RWA decompositions for optimal throughput in reconfigurable optical networks

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1. Introduction

We consider an optical networking architecture with each node consisting of an electronic router (to model Internet Protocol [IP] functionality) overlaying optical crossconnect (OXC), with optical add/drop multiplexers (ADM) interfacing the optical and electronic layers, as depicted in Figure 1. Nodes are interconnected with optical fiber links, and the graph representation of these interconnections is termed the *physical topology*. OXC's and ADM's allow each node to source, terminate, or bypass a wavelength signal. Thus, a path across multiple fibers can be established for information flow between two nodes entirely at the optical layer, with conversion to and from the electronic layer only occuring at the source and termination of the path. Such a path is termed a *lightpath*. The graph consisting of the *logical* interconnections (source-destination pairs) implied by all active lightpaths is termed the *logical topology* of the network. In order to deal effectively with time-varying traffic, we equip our network with configurable components. In particular, we model each node with tunable transceivers and configurable optical switches. These tunable components introduce the flexibility to dynamically configure the logical topology of the network in response to variations in electronic layer queue occupancies. As a result, we can improve the throughput properties of the network by using algorithms employing joint optical layer reconfiguration and electronic layer routing.

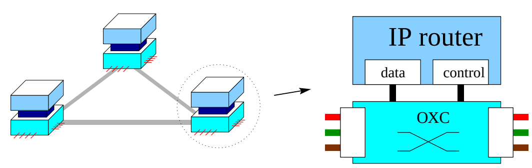

Figure 1: Optical networking architecture, with each node consisting of an optical crossconnect and an electronic router. The network at the left is a 3-node ring physical topology.

The seminal work of Tassiulas and Ephrimedes underlies much of the existing literature in the area of stability of communication networks [3]. Indeed, the network model considered in this paper easily fits into the framework of Tassiulas and Ephrimedes, as does much of the switch scheduling literature. A major accomplishment of this work is a characterization of the capacity region for single-wavelength optical networks through a linkage to the Routing and Wavelength Assignment (RWA) problem for WDM networks. This characterization allows us to derive fundamental geometric properties of the maximum stability region for optical networks of arbitrary topologies. In particular, we consider single-wavelength optical networks. The single wavelength topology is commonly used in traditional metropolitan and access networks operating on one frequency (*e.g.* 1.3nm systems). Moreover, our single-wave-length treatment simplifies the presentation considerably and can be extended, by appropriate scaling of the stability region, to multi-wavelength optical networks.

Our work is conceptually related to Birkhoff-von Neumann (BvN) decompositions, particularly as applied to switching theory [1]. The set of switch configurations (or *service configurations*) available to an $N \times N$ input-queued switch is typically represented by the set of permutation matrices of size N. The result of [3] implies that the convex hull of these service configurations equals the maximum stability region of the input-queued switch. BvN decompositions draw on these concepts to express any stabilizable rate matrix as a convex combination of permutation matrices (service configurations) [1]. Like BvN decompositions for input-queued switches, our work seeks to express any stabilizable rate matrix as a convex combination of service configurations. Unlike input-queued switches, our optical networking architecture has physical constraints, such as port and wavelength limitations, that affect the set of service configurations. For example, the set of service configurations may not include the full set of permutation matrices, and may include non-permutation matrices. Thus, while the work of [3] allows us to express the capacity region as the convex hull of available service configurations, this description can have limited value in providing an understanding of the geometric properties of the capacity region. This is in contrast to the case of the input-queued switch, where a result of Birkhoff has been applied to demonstrate that the convex hull of the service matrices equals the doubly substochastic region [2]. Here, we develop a theory of *RWA decompositions* that enables us to elicit geometric properties of the capacity region of general single-wavelength optical networks.

2. Key results

Here, we demonstrate that in any optical network having a single wavelength per physical fiber link, the question of stability for a particular arrival rate matrix can be directly tied to the RWA problem on the same physical topology graph. Note that our work considers stability properties of single-wavelength optical networks. Yet, we use properties of the RWA for multi-wavelength optical networks to characterize the stability region of single-wavelength optical networks. We directly relate the RWA problem with no wavelength conversion to the set of achievable rates using only single-hop electronic routing, and the RWA problem with wavelength conversion to the set of achievable rates using multi-hop electronic layer routes. Let the number of nodes in the network equal N.

