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# Cross-Layer Survivability in WDM-Based Networks

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***Abstract*—In layered networks, a single failure at a lower layer may cause multiple failures in the upper layers. As a result, traditional schemes that protect against single failures may not be effective in cross-layer networks. In this paper, we introduce the problem of maximizing the connectivity of layered networks. We show that connectivity metrics in layered networks have significantly different meaning than their single-layer counterparts. Results that are fundamental to survivable single-layer network design, such as the Max-Flow Min-Cut theorem, are no longer applicable to the layered setting. We propose new metrics to measure connectivity in layered networks and analyze their properties. We use one of the metrics, Min Cross Layer Cut, as the objective for the survivable lightpath routing problem, and develop several algorithms to produce lightpath routings with high survivability. This allows the resulting cross-layer architecture to be resilient to failures.**

## I. Introduction

Modern communication networks are constructed using a layered approach, as shown in Figure 1. Such a network typically consists of an electronic packet switched network (such as IP); often this packet-switched network is built on top of one or more electronic circuit switched transport networks (e.g., ATM, SONET; sometimes neither or both); and these in turn are built upon a fiber network. This multitude of layers is used in order to simplify network design and operations. However, this layering also leads to certain inefficiencies and interoperability issues. In this paper, we focus on the impact of layering on network survivability.

We examine this problem in the context of Wavelength Division Multiplexing (WDM) based networks; although the concepts discussed are equally applicable to other layered architectures (e.g., IP over ATM, ATM over SONET, etc.). In a WDM-based network the logical topology is defined by a set of nodes and lightpaths connecting the nodes, while the physical topology is defined by a (possibly different) set of nodes and the fibers connecting them. For example, an IP-over-WDM network consists of IP routers that are connected using optical (WDM) lightpaths as shown in Figure 1. These lightpaths are routed over the physical fiber topology. Networks often rely on the logical layer for providing protection and restoration services. However, even when the logical topology is designed to tolerate single logical link failures, once the logical topology is embedded on the physical topology, the logical topology may no longer be survivable to single physical

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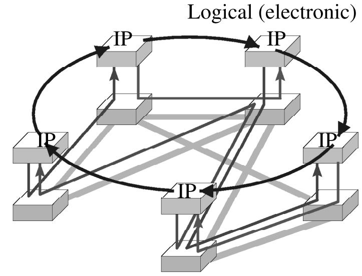


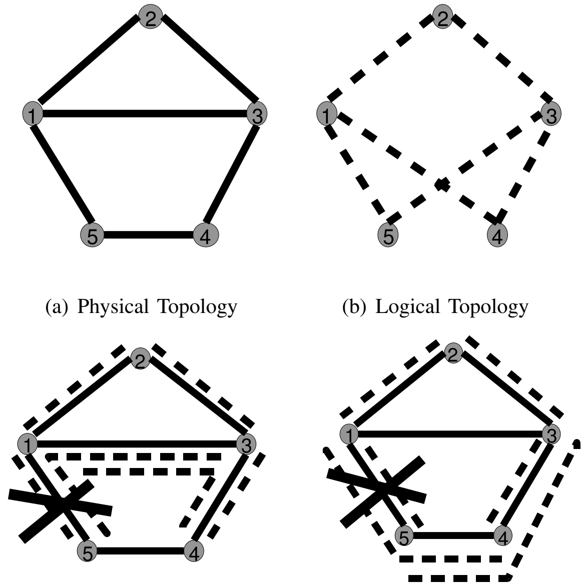

Fig. 1. An IP-over-WDM network where the IP routers are connected using optical lightpaths. The logical links (arrowed lines on top) are formed using lightpaths (arrowed lines at the bottom) that are routed on the physical fiber (thick gray lines at the bottom). In general, the logical and physical topologies are not the same.

(fiber) link failures. This is because each physical fiber link may carry multiple lightpaths. Hence, the failure of a single fiber link can lead to the failure of multiple links in the logical topology, which may subsequently leave the logical topology disconnected.

As a simple illustrative example, consider the physical and logical topologies shown in Figures 2(a) and (b). The lightpaths in the logical topology are routed over the physical topology in two different ways in Figures 2(c) and (d). In Figure 2(c), a failure of physical fiber $(1,5)$ would cause lightpaths $(1,5)$ and $(3,5)$ to fail. Consequently, node 5 will be disconnected from other nodes in the logical topology. On the other hand, in Figure 2(d), the logical topology will remain connected even if one of the fibers fails. The above example demonstrates that in a multi-layer network, a physical link failure can result in multiple logical link failures, and that the routing of the logical links on the physical topology has a big impact on the connectivity of the multi-layer network.

In contrast to the simplified example of Figure 2, real-life networks are highly intertwined and layered. However, due to the lack of general understanding of the issues in cross-layer survivability, most existing protection and restoration mechanisms are based on principles that are applicable only to single-layer network environments, and are subject to cross-layer issues as illustrated above.

Nearly all previous literature in cross-layer survivability focused on single fiber failure [2], [6], [7], [12], [13], [17], [20], [21], [22]. The notion of survivable lightpath routing was first introduced in [17]. The same paper also developed an ILP formulation for survivable routing of arbitrary logical



(a) Physical Topology (b) Logical Topology

(c) Unsurvivable Routing (d) Survivable Routing

Fig. 2. Different lightpath routings can affect survivability

topologies, which was subsequently improved in [7] and [22]. The problem of routing logical rings survivably on the physical network was studied in [17], [20], [21]. In [12] the authors introduced the notion of piecewise survivable mapping and developed an algorithm to compute survivable routings based on piecewise survivable components. The same technique was extended to compute lightpath routings that are survivable against $k$ failures, for a fixed value of $k$ [11].

To the best of our knowledge, this is the first paper that formally studies classical survivability theory in the context of layered networks. We show that standard survivability metrics, such as the minimum cut and maximum disjoint paths, that have been widely used in characterizing the survivability properties of single-layer networks, lose much of their meaning in the context of cross-layer architecture. In particular, the Max-Flow Min-Cut Theorem, which constitutes the foundations of network survivability theory and provides the mathematcial justification of the aforementioned metrics, no longer holds in the cross-layer context. Such a fundamental difference suggests that many basic issues of cross-layer survivability are largely not understood.

