Wireless Channel Allocation Using An Auction Algorithm *

Jun Sun
LIDS, MIT
junsun@mit.edu

Lizhong Zheng
LIDS, MIT
lizhong@mit.edu

Eytan Modiano
LIDS, MIT
modiano@mit.edu

Abstract

We develop a novel auction-based algorithm to allow users to fairly compete for a wireless fading channel. We use the second-price auction mechanism whereby user bids for the channel, during each time-slot, based on the fade state of the channel, and the user that makes the higher bid wins use of the channel by paying the second highest bid. Under the assumption that each user has a limited budget for bidding, we show the existence of a Nash equilibrium strategy. And the Nash equilibrium leads to a unique allocation under certain common channel distribution. For uniformly distributed channel state, we establish that the aggregate throughput received by the users using the Nash equilibrium strategy is at least 3/4 of what can be obtained using an optimal centralized allocation that does not take fairness into account. We also show that the allocation resulted from the Nash equilibrium strategy is pareto optimal.

1 Introduction

Network resources such as bandwidth and power are often limited in wireless and satellite networks. When demand exceeds supply, it is desirable to have a systematic procedure in place for fair allocation. However, there is no consensus on the notion of fairness. Any centrally imposed notion of fairness may appear to be unfair from an individual user's perspective. In this paper, we address the problem of fair resource allocation by allowing individual users to compete for resources through bidding for the use of the channel.

A fundamental characteristic of a wireless network is that the channel over which communication takes place is often time-varying. This variation of the channel quality is due to constructive and destructive interference between multipaths and shadowing effects (fading). In a single cell with one transmitter (base station or satellite) and multiple users communicating through fading channels, the transmitter can send data at higher rates to users with better channels. In a time slotted system such as the HDR system, time slots are allocated among users according to their channel qualities.

The problem of resource allocation in wireless networks has received much attention in recent years. In [1] the authors try to maximize the data throughput of an energy and time constrained transmitter communicating over a fading channel. A dynamic programming formulation that leads to an optimal transmission schedule is presented. Other works address the similar problem, without consideration to fairness, include [7] and [8]. In [5],

*This research is supported by NASA Space Communication Project grant number NAG3-2835.

the authors consider scheduling policies for *maxmin fairness* allocation of bandwidth, which maximizes the allocation for the most poorly treated sessions while not wasting any network resources, in wireless ad-hoc networks. In [4], the authors designed a scheduling algorithm that achieves *proportional fairness*, a notion of fairness originally proposed by Kelly [6]. In [9], the authors present a slot allocation that maximizes expected system performance subject to the constraint that each user gets a fixed fraction of time slots. The authors did not use a formal notion of fairness, but argue that their system can explicitly set the fraction of time assigned to each user. Hence, while each user may get to use the channel an equal fraction of the time, the resulting throughput obtained by each user may be vastly different.

The following simple example illustrates the different allocations that may result from the different notions of fairness. We consider the communication system with one transmitter and two users, A and B, and the allocations that use different notions of fairness discussed in the previous paragraph. We assume that the throughput is proportional to the the channel condition. The channel state for user A and user B in the two time slots are (0.1, 0.2) and (0.3, 0.9) respectively (channel coefficient ranges from 0 to 1, and 1 is the best channel condition). The throughput result for each individual user and for total system under different notions of fairness constraint are given in Table I. When there is no fairness constraint, to maximize the total system throughput would require the transmitter to allocate both time slots to user B. To achieve maxmin fair allocation, the transmitter would allocate slot one to user B and slot two to user A, thus resulting in a total throughput of 0.5. If the transmitter wants to maximize the total throughput subject to the constraint that each user gets one time slot (i.e., the approach of [9]), the resulting allocation, denoted as time fraction fair, is to give user A slot one and user B slot two. As a result, the total throughput is 1.0.

	Throughput for A	Throughput for B	Total throughput
No fair constraint	0	1.2	1.2
Maxmin fair	0.2	0.3	0.5
Time fraction	0.1	0.9	1.0

Table 1: Throughput results using different notions of fairness.

In the above example, the transmitter selects an allocation to ensure an artificially chosen notion of fairness. From Table I, we can see that from the user's perspective, no notion is truly fair as both users want slot two. In order to resolve this conflict, we use a new approach which allows users to compete for time slots. In this way, each user is responsible for its own action and resulting throughput. We call the fraction of bandwidth received by each user *competitive fair*. Using this notion of competitive fairness, the resulting throughput obtained for each user can serve as a reference point for comparing various allocations.