The routing and wavelength assignment (RWA) problem takes as input a physical topology $\mathcal{P}$ and the integer traffic matrix T, corresponding to wavelength demands that are to be fully satisfied by a static lightpath configuration on $\mathcal{P}$. The output of the RWA problem is an integer W, which is the minimum number of wavelengths required to service T on physical topology $\mathcal{P}$. We consider two versions of the RWA problem: RWA with no wavelength conversion capability and RWA with full wavelength conversion capability. In the case of no wavelength conversion capability, the RWA is subject to the *wavelength continuity constraint*, which requires that no lightpath makes use of more than a single color from its source to its destination. In this case, for the particular physical topology $\mathcal{P}$, let $W_{\mathcal{P}}^{\mathrm{nc}}(T)$ be the minimum number of wavelengths required to service traffic T with no wavelength conversion. A network node having full wavelength conversion capability can transform any pass-through lightpath, in the optical domain, from its incident wavelength to any other wavelength. In this case, we define $W_{\mathcal{P}}^{\mathrm{c}}(T)$ to be the minimum number of wavelengths required to service traffic T with wavelength conversion on physical topology $\mathcal{P}$.

In the RWA problem, multiple *single-wavelength* logical configurations are *multiplexed* through the use of frequency division (WDM). In our reconfigurable network setting, restricted to a single wavelength per optical fiber, multiple single-wavelength logical configurations are multiplexed through the use of time division (by enabling logical reconfiguration and adjustable electronic-layer routing over time). Through careful interchange of time and frequency, we can conceptually link the RWA problem to the stability issue in our reconfigurable network. Consequently, we can demonstrate how to transform a RWA for a particular wavelength traffic T into a sequence of arrival rate matrices belonging to the capacity region of physical topology $\mathcal{P}$, when $\mathcal{P}$ has a single wavelength per optical fiber. We call this transformation a *RWA decomposition*.

Denote the maximum stability region of arrival rates that can be supported on network $\mathcal{P}$ with joint WDM reconfiguration and multi-hop electronic routing as $\Lambda_{\mathcal{P}}^{\mathrm{mh}}$, and that with exclusively single-hop electronic layer routing as $\Lambda_{\mathcal{P}}^{\mathrm{sh}}$. Using RWA decompositions with no conversion, the integer traffic demand matrix T can be translated to a sequence of arrival rates belonging to $\Lambda_{\mathcal{P}}^{\mathrm{sh}}$. Here, we consider all such arrival rates, gathered over all possible integer traffics T in the RWA problem. Defining $\mathcal{R}_{\mathcal{P}}^{\mathrm{nc}}$ as the set of all such arrival rates, we can show

$$\mathcal{R}_{\mathcal{P}}^{\mathrm{nc}} = \left\{ \lambda = \frac{1}{W} T : T \in \mathbb{Z}_+^M, W \in \mathbb{Z}_+, W \geq W_{\mathcal{P}}^{\mathrm{nc}}(T) \right\}, \tag{1}$$

where $\mathbb{Z}_+$ is the set of non-negative integers, $\mathbb{Z}_+^M$ is the M-fold Cartesian product of $\mathbb{Z}_+$, and $M = N(N-1)$. Similarly, using RWA decompositions with conversion, we can define the set $\mathcal{R}_{\mathcal{P}}^{\mathrm{c}}$.

Theorem 1 *The maximum single-hop stability region is given by the closure of $\mathcal{R}^{\mathrm{nc}}_{\mathcal{P}}$: $\Lambda^{\mathrm{sh}}_{\mathcal{P}} = \mathrm{cl}(\mathcal{R}^{\mathrm{nc}}_{\mathcal{P}})$. The maximum multi-hop stability region is given by the closure of $\mathcal{R}^{\mathrm{c}}_{\mathcal{P}}$: $\Lambda^{\mathrm{mh}}_{\mathcal{P}} = \mathrm{cl}(\mathcal{R}^{\mathrm{c}}_{\mathcal{P}})$.*

Drawing on this connection to the RWA problem, we can then prove the following geometric properties of the maximum stability regions. The first geometric property of interest is the maximum uniform arrival rate θ at every source-destination pair that can be stabilized on $\mathcal{P}$. Let J be the $N \times N$ matrix having each element equal to unity.

