This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE INFOCOM 2008 proceedings.

MAC for Networks with Multipacket Reception Capability and Spatially Distributed Nodes

Guner D. Celik
CNRG
MIT
gcelik@mit.edu

Gil Zussman
Dept. of Electrical Engineering
Columbia University
gil@ee.columbia.edu

Wajahat F. Khan and Eytan Modiano
CNRG
MIT
{wajahat, modiano}@mit.edu

***Abstract*—The physical layer of future wireless networks will be based on novel radio technologies such as UWB and MIMO. One of the important capabilities of such technologies is the ability to capture a few packets simultaneously. This capability has the potential to improve the performance of the MAC layer. However, we show that in networks with spatially distributed nodes, reusing backoff mechanisms originally designed for narrow-band systems (e.g. CSMA/CA) is inefficient. It is well known that when networks with spatially distributed nodes operate with such MAC protocols, the channel may be captured by nodes that are near the destination, leading to unfairness. We show that when the physical layer enables multipacket reception, the negative implications of reusing the legacy protocols include not only such unfairness but also a significant throughput reduction. We present alternative backoff mechanisms and evaluate their performance via Markovian analysis and simulation. We show that our alternative backoff mechanisms can improve both overall throughput and fairness.**

***Index Terms*—Multipacket Reception, Capture, Medium Access Control (MAC), Performance Evaluation.**

I. Introduction

Future wireless communications technologies, such as Ultra-Wideband (UWB), have several characteristics that set them apart from other wireless communications technologies. One of these characteristics is the ability to capture a few packets simultaneously [25], [26]. A *Multipacket Reception* (MPR) capability at the physical layer calls for the design of new Medium Access Control (MAC) protocols, that are tailored for this capability [25]. The combination of MPR at the physical layer and a tailored MAC layer has the potential to significantly improve the network's performance [8]. Yet, recent proposals for MAC protocols for UWB networks (e.g. [7]) do not specifically address the potential MPR capability. Moreover, although a number of previous works provided important insights regarding the effects of MPR, some of the important characteristics of the physical layer and the need for distributed operation have not been fully considered yet. Therefore, in this paper we focus on studying the design considerations of *distributed MAC protocols for wireless networks with spatially distributed nodes and MPR capability*.

The design of MAC protocols for wireless networks has received tremendous attention in the last four decades [4], [23]. A basic underlying assumption in the design and evaluation of legacy MAC protocols (e.g. Aloha) was that any concurrent transmission of two or more packets results in a collision and failure of all packets. This assumption does not reflect the actual situation in many wireless communications systems in which the packet with the strongest power level *can* be received successfully (captured) in the presence of contending transmissions. This *capture effect* has been extensively studied in the past. In particular, [1], [13]–[15], [17], [27] studied the effect of capture on Aloha systems and [12], [18], [21] studied the issue of capture in IEEE 802.11 (CSMA/CA) systems (a review of previous work can be found in [5]).

Some systems are capable of capturing *more than a single packet*. Such a capability is known as *Multipacket Reception* capability. Previous research regarding MPR includes the works of Ghez et al. [10] who studied the stability properties of slotted aloha with MPR; Tong et al. [19], [25], [26] who proposed MAC protocols that take into account MPR; and Nguyen et al. [20] who considered the SINR capture model of [27] in the context of MPR systems. Finally, a number of algorithms that control the transmission attempts were proposed in [6], [16], [22]. Despite the recent interest in the area of MPR, numerous research challenges still exist [8]. In particular, little has been done toward the design of distributed algorithms that work effectively with spatially distributed nodes.

Under existing backoff mechanisms (e.g. CSMA/CA, used in IEEE 802.11), once a node transmits and senses a collision, it increases its contention window (i.e. decreases the transmission probability). On the other hand, following a successful transmission, it decreases the contention window. We refer to such an operation model as the *Standard Model*. It is well-known that the Standard Model may be unfair and can cause starvation to some of the nodes [2], [9]. The unfairness phenomenon is more pronounced in networks with spatially distributed nodes, since the received signals from the distant nodes are usually weaker than the signals from the nearby nodes. This may lead to starvation of distant nodes because once a nearby node captures the channel, it increases its transmission probability hence continuing to capture the channel. On the other hand, the distant nodes that failed to capture the channel, will decrease their transmission probability further reducing their chances to succeed. Moreover, under most spatial distributions the number of distant nodes is usually considerably larger than that of nearby nodes. Hence, a single "powerful" nearby node can block a number of distant nodes which with the MPR capability have the potential to be captured simultaneously. Therefore, the

Standard Model results not only in unfairness but also in a significant throughput reduction.

To overcome this phenomenon, we define an *Alternative Model* in which a node *decreases* its transmission probability following success and *increases* it following failure. In order to evaluate the Standard and Alternative Models, we define a simple MAC protocol, referred to as the Generic Distributed Probabilistic (GDP) protocol.[1] Under this protocol, each node selects between two transmission probabilities according to its success history. We study the performance of the GDP protocol through analysis and simulations. Our model considers a single receiver (e.g. a base-station) and multiple transmitters.

We note that in this setting a distributed solution is not strictly necessary. However, we focus on the design of randomized (and hence distributed) access mechanisms that can ultimately be of use in a multihop setting where a centralized solution is less appealing. We also note that although we focus on controlling the transmission probability, other mechanisms, such as power control, can also be utilized in a MPR setting. However, power control mechanisms require sophisticated feedback and complex transmitters that can adjust the transmit power level dynamically on a per-packet basis. Although such mechanisms are appropriate for cellular networks, they may be more difficult to adapt to wireless (ad hoc and sensor) networks. Moreover, in a multihop setting in which a transmitter may interfere with transmissions destined to a number of receivers, the distributed power control problem is a challenging open problem even for the single packet reception case.

The main contribution of this work is the thorough performance evaluation of simple generic MAC protocols in a way that enables us to gain important insights regarding the correct operating point when the network is capable of MPR. While the idea of using feedback from the channel to tune a backoff algorithm is not new, our work exploits this idea in the context of networks with spatially distributed nodes capable of MPR and shows that the tuning has to be done differently than in legacy systems capable of single-packet reception. In particular, we provide extensive numerical results that demonstrate that the Alternative Model outperforms the Standard Model both in terms of throughput and fairness in a wide range of network scenarios. This implies that one cannot directly reuse backoff mechanisms (e.g. CSMA/CA) designed for narrowband systems in networks with MPR capability.

