Random Access Wireless Networks with Controlled Mobility

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Abstract—This paper considers wireless networks where messages arriving randomly (in time and space) are collected by a mobile receiver. The messages are transmitted to the mobile receiver according to a random access scheme and the receiver dynamically adjusts its position in order to receive these messages in minimum time. We investigate the use of *wireless transmission* and *controlled mobility* to improve the delay performance in such systems. In particular, we characterize the tradeoff between wireless transmission and physical movement of the mobile receiver. We derive a lower bound for the delay in the system and show how it is affected by different communication parameters. We show that the combination of mobility and wireless transmission results in a significant improvement in delay as compared to a system where wireless transmission is not used.

I. INTRODUCTION

Analyzing the performance of mobile wireless networks has received considerable attention in the last decade. Typically throughput and delay performance in wireless networks utilizing some random mobility model were analyzed (e.g., [3], [12], [14], [29]). In this paper we study the use of *controlled mobility* to improve the delay performance of wireless networks. We consider dynamic wireless networks where messages arriving stochastically in time at random locations in the network are transmitted to a mobile receiver that uses a combination of physical movement and wireless reception to receive these messages.

We consider simple channel access schemes where mobile receiver does not have control over message transmissions. In particular the messages are transmitted according to a random access protocol and depart the system upon successful transmission. The mobile receiver adjusts its position in order to receive these messages in minimum time under the interference of neighboring transmissions. This model can represent a sensor network where a mobile base station collects data from a large number of sensors that are deployed at random locations inside the network.

We study the tradeoff between the use of *controlled mobility* and *wireless transmission* in this setting. Namely, the receiver can choose between receiving a message from shorter distance in order to increase the probability of successful reception or from longer distance in order to decrease travel time at the cost of less reliable transmission. This setup is particularly applicable to networks in which data rate is relatively low so that data transmission time is comparable to the mobile receiver travel time or to networks deployed on a large area so that a mobile element is necessary to provide connectivity.

Characterizing this tradeoff, our objective is to first derive a lower bound for the delay in the system and then to study the effect of *reception distance* on the lower bound.

Random mobility models have been studied in communication framework [3], [12], [13], [14], [25], [28], [29] where fundamental results on throughput and delay of wireless networks were presented. Another related body of literature lies in the area of Delay Tolerant Networks (DTN) [8], [15], [18], [20], [21], [22], [26], [31], [33], [34], [35], [40] where special mobile elements are utilized to collect sensor data. These works focus primarily on mobility and usually analyze particular policies under simplistic communication models. In a related problem [32] derived a lower bound on message travel time in a network where mobile nodes with controlled mobility relay the messages of static nodes.

Other related works are in the area of Vehicle Routing Problems (VRPs) [5], [6], [7], [11], [23], [36]. The most common example of a VRP is the Traveling Salesman Problem (TSP) in which a vehicle is to visit each member of a fixed set of locations such that the total travel cost is minimized. In the TSP with neighborhoods (TSPN) problem [2], [10], [27] (a.k.a., The Lawnmover Problem or The Geometric Covering Salesman Problem) the vehicle is to visit a neighborhood of each demand location. A more detailed review of the literature in this field can be found in [6], [23] and [36]. In this context communication capability does not arise due to the nature of the problem.

Among the VRPs, the Dynamic Traveling Repairman Problem (DTRP) due to Bertsimas and van Ryzin [5], [6] is particularly related to our problem. DTRP is a stochastic and dynamic VRP in which a vehicle is to serve demands that arrive randomly in time and space. Fundamental lower bounds on delay were derived and several vehicle routing policies were analyzed for a single server in [5] and for multiple servers in [6] and for general demand and interarrival time distributions in [7]. We apply this model to wireless networks where the demands are messages to be transmitted to a mobile receiver which is capable of wireless communication. The objective in our system is to find the optimal transmission distance to minimize the expected message delay. To the best of our knowledge, this is the first attempt to develop bounds on delay in a system where a mobile receiver is to collect data messages randomly arriving in time and space using wireless communications and controlled mobility.

