# Cross-Layer Survivability in WDM Networks with Multiple Failures

**Kayi Lee and Eytan Modiano**[1]
*Massachusetts Institute of Technology, Cambridge, MA, USA*
*kylee@mit.edu, modiano@mit.edu*

**Abstract:** We study the survivable lightpath routing problem in the context of multiple failures. We define network metrics to quantify the resilience of lightpath routings, and propose lightpath routing algorithms to maximize network survivability.

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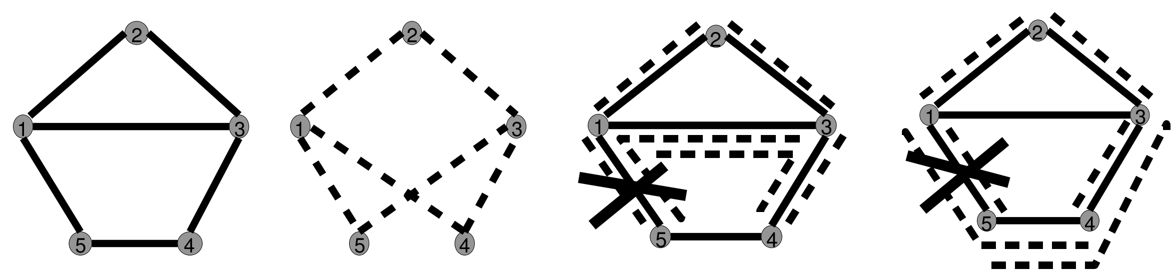
## 1. Cross-Layer Survivability

Survivable routing in Wavelength Division Multiplexing (WDM) networks focuses on designing lightpath routing strategies to improve cross-layer survivability. The primary goal is to route the lightpaths that constitute the logical topology over the fiber network so that fiber failures will not disconnect the logical topology.

Figure 1 gives an example of survivable lightpath routing. Figures 1(a) and (b) show the physical and logical topologies on a 5-node network, and Figures 1(c) and (d) show two different lightpath routings of the logical topology over the physical topology. In Figure 1(c), a failure of physical fiber $(1,5)$ will disconnect lightpaths $(1,5)$ and $(3,5)$. Consequently, node 5 will be isolated from other nodes in the logical topology. On the other hand, in Figure 1(d), the logical topology will remain connected even if one of the fibers fails. This shows that routing lightpaths intelligently can improve the survivability of the logical network.

Previous works on cross-layer survivability focused on single fiber failures [1, 2]. The notion of survivable lightpath routing, introduced in [2], is defined to be a mapping between the logical topology and physical topology such that any single physical link failure leaves the logical network connected. In this paper, we extend the notion of cross-layer survivability to the realm of multiple fiber failures.




(a) Physical Topology (b) Logical Topology (c) Unsurvivable Routing (d) Survivable Routing

Fig. 1. Different lightpath routings can result in various degrees of survivability

## 2. Metrics for Cross-Layer Survivability

Previous work on cross-layer survivability only considered **a single fiber failure**. In order to extend this notion we develop new metrics that measure the resilience of the network to **multiple fiber failures**.

### 2.1 The Min Cross Layer Cut

Given a lightpath routing, the *Min Cross Layer Cut* is the minimum number of fiber failures required to disconnect the logical topology. A routing with high Min Cross Layer Cut suggests that the network remains connected even after a relatively large number of fiber failures. This is a natural generalization of the survivable lightpath routing definition, since a lightpath routing is survivable if and only if its Min Cross Layer Cut is greater than 1.

The Min Cross Layer Cut for a lightpath routing can be formulated as an integer program. Denote the physical and logical topologies as $(V_P, E_P)$, and $(V_L, E_L)$ respectively, where $V_i$ represents the set of nodes and $E_i$

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represents the set of edges in the respective network. Let $k_{ij}^{st}$ be a binary variable representing the lightpath routing, such that logical edge $(s,t)$ uses physical link $(i,j)$ if and only if $k_{ij}^{st}$ = 1. The Min Cross Layer Cut can be obtained by solving the following integer program:

$$\mathsf{ILP_{MCLC}}: \quad \text{Minimize} \sum_{(i,j)\in E_P} y_{ij}, \quad \text{subject to:}$$

$$d_t - d_s \leq \sum_{(i,j)\in E_p} y_{ij} k_{ij}^{st} \qquad \forall (s,t) \in E_L$$

$$d_0 = 0, \sum_{n \in V_L} d_n \geq 1$$

$$d_n, y_{ij} \in \{0,1\} \qquad \forall n \in V_L, (i,j) \in E_P$$

In $\mathsf{ILP_{MCLC}}$, variables $y_{ij}$ indicate whether fiber $(i,j)$ belongs to the Min Cross Layer Cut. Variables $d_n$ represent whether node $n$ is disconnected from node 0 after removing the fibers in the Min Cross Layer Cut.

