Survivable Paths in Multilayer Networks

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***Abstract*—We consider the problem of protection in multilayer networks. In single-layer networks, a pair of disjoint paths can be used to provide protection for a source-destination pair. However, this approach cannot be directly applied to layered networks where disjoint paths may not always exist. In this paper, we take a new approach which is based on finding a *set of paths* that may not be disjoint but together will survive any single physical link failure. We consider the problem of finding the minimum number of survivable paths. In particular, we focus on two versions of this problem: one where the length of a path is restricted, and the other where the number of paths sharing a fiber is restricted. We prove that in general, finding the minimum survivable path set is NP-hard, whereas both of the restricted versions of the problem can be solved in polynomial time. We formulate the problems as Integer Linear Programs (ILPs), and use these formulations to develop heuristics and approximation algorithms.**

I. Introduction

Multilayer network architectures such as IP-over-WDM have played an important role in advancing modern communication networks. Typically, a layered network is constructed by embedding a logical topology onto a physical topology such that each logical link is routed using a path in the physical topology. While such a layering approach makes it possible to take advantage of the flexibility of upper layer technology (e.g., IP) and the high data rates of lower layer technology (e.g., WDM), it raises a number of challenges for efficient and reliable operations. In this paper, we focus on the issue of providing protection in layered networks.

The protection problem in single-layer networks is rather straightforward; namely, providing a pair of disjoint paths (one for primary and one for backup) guarantees a route between two nodes against any single link failure. This approach, however, cannot be directly applied to layered networks, because a pair of seemingly disjoint paths at the logical layer may share a physical link and thus simultaneously fail in the event of a physical link failure. To address this issue [1] introduced the notion of *physically disjoint* logical paths.

In [2], this notion was generalized as Shared Risk Link Group (SRLG) disjoint paths, i.e., two paths between the source and destination nodes that do not share any risk (e.g., fiber and conduit). Nearly all the previous works in the context of layered network protection have focused on finding SRLG-disjoint paths [3], [4], [5], [6].

Although the SRLG-disjoint paths problem has been well studied, there are networks in which the SRLG-disjoint paths do not exist between a source and a destination. In order to address this challenge, we take an alternative approach that is based on finding a set of paths that together will survive any single physical link failure. Thus, in the case that SRLG-disjoint paths do not exist, we may find three or more paths such that in the event of a fiber failure, at least one of the paths remains connected. This notion of *survivable path set* generalizes the traditional notion of SRLG-disjoint paths, and enables us to provide protection for a broader range of scenarios. Our contributions can be summarized as follows:

- We introduce a new notion of survivable path set to provide protection even for the case where SRLG-disjoint paths do not exist.
- We prove the NP-hardness of the minimum survivable path set (MSP) problem.
- We show that under certain practical restrictions, the MSP problem is polynomially solvable.
- We develop heuristics and approximation algorithms for the MSP problem.

In Section II, we present the network model. In Section III, we study the problem of finding a minimum set of paths that will survive any single fiber failure and develop several approximation algorithms. Finally, we provide simulation results in Section IV and conclusions in Section V. Due to the page limit, the proofs of some theorems and lemmas have been omitted, and can be found in [7].

II. Network Model

We consider a layered network that consists of a logical topology $G_L = (V_L, E_L)$ built on top of a physical topology $G_P = (V_P, E_P)$ where V and E are the sets of nodes and links respectively, and $V_L \subset V_P$. Each logical link (i, j) in E_L is mapped onto an $i - j$ path in the physical topology. This is called lightpath routing. Different lightpaths may use the same fiber (physical link), therefore when a fiber fails, all the lightpaths using that fiber will fail. Hence, a logical path survives the failure of any fiber that it does not use.