In this paper we characterize the tradeoff between physi-

cal movement and wireless transmission in mobile networks where messages arriving randomly in time and space are collected by a mobile receiver. We derive a lower bound on the delay in such a system and show how it is affected by different communication parameters. We show that compared to a system where wireless transmission is not used, significant improvements in delay can be obtained by utilizing a combination of controlled mobility and wireless transmission.

This paper is organized as follows. In Section II we describe the system model. We derive bounds on the travel distance and the reception distance in Section III and analyze the system delay in Section IV. We present numerical results in Section V and conclude in Section VI.

II. Model

Consider a region[1] $\Re$ of area A and messages arriving into $\Re$ according to a Poisson process (in time) of intensity λ. The messages are distributed independently and uniformly in $\Re$ and they are transmitted to a mobile receiver with wireless communication capability. The receiver's job is to collect these messages in the least amount of time by receiving these messages via wireless communication.

We assume a slotted system with slot duration, τ, that is equal to the message transmission time. Messages are transmitted to the receiver with probability p in each time slot independently of all other transmissions and until they are successfully received[2]. We utilize the Protocol Model [12], [13], [16], [28], [32] where a transmission at distance r to the receiver is successful, if there are no simultaneous transmissions within a disk of radius $(1 + \triangle) r$, $(\triangle > 0)$ around the receiver.

The receiver travels inside the region on straight lines from the current message reception point to the next at a constant speed v. The receiver knows message locations [2], [5], [23], [32], [36] as typically assumed in vehicle routing problems. We let r be the *reception distance* of the receiver. If the location of the next message to be received is within r, the receiver stops and attempts to receive the message. Otherwise the receiver travels towards the message location until it is within a distance r away from the message. *Note that r is a design variable which essentially captures the trade off between wireless transmission and controlled mobility.* Namely, the receiver can choose between coming close to the location of a message (small r) in order to increase the probability of successful transmission or staying far away (large r) in order to save on travel time at the cost of less reliable transmission.

We define the stable message arrival rate λ as the throughput in the system. Let s_i be the reception time for message i and let d_i be the receiver travel distance from the receiver's reception location for the message served prior to message i to receiver's

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[1] Our results are valid for general network regions of any shape, however, in order to increase the understanding we will assume a disk-shaped network region.

[2] An instant feedback from the receiver is assumed after a successful transmission at which time the message departs from the network.

reception location for message i. Message i's total service time consists of the reception time s_i and the travel time d_i/v. The average of s_i and d_i, denoted by $\bar{s}$ and $\bar{d}$ respectively, are defined as their expectation in steady state and are given by $\bar{s} = \lim_{i \to \infty} E[s_i]$ and $\bar{d} = \lim_{i \to \infty} E[d_i]$ respectively, where the limits are assumed to exist. The fraction of time the server spends in receiving messages is denoted by ρ, and for stable systems $\rho = \lambda \bar{s}$. We define T_i as the time between the arrival of message i and its successful reception. The waiting time of message i is denoted by W_i, hence $W_i = T_i - s_i$. Average system time of messages in steady state is defined by $\overline{T} = \lim_{i \to \infty} E[T_i]$ and $\overline{W} = \overline{T} - \bar{s}$. $\overline{T}^*$ is defined as the optimal system time which is given by the policy that minimizes $\overline{T}$. We define by $\overline{N}$ the steady state expected number of messages in the queue.

III. The Travel Distance and The Reception Distance

We first derive a lower bound on the average travel distance per message $\bar{d}$ in terms of the average number of messages in the system $\overline{N}$ and the reception distance r. We then obtain an expression for r in terms of $\overline{N}$ and the failure probability μ.

Theorem 1: A lower bound on $\bar{d}$ is given by

$$\bar{d} \geq \frac{2\sqrt{A}}{3\sqrt{2\pi(\overline{N} + 1/2)}} - 2r\big(1 - \frac{8\pi(\overline{N} + 1/2)r^2}{3A}\big). \quad (1)$$