In our model, users compete for time-slots. For each time-slot, each user has a different valuation (i.e., its own channel condition). And each user is only interested in getting a higher throughput for itself. Naturally, these characteristics give rise to an auction. In this paper we consider the second-price auction mechanism. Using the second-price auction mechanism, users submit a "bid" for the time-slot and the transmitter allocates the slot to the user that made the highest bid. Moreover, in the second-price auction mechanism, the winner only pays the second highest bid [13]. The second-price auction mechanism is used here due to its "truth telling" nature (i.e., it is optimal for user to bid its true value). Each user is assumed to have an initial amount of money. The money possessed by each user can be viewed as fictitious money that serves as a mechanism to differentiate the QoS given to the various users. This fictitious money, in fact, could correspond to a certain QoS for which the user paid in real money. As for the solution of the slot auction game, we use the concept of Nash equilibrium, which is a set of strategies (one for each player) from which there are no profitable unilateral deviation.

In this paper, we consider a communication system with one transmitter and two users. For each time slot, channel states are independent and identically distributed with known probability distribution. Each user wants to maximize its own *expected* total throughput subject to an average money constraint. Our major results include:

- We find the Nash equilibrium strategy for general channel state distribution.
- We show that the Nash equilibrium strategy pair leads to a unique allocation for certain channel state distribution, such as the exponential distribution and the uniform distribution over $[0, 1]$.
- We show that the Nash equilibrium strategy of this auction leads to an allocations at which total throughput is no worse than $3/4$ of the throughput obtained by an algorithm that attempts to maximize total system throughput without a fairness constraint under uniform distribution.
- We show that the Nash equilibrium strategy leads to an allocation that is pareto optimal (i.e., it is impossible to make some users better off without making some other users worse off).

Game theoretical approaches to resource allocation problems have been explored by many researchers recently (e.g., [2][12]). In [2], the authors consider a resource allocation problem for a wireless channel, without fading, where users have different utility values for the channel. They show the existence of an equilibrium pricing scheme where the transmitter attempts to maximize its revenue and the users attempt to maximize their individual utilities. In [12], the authors explore the properties of a congestion game where users of a congested resource anticipate the effect of their action on the price of the resource. Again, the work of [12] focuses on a wireline channel without the notion of wireless fading. Our work attempts to apply game theory to the allocation of a wireless fading channel. In particular, we show that auction algorithms are well suited for achieving fair allocation in this environment. Other papers dealing with the application of game theory to resource allocation problems include [3][16][17].

This paper is organized as follows. In Section 2, we describe the communication system and the auction mechanism. In Section 3, the Nash equilibrium strategy pair is presented for general channel distribution. We also show the uniqueness of the allocation scheme derived from the Nash equilibrium when the channel state has the exponential or the uniform $[0, 1]$ distribution. In Section 4, we compare the throughput results of the Nash equilibrium strategy with three other centralized allocation algorithms and show that the Nash equilibrium strategy leads to a pareto optimal allocation. Finally, Section 5 concludes the paper.

2 Problem Formulation

We consider a communication environment with a single transmitter sending data to two users over two independent fading channels. We assume that there is always data to be sent to the users. Time is assumed to be discrete, and the channel state for a given channel changes according to a known probabilistic model independently over time. The transmitter can transmit to only one user during a particular slot with a constant power P. The channel fade state thus determines the throughput that can be obtained.

For a given power level, we assume for simplicity that the throughput is a linear function of the channel state. This can be justified by the Shannon capacity at low signal-to-noise ratio, or by using a fixed modulation scheme [1]. For general throughput function, the method used in this paper applies as well. Let X_i be a random variable denoting the channel state for the channel between the transmitter and user i, $i = 1, 2$. When transmitting to user i, the throughput will then be $P \cdot X_i$. Without loss of generality, we assume $P = 1$ throughout this paper.

We now describe the second-price auction rule used in this paper. Let α and β be the *average* amount of money available to user 1 and user 2 respectively during each time slot. We assume that the values of α and β are known to both users. Both users know the distribution of X_1 and X_2. We also assume that the exact value of the channel state X_i is revealed to user i only at the beginning of each time slot. During each time slot, the following actions take place: 1) Each user submits a bid according to the channel condition revealed to it. 2) The transmitter chooses the one with higher bid to transmit. 3) The price that the winning user pays is the second-highest bidder's bid. The user who losses the bid does not pay.

