Physical Topology Design for Survivable Routing of Logical Rings in WDM-based Networks

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***Abstract*— In a WDM-based network, a single physical link failure may correspond to multiple logical link failures. As a result, 2-connected logical topologies, such as rings routed on a WDM physical topology, may become disconnected after a single physical link failure. We consider the design of physical topologies that ensure logical rings can be embedded in a survivable manner. First, we develop necessary conditions on the physical topology to be able to embed all logical rings in a survivable manner. We then use these conditions to provide lower bounds on the number of physical links that an N-node physical topology must have in order to support all logical rings for even sizes K. For example, we show that when $K \geq 4$ the physical topology must have at least $4N/3$ links, and that when $K \geq 6$ the physical topology must have at least $3N/2$ links, and when $K \geq 8$ the physical topology must have at least $1.6N$ links. Furthermore, we show that for $K \geq N-2$ the physical topology must have at least $2N-4$ links. Finally, we design a physical topology that meets the above bound for $K = N-2$. We then modify this physical topology to embed rings of size $K = N-1$ and $K = N$.**

I. Introduction

The ring is the simplest network topology that remains connected in the event of a single link failure. Hence it is also one of the most widely used topologies. In Wavelength Division Multiplexed (WDM) networks, logical ring topologies (nodes connected by lightpaths) can be embedded on arbitrary connected physical topologies (nodes connected by fiber links) to provide protection at the logical layer against physical layer failures. However, since multiple logical links can be carried by a single physical link, the failure of a single fiber link can lead to multiple logical link failures. Hence, although both the logical and physical topologies are independently tolerant to single link failures, once the logical topology is embedded on the physical topology, the logical topology may no longer be survivable to single physical link failures. In order to retain its protection capabilities, each logical ring must be routed on the physical topology in a manner that ensures that each physical link carries only one link of each logical ring [1], [8].

One of the key results observed in [1] is that for many physical topologies it is not possible to embed ring logical topologies in a survivable manner. For example, almost 50% of 9-node rings cannot be embedded in a survivable manner in the 11-node NJLATA network. Similar results were also obtained for other commonly used physical topologies. In this paper we focus on designing the physical topology so that it can support logical rings in a survivable manner. In particular, we investigate properties of physical topologies that enable multiple logical rings to be embedded in a survivable manner and use these properties to design suitable physical topologies. Such a design is particularly useful for service providers that design their network infrastructure in order to serve customer requests for lightpath connections.

While there has been a great deal of work in the area of optical layer protection [2], [3], [4], [5], [6], [7], [8], [9], this survivable routing formulation is a new approach to network protection that has significant implications on the design of future WDM-based networks. Most previous work in WDM network protection is focused on restoration mechanisms that restore all lightpaths in the event of a physical link failure. Link based restoration recovers from a link failure by restoring the failed physical link, hence simultaneously restoring all of the associated lightpaths [3], [4], [7]. This is often done using optical loop-back protection [3], [4], [6]. In contrast, path based protection restores each of the lightpaths independently, by finding an alternative end-to-end path for each lightpath [3], [4], [9]. In many cases it is indeed necessary to restore all failed lightpaths. However, in other cases some level of protection is provided in the electronic layer and restoration at the physical layer may not be necessary. For example, when the electronic layer consists of SONET rings, single link failures can be recovered through loopback protection at the electronic layer. In this case, providing protection at both the optical and electronic layers is somewhat redundant. Another less obvious example is that of packet traffic in the Internet where multiple electronic layer paths exist between the source and destination and the Internet Protocol (IP) automatically recovers from link failures by rerouting packets. In such cases, a less stringent requirement may be imposed on the network. For example, we may require that the network remain connected in the event of a physical link failure. This approach is, of course, not suitable for all situations. For instance, when a network is carrying high priority traffic with Quality of

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Service and protection guarantees, it may still be necessary to provide full restoration. However, when a network is used to support best effort internet traffic, guaranteeing connectivity may suffice.