Proof: See Appendix. ∎

The intuition behind the proof is based on a nearest neighbor argument. Namely, the per-message distance the receiver moves for a given message i is at least as big as the distance from the closest of all messages (and the location of the receiver) to the location of message i less $2r$. The $2r$ distance is the *maximum* possible per-message reduction in travel distance due to wireless transmission. Therefore the average reduction is expected to be less than $2r$ as given in (1). Note that $r = 0$ corresponds to a system without wireless transmission (i.e., the receiver visits the message locations). In that case (1) collapses to the lower bound $\bar{d} \geq 2\sqrt{A}/(3\sqrt{2\pi(\overline{N} + 1/2)})$ given in [5], which is a lower bound on the average nearest neighbor distance. The right hand side of (1) takes its minimum value of 0 at $\hat{r} \triangleq \sqrt{A}/(2\sqrt{2\pi(\overline{N} + 1/2)})$, i.e., at $3/4^{\text{th}}$ of the average nearest neighbor distance lower bound. Therefore r is limited to the range $0 \leq r \leq \hat{r}$, or to $O(1/\sqrt{\overline{N}})$. As shown below $r \propto O(1/\sqrt{\overline{N}})$ is indeed necessary in order to have a probability of success that is non-decreasing in $\overline{N}$ under the interference of $\overline{N}$ concurrent transmissions. For $0 < r < \hat{r}$, the second part of the lower bound in (1) , i.e., $2r\big(1 - \frac{4\pi(2N+1)r^2}{3A}\big)$, varies between 0 and $2r$ and corresponds to the *reduction* in the average travel distance due to wireless transmission.

A. The Failure Probability and The Reception Distance

The new arrivals into the network are distributed according the uniform distribution over the network region, however, the distribution of messages waiting for service is a *perturbed* uniform distribution as it may depend on mobile receiver's policy.
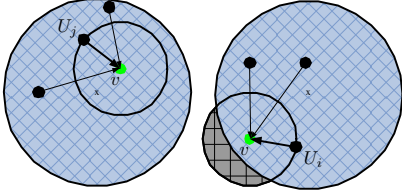
Fig. 1. An example illustrating the reasoning behind (2) and (11). For (2) v denotes the receiver location and for (11) it denotes the location of a tagged message. U_i and U_j are locations of messages i and j respectively.

However it can be well approximated by a uniform distribution for a stable system. For the analysis of the similar system but without wireless transmission capability (i.e., the system where the receiver visits each message location) in [5], the delay lower bound with and without the uniformity assumption is essentially the *same*. Furthermore, as the network load ρ increases, the distribution of message locations approaches the uniform distribution. This is because as the arrival rate increases, the relative impact of the receiver's perturbation on the distribution decreases. Therefore in order to be able to derive analytical results, we assume that the distribution of the message locations are i.i.d. uniform in this section.

Consider an arbitrary message (designated by message 0) at location U_0 and at a distance r from the receiver. Suppose in addition to message 0 there are N_s other messages waiting for service in the network. Let $\tilde{N}_s$ be the number of messages that are being transmitted during message 0's reception, enumerated from 1 to $\tilde{N}_s$. It is easy to see that $\tilde{N}_s$ is binomial with parameters N_s and p. Let $U_j, j \in [1, 2, ..., \tilde{N}_s]$ denote the location of interferer j. Note that throughout the rest of this section the probabilities are conditional on the receiver being at position v and for notational simplicity we will not express this conditioning in the equations. Applying the Protocol Model, message 0 is successfully received if for given N_s and $\tilde{N}_s$

$$|U_i - v| \geq (1 + \triangle)|U_0 - v|, \ \forall \ i \in [1, 2, ..., \tilde{N}_s].$$

We are given that $|U_0 - v| = r$. Hence we have

$$1 - \frac{\pi(1+\triangle)^2 r^2}{A} \leq \Pr\big(|U_i - v| \geq (1+\triangle)r\big) \leq 1 - \frac{\pi(1+\triangle)^2 r^2}{4A}, \forall i. \tag{2}$$