This paper is organized as follows. In Section II we describe the system model and the generic protocol. In Section III we present an analysis of the GDP protocol in a fading-free environment, and in Section IV we extend our analysis to a fading environment. We use the GDP protocol to demonstrate the basic need to redesign the backoff mechanisms for networks with MPR capabilities. However, the GDP protocol is rather simplistic. Hence, in Section V we briefly discuss a more practical implementation using a simple contention window backoff mechanism similar to that used in IEEE 802.11 systems. We present numerical results in Section VI.

[1] We note that our objective is to gain understanding of the effects of MPR. Therefore, this protocol does not deal with many practical problems.

II. Model and Protocol

Consider n spatially distributed nodes that transmit to a single receiver (e.g. a base station or an access point) over a slotted channel. We assume that the packets are one-slot long, that the transmit power is constant (denoted by P_T), and that the propagation model includes path loss and Rayleigh fading (i.e. there exist independent and identically distributed Rayleigh fading channels between the nodes and the receiver). The received power of a transmission from node i, located at distance r_i from the receiver, is given by [13], [27]:

$$P_R(i) = R^2 K r_i^{-\beta} P_T, \tag{1}$$

where R is a Rayleigh distributed random variable with unit power ($R^2 \sim \text{Exponential}(1)$), β is the power loss exponent (typically between 2 and 6), and K is the attenuation constant. When we analyze systems without fading, we set $R = 1$.

We use the SINR capture model [15], [17], [20], [27], defined below (also known as the Physical Model [11] and the power capture model [13]).

Definition 1 (SINR Capture Model): Given k simultaneous transmissions, the packet of user i is captured (successfully decoded) at the receiver if

$$SINR(i) = \frac{P_R(i)}{N + \sum_{j=1,\, j\neq i}^{k} P_R(j)} > z, \tag{2}$$

where z is the capture threshold ratio and N is the background noise.

The background noise power level is usually much lower than the power level of the interference, and therefore, it will be neglected. For single packet reception narrow-band systems $1 \leq z \leq 10$, whereas for wideband Multipacket Reception systems (e.g. CDMA and UWB) $z < 1$ [13], [20]. Since we focus on the MPR capability, we assume that $z < 1$. The *maximum* number of simultaneously successful transmissions is denoted by c. From (2), it can be seen that the maximum number of packets will be captured, if there are c equal received-power packets at the receiver. Hence, $c = \lceil 1/z \rceil$.[2]

We analyze the performance of the system in various scenarios. In particular, we consider the node distribution models, defined below.[3] We note that Fig. 1 illustrates an example of nodes deployed according to the rings model.

Definition 2 (Rings Model): $n_1, n_2, \ldots, n_L$ nodes are located on L rings around the receiver ($\sum_{i=1}^{L} n_i = n$). The radii of these rings are denoted by $r_1, r_2, \ldots, r_L$, respectively.

Definition 3 (Disk Model [15], [17], [27]): n nodes are randomly distributed in a disk of radius r_d and the Probability Density Function (PDF) of a node's distance from the receiver (r) is denoted by $f(r)$.

[2] In [13], the capture equation (2) is defined with an equality, and therefore, there: $c = 1 + \lfloor 1/z \rfloor$.

[3] Since nodes cannot get arbitrarily close to the receiver, when considering the SINR model, we assume that nodes are distributed such that $\min(r) \geq 1$. Specifically, in the disk model, we consider a punctured disk [14], [15].

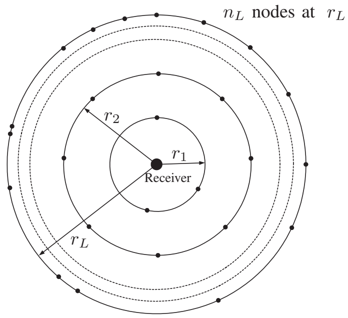

Fig. 1. Nodes deployed according to the rings model on rings whose radii are $r_1, r_2, \ldots, r_L$.

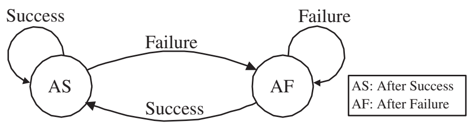

Fig. 2. The state diagram of a node under the generic protocols.

In Section II-A we will present a generic protocol in which the transmission attempts are random. When analyzing this protocol, we denote the failure probability of a packet transmitted by a node located at distance r from the receiver by $p_{\text{f}}(r)$ and the probability of transmission by $\tau(r)$. The throughput of a node at distance r is denoted by $S(r)$ and is defined as the expected number of successfully received packets per slot for that node. The overall throughput is the expected number of successfully received packets per slot throughout the system and is denoted by S.

In order to enable analytic performance evaluation, we make two assumptions that have been extensively used in the past (see [4], [23]): (i) there exists a simple immediate and reliable feedback mechanism that provides the node an acknowledgement if its attempt succeeded and (ii) the nodes are saturated (i.e. at each node, there is always a packet to send).

A. Generic Distributed Probabilistic (GDP) Protocol

We now define the Generic Distributed Probabilistic (GDP) protocol. The analysis and performance evaluation of this protocol is later used to demonstrate the need to redesign MAC protocols for networks with MPR capability. Since we focus on the effects of MPR, the protocol does not deal with many practical problems and is far from providing a complete MAC solution (such as provided, for instance, by IEEE 802.11).

The operation of a node under the GDP protocol is described in the state diagram in Fig. 2. A node can be in two states: AS - After Success and AF - After Failure. Transitions may take place after a transmission attempt. A node moves into the AS (AF) state following a successful (failed) transmission of a packet by that node. Since transitions take place after an attempt, a node does not change its state following an idle slot.

The GDP protocol can be viewed as a two-state Aloha system that dynamically adjusts the transmission probability of *each node* according to its success history. When a node is in the AS (AF) state, it transmits with probability p_{ts} (p_{tf}) at each slot, disregarding the status of the channel. The protocol is generic in the sense that it can be evaluated under any combination of p_{ts} and p_{tf}. A high p_{ts} value corresponds to maintaining a small contention window following a successful attempt in the traditional backoff mechanisms such as the one used in IEEE 802.11. Similarly, a low p_{tf} value corresponds to maintaining a large contention window following a failure. Hence, for a large value of p_{ts} and a small value of p_{tf}, the protocol captures, *in a very simplistic sense*, the dynamic and adaptive operation of traditional backoff mechanisms. Accordingly, we refer to the operation of the GDP protocol with $p_{\text{ts}} > p_{\text{tf}}$ as the *Standard Model* and to the operation with $p_{\text{tf}} > p_{\text{ts}}$ as the *Alternative Model*. We will show that although the Standard Model may achieve good throughput in networks with single packet reception, in networks with MPR capability and spatially distributed nodes, the *Alternative Model usually yields higher throughput than the Standard Model*.[4]

III. Rings Model without Fading - Analysis

In this section we analyze the GDP protocol under the rings model in a system without fading. We provide exact and approximate results. We have used the exact results in order to check the accuracy of the approximation method and of our simulation model (described in Section VI).