We consider the design of N-node physical topologies that can support survivable routings of ring logical topologies of size $K \leq N$. Since rings of size 3 can be trivially embedded in a survivable manner on any 2-connected physical topology, we focus on the problem of embedding rings of size $K \geq 4$. We begin by developing necessary conditions on the physical topology for ensuring all K node ring permutations can be embedded in a survivable manner. These conditions lead to lower bound requirements on the number of physical links. We then design physical topologies that can support all ring permutations in a survivable manner using the minimum number of physical links. These designs are all hub based.

II. Lower Bounds on Physical Link Requirements

We consider a bidirectional physical topology with nodes $\mathcal{N}$ and edges $\mathcal{E}$ (we define $N \equiv |\mathcal{N}|$ as the number of nodes in the physical topology). Similarly, each bidirectional logical topology consists of a set of nodes $\mathcal{N}_L$ and edges $\mathcal{E}_L$. A cut is a partition of the set $\mathcal{N}$ into subsets $\mathcal{S}$ and $\mathcal{N} \setminus \mathcal{S}$.[1] The cut-set corresponds to the set of edges in $\mathcal{E}$ that have one endpoint in $\mathcal{S}$ and the other in $\mathcal{N} \setminus \mathcal{S}$. For any cut $\{\mathcal{S}, \mathcal{N} \setminus \mathcal{S}\}$ of the physical topology, let $|CS_P(\mathcal{S}, \mathcal{N} \setminus \mathcal{S})|$ be the number of physical links along this cut and $|CS_L(\mathcal{S}, \mathcal{N} \setminus \mathcal{S})|$ be the number of logical links traversing the same cut.

In [1] we showed that the routing of a logical topology is survivable if and only if no single physical link is shared by all logical links belonging to a cut-set of the logical topology. For ring logical topologies, each pair of logical links is a cut-set, thus no pair of logical links can be routed on the same physical link, or equivalently each logical link must be routed on a separate physical link. In order to be able to route each logical link along disjoint physical paths the number of physical links in each cut must be greater than the number of logical links in each cut, i.e., $|CS_P(\mathcal{S}, \mathcal{N} \setminus \mathcal{S})| \geq |CS_L(\mathcal{S}, \mathcal{N} \setminus \mathcal{S})|$ for each cut. Hence, as proven in [1], the routing requirement leads to the following necessary condition on a physical topology capable of embedding all possible K-node rings in in a survivable manner.

Theorem 2.1: For a physical topology to support any possible K-node ring logical topology in a survivable manner the following must hold. For any cut $\{\mathcal{S}, \mathcal{N} \setminus \mathcal{S}\}$ of the physical topology, $|CS_P(\mathcal{S}, \mathcal{N} \setminus \mathcal{S})| \geq 2 \min\{|\mathcal{S}|, |\mathcal{N} \setminus \mathcal{S}|, \lfloor K/2 \rfloor\}$.

In other words, for all cuts of the physical topology, the number of physical links in the cut set must be greater than or equal to twice the minimum of the number of nodes on the smaller side of the cut and $\lfloor K/2 \rfloor$, where $\lfloor K/2 \rfloor$ corresponds to the maximum number of nodes in a K-node ring logical topology that can be on both sides of the cut. Note that this is a necessary *but not sufficient* condition.

[1]For sets $\mathcal{A}$ and $\mathcal{B}$, the set $\mathcal{A} \setminus \mathcal{B}$ is defined as $\mathcal{A} \cap \mathcal{B}^C$, where $\mathcal{B}^C$ is the complement of the set $\mathcal{B}$

TABLE I
Lower bounds on the number of physical links required to embed logical topologies of size K.

Logical Ring Size	Physical Link Requirement	Result
$K = 4$	$4N/3$	Theorem 2.2
$K = 6$	$3N/2$	Theorem 2.2
$K = 8$	$1.6N$	Theorem 2.2
$K = 10$	$1.625N$	Theorem 2.2
$K = N - 2$	$2N - 4$	Theorem 2.3

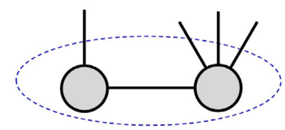


Fig. 1. Cut-set consisting of a degree 2 node connected to a degree 4 node.