In Section II, we investigate some of these cross-layer survivability issues in greater depth and highlight the major challenges. In Section III, we specify the requirements for cross-layer survivability metrics, and propose two new metrics, Min Cross Layer Cut and Min Weighted Load Factor, that measure the connectivity of multi-layer networks. In Section IV, we consider the survivable lightpath routing problem using the Min Cross Layer Cut as the objective, and develop several lightpath routing algorithms based on the multi-commodity flow formulation in order to maximize the cross-layer connectivity of the network. In Section V, we present the simulation results for these algorithms, along with some empirical studies of the metrics introduced in Section III. Part of the results in this paper on survivability metrics and lightpath routing algorithms were presented in [15].

## II. New Issues in Cross-Layer Survivability

In order to understand the cross-layer survivability issues, in this section we will focus on some results of fundamental importance to survivability of single-layer networks, and revisit them in the context of multi-layer networks. We will show that fundamental survivability results, such as the Max Flow Min Cut Theorem, are no longer applicable to multi-layer networks. Consequently, metrics such as "connectivity" have significantly different meanings in the cross-layer setting. Such fundamental differences make it much more challenging to design survivable multi-layer networks.

### A. Max Flow vs Min Cut

For single-layer networks, the Max-Flow Min-Cut Theorem [1] states that the maximum number of disjoint paths between two nodes $s$ and $t$ is always the same as the minimum number of edges that need to be removed from the network in order to disconnect the two nodes. Consequently, the term *connectivity* between two nodes can be used unambiguously to refer to different measures such as maximum disjoint paths or minimum cut, and this makes it a natural choice as the standard metric for measuring network survivability.

Because of its fundamental importance, we would like to investigate the Max-Flow Min-Cut relationship for multi-layer networks. We first generalize the definitions of *Max Flow* and *Min Cut* for layered networks:

*Definition 1:* In a multi-layer network, the *Max Flow* between two nodes $s$ and $t$ in the logical topology is the maximum number of $s-t$ paths in the logical topology that do not share any common physical link. The *Min Cut* between two nodes $s$ and $t$ in the logical topology is the minimum number of physical links that need to be removed in order to disconnect the two nodes in the logical topology.

We model the physical topology as $G_P = (V_P, E_P)$, where $V_P$ and $E_P$ are the nodes and links in the physical topology. The logical topology is modelled as $G_L = (V_L, E_L)$ in a similar fashion. The lightpath routing is represented by a set of binary variables $f_{ij}^{st}$, where a logical link $(s,t)$ uses physical fiber $(i,j)$ if and only if $f_{ij}^{st}$ = 1. Let $\mathcal{P}_{st}$ be the set of all $s-t$ paths in the logical topology. For each path $p \in \mathcal{P}_{st}$, let $L(p)$ be the set of physical links used by the logical path $p$, that is, $L(p) = \cup_{(s,t)\in p}\left\{(i,j)|f_{ij}^{st}=1\right\}$. Then the Max Flow and Min Cut between nodes $s$ and $t$ can be formulated mathematically as follows:

$$\mathsf{MaxFlow_{st}}: \quad \text{Maximize} \sum_{p\in\mathcal{P}_{st}} f_p, \quad \text{subject to:}$$

$$\sum_{p:(i,j)\in L(p)} f_p \leq 1 \qquad \forall (i,j) \in E_P \qquad (1)$$

$$f_p \in \{0,1\} \qquad \forall p \in \mathcal{P}_{st}$$

$$\mathsf{MinCut}_{\mathsf{st}}: \quad \text{Minimize} \sum_{(i,j)\in E_P} y_{ij}, \quad \text{subject to:}$$

$$\sum_{(i,j)\in L(p)} y_{ij} \geq 1 \qquad\qquad \forall p \in \mathcal{P}_{st} \qquad (2)$$

$$y_{ij} \in \{0,1\} \qquad\qquad \forall (i,j) \in E_P$$

The variable $f_p$ in the formulation $\mathsf{MaxFlow}_{\mathsf{st}}$ indicates whether the path $p$ is selected for the set of $(s,t)$-disjoint paths. The constraint (1) requires that no selected logical paths share a physical link. Similarly, in the formulation $\mathsf{MinCut}_{\mathsf{st}}$, the variable $y_{ij}$ indicates whether the physical fiber $(i,j)$ is selected for the minimum $(s,t)$-cut. The constraint (2) requires that all logical paths between $s$ and $t$ traverse some physical fiber $(i,j)$ with $y_{ij}=1$.

Note that the above formulations generalize the the Max Flow and Min Cut for single-layer networks. In particular, the formulations model the classical Max Flow and Min Cut of a graph $G$ if both $G_P$ and $G_L$ are equal to $G$, and $f^{st}_{ij}=1$ if and only if $(s,t)=(i,j)$.

Let $\mathsf{MaxFlow}_{\mathsf{st}}$ and $\mathsf{MinCut}_{\mathsf{st}}$ be the optimal values of the above Max Flow and Min Cut formulations. We also denote $\mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$ and $\mathsf{MinCut}^{\mathsf{R}}_{\mathsf{st}}$ to be the optimal values to the linear relaxations of above Max Flow and Min Cut formulations. The Max-Flow Min-Cut Theorem for single-layer networks can then be written as follows:

$$\mathsf{MaxFlow}_{\mathsf{st}} = \mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}} = \mathsf{MinCut}^{\mathsf{R}}_{\mathsf{st}} = \mathsf{MinCut}_{\mathsf{st}}.$$

The equality among these values has profound implications on survivable network design for single-layer networks. Because all these survivability measures converge to the same value, it can naturally be used as the standard survivability metric that is applicable to measuring both disjoint paths or minimum cut. Another consequence of this equality is that linear programs (which are polynomial time solvable) can be used to find the minimum cut and disjoint paths in the network.