**Theorem 1** *Computing the Min Cross Layer Cut of a lightpath routing is NP-hard.*

The above Theorem implies that the survivable routing problem with a goal of maximizing the Min Cross Layer Cut is likely to be a computationally complex problem. In the next section we will describe efficient algorithms based on a multi-commodity flow formulation.

### *2.2 The Min Weighted Load Factor*

The *Min Weighted Load Factor (MWLF)* is an extension of the metric *Load Factor* introduced in [3].

$$MWLF = \max_{w} \min_{S \subset N_L, (i,j)\in E_P} \frac{\sum_{(s,t)\in\delta(S)} w_{st}}{\sum_{(s,t)\in\delta(S)} w_{st} k_{ij}^{st}}, \quad \text{where} \qquad (1)$$

- $w$ is a weight function of $E_L$, subject to the constraint $\sum_{(s,t)\in\delta(S)} w_{st} > 0$ for all cut sets $\delta(S), S \subset N_L$.
- $k_{ij}^{st}$ is a binary variable which equals 1 if and only if fiber $(i,j)$ carries lightpath $(s,t)$.

The idea behind the Min Weighted Load Factor is that a good lightpath routing should spread the load across fibers as much as possible, in order to minimize the impact of each fiber failure. If we interpret the weight function $w$ as traffic on the logical network, the Min Weighted Load Factor measures the maximum fraction of traffic across any cut that can potentially be affected by a fiber failure. In particular, if a lightpath routing is not survivable, there is a fiber in the network whose failure will disconnect the logical topology, and the Min Weighted Load Factor would be 1.

Our next result shows that the Min Weighted Load Factor is a good approximation of the Min Cross Layer Cut. Let $OPT_{MCLC}$ and $OPT_{MCLC}^R$ be the optimal objective values for $\mathsf{ILP_{MCLC}}$ and its relaxation respectively. Also, let $OPT_{MWLF}$ be the Min Weighted Load Factor of the lightpath routing. We have the following relationship:

**Theorem 2** $OPT_{MCLC}^R \leq OPT_{MWLF} \leq OPT_{MCLC}$.

## 3. Lightpath Routing Algorithm to Minimize Min-Cross Layer Cut

We propose a multi-commodity flow based lightpath routing algorithm to maximize the Min Cross Layer Cut. Multi-commodity flow has been used to solve the Routing and Wavelength Assignment (RWA) problem of WDM networks. Here we use multicommdity flow as a framework to design survivable lightpath routing algorithms. The novelty is to apply weights $w_{st}$ to the lightpaths to capture the connectivity structure of the physical and logical topologies with a polynomial number of constraints. The algorithm finds a lightpath routing by solving the following multi-commodity flow problem:

$$\mathsf{ILP_{MCF_w}}: \quad \text{Minimize } \rho, \quad \text{subject to:}$$

$$\rho \geq \sum_{(s,t)\in E_L} w_{st} f_{ij}^{st} \qquad \forall (i,j) \in E_P$$

$$\{f_{ij}^{st} : (i,j) \in E_P\} \text{ is an } (s,t)\text{-flow of size 1} \qquad \forall (s,t) \in E_L,$$

where $E_P$ and $E_L$ are edges in the physical and logical networks respectively, $w$ is a function on $E_L$, and $f_{ij}^{st}$ is a binary variable which equals 1 if and only if lightpath $(s,t)$ uses fiber $(i,j)$.

The formulation $\mathsf{ILP_{MCF_w}}$ minimizes the maximum weight carried by a fiber. Note that when the weight of all lightpaths is 1, the above formulation reduces to the traditional RWA problem where the goal is to minimize the number of lightpaths traversing a fiber.

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We propose a different weight function $w_{MC}$ for $\mathsf{ILP}_{\mathsf{MCF_w}}$ that captures the connectivity of the logical network. For each edge $(s,t) \in E_L$, we define $w_{MC}(s,t) = \frac{1}{MinCut_L(s,t)}$, where $MinCut_L(s,t)$ is the minimum $(s,t)$-cut in the logical topology. As a result, an edge that belongs to a smaller cut will be assigned a higher weight. The algorithm will therefore try to avoid routing these small cut edges on the same fiber. The following Theorem implies that the formulation $\mathsf{ILP}_{\mathsf{MCF_w}}$ with weight function $w_{MC}$ maximizes a lower bound for the Min Cross Layer Cut.