Using this necessary condition we establish requirements on the physical topology which show that nodes of low degree must be connected to nodes of larger degree. These requirements are summarized in the lemmas below and are used to design appropriate physical topologies in Section III. Furthermore, using these lemmas we develop lower bounds on the number of physical links needed to embed rings of size K for even-valued $K \geq 4$. We also show in Theorem 2.3, that to embed rings of size $K = N - 2$, a minimum of $2N - 4$ physical links are needed. A summary of several of the lower bound results is given in Table I.

Lemma 2.1: Any node of degree D must have physical links to nodes having degree at least 4 for $D = 2$ and at least 3 for $D = 3$ when $K \geq 4$ and $N \geq 4$.

Proof: To prove the lemma, we consider 2-node cuts where one of the two nodes has degree D. Theorem 2.1 requires that there are at least 4 edges crossing the cut (this follows since $|\mathcal{S}| = 2$ implies $2 \min\{|\mathcal{S}|, |\mathcal{N} \setminus \mathcal{S}|, \lfloor K/2 \rfloor\} = 4$). Consider first the case $D = 2$; the other node in the cut must have degree at least 4 to satisfy this requirement. This is illustrated in Fig. 1: note that if the node of degree 2 were connected instead to a node of degree 2 or 3, then an insufficient number of links would cross the cut. Next consider the case $D = 3$. The same requirement of 4 edges crossing the cut holds here, which implies that the second node must have degree at least 3. □

Next we introduce a lemma that restricts the interconnections allowed between groups of nodes. Following the lemma, we provide three clarifying examples.

Definition 2.1: Define a *grouping* as a set of $k_2 + k_3$ nodes, k_2 of which have degree 2 and k_3 of which have degree 3. Further, these nodes may be interconnected, but each node must have at least one single link free to connect to nodes outside of the grouping. For consistency in naming, we define the degree of this object by the pair (k_2, k_3). □

Lemma 2.2: Suppose a node of degree D connects to n groupings, each of degree (k_2, k_3), in the sense that the node of degree D has a physical link to every node in each grouping.

For K and N sufficiently large,

$$n \leq \left\lfloor \frac{D-2}{2k_2+k_3} \right\rfloor. \quad (1)$$

Proof: The total number of nodes in a cut including n groupings and the node of degree D is $|\mathcal{S}| = n(k_2 + k_3) + 1$. Assume $|\mathcal{S}|$ achieves the minimum value in the set $\{|\mathcal{S}|, |\mathcal{N} \setminus \mathcal{S}|, \lfloor K/2 \rfloor\}$. Then Theorem 2.1 requires that

$$2(1 + nk_2 + nk_3) \leq |CS_P(\mathcal{S}, \mathcal{N} \setminus \mathcal{S})| \quad (2)$$

$$\leq (nk_2 + 2nk_3) + (D - nk_2 - nk_3). \quad (3)$$

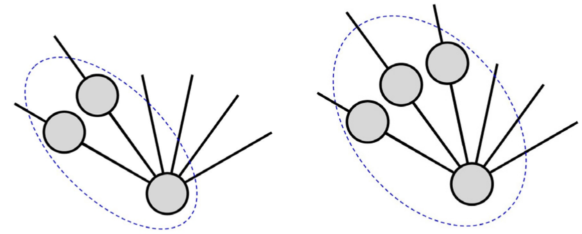
Here, the right hand side of (3) is obtained by considering the maximum number of edges crossing the cut. This value occurs when all edges, excluding the edges connecting the node of degree D to the nodes of the groupings, cross the cut. Thus, the node of degree D contributes $D - n(k_2 + k_3)$ edges that cross the cut (this is the first term in (3)). The n groupings contribute an additional $n(k_2 + 2k_3)$ edges that cross the cut, since nodes of degree 2 contribute one edge each, and nodes of degree 3 contribute 2 edges each (this is the second term in (3)). Simple algebraic manipulation of (3) yields $n \leq \frac{D-2}{2k_2+k_3}$. Since n is an integer, the bound in (1) is established.