It is therefore interesting to see whether the same relationship holds for multi-layer networks. First, it is easy to verify that the linear relaxations for the formulations $\mathsf{MaxFlow}_{\mathsf{st}}$ and $\mathsf{MinCut}_{\mathsf{st}}$ maintain a primal-dual relationship, which, by Duality Theorem [5], implies that $\mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$=$\mathsf{MinCut}^{\mathsf{R}}_{\mathsf{st}}$. In addition, since any feasible solution to an integer program is also a feasible solution to the linear relaxation, we can establish the following relationship:

*Observation 1:* $\mathsf{MaxFlow}_{\mathsf{st}} \leq \mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}} = \mathsf{MinCut}^{\mathsf{R}}_{\mathsf{st}} \leq \mathsf{MinCut}_{\mathsf{st}}$.

Therefore, like single-layer networks, the maximum number of disjoint paths between two nodes cannot exceed the minimum cut between them in a multi-layer network.

However, unlike the single-layer case, the values of $\mathsf{MaxFlow}_{\mathsf{st}}, \mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$ and $\mathsf{MinCut}_{\mathsf{st}}$ are not always identical, as illustrated in the following example. In our examples throughout the section, we use a logical topology with two nodes $s$ and $t$ that are connected by multiple lightpaths. For simplicity of exposition, we omit the complete lightpath routing and only show the physical links that are shared by multiple lightpaths. Theorem 1 states that this simplification can be made without loss of generality.

*Theorem 1:* Let $G_L$ be a logical topology with two nodes $s$ and $t$, connected by $n$ lightpaths $E_L = \{e_1, e_2, \ldots, e_n\}$, and let $\mathcal{R} = \{R_1, R_2, \ldots, R_k\}$ be a family of subsets of $E_L$, where each $|R_i| \geq 2$, that captures the fiber-sharing relationship of the logical links. There exist a physical topology $G_P = (V_P, E_P)$ and lightpath routing of $G_L$ over $G_P$, such that:

1) there are exactly $k$ fibers in $E_P$, denoted by $F = \{f_1, f_2, \ldots, f_k\}$, that are used by multiple lightpaths;
2) for each fiber $f_i \in F$, the set of lightpaths using the fiber $f_i$ is $R_i$.

*Proof:* Omitted for brevity. See [14]. ∎

Theorem 1 implies that for a two-node logical topology, any arbitrary fiber-sharing relationship $\mathcal{R}$ can be realized by reconstructing a physical topology and lightpath routing based on $\mathcal{R}$. Therefore, in the following discussion, we can simplify our examples by only giving the fiber-sharing relationship of our two-node logical topology without showing the details of the lightpath routing.

In Figure 3, the two nodes in the logical topology are connected by three lightpaths. The logical topology is embedded on the physical topology in such a way that each pair of lightpaths share a fiber. It is easy to see that no single fiber can disconnect the logical topology, and that any pair of fibers would. Hence, the value of $\mathsf{MinCut}_{\mathsf{st}}$ is 2 in this case. On the other hand, the value of $\mathsf{MaxFlow}_{\mathsf{st}}$ is only 1, because any two logical links share some physical fiber, so none of the paths in the logical network are physically disjoint. Finally, the value of $\mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$ is 1.5 because a flow of 0.5 can be routed on each of the lightpaths without violating the capacity constraints at the physical layer. Therefore, all three quantities are different in this example.

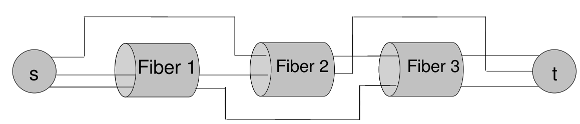



Fig. 3. A logical topology with 3 links where each pair of links shares a fiber in the physical topology

In fact, the above example can be generalized to make the integrality gap between $\mathsf{MaxFlow}_{\mathsf{st}}$ and $\mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$ (and consequently the gap between $\mathsf{MaxFlow}_{\mathsf{st}}$ and $\mathsf{MinCut}_{\mathsf{st}}$) arbitrarily large. We can construct an instance of lightpath routing in a way that the two nodes in the logical network are connected by $n$ lightpaths, and every pair of lightpaths share a separate fiber. In this case, the value of $\mathsf{MaxFlow}_{\mathsf{st}}$ will be 1, and the value of $\mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$ will be $\frac{n}{2}$, using the same arguments as above. We summarize our observation as follows:

*Observation 2:* In a layered network, the values of $\mathsf{MaxFlow}_{\mathsf{st}}, \mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$ and $\mathsf{MinCut}_{\mathsf{st}}$ can be all different.

In addition, the integrality gap between $\mathsf{MaxFlow}_{\mathsf{st}}$ and $\mathsf{MaxFlow}^{\mathsf{R}}_{\mathsf{st}}$ can be arbitrarily large.

Therefore, a multi-layer network with high connectivity value (i.e. that tolerates a large number of failures) does not guarantee existence of physically disjoint paths. This is in sharp contrast to single-layer networks where the number of disjoint paths is always equal to the minimum cut.

It is thus clear that network survivability metrics across layers are not trivial extensions of the single layer metrics. New metrics need to be carefully defined in order to measure cross-layer survivability in a meaningful manner. In Section III, we will specify the requirements for cross-layer survivability metrics, and propose two new metrics that can be used to measure the connectivity of multi-layer networks.

### B. Computational Complexity

For single-layer networks, because the integral Max Flow and Min Cut values are always identical to the optimal relaxed solutions, these values can be computed in polynomial time [1]. However, computing and approximating their cross-layer equivalents turns out to be much more difficult. Theorem 2 describes the complexity of computing the Max Flow and Min Cut for multi-layer networks.

*Theorem 2:* Computing Max Flow and Min Cut for multi-layer networks is NP-hard. In addition, both values cannot be approximated within any constant factor, unless P=NP.

*Proof:* The Max Flow can be reduced from the NP-hard Maximum Set Packing problem [8]:

**Maximum Set Packing:** Given a set of elements $E = \{e_1, e_2, \ldots, e_n\}$ and a family $\mathcal{F} = \{C_1, C_2, \ldots, C_m\}$ of subsets of $E$, find the maximum value $k$ such that there exist $k$ subsets $\{C_{j_1}, C_{j_2}, \ldots, C_{j_k}\} \subseteq \mathcal{F}$ that are mutually disjoint.

