*}_{\mathcal{B}}$ of (24), obtained numerically. We observe that the SPP nearly attains the optimal peak age for buffered node in (24) in all three cases.

This can be seen from our observation in Figure 2. In Figure 2, we observe that the age difference between optimal age and the age with rate control $\rho_e = 1/2$, which is Δ, diminishes drastically as link activation frequency decreases. When the interference in the network is large, the link activation frequencies are bound to be small. This essentially results in close proximity of our separation principle policy with the optimal.

In Figure 5, we also plot (in blue) peak age if the network had active sources instead of buffered sources. We observe that optimal peak age for the buffered case is about 4 times that in the active source case. This shows that the cost of not being able to control the MAC layer queue can be as large as a 4 fold increase in age.

VI. Conclusion

We considered the problem of minimizing age of information in wireless networks, under general interference constraints. We first considered a network with active source, where fresh updates are available for every transmission, and showed that a stationary policy is peak age optimal. We also showed that this policy achieves average age that is within a factor of two of the optimal average age. For a network with buffered sources, in which the generated update packets are queued at the MAC layer queue for transmission, we proved an important separation principle wherein it suffices to design scheduling and rate control separately. Numerical evaluation suggest that this proposed separation principle policy is nearly indistinguishable from the optimal. We also derived peak age and average age for discrete time FIFO G/Ber/1 queue, which may be of independent interest.

References

[1] R. Talak, S. Karaman, and E. Modiano, "Minimizing age-of-information in multi-hop wireless networks," in *Proc. Allerton*, Oct. 2017.

[2] I. Bekmezci, O. K. Sahingoz, and S. Temel, "Flying ad-hoc networks (FANETs): A survey," *Ad Hoc Networks*, vol. 11, pp. 1254–1270, May 2013.

[3] R. Talak, S. Karaman, and E. Modiano, "Speed limits in autonomous vehicular networks due to communication constraints," in *Proc. CDC*, pp. 4998–5003, Dec. 2016.

[4] K. D. Kim and P. R. Kumar, "Cyber-physical systems: A perspective at the centennial," *Proceedings of the IEEE*, vol. 100, pp. 1287–1308, May 2012.

[5] S. Guharoy and N. B. Mehta, "Joint evaluation of channel feedback schemes, rate adaptation, and scheduling in OFDMA downlinks with feedback delays," *IEEE Transactions on Vehicular Technology*, vol. 62, pp. 1719–1731, May 2013.

[6] S. Sesia, I. Toufik, and M. Baker, *LTE, The UMTS Long Term Evolution: From Theory to Practice*. Wiley Publishing, 2009.

[7] S. Kaul, M. Gruteser, V. Rai, and J. Kenney, "Minimizing age of information in vehicular networks," in *Proc. SECON*, pp. 350–358, Jun. 2011.

[8] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?," in *Proc. INFOCOM*, pp. 2731–2735, Mar. 2012.

[9] L. Huang and E. Modiano, "Optimizing age-of-information in a multi-class queueing system," in *Proc. ISIT*, pp. 1681–1685, Jun. 2015.

[10] C. Kam, S. Kompella, and A. Ephremides, "Age of information under random updates," in *Proc. ISIT*, pp. 66–70, Jul. 2013.

[11] K. Chen and L. Huang, "Age-of-information in the presence of error," *ArXiv e-prints*, May 2016.

[12] S. K. Kaul, R. D. Yates, and M. Gruteser, "Status updates through queues," in *Proc. CISS*, pp. 1–6, Mar. 2012.

[13] E. Najm and R. Nasser, "Age of information: The gamma awakening," *ArXiv e-prints*, Apr. 2016.

[14] M. Costa, M. Codreanu, and A. Ephremides, "Age of information with packet management," in *Proc. ISIT*, pp. 1583–1587, Jun. 2014.

[15] Q. He, D. Yuan, and A. Ephremides, "Optimizing freshness of information: On minimum age link scheduling in wireless systems," in *Proc. WiOpt*, pp. 1–8, May 2016.

[16] I. Kadota, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Minimizing the age of information in broadcast wireless networks," in *Proc. Allerton*, pp. 844–851, Sep. 2016.

[17] Y.-P. Hsu, E. Modiano, and L. Duan, "Age of information: Design and analysis of optimal scheduling algorithms," in *Proc. ISIT*, pp. 1–5, Jun. 2017.

[18] S. K. Kaul and R. D. Yates, "Status updates over unreliable multiaccess channels," *ArXiv e-prints*, May 2017.

