Robust Design of Cognitive Radio Networks

Hyang-Won Lee
Department of Internet and Multimedia Engineering
Konkuk University, Seoul 143-701, Korea
Email: leehw@konkuk.ac.kr

Eytan Modiano
Laboratory for Information and Decision Systems
MIT, Cambridge, MA 02139
Email: modiano@mit.edu

***Abstract*—We develop a new framework for designing robust cognitive radio networks using white channels. Our framework assigns white channels to wireless links and provisions spare channel capacity that will used in the event of channel preemption. We characterize the optimal white channel assignment that minimizes the spare capacity while guaranteeing survivability against white channel preemptions.**

I. INTRODUCTION

There has been an increasing concern about wireless radio spectrum congestion due to the explosive growth of wireless communication devices such as smartphones. On the other side, recent studies show that the licensed spectrum is significantly underutilized [1], and hence, a significant amount of (underutilized) spectrum can possibly be used for enhancing the capacity of wireless networks. *Cognitive radio* has emerged as a promising solution for enabling efficient utilization of spectrum resources [2]–[5].

In this paper, we study the problem of constructing a reliable cognitive radio network using white channels. While such a network can be constructed at low cost (as white channels can be used for free), any white channel should be preempted whenever a primary user joins the channel. This can significantly deteriorate the network performance, and therefore, it is necessary to provide recovery mechanisms so as to continue data transmissions in the event of a channel preemption.

One way to address this issue is to let the disrupted links scan and find another idle white channel on the fly (see [6] and references therein). While this approach provides an economical solution, it may lead to significant delay until it finds an available channel. More importantly, there may not always be an idle channel, hence it is impossible to provide "guaranteed recovery" depending solely on white channels. In this paper, we take an alternative approach to providing guaranteed reliability for cognitive radio networks. In our approach, a *dedicated* spare channel is provisioned so that it can be used whenever a white channel preemption occurs in the network.

II. MODEL AND PROBLEM DESCRIPTION

We assume that there is a set W of white channels. A white channel is busy while it is being occupied by a primary user, and becomes idle once the primary user leaves the channel. When a white channel is idle, it can be used by secondary users, but the channel should be vacated if a primary user joins the channel to use it. This event of channel preemption is called a *channel failure* in this paper.

Consider a wireless network represented by an undirected graph $G = (V, E)$ where V is the set of nodes and E is the set of links. There is a link between two nodes if they can directly communicate with each other. We assume the 1-hop interference model where a node can either transmit or receive the data at the same time. Let r_e be the traffic load on link e. The network G is built using white channels W such that it can support the network load $r = [r_e, \forall e \in E]$. Specifically, each link e is assigned a white channel in W, and it transmits its data while the white channel is idle. Let R_w be the achievable rate of white channel w. Assume $R_w \geq r_e, \forall e, w$, i.e., the load on a link is no greater than any white channel capacity. Let $y_e^w = 1$ if a white channel w is assigned to link e, and 0 otherwise. Note that if link e is assigned white channel w, then link e should be scheduled for at least $\frac{r_e}{R_w}$ fraction of time in order to support the demand r_e. Furthermore, if there is an interfering link using the same white channel, then they cannot transmit simultaneously. Therefore, the schedulability is determined by the channel assignment.

Since a feasible schedule under the 1-hop interference model is a matching, the condition for the schedulability can be expressed using the matching polytope [7]:

$$\sum_{e \in \delta(v)} r_e y_e^w \leq R_w, \forall v \in V, w \in W \tag{1}$$

$$\sum_{e \in E(U)} r_e y_e^w \leq \alpha(U) R_w, \forall U \subseteq V : |U| \text{ odd } , w \in W \tag{2}$$

$$\sum_{w \in W} y_e^w = 1, \forall e \in E \tag{3}$$

$$y_e^w \in \{0, 1\}, \forall e \in E, w \in W,$$

where $\alpha(U) = \frac{|U|-1}{2}$ and $E(U)$ is the set of links whose both ends are in U. For each w, the constraints (1) and (2) represent the matching polytope over the subgraph of G using white channel w. Hence, this is an extension of the matching polytope to the multi-channel setting, and it requires that the links over each white channel be schedulable under the 1-hop interference model.The constraints in (3) require that only one white channel can be assigned to a link.

A channel assignment y is said to be *feasible* if it satisfies the above constraints. Given a set W of white channels, $F(W)$ is defined to be the set of feasible channel assignments using

This work was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Science, ICT & Future Planning (2012R1A1A1012610).

978-1-4799-6786-5/14/$31.00 ©2014 IEEE

(some or all) white channels in W. We assume that $F(W)$ is nonempty, i.e., there exists a feasible channel assignment. As a consequence, the network load can always be supported as long as the white channels are all idle.

III. SURVIVING k FAILURES

Suppose that we want to survive any k white channel failures, i.e., the spare channel should be able to support the demands on the links disrupted by any k white channel failures. Hence, the goal is to find a white channel assignment requiring minimum spare capacity to protect against k channel failures. The problem can be formulated as follows: DetRec:

$$\begin{aligned} \min_{C,y} \quad & C \\ \text{s.t.:} & \sum_{w\in S}\sum_{e\in\delta(v)} r_e y_e^w \le C, \forall v \in V,\ \forall S \in \mathcal{W}(k) && (4) \\ & \sum_{w\in S}\sum_{e\in E(U)} r_e y_e^w \le \alpha(U)C, \forall U \in \mathcal{V}, S \in \mathcal{W}(k) && (5) \\ & y \in F(W), \end{aligned}$$

where $\alpha(U) = \frac{|U|-1}{2}$, $\mathcal{V} = \{U \subseteq V : |U| \text{ odd } \ge 3\}$ and $\mathcal{W}(k) = \{S \subseteq W : |S| = k\}$. Similar to (1) and (2), the constraints in (4) and (5) require that after any k channel preemptions, the traffic demands on the disrupted links should be supported by the spare channel with capacity C. Hence, the optimal solution to the problem is a feasible white channel assignment with minimum spare channel capacity.