Given an instance of Maximum Set Packing that contains a ground set $E$ and a family $\mathcal{F}$ of subsets of $E$, we construct a logical topology with two nodes, $s$ and $t$, connected by $|\mathcal{F}|$ logical links, where each logical link corresponds to a subset in $\mathcal{F}$. The logical links are embedded on the physical network in a way that two logical links share a physical fiber if and only if their corresponding subsets share a common element in the Maximum Set Packing instance. It immediately follows that a set of $s-t$ paths in the logical topology that are physically disjoint corresponds to a family of mutually disjoint subsets of $E$.

Similarly, the Min Cut can be reduced from the NP-hard Minimum Set Cover problem [8]:

**Minimum Set Cover:** Given a set of elements $E = \{e_1, e_2, \ldots, e_n\}$ and a family $\mathcal{F} = \{C_1, C_2, \ldots, C_m\}$ of subsets of $E$, find the minimum value $k$ such that there exist $k$ subsets $\{C_{j_1}, C_{j_2}, \ldots, C_{j_k}\} \subseteq \mathcal{F}$ that cover $E$, i.e., $\bigcup_{l \in \{1 \ldots k\}} C_{j_l} = E$.

Given an instance of Minimum Set Cover with ground set $E$ and family of subsets $\mathcal{F}$, we construct a logical topology that contains two nodes connected by a set of $|E|$ logical links, where each logical link $l_i$ corresponds to the element $e_i$. The logical links are embedded on the physical network in a way that exactly $|\mathcal{F}|$ fibers, namely $\{f_1, \ldots, f_{|\mathcal{F}|}\}$, are used by multiple logical links, and the logical link $l_i$ uses physical fiber $f_j$ if and only if $e_i \in C_j$. It follows that the minimum number of physical fibers that forms a cut between the two nodes in the logical topology is equal to the size of a minimum set cover.

The inapproximability result follows immediately from the inapproximabilities of the Maximum Set Packing and Minimum Set Cover problems [16], [3], [9]. ■

In summary, the notion of survivability in multi-layer networks bears subtle yet important differences from its single-layer counterpart. Because of that, many new issues arise from defining, measuring as well as optimizing survivability of a multi-layer network. In the rest of the paper, we will focus on designing appropriate metrics for layered networks, and developing efficient algorithms to maximize the cross-layer survivability.

## III. Metrics for Cross-Layer Survivability

### A. Requirements for Cross-Layer Survivability Metrics

The previous section demonstrates some fundamental challenges in designing survivable layered network architectures. In particular, choosing the right metric to quantify survivability becomes an important and non-trivial question. Although the right metric will depend on the particular survivability requirement (e.g., disjoint paths or minimum cut), we believe that any reasonable metric must have at least the following three properties:

- **Consistency:** A network with a higher metric value should be more resilient to failures.
- **Monotonicity:** Any addition of physical or logical links to the network should not decrease the metric value, as long as the embedding of existing logical links is unchanged.
- **Compatibility:** The metric should generalize the connectivity metric for single-layer networks. In particular, when applied to the degenerated case where the physical and logical topologies are identical, the metric should be equivalent to the connectivity (in the single-layer sense) of the topology.

Next, we introduce two metrics that measure the capability of a layered network, as a whole, to withstand multiple physical failures. It can be shown that both metrics have the above properties. Although the two metrics appear to measure different aspects of network connectivity, we will show that they are in fact closely related.

### B. Min Cross Layer Cut

The *Min Cross Layer Cut (MCLC)* is a natural generalization of *global min-cut* in single-layer networks. Similar to the way $\mathsf{MinCut}_{\mathsf{st}}$ is defined in Section II between two given nodes $s$ and $t$ in the network, the Min Cross Layer Cut of a layered network is defined to be the smallest set of physical links whose removal will *globally* disconnect the logical network. A lightpath routing with high $MCLC$ value implies that the network remains connected even after a relatively large number of physical failures. It also generalizes the survivable lightpath routing definition in [17], since a lightpath routing is survivable if and only if its $MCLC$ is greater than 1.

Using a similar proof to the one for $\mathsf{MinCut_{st}}$ in Theorem 2, we can prove that computing the $MCLC$ for a multi-layer network is NP-hard. However, for practical purposes, we can compute the $MCLC$ of a large multi-layer network (e.g. 100 nodes) reasonably fast by solving an integer linear program.

Given the physical and logical topologies $(V_P, E_P)$, and $(V_L, E_L)$, let $f_{ij}^{st}$ be binary constants that represent the lightpath routing, such that logical link $(s,t)$ uses physical fiber $(i,j)$ if and only if $f_{ij}^{st}$ = 1. The $MCLC$ can be formulated as the integer program below:

$$\mathsf{M_{MCLC}} : \text{Minimize} \sum_{(i,j)\in E_P} y_{ij}, \quad \text{subject to:}$$

$$d_t - d_s \leq \sum_{(i,j)\in E_P} y_{ij} f_{ij}^{st} \qquad \forall (s,t) \in E_L \tag{3}$$

$$\sum_{n\in V_L} d_n \geq 1 \tag{4}$$

$$d_0 = 0$$

$$d_n, y_{ij} \in \{0,1\} \qquad \forall n \in V_L, (i,j) \in E_P$$

The integer program contains a variable $y_{ij}$ for each physical link $(i,j)$, and a variable $d_k$ for each logical node $k$. Constraint (3) maintains the following property for any feasible solution: if $d_k = 1$, the node $k$ will be disconnected from node 0 after all physical links $(i,j)$ with $y_{ij} = 1$ are removed. To see this, note that since $d_k = 1$ and $d_0 = 0$, any logical path from node 0 to node $k$ contains a logical link $(s,t)$ where $d_s = 0$ and $d_t = 1$. Constraint (3) requires that such a logical link traverse at least one of the fibers $(i,j)$ with $y_{ij} = 1$. As a result, all paths from node 0 to node $k$ must traverse one of these fibers, and node $k$ will be disconnected from node 0 if these fibers are removed from the network. Constraint (4) requires node 0 to be disconnected from at least one node, which ensures that the set of fibers $(i,j)$ with $y_{ij} = 1$ forms a global Cross Layer Cut.

In Section IV, we will use $MCLC$ as the objective for the survivable lightpath routing problem, and develop algorithms to maximize such an objective.

### C. Weighted Load Factor

Another way to measure the connectivity of a layered network is by quantifying the "impact" of each physical failure. The *Weighted Load Factor (WLF)*, an extension of the metric *Load Factor* introduced in [10], provides such a measure of survivability.