[19] A. Kumar, D. Manjunath, and J. Kuri, *Wireless Networking*. Morgan Kaufmann, 2008.

[20] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[21] B. Korte and J. Vygen, *Combinatorial Optimization: Theory and Algorithms*. Springer Publishing Company, Incorporated, 4th ed., 2007.

[22] D. Garber and E. Hazan, "A linearly convergent variant of the conditional gradient algorithm under strong convexity, with applications to online and stochastic optimization," *SIAM J. on Opt.*, vol. 26, no. 3, pp. 1493–1528, 2016.

[23] B. Hajek and G. Sasaki, "Link scheduling in polynomial time," *IEEE Trans. Inf. Theory*, vol. 34, pp. 910–917, Sep. 1988.

[24] R. Durrett, *Probability: Theory and Examples*. Cambridge University Press, 4 ed., 2010.

[25] R. W. Wolff, *Stochastic Modeling and the Theory of Queues*. Prentice Hall, 1 ed., 1989.

[26] S. M. Ross, *Stochastic Processes*. John Wiley & Sons, Inc., 2 ed., 1996.

Appendix

A. Proof of Theorem 1

Let π be a policy in Π, and $T_e(i)$ be the time of ith successful activation for link e. Then $S_e(i) = T_e(i) - T_e(i-1)$, for all $i \geq 1$, is the inter-(successful) activation time for link e, where $T_e(0) = 0$. Note that $S(i) = A_e(T_e(i))$ for all age update instances i. This implies that the peak age is given by

$$\limsup_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} A_e\left(T_e(i)\right) = \limsup_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} S_e(i), \tag{48}$$

$$= \limsup_{N \to \infty} \frac{T_e(N)}{N}. \tag{49}$$

Notice that the time $T_e(N) \to \infty$ as $N \to \infty$. We, therefore, have

$$\frac{1}{A^{\text{p}}_e(\pi)} = \liminf_{N \to \infty} \frac{N}{T_e(N)} = \liminf_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} U_e(t) R_e(t), \tag{50}$$

where $U_e(t) = \mathbb{I}_{\{e \in m(t), m(t) \in \mathcal{A}\}}$ and $R_e(t)$ is the channel process. Now, notice that the process $\left(U_e(t), R_e(t)\right)_{t \geq 0}$ is jointly Ergodic. To see this, note that $\{U_e(t)\}_{t \geq 0}$ is an Ergodic processes because $\{U_e(t)\}_{t \geq 0}$ takes values only in $\{0, 1\}$ and the limit in (5) exists for $\pi \in \Pi$. Moreover, since $R_e(t)$ is independent of $U_e(t)$, they are jointly Ergodic. We, therefore, have

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} U_e(t)R_e(t) = \mathbb{E}\left[\liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} U_e(t)R_e(t)\right],$$

$$= \liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[U_e(t)R_e(t)\right], \tag{51}$$

$$= \liminf_{T\to\infty} \gamma_e \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} U_e(t)\right], \tag{52}$$

$$= \gamma_e f_e(\pi), \tag{53}$$

where the first equality follows due to Ergodicity, the second due to the bounded convergence theorem [24], and the third because $R_e(t)$ is i.i.d. across time t with $\gamma_e = \mathbb{E}[R_e(t)]$ and is independent of $U_e(t)$. The last equality follows from (5). Weighted summation over all links $e \in E$ gives the result.

To prove that the peak age $A^{\text{p}}(\pi)$ can be achieved by a stationary centralized policy $\pi_{\text{st}} \in \Pi$, it suffices to show that a stationary centralized policy π_{st} achieve the same link activation frequencies, i.e., $\mathbf{f}(\pi) = \mathbf{f}(\pi_{\text{st}})$.

Let $\pi \in \Pi$ be the policy that achieves link activation frequencies $\mathbf{f} = (f_e | e \in E)$. Then, the policy π activates interference-free sets in $\mathcal{A}$ also with a certain frequency. Let x_m be the frequency of activation for a set $m \in \mathcal{A}$, i.e.,

$$x_m = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}_{\{m(t)=m\}}, \tag{54}$$

where $m(t)$ denotes the set of links activated at time t. Clearly, we should have

$$\sum_{m\in\mathcal{A}} x_m \leq 1. \tag{55}$$

Furthermore, we must have $\mathbf{f}(\pi) = M\mathbf{x}$, where M is given by (8), and $\mathbf{f}(\pi)$ and $\mathbf{x}$ are column vectors of $f_e(\pi)$ and x_m, respectively. Now consider a stationary centralized policy $\pi_{\text{st}} \in \Pi$ for which $m \in \mathcal{A}$ is activated in each slot with probability x_m, independent across slots; we can do this because of the property (55). Then we have $\mathbf{f}(\pi_{\text{st}}) = M\mathbf{x} = \mathbf{f}(\pi)$. This proves the result.