We note that Lemma 2.2 holds for values of K and N that are sufficiently large to test for *and* to find the upper bound on n, while maintaining $|\mathcal{S}|$ as the minimizing element of the set $\{|\mathcal{S}|, |\mathcal{N} \setminus \mathcal{S}|, \lfloor K/2 \rfloor\}$. The details of the conditions on K and N are given in [10]. □

We will now provide three examples that demonstrate the usefulness and flexibility of Lemma 2.2. The first two examples consider the very important cases of groupings of degree $(1, 0)$ and $(0, 1)$. The third example serves to clarify the notion of a grouping. The examples all assume that K and N are sufficiently large for Lemma 2.2 to apply.

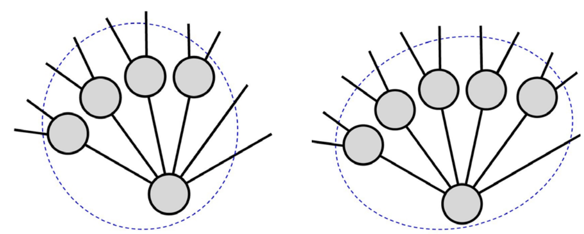
Example 2.1: Consider a grouping of degree $(1, 0)$; each grouping consists of one degree 2 node. For a node of degree $D = 6$, we will use Lemma 2.2 to determine how many of these groupings the degree 6 node may connect to without violating Theorem 2.1. In other words, we will determine the maximum number of nodes of degree 2 that may be connected to a node of degree 6 node. By Lemma 2.2, this value is $\lfloor 4/2 \rfloor = 2$. We demonstrate this bound in Fig. 2: Fig. 2(a) shows that when the degree 6 node connects to two nodes of degree 2, the minimum requirement of 6 edges may cross the cut. Fig. 2(b) adds an additional node of degree 2, which leaves a maximum of 6 edges crossing the cut (as shown). However, Theorem 2.1 requires a minimum of 8 edges crossing a cut of 4 nodes. Thus, we have shown that a degree 6 node may connect to at most two nodes of degree 2. □

Example 2.2: Consider a grouping of degree $(0, 1)$; each grouping consists of one degree 3 node. In this case, Lemma 2.2 sets a limit of $\lfloor 4/1 \rfloor = 4$ nodes of degree 3 that may be connected to a node of degree $D = 6$. This is illustrated in Fig. 3: Fig. 3(a) shows that when a degree 6 node is connected to four nodes of degree 3, the minimum requirement of 10 edges



(a) two degree 2 nodes (b) three degree 2 nodes

Fig. 2. A node of degree 6 connecting to nodes of degree 2.



(a) four degree 3 nodes (b) five degree 3 nodes

Fig. 3. A node of degree 6 connecting to nodes of degree 3.

may cross the cut. Fig. 3(b) adds a fifth degree 3 node to the cut, which leaves a maximum of 11 edges crossing the cut (as shown). However, Theorem 2.1 requires a minimum of 12 edges cross a cut of 6 nodes. Thus, we have shown that a node of degree 6 may connect to at most four nodes of degree 3. □

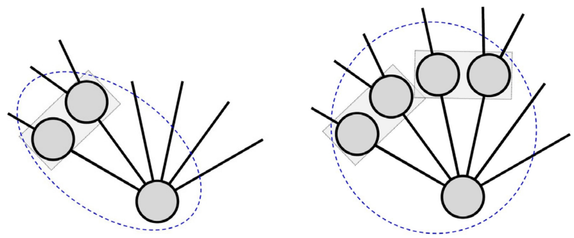
Example 2.3: The last example clarifies the notion of a grouping. Consider a grouping of degree $(1, 1)$; each grouping consists of one node of degree 2 and one node of degree 3. In this case, Lemma 2.2 sets a limit of $\lfloor 4/3 \rfloor = 1$ groupings of degree $(1, 1)$ that a node of degree $D = 6$ may connect to. This is illustrated in Fig. 4 where each grouping is shaded by a box: Fig. 4(a) shows that when the degree 6 node connects to one grouping of degree $(1, 1)$, as many as seven edges may cross the cut. Fig. 4(b) shows that when the degree 6 node connects to two groupings of degree $(1, 1)$, a maximum of eight edges cross the cut. However, Theorem 2.1 requires a minimum of 10 edges crossing a cut of 5 nodes. □

Using Lemma 2.1 and Lemma 2.2, we can establish the following theorem on the number physical links required to support rings of size K.