Let $(V_P, E_P)$, $(V_L, E_L)$ and $f_{ij}^{st}$ represent the physical topology, logical topology and lightpath routing as above. The $WLF$ can be formulated as follows:

$$\mathsf{M_{WLF}} : \text{Maximize} \quad \frac{1}{z}, \quad \text{subject to:}$$

$$z \cdot \sum_{(s,t)\in\delta(S)} w_{st} \geq \sum_{(s,t)\in\delta(S)} w_{st} f_{ij}^{st} \qquad \forall S \subset V_L, (i,j) \in E_P$$

$$\sum_{(s,t)\in\delta(S)} w_{st} > 0 \qquad \forall S \subset V_L$$

$$0 \leq z, w_{st} \leq 1 \qquad \forall (s,t) \in E_L,$$

where $\delta(S)$ is the cut set of $S$, i.e., the set of logical links that have exactly one end point in $S$.

The variables $w_{st}$ are the weights assigned to the lightpaths. Over all possible logical cuts, the variable $z$ measures the maximum fraction of weight inside a cut carried by a fiber. Intuitively, if we interpret the weight to be the amount of traffic in the lightpath, the value $z$ can be interpreted as the maximum fraction of traffic across a set of nodes disrupted by a single fiber cut. The Weighted Load Factor formulation, defined to maximize the reciprocal of this fraction, thus tries to compute the logical edge weights that minimize the maximum fraction. This effectively measures the best way of spreading the weight across the fibers for the given lightpath routing. A lightpath routing with a larger Weighted Load Factor value means that it is more capable of spreading its weight within any logical cut across the fibers.

The Weighted Load Factor also generalizes the survivable lightpath routing defined in [17], since its value will be greater than 1 if and only if the lightpath routing is survivable.

Although the formulation $\mathsf{M_{WLF}}$ contains the quadratic terms $zw_{st}$, the optimal value of $z$ can be obtained by iteratively solving the linear program with different fixed values of $z$. Using binary search over the range of $z$, we can efficiently find the minimum $z$ where a feasible solution exists.

Theorem 3 describes the relationship between the $WLF$ and the $MCLC$. Given a lightpath routing, let $\mathsf{M_{MCLC}}$ be the ILP formation for its Min Cross Layer Cut, and let $MCLC$ and $MCLC^R$ be the optimal values for $\mathsf{M_{MCLC}}$ and its linear relaxation respectively. In addition, let $WLF$ be the Weighted Load Factor of the lightpath routing. Then we have the following relationship:

*Theorem 3:* $MCLC^R \leq WLF \leq MCLC$.

*Proof:* For brevity, we only outline the proof. The complete proof can be found in [14]. Suppose the $WLF$ of a lightpath routing is $k$. This means that for any cut set $\delta(S)$ of the logical topology $G_L$, every physical fiber will cover at most $\frac{1}{k}$ of the total weight in the cut. Therefore, the lightpaths in the cut are spread across at least $k$ physical fibers. In other words, it requires removal of at least $k$ fibers in order to disconnect all the lightpaths in the cut, leading to the upper-bound. To show the lower bound, we consider the dual of the linear relaxation of $\mathsf{M_{MCLC}}$, and show that its optimal value is at most $WLF$. The lower bound follows immediately as a result of the Duality Theorem. ∎

Therefore, although the two metrics appear to measure different aspects of network connectivity, they are inherently related. In fact, as we will show in Section V, the two values are often identical.

## IV. Lightpath Routing Algorithms for Maximizing MCLC

In this section, we consider the survivable lightpath routing problem using the Min Cross Layer Cut as the objective. All the algorithms introduced in this section are based on multi-commodity flows, where each lightpath is considered a commodity to be routed over the physical network. Given the physical network $G_P = (V_P, E_P)$ and the logical network $G_L = (V_L, E_L)$, the multi-commodity flow for a lightpath routing can be generally formulated as follows:

$$\mathsf{MCF}_{\mathcal{X}}: \quad \text{Minimize } \mathcal{X}(f), \quad \text{subject to:}$$
$$f_{ij}^{st} \in \{0,1\}$$
$$\{(i,j) : f_{ij}^{st} = 1\} \text{ forms an } (s,t)\text{-path in } G_P, \;\; \forall (s,t) \in E_L,$$

where $f$ is the variable set that represents the lightpath routing, such that $f_{ij}^{st} = 1$ if and only if lightpath $(s,t)$ uses physical fiber $(i,j)$ in its route. $\mathcal{X}(f)$ is an objective function that depends on $f$.

The multi-commodity flow formulation has been used to solve the Routing and Wavelength Assignment problem of WDM networks [4], [18], where the objective function $\mathcal{X}$ is defined in a way such that the number of lightpaths that traverse the same fiber is minimized. In this section, we will use the multi-commodity flow formulation with a different objective, which attempts to maximize the Min Cross Layer Cut of the lightpath routing.

### A. Integer Programming Formulation

Ideally, to ensure that the lightpath routing is survivable against the largest number of failures, the objective function $\mathcal{X}(f)$ should express the $MCLC$ value of the lightpath routing given by $f$. However, since there is no simple way to express the lightpath routing problem that maximizes the $MCLC$ as an integer linear program, we use an objective function that approximates the $MCLC$ value.

In our formulation, each lightpath is assigned a weight $w$. The objective function $\rho_w$ measures the maximum *load* of the fibers, where the *load* is defined to be the total lightpath weight carried by the fiber. The intuition is that the multi-commodity flow formulation will try to spread the weight of the lightpaths across multiple fibers, thereby minimizing the impact of any single fiber failure.

We can formulate an integer linear program with such an objective as follows:

$$\mathsf{MCF}_w: \quad \text{Minimize } \rho_w, \quad \text{subject to:}$$
$$\rho_w \geq \sum_{(s,t)\in E_L} w(s,t) f_{ij}^{st} \qquad \forall (i,j) \in E_P$$
$$f_{ij}^{st} \in \{0,1\}$$
$$\{(i,j) : f_{ij}^{st} = 1\} \text{ forms an } (s,t)\text{-path in } G_P, \;\; \forall (s,t) \in E_L.$$

As we will prove in Theorem 4, with a careful choice of the weight function $w$, the value $\frac{1}{\rho_w}$ gives a lower bound on the $MCLC$. Therefore, a lightpath routing with a low $\rho_w$ value is guaranteed to have a high $MCLC$.