A. Two Rings - Exact Analysis

Consider n_1 and n_2 nodes distributed on two rings whose radii are r_1 and r_2. Using (1) and (2), it can be seen that if $r_1^{-\beta} > z((c-1)r_1^{-\beta} + n_2 r_2^{-\beta})$, the n_2 nodes on the ring at r_2 cannot generate enough interference to cause failure to a transmission from a node on the ring at r_1 (even if there are $c-1$ other transmissions from r_1). A simplification yields the following observation.

Observation 1: If $n_2 < (r_2/r_1)^\beta (z^{-1} - c + 1)$, then transmissions from the nodes on the ring at r_2 cannot cause failures to packets transmitted from nodes on the ring at r_1.

The following can also be obtained from (1) and (2).

Observation 2: If $(r_1/r_2)^\beta < z$, a transmission from a node at r_2 can be captured only when *all the nodes at r_1 are silent.* For example, for $z = 0.2, \beta = 4, r_1 = 1$, and $r_2 = 2$, the condition in Observation 1 implies that $n_2 < 16$. In order to facilitate the exact analysis, we assume that the conditions presented in the observations above hold.

We describe the evolution of the system under the GDP protocol by a two dimensional Markov chain whose states are denoted by (i, j). i and j denote the number of nodes on the rings at r_1 and at r_2 (respectively) that are in the AF state. Accordingly, the number of states in the chain is $(n_1+1)(n_2+1)$. An example of part of such a Markov chain is illustrated in Fig. 3. Each arrow represents a possible transition between states (i, j) and (k, l) that has some transition probability associated with it. We group the transition probabilities according to the type of change (positive, negative, or none) in each of the two dimensions. For example, one group includes

[4] There are some cases in which the Standard Model outperforms the Alternative Model. For example, when there is a single node in the network the Standard Model is preferable.

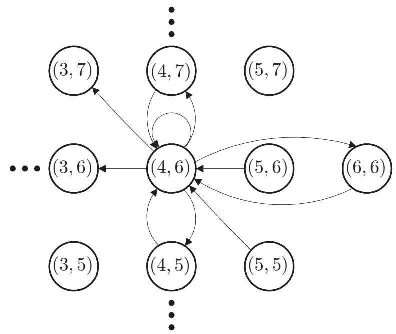

Fig. 3. Part of the Markov chain characterizing a system with two rings in which $n_1 = 6$ and $n_2 = 10$ (only some of the possible transitions to and from state $(4, 6)$ are shown).

the transition probabilities for which $k > i$ and $l > j$, while another group includes the probabilities for which $k < i$ and $l = j$. All the possible combinations of changes results in 9 groups.

Due to space constraints, we omit the equations describing the transition probabilities (they can be found in [5, Chapter 5]). Given the transition probabilities, the steady state probabilities (denoted by $p(i, j)$) can be obtained by solving a set of linear equations. Once the steady state probabilities are obtained, the throughput of the system can be calculated as follows. We denote by $S_{\text{rg}}(m, i)$ the overall throughput of the nodes on a ring that has m nodes, of which i are in the AF state. Assuming that there is a single ring:

$$S_{\text{rg}}(m, i) = \sum_{f=0}^{min(c,i)} \sum_{s=0}^{min(c-f,m-i)} (f + s) \cdot \binom{i}{f} p_{\text{tf}}^{f} (1 - p_{\text{tf}})^{i-f} \binom{m-i}{s} p_{\text{ts}}^{s} (1 - p_{\text{ts}})^{m-i-s}. \quad (3)$$

The two summations are for the nodes at the two states (f is the index for the nodes at AF and s is for the nodes at AS). Since at most c packets can be captured simultaneously, the summations are bounded not only by the number of nodes in the relevant state but also by c and $c - f$.

Due to our assumptions, the success of nodes on the ring at r_1 is not affected by transmissions of nodes at r_2, and therefore, the throughput of these nodes can be computed as if there is a single ring. Moreover, nodes on the ring at r_2 succeed only if nodes on r_1 are silent. Therefore, in such a case, the nodes on r_2 can be treated as nodes on a single ring. Accordingly, the overall throughput is given by

$$S = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} p(i, j) \big(S_{\text{rg}}(n_1, i) + (1 - p_{\text{ts}})^{n_1 - i} (1 - p_{\text{tf}})^{i} S_{\text{rg}}(n_2, j) \big), \quad (4)$$

where $(1 - p_{\text{ts}})^{n_1-i}(1 - p_{\text{tf}})^i$ is the probability that the nodes on r_1 are silent.

B. Multiple Rings - Approximate Analysis

The exact analysis is quite cumbersome even for two rings. Hence, we present an approximation method that allows us to obtain results for a large number of rings. When the conditions in observations 1 and 2 are satisfied, the state of the nodes at r_1 is independent of the actions of nodes at r_2. Nodes at r_1 are of course not independent of each other. The dependence of the nodes at r_2 on the nodes at r_1 is limited to the event that "no transmission occurs from a node at r_1". Assuming that such a partial independence between the rings exists even in a system with multiple rings, we now develop a method in which we use different Markov chains to characterize the states of the nodes on the different rings. The state of each Markov chain is the number of nodes at the AF state on that ring. Within the rings these Markov chains capture the dependence between the nodes. The interaction between the rings is captured by the following independence assumption.

Definition 4 (Inter-ring Independence Assumption): For a node on a ring at r_j, the probability that a node on the ring at r_i $(i \neq j)$ transmits ($\tau(r_i)$), is constant and is independent of the states of the nodes on the rings at $r_h (h \neq i)$.

In general, the probability that a node transmits depends on its success history and the actions of the other nodes. Yet, similar independence assumptions have been made in the analyses of IEEE 802.11 (while taking into account the dependence on r_i [18] or ignoring it [3]). We note that since we assume that there *is* dependence between the nodes on each ring, the assumption is weaker than the assumptions made in the past.

We wish to limit the possible interactions between the different rings, and thereby, between the different Markov chains. Namely, we want that for a ring at r_i, the effective $\tau(r_j)\ \forall r_j > r_i$ will be zero. For example, if the condition in Observation 1 holds, the nodes at r_1 are not affected by transmissions at r_2. Therefore, in the derivation of the Markov chain of the ring at r_1, we can assume that $\tau(r_2) = 0$. In order to limit the interactions, the conditions at observations 1 and 2 have to hold in addition to a number of other conditions. In the following observations, we demonstrate the conditions for the case of 3 rings. We then outline the results for this case.