The lower bound above follows since $Pr(|U_i - v| < (1+\triangle)r)$ is upper bounded by the probability that a uniformly distributed random variable lies in a disk of radius $(1 + \triangle)r$ centered at v divided by the total area A. To see this, as illustrated in Fig. 1, if the receiver location v is close to the peripheral of the network area, then $Pr(|U_i - v| < (1+\triangle)r)$ is equal to the *portion* of the area of the disk of radius $(1+\triangle)r$ centered at v that is *inside* the region $\Re$, divided by the total network area A. Similarly, the upper bound is due to the fact that at least a quarter of such a disk centered at v is inside the network region. Note that given that the receiver is at position $V = v$, the random variables $|U_i - v|, \ i \in [1, 2, ..., \tilde{N}_s]$ are i.i.d. with the tail distribution bounded by (2). Moreover the bounds in (2) are valid for all receiver positions. Hence,

we bound the success probability of this transmission from distance r to the receiver as

$$(1 - \frac{\pi(1+\triangle)^2 r^2}{A})^{\tilde{N}_s} \leq \Pr(\text{Success}|N_s, \tilde{N}_s) \leq (1 - \frac{\pi(1+\triangle)^2 r^2}{4A})^{\tilde{N}_s}. \tag{3}$$

These bounds suggest that r should be a decreasing function of $\overline{N}$ in order to have a success probability non-decreasing in $\overline{N}$. This implies that the term $\pi(1 + \triangle)^2 r^2/A$ in (3) is small compared to 1. Therefore we approximate the upper bound in (3) using the first order Taylor series approximation around 0 as given in (4). Relaxing the lower bound in (3) we obtain

$$1 - \frac{\pi(1+\triangle)^2 r^2}{A}\tilde{N}_s \leq \Pr(\text{Success}|N_s, \tilde{N}_s) \leq 1 - \frac{\pi(1+\triangle)^2 r^2}{4A}\tilde{N}_s. \tag{4}$$

Taking the expectation of both sides with respect to $\tilde{N}_s$ for a given value of N_s and noting that $E[\tilde{N}_s] = N_s p$ we obtain

$$1 - \frac{\pi(1+\triangle)^2 r^2}{A}N_s p \leq \Pr(\text{Success}|N_s) \leq 1 - \frac{\pi(1+\triangle)^2 r^2}{4A}N_s p. \tag{5}$$

Now taking the expectation with respect to N_s (note that $E[N_s] = \overline{N}$), and calling

$$\mu \triangleq \frac{(1+\triangle)^2 \pi p(\overline{N} + 1/2)r^2}{A}, \tag{6}$$

we see that the average success probability of a transmission from distance r to the receiver is bounded as[3]

$$1 - \mu \leq \Pr(\text{Success}) \leq 1 - \mu/4. \tag{7}$$

We take the inverse of $\mu(r)$ to find

$$r = (1 + \triangle)^{-1}\sqrt{\frac{\mu A}{\pi p(\overline{N} + 1/2)}}. \tag{8}$$

This implies that with $O(\overline{N})$ concurrent transmissions in the channel, the success probability of a transmission at distance $O(1/\sqrt{\overline{N}})$ from the receiver is independent of $\overline{N}$. A similar phenomenon has been observed by Tse *et al.* [14] and by Hajek *et al.* [17] for different communication channel models. Since the receiver receives each message from distance at most $r(\mu) = (1 + \triangle)^{-1}\sqrt{\mu A/(\pi p(\overline{N} + 1/2))}$, each reception attempt is guaranteed to be successful with probability $1 - \mu$. Recalling that r is allowed to be at most $\hat{r}$, i.e., at most $3/4^{\text{th}}$ of the average nearest neighbor distance, only a small fraction of transmissions occur with reception distances less than r. Therefore we approximate the probability of success for each reception attempt to be $1 - \mu$. Given that r is confined to the interval $0 \leq r \leq \hat{r}$, μ takes values in the interval $0 \leq \mu \leq \hat{\mu}$ where $\hat{\mu} = \min(p(1 + \triangle)^2/8, 1)$.

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[3]For the upper bound we assume $\frac{(\overline{N}+1/2)}{\overline{N}} \cong 1$. Also note that this difference between the upper and lower bounds of Pr(Success) (i.e., the $1/4$ factor) is due to the edge effects in the network.

