Optimizing Age of Information in Wireless Networks with Perfect Channel State Information

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***Abstract*—Age of information (AoI), defined as the time elapsed since the last received update was generated, is a newly proposed metric to measure the timeliness of information updates in a network. We consider AoI minimization problem for a network with general interference constraints, and time varying channels. We propose two policies, namely, virtual-queue based policy and age-based policy when the channel state is available to the network scheduler at each time step. We prove that the virtual-queue based policy is nearly optimal, up to a constant additive factor, and the age-based policy is at-most factor 4 away from optimality. Comparison with previous work, which derived age optimal policies when channel state information is not available to the scheduler, demonstrates a 4 fold improvement in age due to the availability of channel state information.**

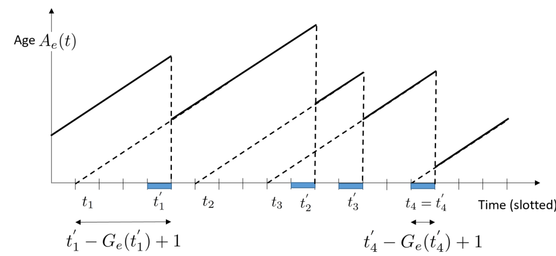

Fig. 1. Time evolution of age, $A_e(t)$, of a link e. Times t_i and t_i' are instances of ith packet generation and reception, respectively. Given the definition $G_e(t_i') \triangleq t_i$, the age is reset to $t_i' - G_e(t_i') + 1$ when the ith packet is received.

I. Introduction

Timely delivery of information updates is gaining increasing relevance with the advent of technologies such as cyber-physical systems, internet of things, and unmanned aerial vehicular networks. In unmanned aerial vehicular networks, timely delivery of status updates, such as vehicle position and velocity, may be critical to network safety [1], [2]. In internet of things or cyber-physical systems, timely delivery of sensor information can significantly improve the overall system performance [3].

Age of information (AoI) is a recently proposed metric that measures the time that elapsed since the last received update was generated by the source [4], [5]. Figure 1 shows the typical evolution of AoI at a destination node, as a function of time. Upon reception of a new update packet AoI drops to the time elapsed since the generation of the packet, and grows linearly otherwise. Therefore, AoI is a destination node centric measure, unlike packet delay, and is better suited for applications involving dissemination of time sensitive information.

In [4], a simulation study considered AoI in a network of vehicles exchanging status updates. Motivated by [4], AoI was analyzed for several queueing models [5]–[11].

However, AoI minimization for a network under general interference constraints and channel uncertainty has received very little attention. A problem of scheduling finitely many update packets under physical interference constraints was shown to be an NP-hard problem in [12]. Age for a broadcast network, where only a single link can be activated at any time, was studied in [13], [14]. Preliminary analysis of age for a slotted ALOHA like random access was done in [15], and a distributed algorithm for age optimal ALOHA was only recently proposed in [16]. Age in multi-hope interference networks has been studied in [17].

We considered the problem of age minimization for a wireless network under general interference constraints, and time varying channel, in [18]. We considered two types of sources: *active sources*, which generate fresh information in every slot, and *buffered sources*, which cannot generate fresh information in every slot. We showed that for a network with active sources, a stationary scheduling policy, which schedules links according to a stationary probability distribution, is peak age optimal and factor-2 average age optimal. We also showed that the same scheduling policy, with a certain packet generation rate control, is nearly optimal in the buffered case.

In [18], however, the space of policies was limited to not using the channel state information. In this paper, we relax this assumption and consider scheduling policies which have perfect channel state information $\mathbf{S}(t)$ at every time slot t. We consider the active sources case, and propose two policies: a virtual-queue based policy and an age-based policy, which uses the current channel state information to make scheduling decisions. We show, via numerical simulations, that the availability of channel state information can significantly improve the AoI performance of the network.

We prove that the virtual-queue based policy is nearly peak age optimal, up to an additive factor, while the age-based policy is at most a factor 4 away from the optimal peak and average age. Similar result has been recently derived for another age-based policy proposed for a broadcast network, in which only a single link can be activated [19]. Numerical simulations suggest that this bound is pessimistic, and that the

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This work was supported by NSF Grants AST-1547331, CNS-1713725, and CNS-1701964, and by Army Research Office (ARO) grant number W911NF-17-1-0508.

proposed scheme performs much better.

In numerical simulations, we observe the benefit/utility of using channel state information in scheduling to minimize age, especially when the network has 'high' level of interference or 'bad' channel quality. We demonstrate, by considering a specific network example, that the gap in age performance between the known channel case and the unknown channel case can be as large as 4 fold. Even though channel state information may not be perfectly available in certain network settings, this work establishes the utility of acquiring such channel state information for scheduling to minimize age.

II. System Model

Consider a wireless network $G = (V, E)$, where V denotes the set of nodes and E the set of directed links. Not all links can be activated simultaneously. Thus, we call a set $m \subset E$ that can be activated simultaneously without interference a *feasible activation set*. We use $\mathcal{A}$ to denote the collection of all feasible activation sets. We consider a slotted time system, where the slot duration is normalized to unity.

We use $S_e(t)$ to denote the channel process, where $S_e(t) = 1$ if the channel is in the ON state at time t and $S_e(t) = 0$ if the channel is in the OFF state at time t. The space of all channel states is given by $\mathbb{S} = \{0, 1\}^{|E|}$. We consider $\{S_e(t)\}_{t\geq 0}$ to be independent and identically distributed (i.i.d.) across time t, with $\gamma_e = \mathbb{P}\left[S_e(t) = 1\right] > 0$, for all $e \in E$. We call this the *i.i.d. channel process*. Note that the channel process is not identically distributed across links, and that γ_e can be different for different links $e \in E$.