The following observation is derived from (1) and (2) in a very similar way to the derivation of Observation 1.

Observation 3: If $n_3 < (r_3/r_2)^{\beta}(z^{-1} - c + 1)$, then transmissions from the nodes on the ring at r_3 cannot cause failures to packets transmitted from the nodes on the ring at r_2.

Using (1) and (2), it can be seen that if $r_1^{-\beta} > z((c-1)r_1^{-\beta} + n_2 r_2^{-\beta} + n_3 r_3^{-\beta})$ holds, the n_2 and n_3 nodes on the rings at r_2 and r_3 cannot generate enough interference to cause failure to a transmission from a node on the ring at r_1 (even if there are $c - 1$ other transmissions from r_1). A simplification yields the following observation.

Observation 4: If $n_2 < (r_2/r_1)^{\beta}(z^{-1} - c + 1) - n_3(r_2/r_3)^{\beta}$, then transmissions from the nodes on the rings at r_2 and r_3 cannot cause failures to packets transmitted from the nodes on the ring at r_1.

The following observation can be obtained in a similar way to Observation 2.

Observation 5: If $(r_2/r_3)^{\beta} < z$, then a transmission from a node at r_3 can be captured only when *all the nodes at r_2 and at r_1 are silent.*

For example, for $z = 0.2$ and $\beta = 4$, $r_1 = 1, r_2 = 2, r_3 = 4, n_3 \leq 15$ and $n_2 \leq 15$ satisfy the conditions in

$$P_{j\to m} = \begin{cases} (1-(1-\tau(r_1))^{n_1})\binom{n_2-j}{k} p_{\text{ts}}^k(1-p_{\text{ts}})^{n_2-j-k} & \text{if } m \leq c \\ (1-(1-\tau(r_1))^{n_1})\binom{n_2-j}{k} p_{\text{ts}}^k(1-p_{\text{ts}})^{n_2-j-k}+ & \\ (1-\tau(r_1))^{n_1}\binom{n_2-j}{k} p_{\text{ts}}^k(1-p_{\text{ts}})^{n_2-j-k}\sum_{v=c+1-k}^{j}\binom{j}{v} p_{\text{tf}}^v(1-p_{\text{tf}})^{j-v} & \text{if } k \leq c \text{ and } m \geq c+1 \\ \binom{n_2-j}{k} p_{\text{ts}}^k(1-p_{\text{ts}})^{n_2-j-k} & \text{if } k \geq c+1 \end{cases} \tag{5}$$

observations 1-5. Assuming that the conditions in observations 1-5 are satisfied, we obtain the transition probabilities of the Markov chains as follows. For a Markov chain of a given ring, there are 3 possible transitions in every time slot: the number of nodes in the AF state can increase, decrease, or stay the same. We associate a transition probability with each of these events. Due to space constraints we provide two demonstrative cases (the complete description can be found in [5]).

The first case is a transition probability for the Markov chain of the nodes at r_1. Recall that due to our assumptions, nodes on the ring at r_1 are not affected by nodes on other rings. Specifically, we denote by $P_{i\to(i-s)}$, the probability that the number of nodes in the AF state decreases from i to $i-s$. In this case, there are $s \leq i$ successful transmissions from the i nodes that are in the AF state (i.e. nodes transmitting with probability p_{tf}). Since the maximum number of simultaneously successful transmissions is c, for $s \geq c+1$, $P_{i\to(i-s)} = 0$. If $s \leq c$, then there should be s transmissions from the i nodes at the AF state and up to $c-s$ transmissions from the n_1-i nodes at the AS state. Accordingly, for $s \leq c$:

$$P_{i\to(i-s)} = \binom{i}{s} p_{\text{tf}}^s(1-p_{\text{tf}})^{i-s} \cdot \sum_{v=0}^{\min(c-s,n_1-i)}\binom{n_1-i}{v} p_{\text{ts}}^v(1-p_{\text{ts}})^{n_1-i-v}.$$

The second case, appearing in (5), is a transition probability for the Markov chain of the ring at r_2. It is the probability that the number of nodes in the AF state increases from j to m. This can happen due to k ($k \triangleq m-j$) failed transmissions from the n_2-j nodes that are at the AS state. If $m \leq c$, nodes at r_2 fail only if there is at least one transmission from the ring at r_1. Under the Inter-ring Independence Assumption this happens with probability $(1-(1-\tau(r_1))^{n_1})$, where $\tau(r_1)$ is the expected transmission probability of a node on the ring at r_1 computed using the Markov chain of that ring. When $m \geq c+1$, in addition to the failures due to the transmissions from the ring at r_1, nodes on the ring at r_2 can fail if there are more than c transmissions from that ring. This second contribution occurs when there is no transmission from the ring at r_1, k transmissions from the n_1-i nodes on the ring at r_2 that are in the AS state, and at least $c+1-k$ transmissions from the i nodes at r_2 that are in the AF state. If $k \geq c+1$, all the transmissions from the ring at r_2 will fail. In this case, the transition probability is the probability that k nodes in the AS state will transmit.

Due to the decoupling between the Markov chains, the approximation method is easier to formulate than the exact method presented in Section III-A. Particularly, the derivations of the transition probabilities in the Markov chains of the different rings are very similar to each other. Given the transition probabilities, the steady state probabilities of each of these Markov chains ($p_1(i), p_2(j)$, and $p_3(h)$) can be obtained by solving a set of linear equations. Once they are obtained, the throughput of the system can be obtained in a similar way to the derivation of (4) and by using $S_{\text{rg}}(m,i)$ (defined in (3)). It is given by the following expression

$$S = \sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{h=0}^{n_3} p_1(i)p_2(j)p_3(h)\big(S_{\text{rg}}(n_1,i) + (1-p_{\text{ts}})^{n_1-i}(1-p_{\text{tf}})^i S_{\text{rg}}(n_2,j) + (1-p_{\text{ts}})^{n_1+n_2-i-j}(1-p_{\text{tf}})^{i+j} S_{\text{rg}}(n_3,h)\big).$$

Due to the Inter-ring Independence Assumption, the probability of having i, j, and h nodes in the AF states on the rings at r_1, r_2, and r_3 (respectively) is $p_1(i)p_2(j)p_3(h)$. Due to our assumptions, the nodes on the ring at r_1 are not affected by the nodes on other rings, and therefore, their throughput can be computed using $S_{\text{rg}}(m,i)$ as if there is a single ring. Nodes on the ring at r_2 succeed, only if the nodes at r_1 are silent (the probability of this event is $(1-p_{\text{ts}})^{n_1-i}(1-p_{\text{tf}})^i$). Similarly, the nodes at r_3 succeed only if the nodes on the rings at r_1 and r_2 are silent (i.e. with probability $(1-p_{\text{ts}})^{n_1+n_2-i-j}(1-p_{\text{tf}})^{i+j}$).