The lightpath routing strategy of the algorithm is determined by the weight function $w$. For example, if $w$ is set to 1 for all lightpaths, the integer program will minimize the maximum number of lightpaths traversing the same fiber. Effectively, this will minimize the number of disconnected lightpaths in the case of a single fiber failure.

In order to customize $\mathsf{MCF}_w$ towards maximizing the $MCLC$ of the solution, we propose a different weight function $w_{MinCut}$ that captures the connectivity structure of the logical topology. For each edge $(s,t) \in E_L$, we define $w_{MinCut}(s,t)$ to be $\frac{1}{|MinCut_L(s,t)|}$, where $MinCut_L(s,t)$ is the minimum $(s,t)$-cut in the logical topology. Therefore, if an edge $(s,t)$ belongs to a smaller cut, it will be assigned a higher weight. The algorithm will therefore try to avoid putting these small cut edges on the same fiber.

If $w_{MinCut}$ is used as the weight function in $\mathsf{MCF}_w$, we can prove the following relationship between the objective value $\rho_w$ of a feasible solution to $\mathsf{MCF}_w$ and the Weighted Load Factor of the associated lightpath routing:

*Theorem 4:* For any feasible solution $f$ of $\mathsf{MCF_w}$ with $w_{MinCut}$ as the weight function, $\frac{1}{\rho_w} \leq WLF$.

*Proof:* By definition of the weight function $w_{MinCut}$, given any $S \subset V_L$, every edge in $\delta(S)$ has weight at least $\frac{1}{|\delta(S)|}$. Therefore, we have:

$$\sum_{(s,t)\in\delta(S)} w(s,t) \geq \sum_{(s,t)\in\delta(S)} \frac{1}{|\delta(S)|} = 1 \tag{5}$$

Now consider the lightpath routing associated with $f$. For any logical cut $\delta(S)$, the maximum fraction of weight inside the cut carried by a fiber is:

$$\begin{aligned}
&\max_{(i,j)\in E_P} \frac{\sum_{(s,t)\in\delta(S)} w(s,t) f_{ij}^{st}}{\sum_{(s,t)\in\delta(S)} w(s,t)} \\
&\leq \max_{(i,j)\in E_P} \sum_{(s,t)\in\delta(S)} w(s,t) f_{ij}^{st}, \qquad \text{by Equation (5)} \\
&\leq \max_{(i,j)\in E_P} \sum_{(s,t)\in E_L} w(s,t) f_{ij}^{st} \\
&\leq \rho_w.
\end{aligned}$$

In other words, no fiber in the network is carrying more than a fraction $\rho_w$ of the weight in any cut. This gives us a feasible solution to the Weighted Load Factor formulation $\mathsf{M_{WLF}}$, where each variable $w_{st}$ is assigned the value of $w_{MinCut}(s,t)$, and the variable $z$ is assigned the value of $\rho_w$. As a result, the Weighted Load Factor, defined to be the maximum value of $\frac{1}{z}$ among all feasible solutions to $\mathsf{M_{WLF}}$, must be at least $\frac{1}{\rho_w}$. ■

As a result of Theorems 3 and 4, the $MCLC$ of a lightpath routing is lower bounded by the value of $\frac{1}{\rho_w}$, which the algorithm will try to maximize.

### *B. Randomized Lightpath Routing Algorithms*

While the multi-commodity flow integer program formulation introduces a novel way to route lightpaths in a survivable manner, such an approach may not scale to large networks, due to the inherent complexity of solving integer programs. In order to circumvent the computational difficulty, we apply the *randomized rounding* technique, which is able to quickly obtain a near-optimal solution to the integer program. Randomized rounding has previously been used to solve multi-commodity flow problems to minimize the link load [4], [19], and its performance guarantee is studied in [19].

Our randomized rounding algorithm, $\mathsf{Random_k}$, involves the following steps:

**Algorithm 1** $\mathsf{RANDOM_k}$

1: Compute the optimal fractional solution $f$ to the linear relaxation of integer program $\mathsf{MCF}_w$. For each lightpath $(s,t)$, the values of $f_{ij}^{st}$ represent a flow from $s$ to $t$ with a total flow value of 1.
2: For each lightpath $(s,t)$, decompose the solution $f_{ij}^{st}$ into flow paths, each with weight equal to the flow value of the path.
3: **for** $i = 1, 2, \ldots, k$, **do:**
   Create a random lightpath routing $R_i$: For each lightpath $(s,t)$, randomly pick one path from the set of flow paths generated in Step 2, using the path weights as the probabilities.
4: Return the $R_i$ with the highest Min Cross Layer Cut value.

The parameter $k$ specifies the number of trials in the process of random lightpath routing generation. The higher the value of $k$, the more likely the algorithm will encounter a lightpath routing with a high $MCLC$ value.

Although the last step requires the $MCLC$ computation of the lightpath routings, the integer program $\mathsf{M_{MCLC}}$ contains only $|E_P|$ binary variables, which is much fewer than the $|E_P||E_L|$ variables contained in $\mathsf{MCF}_w$. Therefore, the randomized algorithm runs considerably faster than the integer program algorithm. In the next section, we will compare the performance of the two algorithms, both in terms of running time and quality of the solution.

## V. SIMULATION

In order to evaluate the performance of the algorithms introduced in the previous section, we generated 350 random logical topologies with connectivity at least 4, and size ranging from 6 nodes to 12 nodes. We ran the lightpath routing algorithms on these topologies and computed the $MCLC$ values of the generated lightpath routings as a measure of their survivability performance. We used an augmented NSFNET (Figure 4) as the physical topology. The augmented NSFNET has connectivity 4, which makes it possible for the algorithms to achieve a higher $MCLC$ value.