We use $U_e(t)$ to denote transmission decision on link e at time t. $U_e(t) = 1$ if link e is scheduled to transmit at time t. Not all transmissions succeed even if the set of activated links is a feasible activation set due to channel uncertainties. A successful transmission occurs over link e, at time t, if and only if $U_e(t)S_e(t) = 1$.

We consider *active nodes*, which transmit fresh information at every transmission opportunity. We define age $A_e(t)$, of a link e at time t, to be the time that elapsed since the last successful activation of link e. Figure 2 shows the evolution of age $A_e(t)$ for a link e. Age $A_e(t)$ drops to 1 upon a successful activation of link e, and increases by 1 in every slot in which there is no successful activation of link e, i.e.,

$$A_e(t+1) = \begin{cases} A_e(t) + 1 & \text{if } U_e(t)S_e(t) = 0 \\ 1 & \text{if } U_e(t)S_e(t) = 1 \end{cases}. \quad (1)$$

This age evolution equation can be written compactly as

$$A_e(t+1) = 1 + A_e(t) - U_e(t)S_e(t)A_e(t), \quad (2)$$

for all $t \geq 0$, and $e \in E$.

We consider two popular age measures, namely, average age and peak age. Average age is the area under the age curve in Figure 2, while peak age is the average of all the peaks of the age curve. More precisely, we define average age of a link e as

$$\overline{A}_e^{\text{ave}} = \limsup_{t\to\infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} A_e(\tau)\right], \quad (3)$$

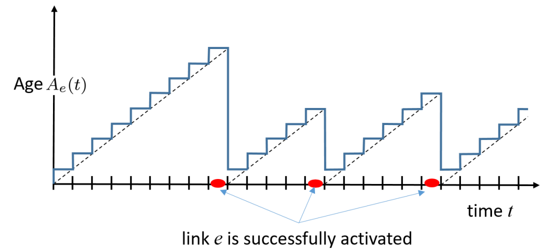

Fig. 2. Evolution of age of link e, namely $A_e(t)$, as a function of time t.

and the average age of the network to be the weighted sum

$$\overline{A}^{\text{ave}} = \sum_{e\in E} w_e \overline{A}_e^{\text{ave}}. \quad (4)$$

Note that the sum of all the peaks, until time t, in the age curve can be expressed as $\sum_{\tau=0}^{t} U_e(\tau)S_e(\tau)A_e(\tau)$. This is because $U_e(\tau)S_e(\tau) = 1$ only at times when age peaks. We, therefore, define the peak age to be

$$\overline{A}_e^{\text{p}} = \limsup_{t\to\infty} \frac{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e(\tau)\right]}{\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)\right]}, \quad (5)$$

for every link $e \in E$, and the peak age of the network to be the weighted sum

$$\overline{A}^{\text{p}} = \sum_{e\in E} w_e \overline{A}_e^{\text{p}}. \quad (6)$$

We are interested in designing policies that minimize peak and average age.

Since both peak and average age are time average measures, the performance of a policy π does not depend on the initial age at time 0. We, therefore, assume that the system starts with $A_e(0) = 0$ for all $e \in E$.

A. Unknown Channel Case

In [18], we considered age minimization under the unknown channel case. Specifically, we considered all policies which scheduled feasible activation set $m_t \in \mathcal{A}$ at time t as a function of the history

$$\hat{\mathcal{H}}(t) = \{\mathbf{U}(\tau), \mathbf{A}(\tau') \big| 0 \leq \tau < t \text{ and } 0 \leq \tau' \leq t\}. \quad (7)$$

We showed in [18] that stationary policies, which schedule links according to a probability distribution that is independent of $\hat{\mathcal{H}}(t)$, is in fact peak age optimal and factor-2 average age optimal.

In stationary scheduling policies, every feasible activation set $m \in \mathcal{A}$ is assigned a fixed probability x_m with which it is activated in slot t, independent across slots. The probability that a link $e \in E$ is activated in a slot is given by

$$f_e = \sum_{m:e\in m} x_m, \quad (8)$$

for all $e \in E$. This set of equations can be compactly written as $\mathbf{f} = M\mathbf{x}$, for a 0-1 matrix M. Note that an activated link may fail in successfully transmitting the packet due to channel

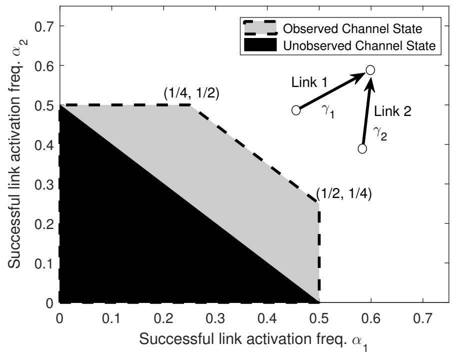

Fig. 3. Plot of achievable successful link activation frequency regions for the two link network, in which only one link can be activated at a time. Shown are regions when channel state is observed (grey) and unobserved (black).

errors. The probability of successful activation of a link e in any slot is $\alpha_e = \gamma_e f_e$, since the scheduling decision is independent of the current channel state.