We verified the accuracy of the approximation by comparing its results for the 2 rings case with the exact results. While the results for the $n_1 = 1$, $n_2 = 5$ and $n_1 = 2$, $n_2 = 10$ cases were same up to three decimal places, the results for the $n_1 = 6$, $n_2 = 10$ case were within 1.6% of the exact results. We also compared the approximate results for the 3 rings case to results obtained by the simulation model described in Section VI. In all the considered cases, the approximate results were within 2.2% of the simulation results.

IV. Rings Model with Fading

We continue to study the performance of the GDP protocol under the rings model. Unlike in the previous section, we now assume that the propagation model includes Rayleigh fading. This assumption allows us to obtain approximate results for a large number of rings without using Markov chains.

We also introduce the following independence assumption.

Definition 5 (Independence Assumption): The probability that a packet transmitted by a node on the ring at distance r_i

$$p_{\mathrm{f}}(r_i) = \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{n_2} \cdots \sum_{k_i=0}^{n_i-1} \cdots \sum_{k_L=0}^{n_L} \binom{n_1}{k_1} \tau(r_1)^{k_1}(1-\tau(r_1))^{n_1-k_1} \binom{n_2}{k_2} \tau(r_2)^{k_2}(1-\tau(r_2))^{n_2-k_2} \cdots$$
$$\binom{n_i-1}{k_i} \tau(r_i)^{k_i}(1-\tau(r_i))^{n_i-k_i-1} \cdots \binom{n_L}{k_L} \tau(r_L)^{k_L}(1-\tau(r_L))^{n_L-k_L} \left(1-p_{c,j}(r_i|r_1,r_2,\ldots,r_L;k_1,k_2,\ldots,k_L)\right) \forall i \in L. \quad (7)$$

is lost, $p_{\mathrm{f}}(r_i)$, is constant and is independent of the number of retransmissions suffered.

In general, when a node transmits a packet, the probability that it is lost (due to collisions) depends on other transmissions during that slot. Yet, as mentioned above, similar independence assumptions have been made in the analyses of IEEE 802.11 [3], [18]. When the number of nodes is large, the independence assumption is likely to hold. Indeed, in [5, Chapter 5], we have shown via exact analysis and simulation that results based on this assumption provide a good approximation.

According to Fig. 2, a node changes its state from AS to AF, if it attempts to transmit and fails. Under the independence assumption, the probability of such a transition for a node on the ring at r_i is $p_{\mathrm{ts}}p_{\mathrm{f}}(r_i)$. Similarly, a node changes its state from AF to AS, if it attempts and succeeds (i.e. with probability $p_{\mathrm{tf}}(1-p_{\mathrm{f}}(r_i))$). Hence, the state of a specific node is described by a 2-state Markov chain with the above transition probabilities. The steady state probabilities of states AS and AF for a node on the ring at distance r_i can be shown to be:

$$p_{\mathrm{AS}}(r_i) = \tfrac{p_{\mathrm{tf}}(1-p_{\mathrm{f}}(r_i))}{p_{\mathrm{ts}}p_{\mathrm{f}}(r_i)+p_{\mathrm{tf}}(1-p_{\mathrm{f}}(r_i))},$$
$$p_{\mathrm{AF}}(r_i) = \tfrac{p_{\mathrm{ts}}p_{\mathrm{f}}(r_i)}{p_{\mathrm{ts}}p_{\mathrm{f}}(r_i)+p_{\mathrm{tf}}(1-p_{\mathrm{f}}(r_i))}.$$

Accordingly, the overall transmission probability of a node on the ring at r_i is given by

$$\tau(r_i) = \frac{p_{\mathrm{ts}}}{1-p_{\mathrm{f}}(r_i)+\frac{p_{\mathrm{ts}}}{p_{\mathrm{tf}}}p_{\mathrm{f}}(r_i)} \quad \forall i \ \ 1 \le i \le L. \qquad (6)$$

We define $p_{c,j}(r_i|r_1,r_2,\ldots,r_L;\,k_1,k_2,\ldots,k_L)$ as the probability that a packet from a node at distance r_i is captured, given that there are j interferers distributed such that there are $k_1,k_2,\ldots,k_i,\ldots,k_L$ interferers at distances $r_1,r_2,\ldots,r_i,\ldots,r_L$, respectively ($j=\sum_{m=1}^{L}k_m$).[5] Using the independence assumption and considering all possible interferer configurations, we derive in (7) the failure probability of a transmission from distance r_i for all the values of i.[6] In order to be able to numerically solve the set of $2L$ non-linear equations, given by (6) and (7), we need to obtain $p_{c,j}(r_i|r_1,r_2,\ldots,r_L;\,k_1,k_2,\ldots,k_L)$. To derive it we apply a result of Zorzi and Rao [27] who analyzed an *Aloha system* with Rayleigh fading and capture. We define $p_{c,j}(r_0|r_1,r_2,\ldots,r_j)$ as the probability that a packet from a node at distance r_0 is captured in the presence of j other transmissions from distances $r_1,r_2,\ldots,r_j$. It is given in [27] as

$$p_{c,j}(r_0|r_1,r_2,\ldots,r_j) = \prod_{m=1}^{j} \frac{1}{1+z\left(\frac{r_0}{r_m}\right)^{\beta}}. \qquad (8)$$

[5]Note that $0 \le k_i \le n_i - 1$ and $0 \le k_m \le n_m \,\forall m \ne i$.

[6]Note that this is a generalization of the derivation in [21] of the failure probability in a system with two power levels.

Using (8) and combining the terms with the same distance, we obtain:

$$p_{c,j}(r_i|r_1,r_2,\ldots,r_L;k_1,k_2,\ldots,k_L) = \prod_{m=1}^{L}\left[\frac{1}{1+z\left(\frac{r_i}{r_m}\right)^{\beta}}\right]^{k_m}. \quad (9)$$

Eq. (9) can now be used within (7). Then, (6) along with (7) can be solved numerically. Using the numerical solutions, one can obtain the overall throughput $S=\sum_{i=1}^{L} n_i\tau(r_i)(1-p_{\mathrm{f}}(r_i))$. We used Matlab to solve the equations and obtain numerical results. We verified their accuracy using the simulation model described in Section VI. While in almost all the cases, the approximate results were within 2.6% of the simulation results, a few cases were within 4.6%.