IV. Lower Bound on Delay

We derive a lower bound on delay by parameterizing the system in terms of the failure probability μ. This ensures analytical tractability and we present the numerical results in Section V in terms of the reception distance r. Substituting (8) into (1) we have $\bar{d} \geq \frac{2\sqrt{A}}{3\sqrt{2\pi(\overline{N}+1/2)}}\kappa(\mu)$ where $\kappa(\mu) \triangleq 1 - 3\frac{\sqrt{2\mu}}{\sqrt{p}}(1+\triangle)^{-1}(1 - \frac{8\mu(1+\triangle)^{-2}}{3p})$. Each message is transmitted with probability p to the receiver in each slot until it is successfully received. The average success probability of each reception attempt is $1 - \mu$. Note that the success probabilities of each of these attempts are approximately independent since each message is transmitted with probability p in each slot independent of other transmitters[4]. In this case the average reception time is a geometric random variable with mean $\bar{s} = \frac{\tau}{(1-\mu)p}$. Note that for $r = 0$ (and therefore $\mu = 0$) the average reception time collapses to τ/p which is the reception time for the corresponding system where the receiver visits each message location. This is because each message is transmitted with probability p in each time slot and each transmission takes time τ.

A necessary condition for stability in the system is given by $\bar{s} + \bar{d}/v \leq 1/\lambda$. To see this, note that $\bar{s} + \bar{d}/v$ is the average time the receiver spends per message. Therefore the average interarrival time $1/\lambda$ has to be greater than this time for the system to be stable. Using the $\bar{s}$ and $\bar{d}$ expressions above we can rewrite the stability condition as

$$\lambda \leq \left(\frac{\tau}{(1-\mu)p} + \frac{2\sqrt{A}}{3v\sqrt{2\pi(\overline{N}+1/2)}}\kappa(\mu)\right)^{-1}. \quad (9)$$

For a stable rate λ, we utilize Little's law to get $\overline{N} = \lambda\overline{W} = \lambda(\overline{T} - \bar{s})$. Substituting this in (9) and rearranging yields

$$\overline{T}^* \geq \kappa(\mu)^2 \frac{2\lambda A}{9\pi v^2(1-\rho)^2} - \frac{1-2\rho}{2\lambda}, \quad (10)$$

where $\rho = \lambda\bar{s} = \lambda\frac{\tau}{(1-\mu)p}$ is the load in the system. Note that $\mu = 0$ ($r = 0$) corresponds to a system without wireless transmission. In this case (10) collapses to the delay lower bound given in [5]. The $\kappa(\mu)$ term in (10) together with $\rho = \lambda\frac{\tau}{(1-\mu)p}$ represent the effect of communication capability on delay. Note that $\kappa(\mu)$ takes values in the interval $[0, 1]$ as μ varies in the interval $[0, \hat{\mu}]$ and hence we obtain smaller message delays by saving on the receiver travel time. Further observing (10), we notice that the communication model parameter $\triangle$ affects the optimal delay only through the term $\kappa(\mu)$.

V. Results

We investigate the delay bound in (10) for different values of the transmission distance r and the arrival rate λ in this Section. Fig. 2 shows the minimum delay versus throughput for a given reception distance r. The minimum delay in the system is considerably less than the minimum delay in

[4]They are approximately independent since there is small dependence from slot to slot due to the locations of transmitters.
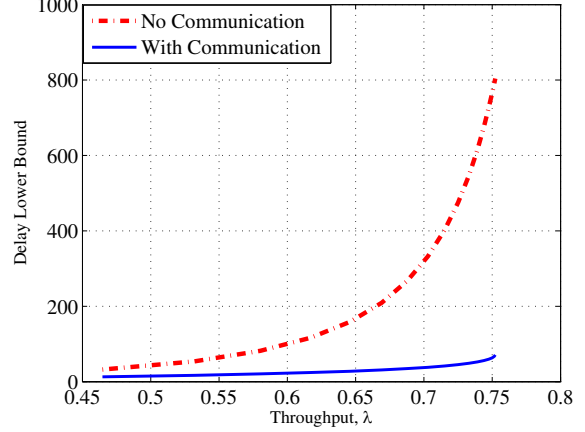

Fig. 2. Delay lower bound vs. throughput for reception distance $r = 0.3$, the network area $A = 200$, the transmission probability $p = 0.85$, the capture parameter $\triangle = 0.2$, the receiver velocity $v = 1$, and the slot time $\tau = 1$.

the corresponding system without wireless transmission. The difference is more significant for high arrival rates. This is due to the fact that with an increased message density in the system, small savings on delay at each message reception add up to make a significant impact on the total delay. To see this, note that the average number of messages served during the waiting time of a message is $\overline{N}$. Heuristically, if we assume that the savings on delay due to wireless transmission is proportional to the reception distance for each transmission, then the total waiting time of a message is decreased by $O(\sqrt{\overline{N}})$.