Further, notice that, if a link e is successfully activated with probability $\alpha_e = \gamma_e f_e$ in each slot, independent across slots, then the time since last transmission, i.e. age $A_e(t)$, is geometrically distributed with rate $\frac{1}{\gamma_e f_e}$. In [18], we showed that this is indeed equal to the peak age of link e, under any stationary policy. As a result, the peak age for the stationary policy, determined by distribution $\mathbf{x}$, is given by $\overline{A}^{\text{p}} = \sum_{e \in E} \frac{w_e}{\gamma_e f_e}$, and thus, the optimal peak age is given by

$$\begin{aligned} \overline{A}^{\text{p}*} = \underset{\mathbf{x}, \mathbf{f}}{\text{Minimize}} \quad & \sum_{e \in E} \frac{w_e}{\gamma_e f_e}, \\ \text{subject to} \quad & \mathbf{f} = M\mathbf{x}, \\ & \mathbf{1}^T \mathbf{x} \leq 1 \text{ and } \mathbf{x} \geq 0. \end{aligned} \tag{9}$$

The peak age optimal stationary policy is obtained by solving (9).

In the next sub-section, we discuss the space of policies considered in this paper, and show how knowing the channel state affects age minimization. We argue that in the case when channel state information is available for scheduling, smaller age than what is given by (9) can be achieved.

B. Scheduling Policies

A scheduling policy determines the set of links $m_t \subset E$ that will be activated at each time t, i.e., $m_t = \{e \in E | U_e(t) = 1\}$. The policy can make use of the past history of link activations and observed channel states to make this decision, i.e., at each time t, the policy π will determine m_t as a function of the set

$$\mathcal{H}(t) = \{\mathbf{U}(\tau), \mathbf{S}(\tau'), \mathbf{A}(\tau') \mid 0 \leq \tau < t, \ 0 \leq \tau' \leq t\}. \tag{10}$$

We consider centralized scheduling policies, in which this information is available to a scheduler, which is also able to implement its scheduling decision.

To see the difference between age minimization under known and unknown channel process consider the two link example shown in Figure 3. In this example, only one link can be activated at a time. Let the weights $w_1 = w_2 = 1$ for the two links, and the channel success probabilities be $\gamma_1 = \gamma_2 = 0.5$. When the channel state $\mathbf{S}(t) = (S_1(t), S_2(t))$ is unavailable the peak age minimization problem is given by (from (9)):

$$\begin{aligned} \overline{A}^{\text{p}*} = \underset{f_1, f_2}{\text{Minimize}} \quad & \frac{1}{\gamma_1 f_1} + \frac{1}{\gamma_2 f_2}, \\ \text{subject to} \quad & f_1 + f_2 \leq 1, \\ & f_1 \geq 0 \text{ and } f_2 \geq 0. \end{aligned} \tag{11}$$

Here, f_1 and f_2 denote the fraction of times link 1 and link 2 are scheduled, respectively. Since $\gamma_1 = \gamma_2 = 0.5$, the optimal solution to (11) is given by $f_1^* = f_2^* = 0.5$, i.e. with probability 0.5 each link gets scheduled in each slot, and as a result, the optimal peak age is $\overline{A}^{\text{p}*} = 8$.

However, if we can observe the channel state $\mathbf{S}(t)$ in every slot before making scheduling decision, we can achieve even smaller age than $\overline{A}^{\text{p}*} = 8$. Consider the following policy: schedule link 1 whenever $S_1(t) = 1$, and otherwise schedule link 2. The successful link activation frequency on link 1 is then $\alpha_1 = \gamma_1 = 0.5$, while on link 2 it is $\alpha_2 = \gamma_2(1 - \gamma_1) = 0.25$. The peak age is given by $\overline{A}^{\text{p}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} = 6 < \overline{A}^{\text{p}*} = 8$. This happens primarily because the set of achievable successful link activation frequencies, namely α_e, is larger in the case when the channel can be observed before deciding on the schedule in each slot. In Figure 3, we show these regions in the observed and unobserved channel state case for the two link example.

This shows that when the channel state is available for making scheduling decisions, the network age performance can be improved upon. In the next sub-section we define a sub-class of policies that make scheduling decision based only on the current channel state $\mathbf{S}(t)$, and not the entire history $\mathcal{H}(t)$. We will see later that these policies can be peak age optimal.

C. $\mathcal{S}$-only policies

Just as the stationary policies turn out to be peak age optimal in the unknown channel case, we define a sub-class of policies that are peak age optimal in the known channel case. These policies do not use any past history, but only the current channel state $\mathbf{S}(t)$, defined as follows [20]:

> **$\mathcal{S}$-only policy:** For each observed channel state $S \in \mathbb{S}$ we assign a probability distribution $p(S, m)$ over the set of feasible activation sets $m \in \mathcal{A}$. If channel state $\mathbf{S}(t)$ is observed then the activation set $m \in \mathcal{A}$ is activated for slot t with probability $p(\mathbf{S}(t), m)$.

For an $\mathcal{S}$-only policy, the rate at which a successful transmission occurs over link e is given by

$$\begin{aligned} \alpha_e = \mathbb{E}\left[U_e(t) S_e(t)\right] &= \mathbb{P}\left[U_e(t) S_e(t) = 1\right], \\ &= \gamma_e \mathbb{P}\left[U_e(t) = 1 | S_e(t) = 1\right], \end{aligned} \tag{12}$$

for all $e \in E$. The space of all such rates $\boldsymbol{\alpha}$ will depend on channel success probabilities γ_e, and thus, we use $\Lambda_{\mathcal{S}}(\boldsymbol{\gamma})$ to denote this space of all feasible $\boldsymbol{\alpha}$ using $\mathcal{S}$-only policies. For the two link example in Figure 3, $\Lambda_{\mathcal{S}}(\boldsymbol{\gamma})$ is exactly the grey region of successful link activation frequencies (α_1, α_2). It is known that if $\Lambda(\boldsymbol{\gamma})$ is the space of rates $\boldsymbol{\alpha}$ achievable under all policies then $\Lambda(\boldsymbol{\gamma}) = \Lambda_{\mathcal{S}}(\boldsymbol{\gamma})$ [20]. This will help us show that an $\mathcal{S}$-only policy is peak age optimal.