Theorem 2.2: Let d_i be the number of degree i nodes in the physical topology. For $K \geq 4$ even, to support all logical rings of size K, the number of physical links L must satisfy

$$L \geq \max\left\{ 2d_2 + \frac{3d_3}{2} + \sum_{\substack{i=4 \\ i \text{ even}}}^{K-1} \left(\frac{i}{4} + \frac{1}{2}\right) d_i + \sum_{\substack{i=5 \\ i \text{ odd}}}^{K-1} \left(\frac{i}{4} + \frac{3}{4}\right) d_i, \right.$$

$$\left. 2d_2 + \frac{5}{2} d_3 + \sum_{i=4}^{K/2} d_i, \quad \frac{KN}{2} - \frac{1}{2} \sum_{i=2}^{K-1} (K - i) d_i \right\}, \quad (4)$$



(a) one grouping having degree $(1, 1)$

(b) two groupings, each having degree $(1, 1)$

Fig. 4. A node of degree 6 connecting to groupings of degree $(1, 1)$.

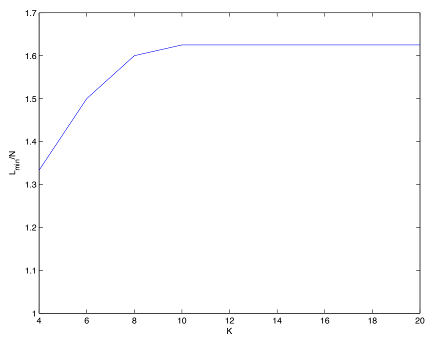


Fig. 5. Plot of lower bound on L (normalized by N) versus K.

for $N \geq K$.

Proof: The proof of Theorem 2.2 may be found in the appendix. □

Equation (4) becomes increasingly complex as K increases. As such, a linear program may be employed to minimize over (4) and establish lower bounds for any even K. Table I summarizes the lower bounds on physical link requirements L for $K = 4, 6, 8, 10$. Of note is that the bound does not change when K is increased from 10 to 12. In fact, for even values of K up to $K = 20$, no change in the lower bound occurs. The values obtained by minimizing (4) up to $K = 20$ are plotted in Fig. 5. Though the bound established in Theorem 2.2 appears to saturate for $K \geq 10$, we have not established that this bound is tight. In [10] we present a class of physical topologies that achieves the bound of $L = 4N/3$ for $K = 4$.

Theorem 2.2 establishes link requirements for embedding survivable rings of constant size K. Next we present a lower bound on link requirements for establishing rings where K is of order N. In particular we consider the case of $K = N - 2$.

Theorem 2.3: The minimum number of physical links necessary to support all logical rings of size greater than or equal to $N - 2$ in a survivable manner is $2(N - 2)$.

Proof: The proof of Theorem 2.3 proceeds by showing that for any physical topology with fewer than $2(N-2)$ links, we can find an $(N-2)$-node ring logical topology where each logical link requires at least two physical links (for a total of $2(N - 2)$ links). Hence a physical topology with fewer than $2(N - 2)$ links cannot support all $(N - 2)$-node logical topologies. The detailed proof may be found in [10]. □

The results of this section provide us with lower bounds on the number of physical links that the physical topology requires. They also give us some insights regarding the structure of the topology. In Section III we use these insights to design physical topologies that meet the above bounds.

III. Physical Topologies That Ensure Survivable Ring Routing

In this section we present two hubbed architectures that were designed based on our observation that nodes of low degree must be connected to nodes of large degree. The Dual Hub Architecture achieves the bound of Theorem 2.3 for $K = N - 2$, and with a single additional link, the Modified Dual Hub Architecture can support rings of size $K = N - 1$ and $K = N$.