Fig. 3 presents the improvement in the minimum average delay as we increase the reception distance for a constant throughput in the system. Initially increasing the reception distance yields lower delay since the mobile receiver saves on travel time. However, as the reception distance increases beyond some optimal point (0.39 in this case), the failure probability dominates and therefore the delay starts to increase. We observe a similar phenomenon in Fig. 4 for the same system parameters. As the failure probability μ increases with r, the average number of messages in the system first decreases since the receiver saves on travel time. After μ reaches its optimal value, the average number of messages in the system increases. Hence we expect an optimal reception distance yielding the minimum delay (or the minimum average number of messages) in the system, and this is confirmed in figures 3 and 4.

VI. Conclusion

In this paper we considered the use of controlled mobility in order to improve the delay performance of wireless networks where messages arriving randomly in time and space are collected by a mobile receiver via wireless reception. We characterized the tradeoff between mobility and wireless transmission and derived bounds for the travel time and the total system time. We analyzed the effects of different
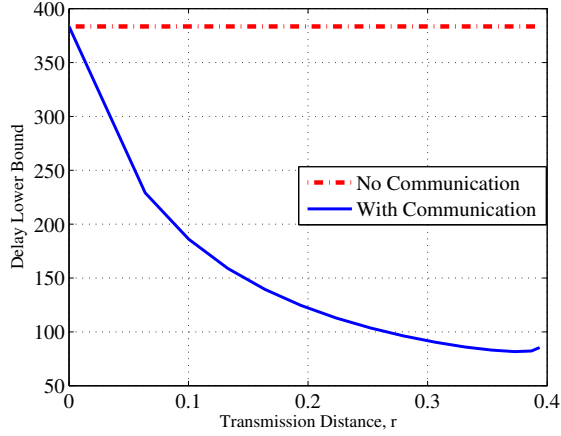
Fig. 3. Delay lower bound vs. transmission distance for arrival rate $\lambda = 0.27$, the network area $A = 200$, the transmission probability $p = 0.9$, the capture parameter $\triangle = 0.2$, the receiver velocity $v = 1$, and the slot time $\tau = 3$.
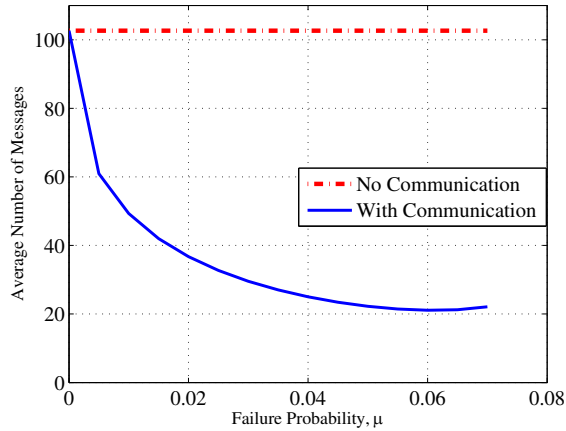

Fig. 4. Average number of messages in the system vs. the failure probability for arrival rate $\lambda = 0.27$, the network area $A = 200$, the transmission probability $p = 0.9$, the capture parameter $\triangle = 0.2$, the receiver velocity $v = 1$, and the slot time $\tau = 3$.

communication parameters on these metrics. Our results show that combined mobility and wireless transmission can improve the delay performance of wireless networks significantly as compared to a system where wireless transmission is not utilized.

This work is our first attempt at utilizing a combination of controlled mobility and wireless transmission for data collection in wireless networks. Therefore there are many related open problems. In the future we intend to optimize the delay bound derived in this paper over the transmission probability p for given values of the arrival rate and the reception distance. We intend to develop policies for the mobile receiver and compare their performance to the delay bound. We also plan to extend the results in this paper to the multiple mobile receivers case. Finally, we intend to study more advanced wireless communication models such as modeling the transmission rate

as a function of the transmission distance and enabling the mobile receiver to receive messages while moving.