B. Proof of Theorem 2

The proof is a direct consequence of the Cauchy-Schwartz inequality. Consider a policy $\pi \in \Pi$ and let $T_e(i)$ be the time of ith successful activation for link e. Then $S_e(i) = T_e(i) - T_e(i-1)$, for all $i \geq 1$, is the inter-(successful) activation time for link e, where $T_e(0) = 0$. Note that $S(i) = A_e(T_e(i))$ for all age update instances i. This implies that the peak age is given by

$$\limsup_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} A_e\left(T_e(i)\right) = \limsup_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} S_e(i). \tag{56}$$

The average is given by

$$A_e^{\text{ave}}(\pi) = \lim_{N\to\infty} \frac{\sum_{i=1}^{N}\sum_{k=1}^{S(i)} k}{\sum_{i=1}^{N} S(i)} = \lim_{N\to\infty} \frac{\frac{1}{2}\sum_{i=1}^{N} S(i)^2}{\sum_{i=1}^{N} S(i)} + \frac{1}{2}. \tag{57}$$

Cauchy-Schwartz inequality gives us

$$\left(\sum_{i=1}^{N} S(i)\right)^2 \leq N \sum_{i=1}^{N} S^2(i). \tag{58}$$

Therefore, we must have

$$\frac{1}{2}\frac{1}{N}\sum_{i=1}^{N} S(i) \leq \frac{0.5\sum_{i=1}^{N} S(i)^2}{\sum_{i=1}^{N} S(i)}. \tag{59}$$

This with (56) and (57) yields $\frac{1}{2}A_e^{\text{p}}(\pi) + \frac{1}{2} \leq A_e^{\text{ave}}(\pi)$. Note that we can claim this because $A^{\text{p}}(\pi)$ is finite for $\pi \in \Pi$ due to (80). Weighted summation over $e \in E$ gives the desired result.

C. Proof of Lemma 1

For a stationary policy, let p be the probability that link e is successfully activated in a time slot, i.e.,

$$p = \Pr\left(e \in m(t),\ m(t) \in \mathcal{A}\right), \tag{60}$$

where $m(t)$ is the set of links activated at time t. Since the policy is stationary, the inter-(successful) activation times $S_e(i)$ would be independent and geometrically distributed with rate $1/p$ given by: $\Pr\left(S_e(i) = k\right) = p\left(1-p\right)^{k-1}$, for all $k \in \{1, 2, \ldots\}$. For this distribution we know that $\mathbb{E}\left[S_e(1)\right] = \frac{1}{p}$ and $\mathbb{E}\left[S_e^2(1)\right] = \frac{2-p}{p^2}$. Using (57) we obtain the average age of link e to be

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} A_e(t) = \lim_{N\to\infty} \frac{\sum_{i=1}^{N}\frac{1}{2}S_e^2(i)}{\sum_{i=1}^{N} S_e(i)} + \frac{1}{2}, \tag{61}$$

$$= \frac{\frac{1}{2}\frac{2-p}{p^2}}{\frac{1}{p}} + \frac{1}{2} = \frac{1}{p}. \tag{62}$$

Using (56) we obtain the peak age of the link e to be

$$\limsup_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} A_e\left(T_e(i)\right) = \limsup_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} S_e(i), \tag{63}$$

$$= \mathbb{E}\left[S_e(1)\right] = \frac{1}{p}. \tag{64}$$

The result can be obtained from (62) and (64) by weighted-averaging.

D. Proof of Theorem 3

Let π_C be the stationary policy that minimizes peak age. We, thus, have

$$A^{\text{p}}(\pi_C) = A^{\text{p}*}, \tag{65}$$

Since π_C is also a stationary policy, Lemma 1 implies

$$A^{\text{ave}}(\pi_C) = A^{\text{p}}(\pi_C). \tag{66}$$

Using (65), (66), and Corollary 1 we obtain

$$A^{\text{ave}}(\pi_C) = A^{\text{p}}(\pi_C) = A^{\text{p}*} \leq 2A^{\text{ave}*} - \sum_{e \in E} w_e. \tag{67}$$

This proves the result.

E. Proof of Theorem 4

Since dependence of γ_e is only in the form w_e/γ_e, we assume $\gamma_e = 1$, for all e, for clarity of presentation. The dependence on γ_e can be re-constructed by substituting w_e/γ_e in place of w_e in the following proof.