As mentioned above, we generalize the traditional notion of SRLG-disjoint paths to account for the case where there does not exist a pair of SRLG-disjoint paths. In a layered network, a set of logical paths is said to be *survivable* if at least one of the paths remain connected after any single physical link failure. Hence, a survivable set consisting of two paths is a pair of SRLG-disjoint paths. Note that there may exist a survivable path set even if SRLG-disjoint paths do not exist. For example, consider the physical and logical topologies in Fig. 1. Each dashed line in Fig. 1(c) shows the lightpath routing of each logical link over the physical topology. Under this lightpath

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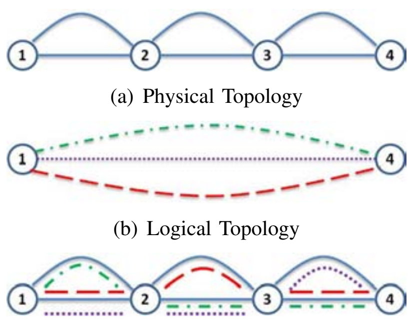
routing, each pair of logical paths between nodes 1 and 4 shares some fiber.

Suppose that we want to find a set of logical paths between nodes 1 and 4 that can survive any single physical link failure. Clearly, there does not exist a pair of SRLG-disjoint paths as each pair of logical paths shares a fiber. However, it is straightforward to check that the set of 3 paths can survive any single fiber cut, although they are not SRLG-disjoint. This example shows that the traditional protection schemes based on SRLG-disjoint paths (such as the ones in [2]) may fail to provide protection against single physical link failures, while there exists a set of paths that can together provide protection. Our goal in this paper is to address the problem of finding a set of survivable paths that together will survive any single fiber failure.



(a) Physical Topology

(b) Logical Topology

(c) Mapping

Fig. 1. Topologies in Multilayer Networks

III. Minimum Survivable Paths Set (MSP)

We start with the problem of finding a minimum survivable path set, i.e., the minimum cardinality set of paths between a pair of nodes s and t that survive any single physical link (fiber) failure. We first present a path-based Integer Linear Program (ILP) formulation for this problem, assuming that the entire set of $s-t$ paths with their routings over fibers is given. For each path j, let P_j be a binary variable which takes the value 1 if path j is selected, and 0 otherwise. The matrix $A \in R^{m \times n}$ refers to the mapping of all n paths over the m fibers such that $a_{ij} = 0$ if path j uses fiber i and $a_{ij} = 1$ otherwise. Let e be a $m \times 1$ vector of ones. The minimum survivable path set problem can be expressed as follows:

$$\text{minimize} \quad \sum_{j=1}^{n} P_j \tag{1}$$

$$\text{subject to} \quad A \times P \geq e \tag{2}$$

$$P_j \in \{0,1\}, \quad j = 1, \cdots, n \tag{3}$$

In the above, the objective function is the number of selected paths. Each row $i \in \{1, \cdots, m\}$ in constraint (2) requires that at least one selected path survives the failure of fiber i, i.e., the selected path set should be survivable. Hence, the optimal solution to the above optimization problem gives a minimum survivable path set whenever exists. Although this formulation requires the knowledge of every path (which is possibly exponential in the number of fibers), the compact and clean expression of the path-based formulation enables us to analyze the useful properties of survivable path sets. Later, we will use this formulation to develop heuristics and approximation algorithms for finding a minimum survivable path set. In addition, the MSP problem can be described by link-based formulation using a polynomial number of constraints and variables without enumerating all of the paths.

A. *MSP in general setting*

In this section, we show that the MSP problem is NP-hard in general and discuss some algorithms that can be used to solve the problem. In Sections III-B and III-C, we will study the MSP problem under practical constraints. Our first result pertains to the complexity of the MSP problem as stated in Theorem 1 below.

Theorem 1: Computing the minimum number of survivable paths in multilayer networks is NP-hard. In addition, this minimum value cannot be approximated within any constant factor, unless $P = NP$.

The proof of Theorem 1 relies on a mapping between the survivable path set problem and the minimum set cover problem. Suppose that each path corresponds to a set of fibers that are not used by that path, i.e., survived. Then, finding a minimum survivable path set is equivalent to finding a minimum path set that survives (covers) all of the fibers. The complete proof can be found in [7].