Dual Hub Architecture: Consider a physical topology with N nodes, two of which are hub nodes. Each non-hub node has degree 2 and is connected to both hub nodes. The hub nodes each have degree $N - 2$. Fig. 6 depicts the physical topology for a Dual Hub Architecture having N nodes. This physical topology has $2N - 4$ links, which is the lower bound on the number of links a physical topology needs to route all logical rings of size $K = N - 2$ established in Theorem 2.3.

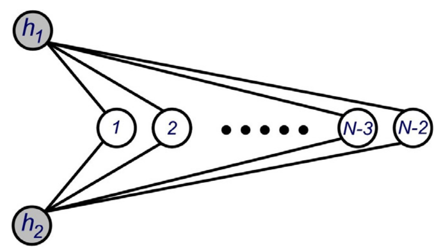


Fig. 6. Dual Hub Architecture.

The Dual Hub Architecture can support survivable routings of all logical rings of size $K = N - 2$ for N even and $K \leq N - 3$ for N odd. To show this we divide the possible logical ring configurations into three cases, corresponding to the number of hubs in the logical ring. The three different cases are considered below.

Case 3.1: Suppose we wish to route the logical ring defined by $(1, 2, \ldots, N - 2)$. That is, all non-hub nodes appear in order on the ring with no hub nodes included. Then, starting at node 1, the logical ring may be routed as follows: node 1 connects to node 2 through hub h_1, and node 2 connects to node 3 through hub h_2. We continue alternating between hub nodes in reaching the remaining nodes. Since N is even, when we reach node $N - 2$, we have reached it from hub h_1, which means we may complete the ring by traversing to node 1 through hub h_2. Clearly any ordering of the non-hub nodes in the ring can be supported. □

Case 3.2: Suppose we wish to route the logical ring defined by $(h_1, 1, 2, \ldots, N - 3)$. Here, only hub h_1 appears on the ring, followed by nodes 1 through $N - 3$ in order. Starting at hub h_1, the logical ring may be routed as follows: hub h_1

connects to node 1 directly, and node 1 connects to node 2 through hub h_2. Continuing to alternate between hub nodes in reaching the remaining nodes as before, we reach node $N-3$ from h_1. Since the direct link back to h_1 has been used, we route the last logical link from node $N-3$ through nodes h_2 and $N-2$ to hub h_1. Again, any ordering of the non-hub and hub nodes can be accomplished in a similar manner. □

Case 3.3: Suppose we wish to route the logical ring defined by $(h_1, h_2, 1, 2, \ldots, N-4)$. Here, the hubs are adjacent in the logical topology, followed by nodes 1 through $N-4$ in order. Starting at hub h_1, the logical ring may be routed as follows: hub h_1 connects to hub h_2 by traversing node $N-3$. Then, node 1 is reached directly from hub h_2, and the remaining nodes are reached in the alternating manner described in Examples 3.1 and 3.2. This implies that node $N-4$ is reached from hub h_1. Thus, the last logical link is routed from node $N-4$ through nodes h_2 and $N-2$ to hub h_1. To complete the proof we must consider the case where there are a number of non-hub nodes between the two hub nodes. The detailed proof may be found in [10]. □

Modified Dual Hub Architecture: Larger rings can be embedded survivably by modifying the Dual Hub Architecture with the addition of a single link directly joining the two hub nodes. The Modified Dual Hub Architecture with N nodes supports all $(N-1)$-node logical rings in a survivable manner. Furthermore, the Modified Dual Hub Architecture with N nodes supports all N-node logical rings in a survivable manner when N is odd. In the case of an even number of nodes, if an odd number of nodes separate h_1 and h_2 in the clockwise direction *and* in the counterclockwise direction, then the Dual Hub Architecture is sufficient to route any such logical ring of size $K = N$. In general however, when N is even, the Modified Dual Hub Architecture is not sufficient to route all rings of size N. It can be shown that adding a *second* disjointly routed physical link connecting the hub nodes allows all logical rings of size N to be embedded survivably. Detailed proofs for these results follow routing arguments similar to those provided in the examples above for the Dual Hub Architecture and may be found in [10].