The peak age minimization problem (15) can be re-written with the objective

$$\sum_{e \in E} \frac{w_e}{\sum_{m \in \mathcal{A}} M_{e,m} x_m}, \tag{68}$$

over variables x_m, for $m \in \mathcal{A}$, with constraints $\sum_{m \in \mathcal{A}} x_m \leq 1$ and $x_m \geq 0$ for all $m \in \mathcal{A}$. The Lagrangian function for this problem is

$$L(\mathbf{x}, \mu, \boldsymbol{\nu}) = \sum_{e \in E} \frac{w_e}{\sum_{m \in \mathcal{A}} M_{e,m} x_m} + \mu \left(\sum_{m \in \mathcal{A}} x_m - 1 \right) + \sum_{m \in \mathcal{A}} \nu_m x_m,$$

for $\mu \geq 0$ and $\nu_m \geq 0$ for all $m \in \mathcal{A}$. The KKT conditions then imply

$$\frac{\partial L}{\partial x_l} = 0, \quad \text{for all } l \in E, \tag{69}$$

$$\mu \left(\sum_{m \in \mathcal{A}} x_m - 1 \right) = 0, \tag{70}$$

$$\nu_m x_m = 0 \quad \text{for all } m \in \mathcal{A}, \tag{71}$$

along with feasibility constraints for $\mathbf{x}$, $\mu \geq 0$, and $\nu_m \geq 0$ for all $m \in \mathcal{A}$. Now (69) implies

$$\frac{\partial L}{\partial x_m} = -\sum_{e \in E} \frac{w_e M_{e,m}}{\left(\sum_{m' \in \mathcal{A}} M_{e,m'} x_{m'} \right)^2} + \mu - \nu_m = 0, \tag{72}$$

which reduces to

$$\mu_m(\mathbf{x}) = \mu - \nu_m, \tag{73}$$

for all $m \in \mathcal{A}$. Using (71) and (73) we get that if $x_m > 0$ then $\nu_m = 0$ which implies $\mu_m(\mathbf{x}) = \mu$, while $\mu_m(\mathbf{x}) \leq \mu$ for all $m \in \mathcal{A}$. This proves conditions 1 and 2 of Theorem 4.

Since $\mathbf{x}$ that satisfy the KKT conditions also solve (15) we should have $f_e = \sum_{m \in \mathcal{A}} M_{e,m} x_m > 0$; otherwise the objective function would be unbounded. Thus, $\mu_m(\mathbf{x}) = \mu - \nu_m > 0$ which implies $\mu > 0$ for all $m \in \mathcal{A}$. Then (70) implies $\sum_{m \in \mathcal{A}} x_m = 1$.

Corollary *5:* The μ defined in Theorem 4 is the optimal peak age $A^{\text{p}*}$.

Proof: Given that $\mathbf{x}$ is the optimal solution to (15) and μ be as defined in Theorem 4, the optimal peak age is given by

$$A^{\text{p}*} = \sum_{e \in E} \frac{w_e}{(M\mathbf{x})_e} = \sum_{e \in E} \frac{w_e}{(M\mathbf{x})_e^2} (M\mathbf{x})_e, \tag{74}$$

$$= \sum_{e \in E} \frac{w_e}{(M\mathbf{x})_e^2} \sum_{m \in \mathcal{A}} M_{e,m} x_m. \tag{75}$$

Exchanging the two summations we get

$$A^{\text{p}*} = \sum_{m \in \mathcal{A}} x_m \sum_{e \in E} \frac{w_e M_{e,m}}{(M\mathbf{x})_e^2}, \tag{76}$$

$$= \sum_{m \in \mathcal{A}, x_m > 0} x_m \sum_{e \in m} \frac{w_e}{(M\mathbf{x})_e^2}, \tag{77}$$

where the last equality follows from the definition of $M_{e,m}$. Notice that $\sum_{e \in m} \frac{w_e}{(M\mathbf{x})_e^2}$ is in fact $\mu(m) = \mu$ as $x_m > 0$. This gives

$$A^{\text{p}*} = \sum_{m \in \mathcal{A}, x_m > 0} x_m \mu = \mu, \tag{78}$$

where the last equality follows from condition 3 in Theorem 4. ■

F. Proof of Lemma 2

Let X_n be the inter-arrival time between the $(n-1)$th and nth update packet at the queue, and N_n be the number of update packets in the queue at the arrival of the nth update packet. If Z_{n+1} be the number of service times we can fit in the duration X_{n+1}, then we have

$$N_{n+1} = \max\{N_n + 1 - Z_{n+1}, 0\}. \tag{79}$$

For this recursion, it is known that the steady state distribution of N_n is geometric [25], i.e.,

$$\mathbf{P}[N = k] = \sigma (1 - \sigma)^k, \tag{80}$$

for all $k \in \{0, 1, 2, \ldots\}$.