Since the problem is computationally hard to solve, we consider heuristics and approximation algorithms that give a set of survivable paths in polynomial time. Owing to the similarity to the set cover problem, the heuristics that have been developed for set cover problems can be used here. In particular, a common approach to solve the set cover problem is the greedy algorithm. In order to apply the greedy algorithm to our setting, one needs to enumerate all of the paths with their routings on the fibers. In general, the number of paths in a multilayer network is exponential in the total number of fibers. Moreover, in each iteration, the greedy algorithm tries to find a path that survives the maximum number of fibers. This is equivalent to the Minimum Color Path problem, which is known to be NP-hard. [8]

Another approach which can be used to approximate the set cover problem is randomized rounding which is based on solving the Linear Program (LP) relaxation of the original ILP formulation, and rounding the solution randomly. Randomized rounding gives an $O(\log m)$ approximation, where m is the number of fibers [9]. This is the best possible approximation for the MSP problem, which is due to the fact that the minimum set cover problem cannot be approximated within better than a $\log m$ factor [10].

Fortunately, practical systems impose certain physical constraints that make the survivable path-set problem easier to solve. For example, due to physical impairments and delay constraints, paths are typically limited in length. Furthermore, in WDM networks, the sharing of a fiber by the logical links is limited by the number of available wavelengths. In the following, we show that these physical limitations make the MSP problem tractable.

B. The Path Length Restricted Version

In this section, we assume that each logical path is restricted to use at most K fibers. Restricting the length of paths (i.e., number of fibers on each path) is a realistic assumption, because each logical link is typically constrained in the number of fibers that it may use, and each logical path is constrained in the number of logical links.

Lemma 1: Under the path length restriction, the optimal number of survivable paths is at most $K+1$.

Proof: By the assumption, each path uses at most K fibers, and thus at least $m-K$ fibers are survived by a path. Suppose that we have selected an arbitrary path, and want to add other paths to form a survivable path set. In the worst case, each of the newly selected paths can survive only a single fiber which is not survived by the previously selected paths. Since there are at most K fibers that are not survived by the first path, we need at most K additional paths to survive the rest of the fibers. Therefore, the total number of paths will not exceed $K+1$. ■

Lemma 2: In the path length restricted version of MSP, the total number of paths is polynomial in the number of fibers m, and can be enumerated in polynomial time.

Proof: Under the assumption, a path can consist of up to K fibers, and thus at most K logical links. In a graph with n nodes there can be $O(n^K)$ paths of length up to K. Since the number of nodes is at most $2m$, the total number of logical paths of length up to K is $O(m^K)$. A simple exhaustive search can be used to enumerate the paths. ■

Theorem 2: The path length restricted version of the MSP problem can be solved in polynomial time.

Proof: By Lemma 1, MSP needs at most $K+1$ paths to survive any single failure. Therefore, one can find the exact solution by searching through all subsets of paths with sizes $2, 3, ..., K+1$. This will take $O(P^{K+1})$ iterations where P is the total number of paths. On the other hand, by Lemma 2, the total number of paths is $O(m^K)$. Therefore, the total running time of exhaustive search is $O(m^{K(K+1)})$ which is polynomial in the total number of fibers. ■

Although this exhaustive search returns an optimal solution, its running time can be prohibitive for large values of m and K. This motivates us to study heuristics and approximation algorithms with better running time. First, we consider a greedy algorithm, followed by a randomized algorithm based on ε-net which is a well-known technique in the area of computational geometry.

1) Greedy Algorithm: The first heuristic we consider is a greedy algorithm which is similar to the greedy algorithm for the minimum set cover problem. The input to the greedy algorithm is the set of paths with the set of fibers used by each path and the set of all fibers. The greedy algorithm is an iterative algorithm that works as follows. In the first step, it selects a path using the minimum number of fibers. In the second step, it selects the path that survives the maximum number of fibers that are not survived yet. The second step is repeated until the selected path set survives all of the fibers. Following the proof of Lemma 1, it can be shown that the greedy algorithm also finds a survivable path set with size at most $K+1$.