III. Problem Formulation

In this section, we formulate the peak and average age minimization problems under a general channel process. To do so in a meaningful way, we restrict our search to a certain reasonable policy space. We consider the following policy spaces:

$$\overline{\Pi}_1 = \left\{ \pi \middle| \; \exists B \text{ s.t. } \mathbb{E}\left[A_e^{\pi}(t)\right] \leq B \;\forall\; t \geq 0 \text{ and } e \in E \right\},$$

and

$$\overline{\Pi}_2 = \left\{ \pi \middle| \; \exists B \text{ s.t. } \mathbb{E}\left[\left(A_e^{\pi}(t)\right)^2\right] \leq B \;\forall\; t \geq 0 \text{ and } e \in E \right\},$$

i.e., the space of all policies that have bounded first and second moments of age. Firstly, note that the constraints that the first and second moment of age $A_e(t)$ should not grow in t is natural, because $A_e(t)$ is the time since last successful transmission on link e. It growing in time would necessarily mean that the transmissions are becoming less frequent as time goes by.

We consider the policy space $\overline{\Pi}_1$ for peak age minimization, while space $\overline{\Pi}_2$ for average age minimization. For a 'good' policy, we anticipate the process $\{\mathbf{A}(t)\}_t$ to be stable (or ergodic), in which case the policy is in $\overline{\Pi}_1$. For 'good' average age policy, it stands to reason that stability (or ergodicity) of $\{\mathbf{A}^2(t)\}_t$ would be required. This is because the average age, being the area under the age curve, depends on $A^2(t)$.

We define optimal peak and average age to be

$$\overline{A}^{\text{p}*} = \min_{\pi \in \overline{\Pi}_1} \overline{A}^{\text{p}}(\pi) \quad \text{and} \quad \overline{A}^{\text{ave}*} = \min_{\pi \in \overline{\Pi}_2} \overline{A}^{\text{ave}}(\pi), \tag{13}$$

where the minimization is over the space $\overline{\Pi}_1$ for peak age and over $\overline{\Pi}_2$ for average age. Note that $\overline{\Pi}_2 \subset \overline{\Pi}_1$ since $\mathbb{E}\left[A_e(t)\right] \leq \sqrt{\mathbb{E}\left[A_e^2(t)\right]}$ by Jensen's inequality.

A. Peak Age Minimization

We first present a lemma that states a conservation law for age. Intuitively, it states that for any policy $\pi \in \overline{\Pi}_1$, the sum of all age peaks is equal to the total time elapsed plus a small insignificant term that goes to 0 as $t \to \infty$.

Lemma *1:* For any policy $\pi \in \overline{\Pi}_1$ we have

$$\lim_{t \to \infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e(\tau)\right] = 1, \tag{14}$$

for all $e \in E$.

Proof: See Appendix A. ■

A direct consequence of Lemma 1 is that the peak age minimization problem $\min_{\pi \in \overline{\Pi}_1} \overline{A}^{\text{p}}(\pi)$ reduces to

$$\begin{aligned} &\underset{\boldsymbol{\alpha} \geq 0, \pi \in \overline{\Pi}_1}{\text{Minimize}} && \sum_{e \in E} \frac{w_e}{\alpha_e}, \\ &\text{subject to} && \liminf_{t \to \infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)\right] \geq \alpha_e \;\; \forall\; e. \end{aligned} \tag{15}$$

We prove this equivalence in Appendix B. This result is significant because it shows that the peak age minimization problem is independent of the age evolution equation (2). For this reason peak age minimization is much simpler than average age minimization. We propose a virtual-queue based algorithm in Section IV to solve this problem.

B. Average Age Minimization

In this section, we provide an equivalent formulation for average age minimization. By definition, we know that the average age for a link e is given by $\overline{A}_e^{\text{ave}} = \limsup_{t \to \infty} \mathbb{E}\left[\frac{1}{t}\sum_{t=0}^{t-1} A_e(\tau)\right]$. The following result provides a different characterization of the average age in terms of $A_e^2(t)$, for all $\pi \in \overline{\Pi}_2$. This result will be useful to get an intuitive grasp over the class of policies proposed in Section V.

Lemma *2:* Define $B_e(t) = A_e^2(t) + \beta A_e(t)$ for all t, $e \in E$, and a given $\beta \in \mathbb{R}$. Then, for $\pi \in \overline{\Pi}_2$, we have

$$\overline{A}_e^{\text{ave}} = \frac{1}{2}\limsup_{t \to \infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)B_e(\tau)\right] + \frac{1-\beta}{2},$$

for all $e \in E$.

Proof: See Appendix C. ■

For an intuitive understanding of Lemma 2, note that average age is essentially the average area of the triangles formed by the age curve in Figure 2. Note that $S_e(t)U_e(t)A_e^2(t)$ are square of age peaks in Figure 2, because $S_e(t)U_e(t) = 1$ only at the instances when there is a successful transmission on link e. The additional term of $\beta A_e(t)$ is due to Lemma 1.

Lemma 2 also implies that average age minimization problem over $\pi \in \overline{\Pi}_2$ can be equivalently posed as minimizing

$$\limsup_{t \to \infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{e \in E} w_e U_e(\tau)S_e(\tau)B_e(\tau)\right], \tag{16}$$

where $B_e(\tau) = A_e^2(\tau) + \beta A_e(\tau)$, for all $\tau \geq 0$, $e \in E$, and any chosen $\beta \in \mathbb{R}$. Since, age reduces to 1 after a link activation it makes intuitive sense to choose $\mathbf{U}(t)$ such that as

$$\mathbf{U}(t) = \arg\max_{\mathbf{U}'(t)} \sum_{e \in E} w_e U_e^{'}(t)S_e(t)\left[A_e^2(t) + \beta A_e(t)\right], \tag{17}$$

in time slot t. This, in the least, should minimize age in the next slot. We analyze such policies in Section V, and show that these policies are within a factor of 4 away from the optimal average age $\overline{A}^{\text{ave}*}$. However, in simulations we observe that these policies are very close to optimal.