Now, the system time for the nth update packet is given by

$$T_n = \sum_{j=1}^{N_n + 1} S_j, \tag{81}$$

where S_j are independent, geometrically distributed random variables over $\{1, 2, \ldots\}$ with rate μ. At steady state, since N_n is geometrically distributed over $\{0, 1, 2, \ldots\}$; see (80), $N_n + 1$ will be geometrically distributed over $\{1, 2 \ldots\}$ with the same rate σ. With $N_n + 1$ and S_j being independent and geometrically distributed, we have from (81) that T_n is also geometrically distributed over $\{1, 2, \ldots\}$ with rate $\sigma\mu$.

Let $\alpha = \sigma\mu$ be the rate of geometrically distributed random variable T_n at steady state. We obtain (18) using the system time recursive equation. We know that the system time T_n follows the recursive equation:

$$T_n = \max\{T_{n-1} - X_n, 0\} + S_n, \tag{82}$$

where S_n is the service time of the nth update packet. Taking expected values on both sides we get

$$\frac{1}{\alpha} = \mathbb{E}[\max\{T_{n-1} - X_n, 0\}] + \frac{1}{\mu}, \tag{83}$$

because T_n and S_n are geometrically distributed random variables over $\{1,2,\ldots\}$ with rates α and μ, respectively. The quantity $\mathbb{E}[\max\{T_{n-1}-X_n,0\}]$ can be computed as follows:

$$\begin{aligned}
&\mathbb{E}[\max\{T_{n-1}-X_n,0\}]\\
&\qquad=\mathbb{E}[\mathbb{E}[\max\{T_{n-1}-X_n,0\}|X_n]],\\
&\qquad=\sum_{m=1}^{\infty}\mathbf{P}[X_n=m]\,\mathbb{E}[\max\{T_{n-1}-m,0\}], \quad (84)
\end{aligned}$$

where the last equality follows from the fact that the system time for the $(n-1)$th update packet T_{n-1} and the inter-generation time between $(n-1)$th and nth update packet X_n are independent. Using the fact that, at steady state, T_{n-1} is geometrically distributed over $\{1,2,\ldots\}$ with rate α, we get

$$\begin{aligned}
\mathbb{E}[\max\{T_{n-1}-m,0\}]&=\sum_{k=1}^{\infty}k\alpha(1-\alpha)^{m+k-1},\\
&=(1-\alpha)^m\sum_{k=1}^{\infty}k\alpha(1-\alpha)^{k-1},\\
&=\frac{(1-\alpha)^m}{\alpha}. \quad (85)
\end{aligned}$$

Substituting (85) in (84) we get

$$\begin{aligned}
&\mathbb{E}[\max\{T_{n-1}-X_n,0\}]\\
&\qquad=\frac{1}{\alpha}\sum_{m=1}^{\infty}\mathbf{P}[X_n=m](1-\alpha)^m,\\
&\qquad=\frac{1}{\alpha}M_X(\log(1-\alpha)). \quad (86)
\end{aligned}$$

Substituting (86) back in (83) we obtain the result of (18).

G. Proof of Theorem 5

The average age is given by [8]

$$A^{\text{ave}}=\lambda\left[\frac{1}{2}\mathbb{E}\left[X_n^2\right]+\mathbb{E}[T_nX_n]\right]+\frac{1}{2}, \quad (87)$$

where T_n is the system time for the nth update packet at steady state, and X_n is the inter-generation time between $(n-1)$th and nth update packets. It is clear that $\mathbb{E}\left[X_n^2\right]=M_X^{''}(0)$. Therefore, it suffices to show that

$$\lambda\mathbb{E}[T_nX_n]=\frac{\lambda}{\alpha^*}M_X^{'}(\log(1-\alpha^*))+\frac{1}{\mu}, \quad (88)$$

at steady state, for α^* given in Lemma 2.