As discussed in Section III-A, the greedy algorithm generally gives an $O(\log m)$ approximation to the minimum survivable path set. However, under the assumption of restricted path length, it provides a better approximation as stated in Theorem 3.

Theorem 3: The greedy algorithm provides an $O(\log K)$ approximation in polynomial time for the path length restricted version of MSP.

Proof: Let ξ be the size of minimum survivable path set. Let n_i be the number of fibers that are not survived after the i^{th} iteration of the greedy algorithm. Clearly, we have $n_1 \leq K$. Now, note that there is a path that survives at least $\frac{n_1}{\xi}$ of the remaining n_1 fibers, because otherwise the size of the optimal path set would be larger than ξ. Hence, in the second iteration, the greedy algorithm would select a path that survives at least $\frac{n_1}{\xi}$ of fibers. Thus,

$$n_2 \leq n_1 - \frac{n_1}{\xi} \leq K(1-\frac{1}{\xi}). \tag{4}$$

Similarly,

$$n_3 \leq n_2 - \frac{n_2}{\xi} \leq K(1-\frac{1}{\xi})^2, \tag{5}$$

and in general,

$$n_i \leq K(1-\frac{1}{\xi})^i. \tag{6}$$

The greedy algorithm will terminate when $n_t < 1$, and this condition is satisfied when

$$K(1-\frac{1}{\xi})^t < 1, \tag{7}$$

where t is the total number of iterations. Since $1-x < e^{-x}$ for $x > 0$, inequality (7) is satisfied when

$$Ke^{-\frac{t}{\xi}} \leq 1 \Leftrightarrow t \leq \xi \times \log K. \tag{8}$$

Therefore, the greedy algorithm provides an $O(\log K)$ approximation.

To prove the polynomial time complexity, note that in each iteration of the greedy algorithm, the best path can be found in $O(m^K)$ by searching through all the paths (see the proof of Theorem 2). Furthermore, as mentioned above, the greedy algorithm terminates in at most $K+1$ iterations. Therefore, the computational complexity of the greedy algorithm is $O(Km^K)$. ■

Although the greedy algorithm runs significantly faster than the exhaustive search algorithm, its running time can still be prohibitive for large K and m. Hence, we develop a novel randomized algorithm which has a considerably better running time. This algorithm builds upon solutions to the closely related Set Cover and Hitting Set problems [11]. In particular, the algorithm is based on ε-net, a concept in computational geometry, which provides an approximation algorithm for the Hitting Set problem.

2) ε-net Algorithm: Our ε-net algorithm is an iterative algorithm which selects each path with some probability. If all the fibers are survived by the selected path set in the first iteration, the algorithm terminates. Otherwise, it changes the probability of selecting each path and selects a new set of paths using the new probabilities, until all fibers are survived.

Let W_j be the weight of path j, initialized as $W_j = 1$. Define the weight of each fiber i to be the sum of the weights of paths surviving fiber i, i.e.,

$$W(f_i) = \sum_{j:a_{ij}=1} W_j. \quad (9)$$

Definition 1: A fiber is said to be *ε-Survivable* if

$$W(f_i) \geq \varepsilon \sum_{j=1}^{n} W_j \text{ for some } \varepsilon \in (0,1), \quad (10)$$

where n is the total number of paths.

Note that when all the paths have the same weight of 1, a fiber is *ε-Survivable* if it is survived by at least $\varepsilon \times n$ paths. Hence, if a fiber is *ε-Survivable* with large ε, then it is likely to be survived by randomly selected paths. This observation is exploited in our ε-net algorithm as discussed below.

By applying the randomized algorithm for the hitting set problem from [12] and [13], we can obtain a path-selection algorithm for selecting a random subset of paths that will survive all of the *ε-Survivable* fibers, with high probability. In particular, the algorithm finds a set of paths via s independent random draws, such that in each draw, a path is selected from the entire path set according to the probability distribution $\mu(P_j) = \frac{W_j}{\sum_{j=1}^{n} W_j}, \forall j$.