IV. Symmetric Physical Topologies

Designing physical topologies to embed survivable logical rings while minimizing the number of physical links required led to the creation of physical topologies with multiple hubs. An additional property of these multiple hub topologies is that the physical topology is now also survivable to node failures. The physical network will always remain connected as long as one of the hub nodes is functioning. Hub physical topologies are generally easier to implement in local and metro area network environments. However, as the physical area of the network increases and due to other physical restrictions (such as right of ways, etc.) it may be impractical to deploy to multiple hubs. In this section we present preliminary results on the design of physical topologies that are more symmetric, *i.e.*, where the degree of each node is similar. This topic remains as an important area of future research.

In [1], a 10-node 4-connected symmetric physical topology was shown that is capable of carrying all rings of size $K \leq 9$ in a survivable manner. This physical topology contains 20 physical links and each node has degree 4. For comparison, the Dual Hub Architecture would require 16 physical links in order to carry all logical rings of size $K \leq 9$ on a 10-node physical topology. Unfortunately, it is not possible to generalize this symmetric physical topology to all values of N such that all rings of size $K \leq N-1$ can be routed survivably [10].

One method of generating physical topologies that are provably capable of supporting survivable rings is to select the physical links to form interconnected Hamiltonian cycles. For example, if the physical topology contains two interconnected Hamiltonian cycles, all rings of size $K \leq 5$ can be supported. One of the Hamiltonian cycles is used to connect the first three nodes and the second Hamiltonian cycle is used to connect the remaining two nodes in the logical topology. This is shown in Fig. 7, with only the nodes included in a 5-node logical topology shown. The solid lines represent logical links mapped on the first Hamiltonian cycle, and the dashed lines represent logical links mapped on the second Hamiltonian cycle. This results in a 4-connected physical topology which uses $2N$ physical links. Comparing this design to our lower bounds on physical links required, we have shown using Theorem 2.2 that to embed all rings of size $K \geq 6$, a minimum of $3N/2$ physical links are required. Thus, the interconnected pair of Hamiltonian cycles may not be a very efficient design.

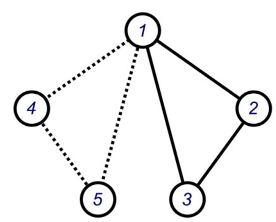


Fig. 7. Two interconnected Hamiltonian cycles can support all logical rings of size 5 in a survivable manner.

In general, designing a physical topology by interconnecting M Hamiltonian cycles results in a $2M$-connected physical topology that is capable of supporting rings of size $2M+1$ in a survivable manner.

V. Conclusions

We have considered the problem of physical topology design for embedding logical rings in a survivable manner. This problem is particularly important for service providers that design their fiber infrastructure in order to support future customer requests for lightpath connections. Since rings are a very commonly used logical topology (due to their ability to recover from failures), we focused in this paper on physical topology design for embedding ring logical topologies. Of course, a natural extension of this work is general design for arbitrary (2-connected) logical topologies.

We obtained some basic necessary conditions on the physical topology in order to be able to route logical rings in a survivable manner. We also developed lower bounds on the number of links that the physical topology must contain in order to be able to support all possible logical links of size K (for various values of K). We designed dual-hub physical topologies to suit these bounds: for the case of $K = N - 2$, the lower bounds are met exactly by the physical topology. Since hub architectures may be impractical in some scenarios, we provided preliminary results relating to physical topologies where each node has equal degree.