We know that the system time follows the following recursive equation [25]:

$$T_n=\max\{T_{n-1}-X_n,0\}+S_n, \quad (89)$$

where S_n is the service time of the nth update packet, and is geometrically distributed over $\{1,2,\ldots\}$ with rate μ. Therefore, $\mathbb{E}[T_nX_n]$ can be computed as

$$\begin{aligned}
\mathbb{E}[T_nX_n]&=\mathbb{E}[\mathbb{E}[T_nX_n|X_n]],\\
&=\sum_{m=1}^{\infty}m\mathbb{E}[T_n|X_n=m]\,\mathbf{P}[X_n=m]. \quad (90)
\end{aligned}$$

Now, $\mathbb{E}[T_n|X_n=m]$ can be evaluated using the recession in (89) as

$$\begin{aligned}
\mathbb{E}[T_n|X_n=m]&=\mathbb{E}[\max\{T_{n-1}-m,0\}+S_n|X_n=m],\\
&=\mathbb{E}[\max\{T_{n-1}-m,0\}]+\mathbb{E}[S_n],
\end{aligned}$$

where the last equality follows because the service time S_n is independent of inter-generation time of update packets X_n. Since $\mathbb{E}[S_n]=\frac{1}{\mu}$ we get

$$\begin{aligned}
\mathbb{E}[T_n|X_n=m]&=\mathbb{E}[\max\{T_{n-1}-m,0\}]+\frac{1}{\mu},\\
&=\sum_{k=1}^{\infty}k\alpha^*(1-\alpha^*)^{k+m-1}+\frac{1}{\mu}, \quad (91)\\
&=(1-\alpha^*)^m\sum_{k=1}^{\infty}k\alpha^*(1-\alpha^*)^{k-1}+\frac{1}{\mu},\\
&=\frac{1}{\alpha^*}(1-\alpha^*)^m+\frac{1}{\mu}, \quad (92)
\end{aligned}$$

where (91) follows because at steady state, T_{n-1} is geometrically distributed over $\{1,2,\ldots\}$ with rate α^*, by Lemma 2. Substituting (92) in (90) we obtain

$$\begin{aligned}
\mathbb{E}[T_nX_n]&=\sum_{m=1}^{\infty}m\left(\frac{1}{\alpha^*}(1-\alpha^*)^m+\frac{1}{\mu}\right)\mathbf{P}[X_n=m],\\
&=\frac{1}{\alpha^*}M_X^{'}(\log(1-\alpha^*))+\frac{1}{\lambda}\frac{1}{\mu}, \quad (93)
\end{aligned}$$

where the last equality follows from

$$M_X^{'}(\log(1-\alpha^*))=\sum_{m=1}^{\infty}m(1-\alpha^*)^m\,\mathbf{P}[X_n=m], \quad (94)$$

which can be obtained using standard properties of moment generating function. This proves (88), and therefore, the result follows.

H. Derivation of Peak and Average Age for Ber/Ber/1 Queue

When update packet generation is Bernoulli with rate λ, the inter-generation time X is geometrically distributed with rate λ. Thus, the moment generating function M_X is given by [26]

$$M_X(t)=\frac{\lambda e^t}{1-(1-\lambda)e^t}. \quad (95)$$

Therefore, (18) of Lemma 2 becomes

$$\begin{aligned}
\alpha&=\mu-\mu M_X(\log(1-\alpha)),\\
&=\mu-\mu\left(\frac{\lambda(1-\alpha)}{1-(1-\lambda)(1-\alpha)}\right),
\end{aligned}$$

which upon solving for α yields

$$\alpha^*=\frac{\mu-\lambda}{1-\lambda}. \quad (96)$$

Peak Age: Substituting this α^*, given by (96), in (19) of Theorem 5 we get peak age to be

$$A^{\text{p}}=\frac{1}{\alpha^*}+\frac{1}{\lambda}=\frac{1-\lambda}{\mu-\lambda}+\frac{1}{\lambda},$$

which can be written as

$$A^{\mathsf{p}} = \frac{1}{\mu}\left[\frac{1}{1-\rho} + \frac{1}{\rho}\right] - \frac{\rho}{1-\rho}, \tag{97}$$

where $\rho = \frac{\lambda}{\mu}$.

Average Age: To compute average age using Theorem 5, need to obtain expressions for $M_X^{''}(0)$, which is $\mathbb{E}\left[X^2\right]$, and $M_X^{'}\left(\log(1-\alpha^*)\right)$. Given that X is geometrically distributed random variable over $\{1, 2, \ldots\}$ with rate λ, we know that [26]

$$M_X^{''}(0) = \mathbb{E}\left[X^2\right] = \frac{2-\lambda}{\lambda^2}. \tag{98}$$

Furthermore,

$$M_X^{'}(t) = \frac{\lambda e^t}{\left(1-(1-\lambda)e^t\right)^2}, \tag{99}$$

and thus,

$$M_X^{'}\left(\log(1-\alpha^*)\right) = \frac{\lambda(1-\alpha^*)}{\left(1-(1-\lambda)(1-\alpha^*)\right)^2}. \tag{100}$$

Substituting $1-\alpha^* = \frac{1-\mu}{1-\lambda}$ from (96) in (100) we obtain

$$M_X^{'}\left(\log(1-\alpha^*)\right) = \frac{\lambda}{\mu^2}\frac{1-\mu}{1-\lambda}. \tag{101}$$

Using (96), (98) and (101) in (20) of Theorem 5 we obtain the result.