Our ε-net algorithm iteratively applies this random path selection as follows. After each iteration, it checks the survivability of the selected path set. If not all fiber failures are survived, the algorithm doubles the weight of all paths that survive the failure of fibers in $\bar{S}$, where $\bar{S}$ is the set all the fibers that are not survived yet (so that such fibers are more likely to be survived by the new selected paths). The random path selection is repeated with the new probability distribution.

Let ξ be the optimal value of the MSP problem. By applying the results in [14], [15], the following theorem can be proved.

Theorem 4: Let $s = c\frac{\log K}{\varepsilon} \log \frac{\log K}{\varepsilon}$, where c is a constant. The ε-net algorithm finds a set of survivable paths of size $O(\log K \log \xi)\xi$, with high probability.

Finally, we propose another algorithm which we call the Random-Sweep greedy algorithm. Although, we could not quantify the performance of this algorithm, its performance is nearly optimal in many scenarios as will be shown in Section IV.

3) Random-Sweep Greedy (RSG) Algortihm: Random Sweep Greedy algorithm is a modified version of the greedy algorithm. Here, the RSG removes a path (from the selected path set) which survives the fibers covered by other selected paths; so that the size of the selected path set can be further reduced while maintaining the survivability.

The RSG algorithm also requires the knowledge of the set of paths and associated fibers. Let P_i be the set of selected paths in the first i iterations, and S_j be the set of fibers that are survived by path j. Define the amortized cost AC_j as the inverse of the number of newly survived fibers by path j. The first two iterations of RSG are the same as the greedy algorithm. That is, in each iteration, it selects a path with minimum amortized cost. If the first two paths survive all of the fibers, the algorithm terminates. Otherwise, it continues as follows.

Suppose the RSG algorithm is in the i^{th} iteration. First, find a path, say i, with minimum amortized cost among the remaining paths. Then, pick a path, say j, randomly from the previously selected path set P_{i-1}, and find $S^* = \cup_{k \in P_i, k \neq j} S_k$, which is the set of fibers that are survived by any of the selected paths other than path j. If $S_j \subset S^*$, remove path j from the set P_i. Note that removing such a path does not affect the survivability of the selected path set, i.e., the same set of fibers are still survived after the removal. However, it will decrease the number of selected paths by one. Repeat this procedure for all paths j, and check if they can be removed from the selected path set P_i.

Table I summarizes the performance of each algorithm under the path length restriction.

Method	Approximation	Running Time	T
ExS	Exact Solution	$O(m^{K(K+1)})$	D
Greedy	$O(\log K)$	$O(Km^K)$	D
ε-net	$O(\log K \log \xi)$	$O(K \log(K) \log(m) \log(\log(K)))$	P

TABLE I
PERFORMANCE BOUNDS UNDER PATH LENGTH RESTRICTED VERSION: EXS-EXHAUSTIVE SEARCH, T-TYPE, D-DETERMINISTIC, P-PROBABILISTIC

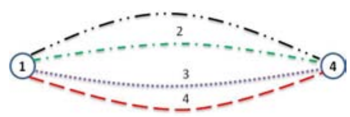
C. Wavelength Restricted version

Another important practical constraint is that in WDM-based networks, the number of lightpaths using a fiber is limited to say W, which is the number of wavelengths supported over a fiber. In this section, we assume that a set of logically disjoint paths with their mapping on the physical topology is given, and the goal is to find a minimum survivable path set among those paths under the WDM restriction. Note that the set of logically disjoint paths can be abstracted as a logical topology with two nodes and parallel links (e.g., the one in Fig. 2(a)). Clearly, under this assumption, the WDM restriction implies that *each fiber can be used by at most W paths*. Using this property, it can be shown that the MSP problem under the WDM restriction can be solved in polynomial time. To prove this, we need the following lemma.