The formulation of our auction is different from the type of auction used in economic theory in several ways. First, we look at a case where the number of object (time slots) in the auction goes to infinity (average cost criteria). While in the current auction research, the number of object is finite [13][14][15]. Second, in our auction formulation, the money used for bidding does not have a direct connection with the value of the time slot. Money is merely a tool for users to compete for time slots, and it has no value after the auction. Therefore, it is desirable for each user to spend all of its money. However, in auction theory, an object's value is measured in the same unit as the money used in the bidding process, hence their objective is to maximize the difference between the object's value and its cost. Lastly, in our formulation, the valuation of each commodity (time-slot) changes due to the fading channel model; a notion that is not common in economic theory.

Besides the second-price auction, the *first-price* auction and the *all-pay* auction are two other commonly used auction mechanisms. In the first-price auction, each bidder submits a single bid without seeing the others' bids, and the object is sold to the bidder who makes the highest bid. In the all-pay auction, each user independently submits a single bid without seeing the others' bids, and the object is sold to the bidder who makes the highest bid. However, both users pay their bid regardless of whether they win or loss [13]. We choose to use the second-price auction in this paper to illustrate the auction approach to resource allocation in wireless networks. Also, second price auction results in an allocation that is efficient. More specifically, it is pareto optimal.

The objective for each user is to design a bidding strategy, which specifies how a user will act in every possible distinguishable circumstance, to maximize its *own* expected throughput per time slot subject to the expected or average money constraint. Once a user, say user 1, chooses a function, say f_1, to be its strategy, it bids an amount of money equal to $f_1(x)$ when it sees its channel condition is $X_1 = x$.

3 Nash Equilibrium under Second-Price Auction

We present in this section a Nash equilibrium strategy pair (f_1^*, f_2^*) for the second-price bidding under general channel distribution. A strategy pair (f_1^*, f_2^*) is said to be in Nash equilibrium if f_1^* is the best response for user 1 to user 2's strategy f_2^*, and f_2^* is the best response for user 2 to user 1's strategy f_1^*. We consider here the case where users choose their strategies from the set F_1 and F_2 respectively. Each user's strategy is a function of its own channel state X_i. Thus, F_i is defined to be the set of continuous real-valued functions over the support of X_i. Without loss of generality, we further assume functions in F_i to be strictly increasing and bounded. We define $A : (x_1, x_2) \to \{1, 2\}$ to an *allocation* that maps the possible channel state realization, (x_1, x_2), to either user 1 or user 2. Here we are interested in the allocation resulted from the Nash equilibrium strategies.

We first consider a channel state X_i that is continuously distributed over a finite interval $[l_i, u_i]$ where l_i and u_i are nonnegative real number with $u_i > l_i$. Later we will consider the case that u_i is infinite (e.g., when X_i is exponentially distributed).

We now illustrate our approach in finding the Nash equilibrium strategy pair. Given user 1's strategy $f_1 \in F_1$ with range from $f_1(l_1) = a$ to $f_1(u_1) = b$, user 2 wants to maximize its own expected throughput while satisfying its expected budget constraint. For a given f_1, if user 2 chooses a bidding function f_2, the expected throughput or payoff function for user 2 is given by

$$G_2 = E_{X_1,X_2}[X_2 \cdot 1_{f_2(X_2)\geq f_1(X_1)}]$$

where

$$1_{f_2(X_2)\geq f_1(X_1)} = \begin{cases} 1 & \text{if} \quad f_2(X_2) \geq f_1(X_1) \\ 0 & \text{otherwise} \end{cases}$$

Recall that in the second-price auction rule, the price that the winner pays is actually the second highest bid. Therefore, the constraint that f_2 must satisfy is given by:

$$E_{X_1,X_2}[f_1(X_1) \cdot 1_{f_2(X_2)\geq f_1(X_1)}] = \beta$$

We first note that the inverse function $f_1^{-1}(y)$ is well defined for $y \in [a, b]$ since f_1 is strictly increasing over $[a, b]$. Therefore, we are able to define the following function:

$$h(y) = \begin{cases} l_1 & \text{if} \quad y \leq a \\ f_1^{-1}(y) & \text{if} \quad a < y < b \\ u_1 & \text{if} \quad y \geq b \end{cases}$$

Given user 1's bidding strategy f_1 and user 2's bid at a particular time slot is y, the probability that user 2 wins this slot, denoted as $P_{2\,win}(y)$, is given by $P_{2\,win}(y) = \int_{l_1}^{h(y)} p_{X_1}(x_1)\, dx_1$.