I. Proof for Peak Age bound in (28)

When $\gamma_e f_e < 1$, the peak age $A_e^{\mathsf{p}}(f_e, \rho_e)$ in (22), is minimized when

$$\rho_e^* = \frac{1}{1+\sqrt{1-\gamma_e f_e}}. \tag{102}$$

As a result, we have

$$\min_{\rho\in[0,1]} A_e^{\mathsf{p}}(f_e, \rho) = \frac{2}{\gamma_e f_e}\left[1+\sqrt{1-\gamma_e f_e}\right]. \tag{103}$$

Note that $A_e^{\mathsf{p}}\left(f_e, 1/2\right) = \frac{4}{\gamma_e f_e} - 1$. Therefore, taking the difference we have

$$\begin{aligned} A_e^{\mathsf{p}}\left(f_e, 1/2\right) - &\min_{\rho\in[0,1]} A_e^{\mathsf{p}}(f_e, \rho) \\ &= \frac{2}{\gamma_e f_e}\left[1-\sqrt{1-\gamma_e f_e}\right] - 1 \leq 1, \end{aligned} \tag{104}$$

where the last inequality follows by noting that $1-\gamma_e f_e \leq \sqrt{1-\gamma_e f_e}$. A weighted sum of (104) gives the result.

J. Proof of Theorem 6

Peak age: Using Result 1, we obtain

$$\sum_{e\in E} w_e A_e^{\mathsf{p}}\left(f_e, 1/2\right) \leq \min_{\boldsymbol{\rho}\in[0,1]^{|E|}} \sum_{e\in E} w_e A_e^{\mathsf{p}}(f_e, \rho_e) + \sum_{e\in E} w_e. \tag{105}$$

Minimizing over $\mathbf{f} \in \mathcal{F}$ we get

$$\underset{\mathbf{f}\in\mathcal{F}}{\text{Minimize}} \sum_{e\in E} w_e A_e^{\mathsf{p}}\left(f_e, 1/2\right) \leq A_{\mathcal{B}}^{\mathsf{p}*} + \sum_{e\in E} w_e, \tag{106}$$

since

$$A_{\mathcal{B}}^{\mathsf{p}*} = \underset{\mathbf{f}\in\mathcal{F}, \boldsymbol{\rho}\in[0,1]^{|E|}}{\text{Minimize}} \sum_{e\in E} w_e A_e^{\mathsf{p}}(f_e, \rho_e).$$

Now, notice that

$$\sum_{e\in E} w_e A_e^{\mathsf{p}}\left(f_e, 1/2\right) = \sum_{e\in E}\frac{4w_e}{\gamma_e f_e} - \sum_{e\in E} w_e, \tag{107}$$

which has the same form as the objective function in (15), except for a constant multiple and a additive factor. As a result, stationary policy π_C minimizes $\sum_{e\in E} w_e A_e^{\mathsf{p}}\left(f_e, 1/2\right)$. Thus, given $\mathbf{f}^*$ as the vector of link activation frequencies for policy π_C, we have

$$A^{\mathsf{p}}\left(\mathbf{f}^*, \mathbf{1}/2\right) = \underset{\mathbf{f}\in\mathcal{F}}{\text{Minimize}} \sum_{e\in E} w_e A_e^{\mathsf{p}}\left(f_e, 1/2\right). \tag{108}$$

Substituting (108) in (106) gives the result. The proof for the average age follows the same line of argument.