C. Bounds on Peak and Average Age

In this sub-section, we provide a characterization of optimal peak age $\overline{A}^{\text{p}*}$ and a lower-bound on average age. We first characterize the optimal peak age by showing that a $\mathcal{S}$-only policy is peak age optimal.

Theorem *1:* The optimal peak age $\overline{A}^{\text{p}*}$ is given by

$$\begin{aligned} \overline{A}^{\text{p}*} = \ & \underset{\boldsymbol{\alpha}}{\text{Minimize}} && \sum_{e\in E} \frac{w_e}{\alpha_e}, \\ & \text{subject to} && \boldsymbol{\alpha} \in \Lambda_{\mathcal{S}}\left(\boldsymbol{\gamma}\right), \end{aligned} \tag{18}$$

and as a consequence, there exists a $\mathcal{S}$-only policy that minimizes peak age, and it can be obtain by solving (18).

Proof: The optimality of $\mathcal{S}$-only policies in solving the problem (15) follows from Theorem 4.5 in [20]. We show that the peak age minimization problem over the space of $\mathcal{S}$-only policies can be written as (18) in [21]. ■

Theorem 1 can be used to obtain a peak age optimal $\mathcal{S}$-only policy. However, the search space $\Lambda_{\mathcal{S}}\left(\boldsymbol{\gamma}\right)$ is usually difficult to characterize for general interference constraints. Another issue is that, to solve (18), requires exact knowledge of the channel statistics γ_e. Our proposed policies, in the next two sections, do not require apriori knowledge of the channel statistics.

We now proceed to derive a lower-bound on average age.

Lemma *3:* For any policy $\pi \in \overline{\Pi}_2$, we have

$$\overline{A}^{\text{p}}\left(\pi\right) \leq 2\overline{A}^{\text{ave}}\left(\pi\right) - \sum_{e\in E} w_e. \tag{19}$$

And as a consequence the same relation also holds at optimality, namely, $\overline{A}^{\text{p}*} \leq 2\overline{A}^{\text{ave}*} - \sum_{e\in E} w_e$.

Proof: See Appendix D. ■

Lemma 3 provides us with a natural lower-bound on the optimal average age $\overline{A}^{\text{ave}*}$ in terms of the optimal peak age. Since, the optimal peak age can be obtained from Theorem 1 we get

$$\frac{1}{2}\sum_{e\in E} \frac{w_e}{\alpha_e^*} + \frac{1}{2}\sum_{e\in E} w_e \leq \overline{A}^{\text{ave}*}, \tag{20}$$

where $\boldsymbol{\alpha}^*$ is a solution to the optimization problem (18).

IV. Virtual-Queue Based Policy

We now propose a policy that solves the peak age minimization problem (15). Note that a policy π can decide on the activation set m_t, at time t, based on the entire history $\mathcal{H}(t)$. However, we do not need the entire history to make a choice at time t but only a representation of it.

To do so, we construct virtual queue $Q_e(t)$, which decreases by (at most) 1 upon a successful transmission over link e, and increases otherwise. These queue lengths determine the 'value' of scheduling link e in time slot t. Therefore, a set $m_t \in \mathcal{A}$ that maximizes $\sum_{e\in m} w_e Q_e(t) S_e(t)$ is activated in slot t. This virtual-queue based policy, π_Q, is described below. Here, $V > 0$ is any chosen constant.

Virtual queue based policy π_Q Start with $Q_e(0) = 1$ for all $e \in E$. At time t,

1) Schedule activation set m_t given by

$$m_t = \arg\max_{m\in\mathcal{A}} \sum_{e\in m} w_e Q_e(t) S_e(t), \tag{21}$$

2) Update $Q_e(t)$ as

$$Q_e(t+1) = \left[Q_e(t) + \sqrt{\frac{V}{Q_e(t)}} - U_e(t)S_e(t)\right]_{+1},$$

for all $e \in E$, where $[x]_{+1} = \max\{x, 1\}$.

The policy π_Q is nearly peak age optimal up to an additive factor.

Theorem *2:* The peak age for policy π_Q is bounded by

$$\overline{A}^{\text{p}}(\pi_Q) \leq \overline{A}^{\text{p}*} + \frac{1}{2}\sum_{e\in E} w_e + \frac{1}{2V}\sum_{e\in E} w_e, \tag{22}$$

where $\overline{A}^{\text{p}*}$ is the optimal value of (18).

Proof: Let $\alpha_e(t) = \sqrt{\frac{V}{Q_e(t)}}$ and $\overline{\alpha}_e(t) = \frac{1}{t}\sum_{\tau=0}^{t-1} \alpha_e(\tau)$ for all $t \geq 0$ and $e \in E$. Also, let $g(\alpha) = \sum_{e\in E} \frac{w_e}{\alpha_e}$ be the objective function in our optimization problem (15). The proof is divided into three parts:

Part A: For all time t, we have

$$\limsup_{t\to\infty} \mathbb{E}\left[g\left(\overline{\boldsymbol{\alpha}}(t)\right)\right] \leq \overline{A}^{\text{p}*} + \frac{1}{2}\sum_{e\in E} w_e + \frac{1}{2V}\sum_{e\in E} w_e. \tag{23}$$

Part B: The virtual queue $\mathbf{Q}(t)$ is mean rate stable, i.e., for all $e \in E$ we have $\limsup_{t\to\infty} \frac{1}{t}\mathbb{E}\left[Q_e(t)\right] = 0$.