Theorem 2 *For physical topology $\mathcal{P}$, the maximum stabilizable uniform arrival rates $\theta^{\mathrm{sh}}_{\mathcal{P}}, \theta^{\mathrm{mh}}_{\mathcal{P}}$, corresponding respectively to exclusively single-hop and to multi-hop routing are given by*

$$\theta^{\mathrm{sh}}_{\mathcal{P}} = \sup_{l \in \mathbb{Z}_+} \frac{l}{W^{\mathrm{nc}}_{\mathcal{P}}(lJ)}, \;\; \theta^{\mathrm{mh}}_{\mathcal{P}} = \sup_{l \in \mathbb{Z}_+} \frac{l}{W^{\mathrm{c}}_{\mathcal{P}}(lJ)}. \tag{2}$$

Next, we consider the maximum scaling on the doubly substochastic region, such that the scaled region is a subset of the stability region of $\mathcal{P}$. The integer matrix $T \in \mathbb{Z}^M$ is called k-allowable if it satisfies $T_{i,j} \in \mathbb{Z}_+, \forall i,j,\; \sum_j T_{i,j} \le k, \forall\, i,\; \sum_i T_{i,j} \le k, \forall j$. We denote by $\mathcal{K}_k$ the set of all k-allowable matrices. Let $\mathcal{W}^{\mathrm{nc}}_{\mathcal{P}}(k) = \max_{T \in \mathcal{K}_k} W^{\mathrm{nc}}_{\mathcal{P}}(T)$, and let $\mathcal{W}^{\mathrm{c}}_{\mathcal{P}}(k)$ be defined similarly. Let the set $\mathcal{D}_s$ denote the doubly substochastic region, scaled by factor s,

$$\mathcal{D}_s = \Big\{ \lambda \in \mathbb{R}^M : \lambda_{i,j} \ge 0\, \forall i,j,\; \textstyle\sum_j \lambda_{i,j} \le s\, \forall i,\; \sum_i \lambda_{i,j} \le s\, \forall j \Big\}. \tag{3}$$

Theorem 3 *Achievability: $\mathcal{D}_{\alpha^{\mathrm{sh}}_{\mathcal{P}}} \subseteq \Lambda^{\mathrm{sh}}_{\mathcal{P}}$ and $\mathcal{D}_{\alpha^{\mathrm{mh}}_{\mathcal{P}}} \subseteq \Lambda^{\mathrm{mh}}_{\mathcal{P}}$, where*

$$\alpha^{\mathrm{sh}}_{\mathcal{P}} = \limsup_{k \to \infty} \frac{k}{\mathcal{W}^{\mathrm{nc}}_{\mathcal{P}}(T)}, \;\; \alpha^{\mathrm{mh}}_{\mathcal{P}} = \limsup_{k \to \infty} \frac{k}{\mathcal{W}^{\mathrm{c}}_{\mathcal{P}}(T)}. \tag{4}$$

Converse: For $\alpha > \alpha^{\mathrm{sh}}_{\mathcal{P}}$, there exists $\lambda \in \mathcal{D}_\alpha$ such that $\lambda \notin \Lambda^{\mathrm{sh}}_{\mathcal{P}}$. Similarly, for $\alpha > \alpha^{\mathrm{mh}}_{\mathcal{P}}$, there exists $\lambda \in \mathcal{D}_\alpha$ such that $\lambda \notin \Lambda^{\mathrm{mh}}_{\mathcal{P}}$.

Using the above results, we can determine from the RWA literature exact geometric properties of single-wavelength tree, ring, torus, and hypercube networks. As an example, we reproduce the result for bidirectional rings in Table 1. Note that the quantities $\theta^{\max}_{\mathcal{P}}, \alpha^{\max}_{\mathcal{P}}$ represent the wavelength-unconstrained maximum uniform arrival rate and maximum scaling on the doubly substochastic region, respectively. Our results provide a measure of the throughput penalty of employing single-hop versus multi-hop routing, and also of wavelength-constrained versus unconstrained networks.

Table 1: Geometric properties for the bidirectional ring (N even)

$W^{\mathrm{nc}}_{\mathcal{P}}(lJ) = W^{\mathrm{c}}_{\mathcal{P}}(lJ)$	$\theta^{\mathrm{sh}}_{\mathcal{P}} = \theta^{\mathrm{mh}}_{\mathcal{P}}$	$\theta^{\mathrm{mh}}_{\mathcal{P}}/\theta^{\max}_{\mathcal{P}}$	$\mathcal{W}^{\mathrm{nc}}_{\mathcal{P}}(k)$	$\alpha^{\mathrm{sh}}_{\mathcal{P}}$	$\mathcal{W}^{\mathrm{nc}}_{\mathcal{P}}(k)$	$\alpha^{\mathrm{mh}}_{\mathcal{P}}$	$\alpha^{\mathrm{mh}}_{\mathcal{P}}/\alpha^{\max}_{\mathcal{P}}$
$\lceil lN^2/8 \rceil$	$8/N^2$	$2/(N-1)$	$\lceil Nk/3 \rceil$	$3/N$	$\lceil Nk/4 \rceil$	$4/N$	$2/N$

References

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