APPENDIX-PROOF OF THEOREM 1

The proof is based on a nearest neighbor argument. The methodology of the proof is similar to the proof of the $\bar{d}$-lower bound in [5] but with simpler techniques and with the added complexity of communication capability in the system. We consider an arbitrary message (henceforth denoted as the tagged message) and define:

$\Omega_p \equiv$ The set of locations of the messages that are in queue at the time of the tagged message's arrival plus the location of the receiver;

$\Omega_f \equiv$ The set of locations of the messages that arrive during the tagged message's waiting time ordered by their time of arrival;

$Y_0 \equiv$ The tagged message's location;

$N_p \equiv |\Omega_p|$ and $N_f \equiv |\Omega_f|$.

We next define $Z_i \equiv \parallel Y_i - Y_0 \parallel$ where Y_i is the location of the ith message that will arrive after the tagged message, i.e., $\Omega_f = \{Y_1, Y_2, ..., Y_{N_f}\}$. Note that for a given tagged message location $Y_0 = y_0$, $\{Z_i; i \geq 1\}$ are i.i.d. with

$$Pr(Z_i \leq z) \leq \frac{\pi z^2}{A}. \qquad (11)$$

To see this, given y_0, $Pr(Z_i \leq z)$ is upper bounded by the probability that a uniformly distributed random variable lies in a disk of radius z centered at y_0. This is illustrated in Fig. 1, namely, if y_0 is close to the peripheral of the network area, then $Pr(Z_i \leq z)$ is equal to the *portion* of the area of the disk of radius z centered at v that is *inside* the region $\Re$, divided by the total area A.

We define the minimum of the sequence of random variables $\{Z_i; 1 \leq i \leq N_f\}$ as $Z_f \equiv \min_{y \in \Omega_f} \parallel y - Y_0 \parallel$. Note that N_f is a random variable with an unknown distribution and expectation $\overline{N}$. Similarly we define $Z_p \equiv \min_{y \in \Omega_p} \parallel y - Y_0 \parallel$. The reason we treat the message locations in the set Ω_p differently is because the distribution of the locations in Ω_p which depends on the receiver policy may not be uniform. Note that in a system *without* communication capability (where the receiver has to visit the message locations) $\bar{d}$ is lower bounded by $E[\min\{Z_f, Z_p\}]$. This is because the tagged message has to be served after the receiver is at one of the locations in the set $\Omega_p \bigcup \Omega_f$. When there is communication capability in the system, we define the random variable Z_f^c corresponding to Z_f (Z_p^c is defined similarly) as

$$Z_f^c = \begin{cases} Z_f - 2r & \text{if } Z_f \geq 2r \\ 0 & \text{otherwise.} \end{cases} \qquad (12)$$

The receiver message reception policy in Section II implies that the receiver attempts to receive the next message when its on the circle of radius r around the next message location (unless the receiver is already inside that circle when it finishes

serving the previous message). Therefore compared to a system without wireless transmission capability, the distance the receiver travels between two message locations is decreased the most when the receiver is on the line connecting the previous and the next message locations. This maximum difference is $2r$, i.e., two times the reception distance. Hence in our system a lower bound on $\bar{d}$ is given as $\bar{d} \geq E[\min\{Z_f^c, Z_p^c\}]$. In order to bound this expectation, we have

$$Pr(\min\{Z_f^c, Z_p^c\} > z) = 1 - Pr(\min\{Z_f^c, Z_p^c\} \leq z)$$
$$\geq 1 - Pr(Z_f^c \leq z) - Pr(Z_p^c \leq z). \quad (13)$$

Note that given y_0 the random variables Z_i are independent conditioned on N_f. Therefore using (11) we obtain

$$Pr(Z_f > z|N_f) = Pr(Z_1 > z, Z_2 > z, ..., Z_{N_f} > z|N_f)$$
$$\geq \left(1 - \frac{\pi z^2}{A}\right)^{N_f} \geq 1 - \frac{\pi z^2}{A} N_f. \quad (14)$$