A. Properties of Optimal Channel Assignment

The optimal channel assignment can be shown to have the following properties.

Theorem 1: It is NP-complete to find a white channel assignment achieving minimum spare channel capacity.

Denote by $y^*(W,k)$ the optimal channel assignment for protection against k preemptions, given a set of white channels W, i.e., $y^*(W,k)$ is an optimal solution to the problem DetRec.

Observation 1: Given W and k, $C(y^*(W,k),k) \le C(y^*(W',k),k)$ for any $W' \subset W$.

Observation 2: Given W and k, $C(y^*(W,k),k) \le C(y^*(W',k+1),k+1)$.

Observation 3: Given W and any $y \in F(W)$, we have $C(y,k) = C_{\max}, \forall k \ge |y|$, where $|y|$ is the number of white channels used in the assignment y, and

$$C_{\max} = \max\left\{ \max_{v\in V} \sum_{e\in\delta(v)} r_e, \max_{U\in\mathcal{V}} \frac{1}{\lfloor\alpha(U)\rfloor} \sum_{e\in E(U)} r_e \right\}.$$

A feasible channel assignment y is said to be *interference-free* if each channel is assigned to a set of links that can be activated simultaneously.

Theorem 2: There exists an interference-free channel assignment if the number of white channels is greater than the maximum node degree, i.e., $|W| > d_{\max}$.

For given y and k, define $M_1(y,k)$ and $M_2(y,k)$ as follows:

$$M_1(y,k) = \max_{v\in V, S\in\mathcal{W}(k)} \sum_{w\in S}\sum_{e\in\delta(v)} r_e y_e^w, \tag{6}$$

$$M_2(y,k) = \max_{U\in\mathcal{V}, S\in\mathcal{W}(k)} \frac{1}{\alpha(U)} \sum_{w\in S}\sum_{e\in E(U)} r_e y_e^w. \tag{7}$$

The spare channel capacity is then given by

$$C(y,k) = \max\left\{M_1(y,k), M_2(y,k)\right\}. \tag{8}$$

Hence, it is optimal to minimize the maximum of $M_1(\cdot,\cdot)$ and $M_2(\cdot,\cdot)$. As mentioned above, the interference-free channel assignment is likely to require less spare capacity. In fact, any interference-free assignment minimizes $M_1(\cdot,\cdot)$.

Lemma 1: Consider any two interference-free channel assignments $y, \tilde{y}$. Then, $M_1(y,k) = M_1(\tilde{y},k)$. Furthermore, it is minimum of all feasible channel assignments, i.e., $M_1(y,k) \le M_1(x,k), \forall x \in F(W)$.

Theorem 3: Assume $r_e = r, \forall e \in E$. Then, any feasible interference-free channel assignment is an optimal solution to the problem DetRec for $k \le d_{\max}$.

For the case of non-uniform network load, we can also prove the same result for $k = 1$. Let $r_{\max} = \max_e r_e$.

Lemma 2: For any feasible white channel assignment y, we have $C(y,k) \ge r_{\max}, \forall k \ge 1$.

Theorem 4: Consider two feasible channel assignments $y, \tilde{y} \in F(W)$ such that y is interference-free and $\tilde{y}$ is not. Then, we have $C(y,1) \le C(\tilde{y},1)$.

IV. CONCLUSION

In this work, we studied the problem of providing survivability for cognitive radio networks built upon white channels. We analyzed the properties of the optimal white channel assignment requiring minimum spare channel capacity. There are several issues for future studies, including approximation algorithms for white channel assignment (based on interference-free assignment) and survivability guarantee for random white channel failures.

REFERENCES

[1] S. S. Company, "General survey of radio frequency bands 30 mhz to 3 ghz," *www.sharedspectrum.com*, September 2010.

[2] J. Mitola III, "Cognitive radio for flexible mobile multimedia communications," *ACM/Kluwer MONET*, vol. 6, no. 5, pp. 435–441, Sep. 2001.

[3] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, vol. 50, no. 13, pp. 2127 – 2159, 2006.

[4] I. F. Akyildiz, W.-Y. Lee, and K. R. Chowdhury, "Crahns: Cognitive radio ad hoc networks," *Ad Hoc Networks*, vol. 7, no. 5, pp. 810 – 836, 2009.

[5] S. Srinivasa and S. Jafar, "Cognitive radios for dynamic spectrum access - the throughput potential of cognitive radio: A theoretical perspective," *Communications Magazine, IEEE*, vol. 45, no. 5, pp. 73–79, 2007.

[6] H. Kim and K. Shin, "Fast discovery of spectrum opportunities in cognitive radio networks," in *New Frontiers in Dynamic Spectrum Access Networks, 2008. DySPAN 2008. 3rd IEEE Symposium on*, Oct 2008, pp. 1–12.

[7] J. Edmonds, "Maximum matching and a polyhedron with (0,1) vertices," *J. Res. Nat. Bur. Standards*, vol. 69, pp. 125–130, 1965.