Part C: If $\mathbf{Q}(t)$ is mean rate stable then

$$\alpha_e^V \triangleq \liminf_{t\to\infty} \mathbb{E}\left[\overline{\alpha}_e(t)\right] \leq \liminf_{t\to\infty} \frac{1}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)\right],$$

and

$$\overline{A}^{\text{p}}\left(\pi_Q\right) \leq \limsup_{t\to\infty} \mathbb{E}\left[g\left(\overline{\boldsymbol{\alpha}}(t)\right)\right]. \tag{24}$$

From (23) and (24) we get the result in (22). Further, it can be shown that $\boldsymbol{\alpha}^V = \left(\alpha_e^V\right)_{e\in E}$, with policy π_Q, solves (15), up to an additive factor given in (22). Due to space constraints, the proofs are given in [21]. ■

Theorem 2 shows that even when the channel statistics are not known the optimal peak age $A^{\text{p}*}$ can be achieved, up to an additive factor of $\frac{1}{2}\sum_{e\in E} w_e$, with arbitrary precision. The precision can be set by choosing a large enough V. For example, we may obtain peak age of at most $\overline{A}^{\text{p}*} + \frac{1}{2}\sum_{e\in E} w_e + \epsilon$ by setting $V = \frac{1}{2\epsilon}\sum_{e\in E} w_e$.

V. AGE-BASED POLICY

We now consider an age based policy, which schedule links as a function of links' age $A_e(t)$. Lemma 2 provided an alternate characterization of average age A^{ave}, which motivated the age-based policy that schedules set $m_t \in \mathcal{A}$ with maximum weight $\sum_{e\in m} w_e S_e(t) \left[A_e^2(t) + \beta A_e(t)\right]$:

Age-based Policy π_A The policy activates links

$$m_t = \arg\max_{m\in\mathcal{A}} \sum_{e\in m} w_e S_e(t) \left[A_e^2(t) + \beta A_e(t)\right], \quad (25)$$

in each slot $t \geq 1$.

The following theorem states that the average and peak age of policy π_A is within a factor of 4 from the respective optimal values. Due to space constraints, a detailed proof is given in [21].

Theorem *3:* The age-based policy policy π_A is at most factor-4 peak and average age optimal, i.e.,

$$\overline{A}^{\text{ave}}(\pi_A) \leq 4\overline{A}^{\text{ave}*} - c_1(\beta) \sum_{e\in E} w_e, \quad (26)$$

and

$$\overline{A}^{\text{p}}(\pi_A) \leq 4\overline{A}^{\text{p}*} - c_2(\beta) \sum_{e\in E} w_e, \quad (27)$$

where $c_1(\beta) = \frac{10+2\beta-\beta^2}{4}$ and $c_2(\beta) = \frac{4+2\beta-\beta^2}{2}$.

We note that $\beta \in \mathbb{R}$ can be chosen to improve the additive factor of optimality. The best bounds, for both peak and average age, occur when $\beta = 1$, for which both $c_1(\beta)$ and $c_2(\beta)$ are maximized. In the next section, we evaluate the age-based policy for different choices of β. We also compare it with the virtual-queue based policy π_Q from Section IV.

VI. NUMERICAL RESULTS

Consider a network of $N = 20$ links, in which at most K links can be activated at any given time. We numerically study the performance of our proposed scheduling policies for this network. We set $w_e = 1$ for all links e. We assume links to be either 'good', in which case $\gamma_e = \gamma_{\text{good}} = 0.9$, or 'bad' in which case $\gamma_e = \gamma_{\text{bad}} = 0.1$. We use n_{bad} to denote the number of bad links in the network. We simulate the policies π_Q, π_A, and the optimal policy for the unknown channel case, proposed in [18], over 10^5 time slots.

In Figure 4 and 5, we plot per-link peak and average age, namely A^{p}/N and A^{ave}/N, as a function of K. Here, we have chosen the parameters $V = 1$ for the virtual-queue policy π_Q, and $\beta = 1$ for the age-based policy π_A. We observe that the peak and average age of the virtual-queue based policy π_Q and the age-based policy π_A nearly coincide.

Also plotted in Figures 4 and 5, is the case when the channel state is not observed, i.e., scheduling decisions are made only using history $\hat{\mathcal{H}}(t)$. We plot the peak age optimal policy π_C

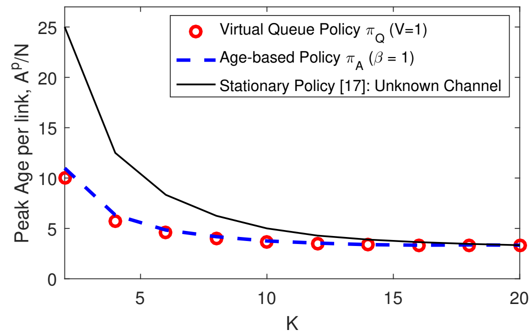

Fig. 4. Per-link peak age, A^{p}/N, for various policies as a function of K.