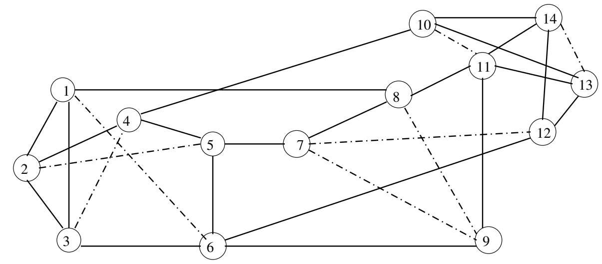



Fig. 4. The augmented NSFNET. The dashed lines are the new links.

*1) Integer Program Lightpath Routing Algorithms:* We first present the simulation result of the lightpath routing algorithms based on the integer multi-commodity flow. We apply different weight functions to the lightpaths to facilitate different routing strategies. In Figure 5, the survivability performance of the following three algorithms are presented:

1) Shortest-Path Routing (Algorithm SP)
2) Algorithm based on $\mathsf{MCF}_w$, with weight 1 for each lightpath (Algorithm $\mathsf{ILP_{Identity}}$)
3) Algorithm based on $\mathsf{MCF}_w$, with weight $\frac{1}{|MinCut_{st}|}$ for lightpath $(s,t)$, where $MinCut_{st}$ is the minimum cut between $s$ and $t$ in the logical topology (Algorithm $\mathsf{ILP_{MinCut}}$).

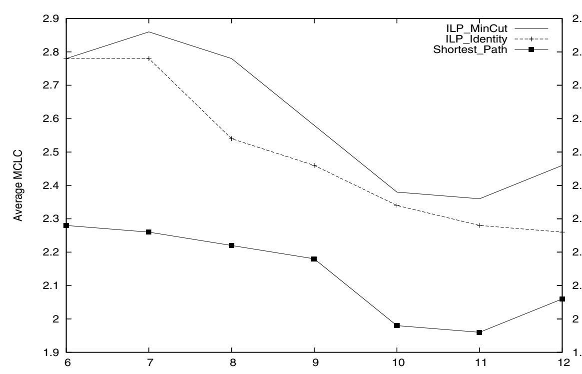



Fig. 5. Average $MCLC$ Performance of Integer Multi-Commodity Flow Lightpath Routing Algorithms on Logical Networks with Different Size

Each data point in the figure is the $MCLC$ average of 50 random instances with the given logical network size. Overall, both $\mathsf{ILP_{MinCut}}$ and $\mathsf{ILP_{Identity}}$ are able to provide lightpath routings with higher survivability than SP. Because the multi-commodity flow formulation tends to load balance lightpaths across fibers, this approach is more suitable for survivable lightpath routing. Even though the lightpath routings given by these algorithms may go through more physical hops than the one generated by the Shortest Path algorithm, they are usually more diverse and therefore can achieve higher survivability.

The quality of the lightpath routing can be further improved with a careful choice of the weight function. For example, in the case of $\mathsf{ILP_{MinCut}}$, the weight function captures certain important structural properties of the logical topology: if an edge belongs to a smaller cut in the logical topology, it will be assigned a higher weight. Consequently, the algorithm will try to avoid putting such high-weight edges on the same fiber, thereby minimizing the impact of a fiber on these high-weight, critical edges. This allows the algorithm $\mathsf{ILP}_{\mathsf{MinCut}}$ to produce lightpath routings with higher $MCLC$ values than $\mathsf{ILP}_{\mathsf{Identity}}$.

*2) Randomized Lightpath Routing Algorithms:* While the integer program algorithm provides a way to compute lightpath routings with high $MCLC$ values, it is often too computationally intensive for large networks. To circumvent the scalability issue, the randomized rounding algorithm provides a light-weight alternative to compute survivable lightpath routings. Table I compares the average running time between the algorithms $\mathsf{ILP}_{\mathsf{MinCut}}$ and $\mathsf{RANDOM}_{10}$ on various logical topology size. All simulations are run on a Xeon E5420 2.5GHz workstation with 4GB of memory. The integer and linear programs are solved using the CPLEX software package. As the number of nodes increases in the logical topology, the running time for the integer program algorithm $\mathsf{ILP}_{\mathsf{MinCut}}$ increases tremendously. On the other hand, there is no observable growth in the average running time for the algorithm $\mathsf{RANDOM}_{10}$, which is less than a minute. As the network size goes beyond 12 nodes, the algorithm $\mathsf{ILP}_{\mathsf{MinCut}}$ often failed to terminate within a day. On the other hand, the algorithm $\mathsf{RANDOM}_{10}$ was able to terminate consistently within 2 hours for very large instances with a 100-node physical topology and 50-node logical topology. This shows that the randomized approach is a much more scalable solution to compute survivable lightpath routings.

| Logical Topology Size | Average Running Time (seconds) | |
|---|---|---|
| | $\mathsf{ILP}_{\mathsf{MinCut}}$ | $\mathsf{RANDOM}_{10}$ |
| 6 | 33.2 | 31.9 |
| 7 | 50.5 | 33.9 |
| 8 | 660.0 | 30.1 |
| 9 | 1539.0 | 26.4 |
| 10 | 3090.6 | 32.3 |
| 11 | 8474.5 | 32.0 |
| 12 | 15369.7 | 29.7 |

TABLE I. Average Running Time of $\mathsf{ILP}_{\mathsf{MinCut}}$ and $\mathsf{RANDOM}_{10}$

In Figure 6, the survivability performance of the randomized algorithm $\mathsf{Random}_{\mathsf{k}}$ with various values of $k$ is compared with its integer program counterpart $\mathsf{ILP}_{\mathsf{MinCut}}$. With only one random trial (i.e., $k = 1$), the average survivability performance of $\mathsf{RANDOM}_1$ is slightly worse than $\mathsf{ILP}_{\mathsf{MinCut}}$. However, if the number of trials $k$ is increased to 10, the randomized algorithm outperforms the integer program algorithm. This is because the algorithm $\mathsf{ILP}_{\mathsf{MinCut}}$ creates the lightpath routing by solving the integer program $\mathsf{MCF}_{\mathsf{w}}$, which maximizes a *lower bound* of $MCLC$. As we will see in Section V-3, this lower bound is often not tight enough to accurately reflect the $MCLC$ value of the lightpath routing. Therefore, in many cases, the algorithm $\mathsf{ILP}_{\mathsf{MinCut}}$ would provide a sub-optimal solution.