The last inequality is due to the fact that $g(\epsilon) = (1 - \epsilon)^{N_f} - 1 + \epsilon N_f \geq 0$ where $0 < \epsilon = \pi z^2/A < 1$. To see this, $g'(\epsilon) = N_f(1 - (1 - \epsilon)^{N_f - 1}) > 0$ for $\epsilon \in (0,1)$. Therefore g is an increasing function of ϵ. Finally noting that $g(\epsilon = 0) = 0$ proves that $g(\epsilon) \geq 0$. Since the above bound is true for all $Y_0 = y_0$, and since Y_0 is independent of future arrivals, it holds without the conditioning on Y_0. Taking its expectation with respect to N_f we obtain

$$Pr(Z_f > z) \geq 1 - \frac{\pi z^2}{A} \overline{N}. \quad (15)$$

In order to bound $Pr(Z_p > z)$, we have the following lemma given in [6].

Lemma 1:

$$Pr(Z_p \leq z) \leq \frac{\pi z^2}{A}(\overline{N} + 1). \quad (16)$$

Proof: Let Ω be any set of k points in a region $\Re$ with area A. Let Y be a uniformly distributed location in $\Re$ independent of Ω and define $Z^* \triangleq \min_{y \in \Omega} \| Y - y \|$. For each point in Ω, we construct a circle of radius z centered at the point. Let A_Ω denote the total area in $\Re$ covered by the union of these circles. Then the event $\{Z^* \leq z\}$ happens if and only if Y lies somewhere inside the region A_Ω. Hence

$$Pr(Z^* \leq z) = \frac{A_\Omega}{A} \leq \frac{\pi z^2}{A} k.$$

Now we apply this result to Z_p by conditioning $Pr(Z_p \leq z)$ on N_p, i.e., the size of Ω_p. Note that Y_0 is independent of Ω_p under conditioning on N_p. Therefore we obtain $Pr(Z_p \leq z|N_p) \leq \frac{\pi z^2}{A} N_p$. Averaging over N_p and observing that $E[N_p] = (\overline{N} + 1)$ we obtain (16). ∎

It is easy to show for $z > 0$ that

$$Pr(Z_f^c > z) = Pr(Z_f^c > z|Z_f \geq 2r)Pr(Z_f \geq 2r)$$
$$+ Pr(Z_f^c > z|Z_f < 2r)Pr(Z_f < 2r)$$
$$= Pr(Z_f > z + 2r|Z_f \geq 2r)Pr(Z_f \geq 2r)$$
$$= Pr(Z_f > z + 2r).$$

A similar expression is valid for Z_p^c. Therefore from (15) and (16) we have

$$Pr(Z_f^c > z) \geq 1 - \frac{\pi(z + 2r)^2}{A} \overline{N},$$

and

$$Pr(Z_p^c > z) \geq 1 - \frac{\pi(z + 2r)^2}{A}(\overline{N} + 1).$$

Then (14) yields

$$Pr(\min\{Z_f^c, Z_p^c\} > z) \geq 1 - (2\overline{N} + 1)\frac{\pi(z + 2r)^2}{A}. \quad (17)$$

Now $Pr(.) \geq 0$ gives $z \leq \frac{\sqrt{A}}{\sqrt{\pi(2\overline{N}+1)}} - 2r \triangleq \hat{z}$ and since Z_f^c and Z_p^c are nonnegative random variables we have[5] $z \geq 0$. Hence

$$E[\min\{Z_f^c, Z_p^c\}] \geq \int_{0^+}^{\hat{z}} Pr(\min\{Z_f^c, Z_p^c\} > z)\, dz$$
$$\geq \int_{0^+}^{\hat{z}} 1 - \frac{\pi(z + 2r)^2}{A}(2\overline{N} + 1)dz,$$

which yields (1) after a few manipulations.

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[5]These two bounds on z imply that $r \leq \frac{\sqrt{A}}{2\sqrt{\pi(2\overline{N}+1)}}$. This is critical since it gives a natural bound on the maximum reception distance on r for which the analysis is valid. We will discuss this further in Section IV.

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