**Theorem 3** *Let $\rho^*$ be the optimal objective value for $\mathsf{ILP}_{\mathsf{MCF_w}}$ using weight function $w_{MC}$. Then, $\frac{1}{\rho^*}$ gives a lower bound for the Min Weighted Load Factor.*

## 4. Simulation

To evaluate the performance of algorithm $\mathsf{MCF}_{\mathsf{MinCut}}$, we have generated 50 instances in our simulation. For each instance, we generate a pair of random 13-node physical and logical topologies, each with global connectivity at least 5. We run three different lightpath routing algorithms and compute the Min Cross Layer Cut for each:

- SP: Each lighpath is routed using the path with minimum number of hops on the physical network.
- $\mathsf{MCF}_{\mathsf{Identity}}$: The lightpath routing is based on $\mathsf{ILP}_{\mathsf{MCF_w}}$, with weight $w_{st} = 1$ for all logical link $(s,t)$.
- $\mathsf{MCF}_{\mathsf{MinCut}}$: The lightpath routing is based on $\mathsf{ILP}_{\mathsf{MCF_w}}$, using the weight function $w_{MC}$ defined in Section 3.

Table 1 summarizes the survivability performance of the algorithms. Overall, algorithm $\mathsf{MCF}_{\mathsf{MinCut}}$ does best in maximizing Min Cross Layer Cut, whereas SP gives the worst performance. Compared with SP, $\mathsf{MCF}_{\mathsf{MinCut}}$ performs better in 35 instances, equally well in 13 instances, and worse in 2 instances. Compared with $\mathsf{MCF}_{\mathsf{Identity}}$, $\mathsf{MCF}_{\mathsf{MinCut}}$ performs better in 19 instances, equally well in 27 instances, and worse in 4 instances.

| Min Cross Layer Cut | | | Number of Instances |
|---|---|---|---|
| $\mathsf{MCF}_{\mathsf{MinCut}}$ | $\mathsf{MCF}_{\mathsf{Identity}}$ | SP | |
| 3 | 3 | 2 | 3 |
| 3 | 3 | 3 | 9 |
| 3 | 3 | 4 | 1 |
| 3 | 4 | 3 | 3 |
| 3 | 4 | 4 | 1 |
| 4 | 3 | 2 | 2 |
| 4 | 3 | 3 | 11 |
| 4 | 4 | 2 | 5 |
| 4 | 4 | 3 | 8 |
| 4 | 4 | 4 | 1 |
| 5 | 3 | 3 | 3 |
| 5 | 4 | 3 | 3 |

Table 1. Survivability Performance of Algorithms $\mathsf{MCF}_{\mathsf{MinCut}}$, $\mathsf{MCF}_{\mathsf{Identity}}$, and SP

The multi-commodity flow based algorithms $\mathsf{MCF}_{\mathsf{Identity}}$ and $\mathsf{MCF}_{\mathsf{MinCut}}$ perform better than SP because the algorithms try to spread the lightpaths across different fibers. This effectively minimizes the impact of any fiber failure. In addition, choosing a good weight function for $\mathsf{ILP}_{\mathsf{MCF_w}}$ leads to a better result. In the case of $\mathsf{MCF}_{\mathsf{MinCut}}$, the weight function $w_{MC}$ captures the connectivity structure of the logical topology, which allows the algorithm to diversely route lighpaths that belong to small cuts.

We also compared the values of the Min Cross Layer Cut and the Min Weighted Load Factor in our simulation results. Out of the 150 lightpath routings generated, there are only 13 instances where the Min Weighted Load Factor is smaller than the Min Cross Layer Cut. This indicates that the Min Cross Layer Cut and the Min Weighted Load Factor are closely connected.

## 5. References


[1] O. Crochat and J.-Y. Le Boudec. Design protection for WDM optical networks. *IEEE Journal on Selected Areas in Communications*, 16(7), September 1998.

[2] Eytan Modiano and Aradhana Narula-Tam. Survivable lightpath routing: A new approach to the design of wdm-based networks. *IEEE Journal on Selected Areas in Communications, vol. 20, no. 4, pp. 800-809, May 2002.*, 2002.

[3] Daniel Dao-Jun Kan. Design of survivable ip-over-wdm networks: Providing protection and restoration at the electronic layer. *Master's Thesis, Massachusetts Institute of Technology*, June 2003.