K. Proof of Corollary 3

Peak Age: First, note that the minimum peak age for the buffered sources is lower bounded by the minimum peak age in active source case:

$$A^{\mathsf{p}*} \leq A_{\mathcal{QB}}^{\mathsf{p}*}. \tag{109}$$

This is because fresh information is generated at the beginning of every slot in the active source case. Further, restricting to stationary policies we have an upper bound on $A_{\mathcal{QB}}^{\mathsf{p}*}$:

$$\begin{aligned} A_{\mathcal{QB}}^{\mathsf{p}*} &\leq \underset{\pi \text{ is stationary}, \ \boldsymbol{\rho}}{\text{Minimize}} A^{\mathsf{p}}\left(\mathbf{f}(\pi), \boldsymbol{\rho}\right), \\ &\leq A^{\mathsf{p}}(\mathbf{f}^*, \mathbf{1}/2), \end{aligned} \tag{110}$$

where $\mathbf{f}(\pi)$ denotes the vector of link activation frequencies for stationary policy π, and $\mathbf{f}^*$ is the vector of link activation frequencies for the stationary policy π_C. Note that $A^{\mathsf{p}}(\mathbf{f}^*, \mathbf{1}/2)$ is given by

$$\begin{aligned} A^{\mathsf{p}}(\mathbf{f}^*, \mathbf{1}/2) &= \sum_{e\in E}\frac{4w_e}{\gamma_e f_e^*} - \sum_{e\in E} w_e, \\ &= 4A^{\mathsf{p}*} - \sum_{e\in E} w_e, \qquad (111) \\ &\leq 4A_{\mathcal{QB}}^{\mathsf{p}*} - \sum_{e\in E} w_e, \qquad (112) \end{aligned}$$

where the last equality follows from (109), while (111) follows because $\mathbf{f}^*$ is a solution to (15). This yields the result.

Average Age: Just as for the peak age, the minimum average age for the buffered sources is lower bounded by the minimum average age for the active source, i.e.,

$$A^{\text{ave}*} \leq A_{\mathcal{QB}}^{\text{ave}*}. \tag{113}$$

This is because fresh packets are available at every slot in the active source case. Using Corollary 1 and (113) we get

$$A^{\mathsf{p}*} \leq 2A_{\mathcal{QB}}^{\text{ave}*} - \sum_{e\in E} w_e. \tag{114}$$

Now, an upper bound on $A^{\text{ave}*}_{\mathcal{QB}}$ can be obtained by restricting to stationary policies. We, thus, have

$$A^{\text{ave}*}_{\mathcal{QB}} \leq \underset{\pi \text{ is stationary}, \; \boldsymbol{\rho}\in[0,1]^{|E|}}{\text{Minimize}} A^{\text{ave}}\left(\mathbf{f}(\pi), \boldsymbol{\rho}\right), \tag{115}$$

$$\leq A^{\text{ave}}\left(\mathbf{f}^{*}, \overline{\rho}\mathbf{1}\right), \tag{116}$$

where $\mathbf{f}^{*}$ is the vector of link activation frequencies for the stationary policy π_C and $\overline{\rho}$ minimizes the upper-bound on average age in (27).

Using the upper-bound in (27) we can get

$$A^{\text{ave}}\left(\mathbf{f}^{*}, \overline{\rho}\mathbf{1}\right) \leq \sum_{e\in E} \frac{w_e}{\gamma_e f^{*}_{e}} \left[1 + \frac{1}{\overline{\rho}} + \frac{\overline{\rho}^2}{1-\overline{\rho}}\right], \tag{117}$$

$$= \alpha \sum_{e\in E} \frac{w_e}{\gamma_e f^{*}_{e}} = \alpha A^{\text{p}*}, \tag{118}$$

where $\alpha = 1 + \frac{1}{\overline{\rho}} + \frac{\overline{\rho}^2}{1-\overline{\rho}}$. This with (114) implies

$$A^{\text{ave}}\left(\mathbf{f}^{*}, \overline{\rho}\mathbf{1}\right) \leq 2\alpha A^{\text{ave}*}_{\mathcal{QB}} - \alpha \sum_{e\in E} w_e, \tag{119}$$

which proves the result.

L. Derivation of Peak and Average Age for D/Ber/1 Queue

Let D be the period of the periodic packet generation and μ be the Bernoulli service rate. The probability distribution of inter-arrival time X of update packets is given by $\mathbf{P}\left[X = D\right] = 1$ and $\mathbf{P}\left[X = k\right] = 0$ for all $k \neq D$. Thus, $M_X(t) = e^{Dt}$. Using this, the equation (18) can be written as

$$\alpha^{*} = \mu - \mu M_X\left(\log(1-\alpha^{*})\right), \tag{120}$$

$$= \mu - \mu\left(1-\alpha^{*}\right)^{D}. \tag{121}$$

We let $\sigma = \frac{\alpha^{*}}{\mu}$. Then σ is the solution to

$$\sigma = 1 - \left(1 - \mu\sigma\right)^{D}. \tag{122}$$

Using (122), and the fact that $M^{'}_{X}(t) = De^{Dt}$, in Theorem 5 we obtain the result.