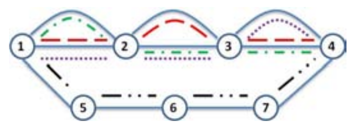
Lemma 3: Under the wavelength restriction, the minimum number of survivable paths is at most $W + 1$.

Theorem 5: Under the wavelength restriction, the MSP problem can be solved in polynomial time.

Although there exists a polynomial time optimal algorithm, it requires excessive computation for large values of W and m. As in the case of restricted path length, we developed approximation algorithms such as greedy and ε-net algorithms which have better running time. Table II shows the summary of



(a) Logical Topology



(b) Routing

Fig. 2. Routing in Multilayer Network

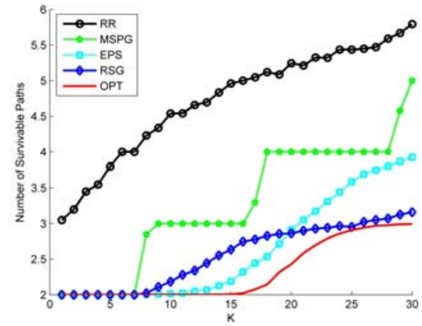


Fig. 3. Comparison of algorithms for MSP under path length restriction

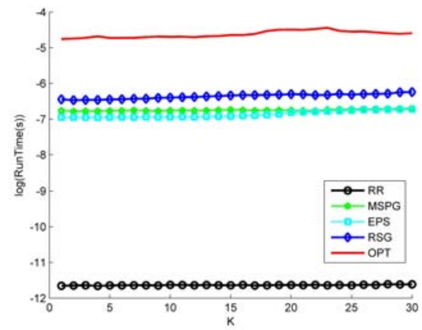


Fig. 4. Run Time Comparison of Different heuristics with respect to Optimal

our approximation algorithms under the wavelength restriction (See [7] for details).

Method	Approximation	Running Time	T
ExS	Exact Solution	$O(W^{W+1}m^{W+1})$	D
Greedy	$O(\log m)$	$O(W^2 m)$	D
ε-net	$O(\log W \log \xi)$	$O(W \log(W) \log(m) \log(\log(W)))$	P

TABLE II
APPROXIMATION BOUNDS UNDER WAVELENGTH RESTRICTED VERSION: EXS-EXHAUSTIVE SEARCH, T-TYPE, D-DETERMINISTIC, P-PROBABILISTIC

IV. SIMULATION RESULTS

We compare the performance of our algorithms using both large-scale random network topologies, as well as the US backbone network topology. In particular, we compare the following algorithms:

- ILP-based optimal algorithm computed by CPLEX; denoted by ILP
- Simple Greedy algorithm from Section III-B1; denoted by MSPG
- Random-Sweep Greedy algorithm from Section III-B3; denoted by RSG
- ε-net algorithm from Section III-B2; denoted by EPS
- Randomized rounding algorithm from Section III-A; denoted by RR

A. Performance in Large-scale Random Topologies with Path Length Restriction

We first consider a random layered network where the logical topology consists of 10 paths between nodes s and t. This layer is mapped onto the physical topology containing 100 fibers, using the mapping structure shown in [16]. In the K restricted version of the problem, each path consists of at most K fibers. For each value of K, we generate 1000 random topologies each with 10 paths routed on the physical topology in a way that each path can select up to K fibers at random, uniformly and independently. We then apply our algorithms to each network in order to find a minimum survivable path set (i.e., to solve the MSP problem). Note that the performances of Randomized Rounding and ε-net algorithms depend on the survivability guarantee of the algorithms, which are 99.9% and 100% respectively for the results shown below.

Fig. 3 compares the average number of survivable paths found by each algorithm. It can be seen that as the value of K increases, the number of paths increases. This is due to the fact that when K is large, logical paths consist of more fibers; therefore, more logical paths are needed since they can share more fibers. Fig. 4 compares the logarithm of the running time of the algorithms. It can be seen that the Randomized Rounding algorithm is the fastest, while the RSG algorithm and the ε-net algorithm have larger running times. Note also that the running times are nearly independent of K.