V. Generic Distributed Contention Window (GDCW) Protocol

We now present the Generic Distributed Contention Window (GDCW) protocol, which is potentially easier to implement compared to the *GDP* protocol and can be operated according to the Standard and Alternative Models. Unlike the GDP protocol, the GDCW protocol does not easily lend itself to analysis. Hence, we evaluate its performance via simulation.

Similarly to the GDP protocol, nodes operate according to the state machine in Fig. 2. Namely, nodes are assigned contention windows of different sizes according to their success or failure in the previous attempt. When a node enters an AS state, it is assigned a contention window of size w_{s} slots. It then selects a backoff interval BO uniformly in $[0, w_{\mathrm{s}}]$ and retransmits after waiting for BO slots, disregarding the status of the channel. Similarly, when a node enters the AF state, it selects a backoff interval uniformly in $[0, w_{\mathrm{f}}]$ and retransmits at the end of this interval.

The GDCW protocol is generic in the sense that it can be evaluated under any combination of w_{f} and w_{s}. For a large value of w_{f} and a small value of w_{s}, the protocol captures, in a very simplistic sense, the operation of backoff mechanisms such as the one used in IEEE 802.11. Hence, we refer to the operation with $w_{\mathrm{f}} > w_{\mathrm{s}}$ as the *Standard Model*. When $w_{\mathrm{s}} > w_{\mathrm{f}}$, we refer to the scheme as the *Alternative Model*.

VI. Performance Evaluation

We used the analytic methods, described in sections III and IV along with extensive simulation experiments to evaluate the performance of the GDP and GDCW protocols under the Standard Model and the Alternative Model. Analytic results can be obtained for the GDP protocol under the rings model, and therefore, we present analytic results for that case. We also present simulation results for both protocols for the more realistic disk model. Extensive additional numerical results can

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE INFOCOM 2008 proceedings.

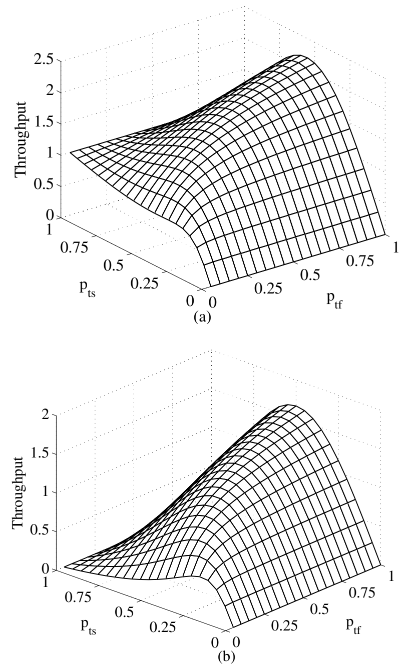

Fig. 4. (a) The overall throughput (S) and (b) the throughput of the distant nodes. The system operates under the GDP protocols in a fading-free environment and the nodes are distributed according to the rings model with $r_1 = 1, r_2 = 2, n_1 = 1$, and $n_2 = 5$.

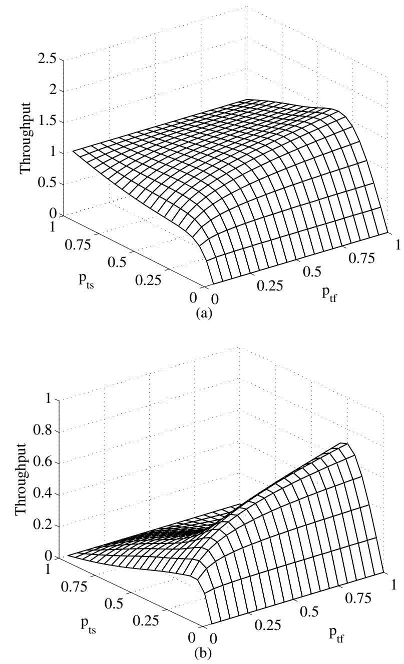

Fig. 5. (a) The overall throughput (S) and (b) the throughput of the nodes at r_3, under the GDP protocol in a fading-free environment. The nodes are distributed according to the rings model with $r_1 = 1, r_2 = 2, r_3 = 4, n_1 = 1, n_2 = 2$, and $n_3 = 4$.

be found in [5]. All the reported results are obtained under the SINR model with the assumption that $N = 0, \beta = 4$, and $z = 0.2$ (i.e. $c = 5$ - at most 5 packets can be captured simultaneously). We verified that the results are not very sensitive to changes in z. As mentioned in Section II, when considering the disk model, we assume that nodes are distributed in a punctured disk and that the nodes' distribution is Uniform. Namely, $f(r) = 2r/(r_d^2 - r_0^2)$, $r_0 \leq r \leq r_d$. In particular, in the reported experiments $r_0 = 1$ and $r_d = 10$.

A. Simulation Model

The simulation model was developed in C. It allows operating the system in different scenarios (e.g. rings model, disk model, with and without fading, etc.) and according to the different protocols. We verified the correctness of the simulation model in a number of ways. For example, we compared results obtained via simulation for the GDP protocol with the rings model to exact results obtained by the method described in Section III-A. In all the cases, the simulation results were within 1.5% of the exact results. We also compared simulation results obtained for the GDP protocol with the disk model and fading to numerical results presented in [14, Figures 4–6] for the special case of $p_{\text{tf}} = p_{\text{ts}}$. Our simulation results were within 2% of the results in [14]. As mentioned in sections III-B and IV, once the performance of the simulation model had been verified, we also used it in order to check the accuracy of the approximation methods.

In the reported simulation results, for each data point the simulation length was 100,000 slots. Since in the disk model nodes are randomly placed, results presented for that model are averaged over 40 different experiments. In each of these experiments nodes are placed differently.

B. Rings Model without Fading

We now present exact and approximate results regarding the performance of the GDP protocol and simulation results regarding the performance of the GDCW protocol.

Fig. 4(a) presents the overall throughput under the GDP protocol with nodes on two rings. The maximum throughput of 2.195 is obtained for $p_{\text{ts}} = 0.55$ and $p_{\text{tf}} = 1$. At that point, p_{tf} is larger than p_{ts}, implying that the Alternative Model outperforms the Standard Model. Fig. 4(b) shows the total throughput obtained in this case by the nodes at distance $r_2 = 2$. While the distant nodes achieve very low throughput in the Standard Model, they manage to achieve high throughput in the Alternative Model. At the operating point in which the overall system throughput is maximized, the throughput obtained by a distant node is 0.33 while the throughput obtained by a nearby node is 0.55. Hence, the Alternative Model not only improves the performance in terms of overall throughput but also provides some degree of fairness.