Appendix

Proof of Theorem 2.2

The proof of Theorem 2.2 will proceed by proving that each individual term from (4) serves as a lower bound on L. We begin by rewriting the expression for number of links in the physical topology L as

$$L = \sum_{i=1}^{K-1} \frac{id_i}{2} + \sum_{i=K}^{N-1} \frac{id_i}{2}. \tag{5}$$

Combining the fact (from Lemma 2.1) that nodes of degree 2 must have physical links to nodes of degree 4 or higher, with the bound of Lemma 2.2 for groupings of degree $(1,0)$, which restricts the number of connections a node of degree D can have to degree 2 nodes, we obtain the following restriction on the number of nodes of degree 2:

$$\begin{aligned} 2d_2 \leq\ & (d_4 + d_5) + 2(d_6 + d_7) + 3(d_8 + d_9) + \cdots \\ & + (K/2 - 2)(d_{K-2} + d_{K-1}) + \sum_{i=K}^{N-1} id_i. \end{aligned} \tag{6}$$

Equation (5) may be used to eliminate the final term of (6), which provides the first restriction on L,

$$L \geq 2d_2 + \frac{3}{2}d_3 + \sum_{\substack{i=4 \\ i \text{ even}}}^{K-1} \left(\frac{i}{4} + \frac{1}{2}\right) d_i + \sum_{\substack{i=4 \\ i \text{ odd}}}^{K-1} \left(\frac{i}{4} + \frac{3}{4}\right) d_i. \tag{7}$$

To achieve this bound, we applied Lemma 2.1 which requires $N \geq 4$, and Lemma 2.2 for $D \in \{1, 2, \ldots, K-1\}$. Using the bounds on K and N, for a grouping of degree $(1,0)$ and K even, we find that (7) holds when and $N \geq K$.

Next we establish an upper bound on the value of $2d_2 + 3d_3$. From Lemma 2.2, we have that a degree D node connects to at most

$$\left\lfloor \frac{D-2}{2k_2 + k_3} \right\rfloor (k_2 + k_3) \tag{8}$$

nodes, when these nodes necessarily belong to groupings of degree (k_2, k_3). Note that $(k_2, k_3) = (0, 1)$ maximizes (8) over all possible groupings. To prove this, suppose that (k_2^*, k_3^*) achieves the maximum in (8). If $k_2^* > 0$, then note that $(0, k_2^* + k_3^*)$ achieves a higher value by decreasing the denominator term while having no effect on the numerator terms of (8). Next, suppose $k_3^* > 1$ and $k_2^* = 0$. Then we have immediately

$$\left\lfloor \frac{D-2}{k_3^*} \right\rfloor k_3^* \leq D - 2.$$

Of course, this inequality is satisfied with equality when $k_3^* = 1$. Then the maximum number of nodes of degree 2 or 3 that a node of degree D can reach is given by $D - 2$. This implies that the following bound holds,

$$\begin{aligned} 2d_2 + 3d_3 \leq\ & d_3 + 2d_4 + 3d_5 + \cdots \\ & + (K/2 - 2)d_{K/2} + \sum_{K/2+1}^{N-1} id_i. \end{aligned} \tag{9}$$

Applying (9) to (5), we obtain the second bound on L,

$$L \geq 2d_2 + \frac{5}{2}d_3 + d_4 + d_5 + \cdots + d_{K/2}. \tag{10}$$

To achieve this bound, we employed Lemma 2.2. Since (9) is derived based on a grouping of degree $(0,1)$ and the assumption that K is even, the bounds on K and N require that $N \geq K$.

Finally, the third bound on L is obtained as follows. We lower bound the second term of (5) as

$$\sum_{i=K}^{N-1} \frac{id_i}{2} \geq K \sum_{i=K}^{N-1} \frac{d_i}{2} \tag{11}$$

Using (5) and combining the fact that $\sum_{i=1}^{N-1} d_i = N$ with (11) provides the lower bound

$$\begin{aligned} L &\geq \frac{1}{2}\sum_{i=2}^{K-1} id_i + \frac{K}{2}\left(N - \sum_{i=2}^{K-1} d_i\right) && (12) \\ &= \frac{KN}{2} - \frac{1}{2}\sum_{i=2}^{K-1}(K - i)d_i. && (13) \end{aligned}$$

The bounds of equations (7), (10), and (13) in combination correspond to the desired bound (4), which holds for all $N \geq K$, as desired. □

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