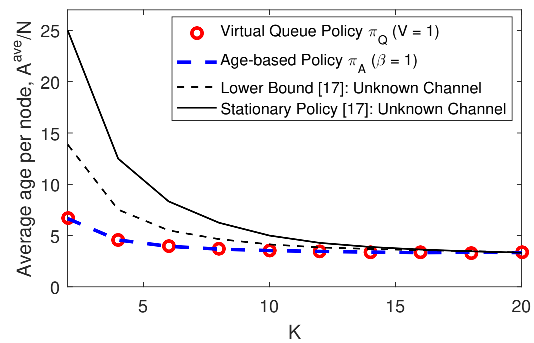

Fig. 5. Per-link average age, A^{ave}/N, for various policies as a function of K.

of [18], while in Figure 5, we also plot a lower-bound on average age that can be achieved by any such policy [18], since π_C is not average age optimal. We observe that the gap between the optimal policy π_C in the unknown channel case and policies π_Q, π_A of the known channel case is large when K is small, and diminishes as K increases. Smaller K implies more network interference, as fewer links can be activated simultaneously. This shows that there is a significant utility, in terms of age reduction, in knowing the channel state, especially when the network suffers from large interference.

In Figure 6 and 7 we plot per-link peak and average age as a function of the fraction of nodes with bad channel, namely $\theta = \frac{n_{\text{bad}}}{N}$. We observe that the gap between the optimal policy π_C in the unknown channel state case, and our policies π_Q and π_A of the known channel case, increases as the fraction θ increases. This indicates that if the channel statistics of the network are poor then there is a significant utility, in terms of age reduction, in knowing the channel state information. For example, when all channels are 'bad', i.e. $\theta = 1$, the gap is as large as 4 fold.

We now analyze performance of our proposed policies π_Q and π_A over the choice of parameters V and β, respectively. Here, we set $K = 5$ and the number of 'bad' channels also to be $n_{\text{bad}} = 5$. For the virtual-queue based policy π_Q, we observe that the parameter V has nearly no effect on convergence time of the algorithm. To illustrate this, in Figure 8, we plot per-link peak age $A^{\text{p}}(\pi_Q)/N$ computed over

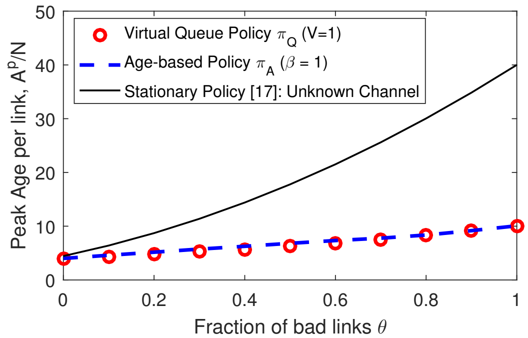

Fig. 6. Per-link peak age, A^{p}/N, for various policies as a function of θ.

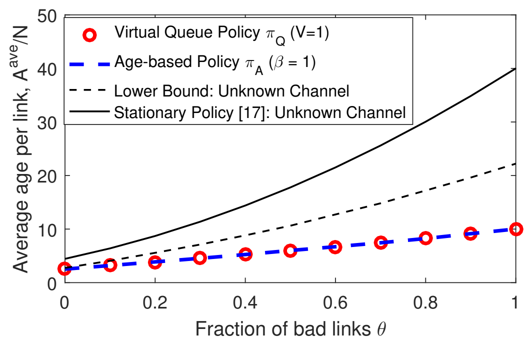

Fig. 7. Per-link average age, A^{ave}/N, for various policies as a function of θ.

the first t time slots, for two different values of $V = 0.1$ and $V = 100$. We observe that the peak age measured over the first t slots converged to the peak age $A^{\mathrm{p}}(\pi_Q)$ at nearly the same time.

For the age-based policy π_A, we again observe no difference in convergence time with respect to β. Theorem 3 guarantees bounds for any $\beta \in \mathbb{R}$. However, in Figure 9, we observe that the peak and average age achieved by π_A gets worse as β becomes negative. This is because $c_1(\beta)$ and $c_2(\beta)$ in Theorem 3 are large and negative when $\beta < 0$.

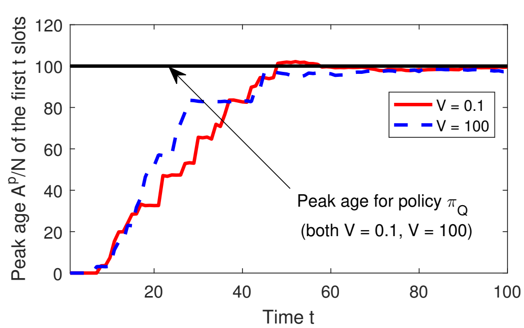

Fig. 8. Per-link peak age, $A^{\mathrm{p}}(\pi_Q)/N$, computed till time t for the virtual-queue policy π_Q for $V = 0.1$ and $V = 100$. Also plotted is the per-link peak age $A^{\mathrm{p}}(\pi_Q)/N$ achieved over a much larger time horizon.

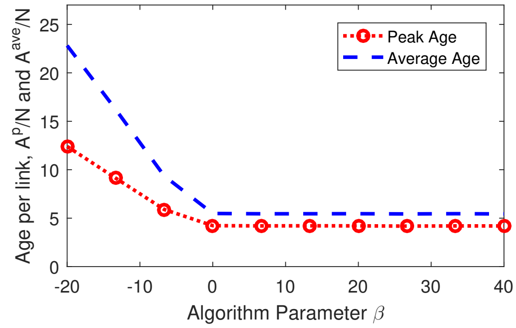

Fig. 9. Per-link average and peak age, $A^{\mathrm{ave}}(\pi_A)/N$ and $A^{\mathrm{p}}(\pi_A)/N$, as a function of parameter β.

VII. Conclusion

We considered the problem of age minimization for a wireless network under general interference constraints and time varying channels, when the channel state information is perfectly available to the scheduler to make scheduling decisions. We proposed a virtual-queue based policy and an age-based policy to minimize age. We proved that the virtual-queue based policy is nearly peak age optimal, up to a constant additive factor, and that the age-based policy is at most a factor 4 away from age optimality.