B. Performance in Large-scale Random Topologies with WDM Restriction

Similar to the previous section, we consider a random layered network with 20 paths between nodes s and t in the logical layer. For each W, we generate 1000 random topologies under the the wavelength restriction where at most W paths can be assigned to each fiber. In order to solve the MSP problem, we have applied our algorithms to each network. The survivability guarantees of the Randomized Rounding and ε-net algorithms are 99.9% and 100% respectively for the results shown below.

Fig. 5 compares the average number of survivable paths found by each algorithm. It can be seen that as the value of W increases, the number of paths increases. This is due to the fact that when W is large, more logical paths can share a fiber, and therefore, more logical paths are needed since a single physical link failure can lead to a large number of logical path failures. Note that the Random-Sweep Greedy (RSG) algorithm is closest to the optimal. Fig. 6 compares

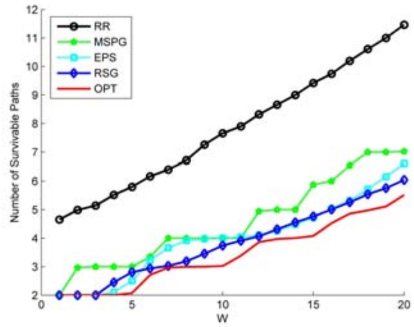


Fig. 5. Comparison of algorithms for MSP under WDM restriction

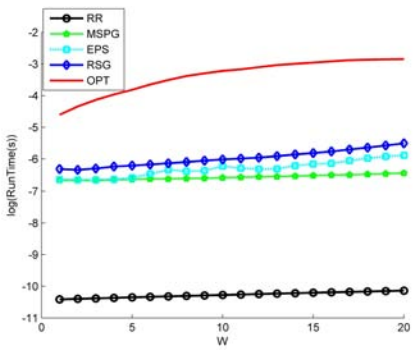


Fig. 6. Run Time Comparison of Different heuristics with respect to Optimal

the running time of each algorithm. Similar to the K-restricted version, we observe that randomized rounding is the fastest.

C. Performance in Real Networks

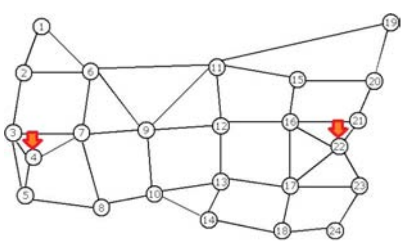


Fig. 7. Physical Topology

Next, we examine the performance of the approximation algorithms over the US backbone topology shown in Fig. 7, with the objective of finding a minimum survivable path set between nodes 4 and 22 [17]. For the logical topology, we generated random graphs with eight nodes (including nodes 4 and 22) each of degree 4. We use shortest path lightpath routing for the logical links.

Table III shows the average number of paths and average running time of each algorithm. It can be seen that the RSG and randomized rounding algorithms are nearly optimal, and furthermore, the randomized rounding gives a solution almost instantly. We also note that the survivability guarantees of the Randomized Rounding and ε-net algorithms are 99.9% and 100% respectively for the results shown in the table.

Method	Number of Paths	Running Time (ms)
ILP	2.0069	7.2133
RSG	2.0160	2.0167
RR	2.0482	0.0272
MSPG	2.2241	0.1911
EPS	2.551	1.6000

TABLE III
COMPARISON OF ALGORITHMS FOR MSP IN REAL NETWORKS

V. CONCLUSION

We considered the problem of finding survivable paths in layered networks. The traditional disjoint paths approach for protection cannot be directly applied to layered networks, since physically disjoint paths may not always exist in such networks. To address this issue, we introduced the new notion of *survivable path set*. We showed that in general the problem of finding the minimum size survivable path set (MSP) is NP-hard and inapproximable. However, under practical constraints, we are able to develop both optimal and approximation algorithms for the MSP problem. An important future direction is to develop backup routing schemes based on survivable path sets.

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