The overall throughput under the GDP protocol with nodes on 3 rings is presented in Fig. 5(a). The maximum throughput is obtained for $p_{\text{ts}} = 0.35$ and $p_{\text{tf}} = 1$. This again demonstrates that the Alternative Model outperforms the Standard Model. Furthermore, at this maximum point, the nodes at different distances are not starved ($S(1) = 0.350, S(2) = 0.295, S(3) = 0.154$). Fig. 5(b) shows the total throughput of the n_3 nodes at r_3. It can be seen that these distant nodes benefit from low values of p_{ts}, which reduce the chances of nearby nodes to

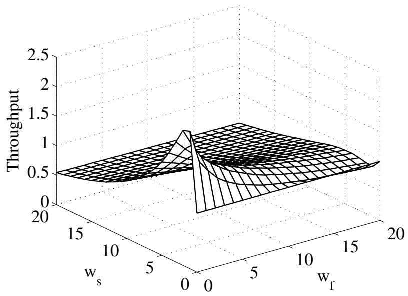

Fig. 6. The throughput (S) under the GDCW protocol in a fading-free environment. The nodes are distributed according to the rings model with $r_1 = 1, r_2 = 2, n_1 = 1$, and $n_2 = 5$.

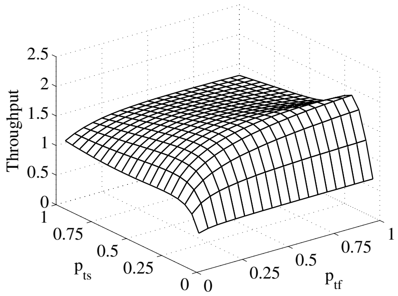

Fig. 7. The throughput (S) under the GDP protocol in a system with fading. The nodes are distributed according to the rings model with $r_1 = 1, r_2 = 2, r_3 = 3, n_1 = 1, n_2 = 3$, and $n_3 = 9$.

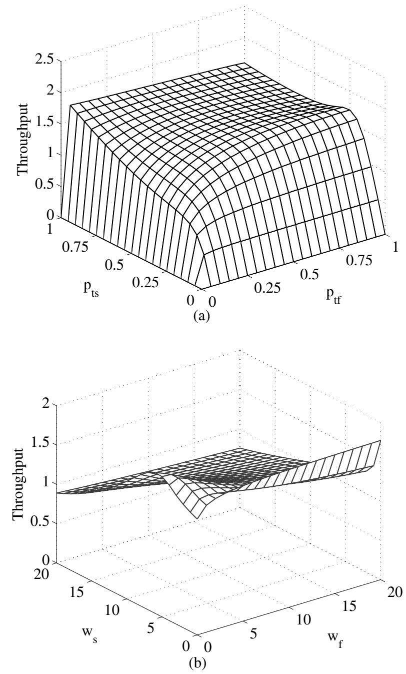

Fig. 8. The throughput (S) under (a) the GDP protocol and (b) the GDCW protocol in a fading-free environment. 10 nodes are distributed according to the disk model.

capture the channel, and high values of p_{tf}, which increase the number of their retransmission attempts.

Up to now we reported analytic results regarding the GDP protocol. We also evaluated the performance of the GDCW protocol via simulation. Such results are shown in Fig. 6. The figure demonstrates that the throughput is maximized when $w_{\text{f}} < w_{\text{s}}$. Namely, the Alternative Model is beneficial for the GDCW protocol as well.

C. Rings Model with Fading

We used the approximation method described in Section IV to obtain numerical results regarding the performance of the GDP protocol in a system with fading in which the nodes are distributed according to the rings model. For example, Fig. 7 shows the overall throughput for a system with 3 rings. As before, the throughput is maximized by large values of p_{tf} and small values of p_{ts}. Once fading is considered, the received power of a transmission from a nearby node is not necessarily stronger than the power of a transmission from a distant node. However, this does not significantly change the fact that the Alternative Model still outperforms the Standard Model.

D. Disk Model

We used the simulation model to obtain numerical results for the case in which nodes are distributed according to the disk model and there is no fading. Figures 8(a) and 8(b) show the throughput under the GDP and GDCW protocols. In both figures there are two local optimums. One of them represents the Alternative model and the other the Standard Model. However, as before, under the Standard model, distant nodes are usually starved. Therefore, the Alternative Model is preferable. It can be seen that the results obtained for the disk model are similar to results obtained for the rings model. Hence, the rings model provides a good approximation, while lending itself to analytic performance evaluation.

In Fig. 9 we present simulation results for a system with fading in which nodes are distributed according to the disk model. The results are similar to the results for the non-fading case. Since the fading effect somehow averages the transmission powers, the nearby nodes do not always succeed and distant nodes may succeed despite transmissions of nearby nodes. Hence, the local optimums are not as strict as in the fading-free case. Moreover, unlike in the fading-free case, there is no local optimum corresponding to the Standard Model.

VII. Conclusions

The Multipacket Reception capability has the potential to significantly improve the performance of wireless networks. In this paper, we focused on the design considerations of MAC protocols for networks of spatially distributed nodes with such capability. It is known that standard backoff mechanisms can lead to unfairness when nodes are spatially distributed. We demonstrated that with MPR capability these mechanisms can also lead to *significant throughput loss*. To deal with this effect, we presented and analyzed an alternative backoff mechanism whereby nodes increase their transmission probability after a failed transmission and decrease it after a success. Our analysis shows that in most cases the Alternative Model outperforms the Standard Model in terms of both throughput and fairness.

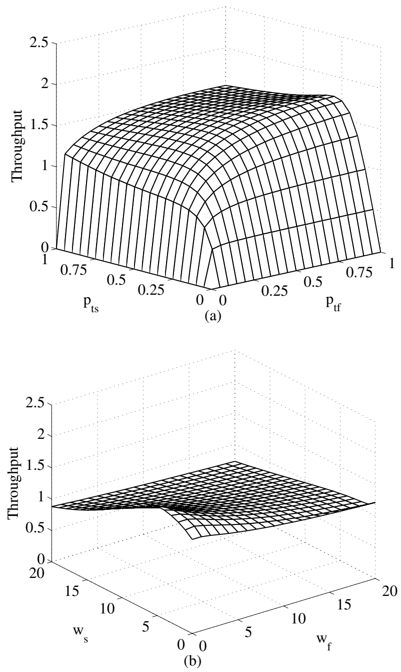

Fig. 9. The throughput (S) under (a) the GDP protocol and (b) the GDCW protocol in a system with fading. 10 nodes are distributed according to the disk model.