On the other hand, the randomized algorithm generates lightpath routings non-deterministically based on the optimal fractional solution that approximates the lightpath routing given by $\mathsf{ILP}_{\mathsf{MinCut}}$. When the randomized rounding process is repeated for enough times, the algorithm often encounters a solution that is better than the one given by the integer program algorithm.

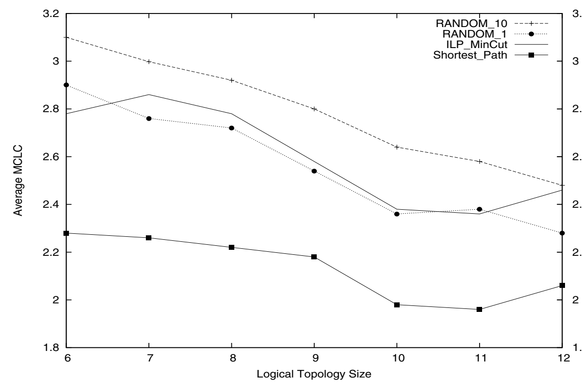



Fig. 6. Average $MCLC$ Performance of Randomized Lightpath Routing Algorithms on Logical Networks with Different Size

Figure 7 illustrates the effect of $k$ on the quality of solution generated by $\mathsf{RANDOM}_{\mathsf{k}}$. We investigated the $MCLC$ values of the lightpath routings given by the randomized algorithm on logical topologies with 8, 10 and 12 nodes, with $k$ ranging from 1 to 60. Not surprisingly, a higher value of $k$ leads to a better solution in general. However, the quality improvement diminishes when $k$ gets larger. Figure 7 suggests that 20 random trials may be sufficient to obtain a solution that is close to the best possible given by the randomized algorithm.

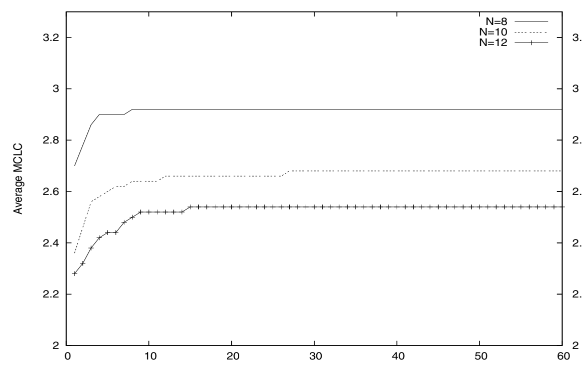



Fig. 7. Effect of $k$ on $MCLC$ values for algorithm $\mathsf{RANDOM}_{\mathsf{k}}$, with various logical topology size $N$

*3) Comparison Among Metrics:* In order to better understand the relationship between the metrics introduced in Section III, we compared the Min Cross Layer Cut and Weighted Load Factor values of the lightpath routings generated by Algorithm $\mathsf{ILP}_{\mathsf{MinCut}}$. In addition, the two metrics were compared with the optimal objective value from $\mathsf{ILP}_{\mathsf{MinCut}}$. As Figure 8 shows, the Weighted Load Factor is a very close approximation of the Min Cross Layer Cut. Among the 350 routings being investigated, the two metrics are identical in 308 cases. This suggests a very tight connection between the two metrics, and the strong correlation between them also justifies the choice of such metrics as survivability measures.

On the other hand, the objective function used by $\mathsf{ILP}_{\mathsf{MinCut}}$, although proven to be a lower bound of the Min Cross Layer Cut, turns out to be quite far away from the exact value. This explains the sub-optimality demonstrated in Section V-2 of the Algorithm $\mathsf{ILP}_{\mathsf{MinCut}}$. An objective function that better approximates the Min Cross Layer Cut, such as the Weighted Load Factor, is likely to improve our existing lightpath routing algorithms.

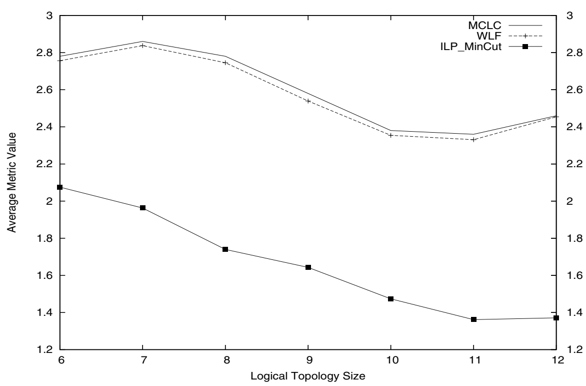



Fig. 8. Comparison among Min Cross Layer Cut (MCLC), Weighted Load Factor (WLF) , and the Optimal Value of $\mathsf{ILP}_{\mathsf{MinCut}}$

## VI. Conclusion

In this paper, we show that classical theory of network survivability, such as the Max-Flow Min-Cut Theorem, does not apply to multi-layer networks. As a result, survivability metrics, such as maximum disjoint paths and minimum cut, have significantly different properties in the multi-layer setting. We propose two cross-layer survivability metrics, the Min Cross Layer Cut and the Weighted Load Factor, that measure the connectivity of a multi-layer network, and develop mathematical formulations to compute these metrics. In addition, we use the metric Min Cross Layer Cut as the objective for the survivable lightpath routing problem, and develop a multi-commodity flow formulation to approximate this objective. We show, through simulations, that our algorithms produce lightpath routings with significantly better Min Cross Layer Cut values compared to the shortest path algorithm.

An important direction for future research is to establish a better formulation for the lightpath routing problem that maximizes the Min Cross Layer Cut. The multi-commodity flow formulation introduced in this paper approximates the Min Cross Layer Cut by using its lower bound as the objective function. However, this lower bound is often not very close to the actual Min Cross Layer Cut value. A better objective function, such as the Weighted Load Factor, would significantly improve the proposed lightpath routing algorithms. We are currently exploring the possibilities in this direction.

The similarity between the Min Cross Layer Cut and the Weighted Load Factor is also intriguing. Our simulation result demonstrated a very tight connection between the two metrics. This observation might reflect certain property of cross-layer network connectivity that are yet to be discovered and formalized. A better understanding of how these metrics relate to each other will possibly lead to important insights into the cross-layer survivability problem.

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