In comparison with our previous work, which derived age optimal policies when the channel state information is not available to the scheduler, we demonstrate a 4 fold improvement in age when the channel state information is available to the schedule in a particular network setting. This work, therefore, establishes the utility in obtaining or using the channel state information in scheduling to minimize age.

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Appendix

A. Proof of Lemma 1

Using the age evolution equation (2), we obtain

$$\frac{1}{t}\mathbb{E}\left[A_e(t)\right] = \frac{1}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1}\left(A_e(\tau+1) - A_e(\tau)\right)\right],$$
$$= 1 - \frac{1}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e(\tau)\right], \quad (28)$$

where the first equality follows because $A_e(0) = 0$. Note that for $\pi \in \overline{\Pi}_1$, we have $\limsup_{t\to\infty} \frac{1}{t}\mathbb{E}\left[A_e(t)\right] = 0$, since $\mathbb{E}\left[A_e(t)\right]$ is bounded. Taking $t \to \infty$ in (28) yields the result.

B. Derivation of the Peak Age Minimization Problem

Using Lemma 1 and the definition of peak age (5), we get

$$\overline{A}_e^{\text{p}}(\pi) = \frac{1}{\liminf_{t\to\infty} \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t}\sum_{e\in E} U_e(\tau)S_e(\tau)\right]}, \quad (29)$$

for all $e \in E$, and any policy $\pi \in \overline{\Pi}_1$. For a detailed argument see [21]. Since $\overline{A}^{\text{p}}(\pi) = \sum_{e\in E} w_e \overline{A}_e^{\text{p}}(\pi)$, the peak age minimization problem $\min_{\pi\in\overline{\Pi}_1} \overline{A}^{\text{p}}(\pi)$ can now be written as

$$\underset{\pi\in\overline{\Pi}_1}{\text{Minimize}} \quad \sum_{e\in E} \frac{w_e}{\liminf_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)}. \quad (30)$$

Using auxiliary variables α_e, this can be written as (15).

C. Proof of Lemma 2

Squaring the age evolution equation (2) on both sides, and using the fact that $U_e^2(t)S_e^2(t) = U_e(t)S_e(t)$ as $U_e(t)S_e(t) \in \{0,1\}$ we obtain

$$A_e^2(t+1) - A_e^2(t) = 1 + 2A_e(t) - U_e(t)S_e(t)A_e^2(t) - 2U_e(t)S_e(t)A_e(t), \quad (31)$$

for all t and e. Since $A_e(0) = 0$, telescoping this over t time slots we get

$$\frac{1}{t}\mathbb{E}\left[A_e^2(t)\right] = \frac{1}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1}\left(A_e^2(\tau+1) - A_e^2(\tau)\right)\right],$$
$$= 1 - \frac{1}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e^2(\tau)\right]$$
$$+ \frac{2}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1} A_e(\tau)\right] - \frac{2}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e(\tau)\right]. \quad (32)$$

For a policy $\pi \in \overline{\Pi}_2$ we must have $\limsup_{t\to\infty} \frac{1}{t}\mathbb{E}\left[A_e^2(t)\right] = 0$, as $\mathbb{E}\left[A_e^2(t)\right]$ is uniformly bounded. Taking the limit $t \to \infty$ in (32), we get

$$2\overline{A}_e^{\text{ave}} = 1 + \limsup_{t\to\infty} \frac{1}{t}\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e^2(\tau)\right], \quad (33)$$

where we have used Lemma 1, as $\pi \in \overline{\Pi}_2 \subset \overline{\Pi}_1$. This proves the Lemma for $\beta = 0$. Adding β times (14) from Lemma 1 to (33) we obtain the result.

D. Proof of Lemma 3

Consider a policy $\pi \in \overline{\Pi}_2$, and a link e. From Cauchy-Schwartz inequality we have

$$\left(\mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e(\tau)\right]\right)^2 \leq \mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)\right] \times \mathbb{E}\left[\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e^2(\tau)\right],$$

since $U_e^2(\tau)S_e^2(\tau) = U_e(\tau)S_e(\tau)$, as $U_e(\tau)S_e(\tau) \in \{0,1\}$. Dividing both sides by t^2 we get

$$\frac{\mathbb{E}\left[\left(\frac{1}{t}\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e(\tau)\right)\right]^2}{\mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)\right]} \leq \mathbb{E}\left[\frac{1}{t}\sum_{\tau=0}^{t-1} U_e(\tau)S_e(\tau)A_e^2(\tau)\right].$$

Taking limsup on both sides and using Lemma 1 and Lemma 2, along with the definitions of $\overline{A}_e^{\text{p}}(\pi)$ and $\overline{A}_e^{\text{ave}}(\pi)$, we get

$$A_e^{\text{p}}(\pi) \leq 2A_e^{\text{ave}}(\pi) - 1. \quad (34)$$

Summing over e with weights w_e we obtain the result in (19).

In order to see that the inequality also holds at optimality, note that

$$\overline{A}^{\text{p}*} = \inf_{\pi\in\overline{\Pi}_1} \overline{A}^{\text{p}}(\pi) \leq \inf_{\pi\in\overline{\Pi}_2} \overline{A}^{\text{p}}(\pi),$$
$$\leq \overline{A}^{\text{p}}(\pi) \leq 2\overline{A}^{\text{ave}}(\pi) - \sum_{e\in E} w_e, \quad (35)$$

for any $\pi \in \overline{\Pi}_2$, where the first inequality follows because $\overline{\Pi}_2 \subset \overline{\Pi}_1$. Taking infimum over $\pi \in \overline{\Pi}_2$ in (35) yields the result.