Hence, new MAC protocols have to be designed for MPR networks and these protocols have to divert from the traditional backoff mechanisms.

This work is the first approach towards the design of distributed MAC protocols for networks with MPR capability. Hence, there are still many open problems to deal with. As mentioned above, since the objective of this work is mainly to provide insight regarding MAC for networks with MPR capability, the studied protocols do not provide a complete solution. We intend to develop more elaborate backoff mechanisms that will utilize feedback from the receiver more effectively. These mechanisms should deal with issues such as power and congestion control that have been ignored in this work. For example, we intend to develop distributed mechanism that would tune the values of the protocol's parameters (e.g. $p_{\text{ts}}, p_{\text{tf}}, w_s, w_f$). Furthermore, estimating the number of contenting nodes (e.g. the work of [24] for the IEEE 802.11 case) is a possible future direction as it could provide important input to the algorithm. Finally, designing an efficient MPR MAC protocol for a multihop setting is a challenging open problem.

Acknowledgments

This work was supported by NSF ITR grant CCR-0325401, ONR grant number N000140610064, a grant from Draper laboratory, a Marie Curie International Fellowship within the 6th European Community Framework Programme, and the MAGNET/ISRC Consortium.

References

[1] N. Abramson, "The throughput of packet broadcasting channels," *IEEE Trans. Commun.*, vol. 25, no. 1, pp. 117–128, Jan. 1977.

[2] B. Bensaou, Y. Wang, and C. C. Ko, "Fair medium access in 802.11 based wireless ad-hoc networks," in *Proc. ACM Mobihoc'00*, Aug. 2000.

[3] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 3, pp. 535–547, Mar. 2000.

[4] M. M. Carvalho and J. J. Garcia-Luna-Aceves, "A scalable model for channel access protocols in multihop ad hoc networks," in *Proc. ACM Mobicom'04*, Sept. 2004.

[5] G. D. Celik, "Distributed MAC protocol for networks with multipacket reception capability and spatially distributed nodes," Master's thesis, MIT, 2007. [Online]. Available: http://web.mit.edu/gcelik/MSThesis/Thesis.pdf

[6] I. Chlamtac and A. Farago, "An optimal channel access protocol with multiple reception capacity," *IEEE Trans. Comput.*, vol. 43, no. 4, pp. 480–484, Apr. 1994.

[7] J. del Prado Pavon, S. Shankar N, V. Gaddam, K. Challapali, and C.-T. Chou, "The MBOA-WiMedia specification for ultra wideband distributed networks," *IEEE Commun.*, vol. 44, no. 6, pp. 128–134, June 2006.

[8] J. Garcia-Luna-Aceves, H. Sadjadpour, and Z. Wang, "Challenges: Towards truly scalable ad hoc networks," in *Proc. ACM Mobicom'07*, Sept. 2007.

[9] M. Garetto, T. Salonidis, and E. Knightly, "Modeling per-flow throughput and capturing starvation in CSMA multi-hop wireless networks," in *Proc. IEEE INFOCOM'06*, Apr. 2006.

[10] S. Ghez, S. Verdu, and S. C. Schwartz, "Stability properties of Slotted Aloha with multipacket reception capability," *IEEE Trans. Autom. Control*, vol. 33, no. 7, pp. 640–649, July 1988.

[11] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[12] Z. Hadzi-Velkov and B. Spasenovski, "On the capacity of IEEE 802.11 DCF with capture in multipath-faded channels," *Int. J. of Wireless Inf. Net.*, vol. 9, no. 3, pp. 191–199, July 2002.

[13] B. Hajek, A. Krishna, and R. O. LaMaire, "On the capture probability for a large number of stations," *IEEE Trans. Commun.*, vol. 45, no. 2, pp. 254–260, Feb. 1997.

[14] A. Krishna and R. O. LaMaire, "A comparison of radio capture models and their effect on wireless lan protocols," in *Proc. IEEE ICUPC'94*, Sept. 1994.

[15] C. T. Lau and C. Leung, "Capture models for model packet radio networks," *IEEE Trans. Commun.*, vol. 40, no. 5, pp. 917–925, May 1992.

[16] N. Likhanov, E. Plotnik, Y. Shavitt, M. Sidi, and B. Tsybakov, "Random access algorithms with multiple reception capability and N-Ary feedback channel," *Problemy Peredachi Informatsii*, vol. 29, no. 1, pp. 82–91, 1993.

[17] J. P. Linnartz, *Narrowband Land-Mobile Radio Networks*. Norwood, MA: Artech House, Inc., 1993.

[18] M. Manshaei, G. R. Cantieni, C. Barakat, and T. Turletti, "Performance analysis of the IEEE 802.11 MAC and physical layer protocol," in *Proc. IEEE WoWMoM'05*, 2005.

[19] G. Mergen and L. Tong, "Receiver controlled medium access in multihop ad hoc networks with multipacket reception," in *Proc. IEEE MILCOM'01*, Oct. 2001.

[20] G. Nguyen, A. Ephremides, and J. Wieselthier, "Comments on "capture and retransmission control in mobile radio"," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 12, pp. 2340–2341, Dec. 2006.

[21] A. Nyandoro, L. Libman, and M. Hassan, "Service differentiation using the capture effect in 802.11 wireless LANs," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2961–2971, Aug. 2007.

[22] M. C. H. Peh, S. V. Hanly, and P. Whiting, "Random access with multipacket reception over fading channels," in *Proc. Australian Commun. Theory Workshop'03*, Feb. 2003.

[23] R. Rom and M. Sidi, *Multiple Access Protocols: Performance and Analysis*. New York: Springer-Verlag, 1990.

[24] A. L. Toledo, T. Vercauteren, and X. Wang, "Adaptive optimization of IEEE 802.11 DCF based on bayesian estimation of the number of competing terminals," *IEEE Trans. Mobile Comput.*, vol. 5, no. 9, pp. 1283–1296, Sept. 2006.

[25] L. Tong, Q. Zhao, and G. Mergen, "Multipacket reception in random access wireless networks: From signal processing to optimal medium access control," *IEEE Commun.*, vol. 39, no. 11, pp. 108–112, Nov. 2001.

[26] Q. Zhao and L. Tong, "A multiqueue service room MAC protocol for wireless networks with multipacket reception," *IEEE/ACM Trans. Netw.*, vol. 11, no. 1, pp. 125–137, Feb. 2003.

[27] M. Zorzi and R. Rao, "Capture and retransmission control in mobile radio," *IEEE J. Sel. Areas Commun.*, vol. 12, no. 8, pp. 1289–1298, Oct. 1994.