Therefore, the optimization problem that user 2 faces can be written as the following:

$$\begin{aligned} &\max \int_{l_2}^{u_2} x_2 p_{X_2}(x_2) P_{2\,win}(f(x_2))\, dx_2 = \max \int_{l_2}^{u_2} x_2 p_{X_2}(x_2) \int_{l_1}^{h(f_2(x_2))} p_{X_1}(x_1)\, dx_1\, dx_2 \\ &\text{subj. to} \int_{l_2}^{u_2} \int_{l_1}^{h(f_2(x_2))} f_1(x_1) p_{X_1}(x_1) p_{X_2}(x_2)\, dx_1\, dx_2 = \beta \end{aligned} \tag{1}$$

where the integration is over the region that user 2's bid is higher than user 1's bid. The constraint term denotes the expected money that user 2 has to pay over the region which it has a higher bid than user 1. To solve the above optimization problem, we use the optimality condition in [11]. First, we write the Lagrangian function below:

$$\begin{aligned}&\int_{l_2}^{u_2}\int_{l_1}^{h(f_2(x_2))} x_2 p_{X_1}(x_1)p_{X_2}(x_2)\,dx_1\,dx_2 - \lambda_2\big(\int_{l_2}^{u_2}\int_{l_1}^{h(f_2(x_2))} f_1(x_1)p_{X_1}(x_1)p_{X_2}(x_2)\,dx_1\,dx_2 - \beta\big)\\ &= \int_{l_2}^{u_2}\big[\int_{l_1}^{h(f_2(x_2))}(x_2-\lambda_2 f_1(x_1))p_{X_1}(x_1)\,dx_1\big]\,p_{X_2}(x_2)\,dx_2 - \lambda_2\beta\end{aligned} \tag{2}$$

We then choose a function f_2 to maximize the above equation. Also, a positive λ_2 is chosen such that the expected money constraint is satisfied. Specifically, for each value x_2, we find

$$\max_{f_2(x_2)}\int_{l_1}^{h(f_2(x_2))}(x_2-\lambda_2 f_1(x_1))p_{X_1}(x_1)\,dx_1 \tag{3}$$

For convenience, we let $y = f_2(x_2)$. Then, Eq. (3) becomes

$$\max_y L_1(y) = \int_{l_1}^{h(y)}(x_2-\lambda_2 f_1(x_1))p_{X_1}(x_1)\,dx_1. \tag{4}$$

For fixed x_2, the term $x_2 - \lambda_2 f_1(x_1)$ is a strictly decreasing function in x_1 since $f_1(x_1)$ is strictly increasing. To maximize $L_1(y)$, the optimal value y^* should be chosen such that $x_2 - \lambda_2 f_1(h(y^*)) = 0$. However, if $x_2 - \lambda_2 f_1(h(y)) > 0$ for all $y \in [a, b]$, we let $y^* = b$. Similarly, if $x_2 - \lambda_2 f_1(h(y)) < 0$ for all $y \in [a, b]$, we let $y^* = a$. Thus, the optimal bidding function has the following form

$$\begin{array}{lll} f_2(x_2) \le a & \text{for} & x_2 \in [l_2, \theta_1] \\ f_2(x_2) = c_2 \cdot x_2 & \text{for} & x_2 \in [\theta_1, \theta_2] \\ f_2(x_2) \ge b & \text{for} & x_2 \in [\theta_2, u_2] \end{array}$$

where $\theta_1, \theta_2 \in [l_2, u_2]$ and $c_2 \cdot \theta_1 = a$, $c_2 \cdot \theta_2 = b$. The above discussion states that for user 2 to maximize its throughput given user 1's strategy f_1, the optimal strategy is not unique. For $x_2 \in [l_2, \theta_1]$, as long as $f_2(x_2) \le a$, user 2 always losses the bid, and the throughput for user 2 does not change. However, from second-price bidding rule, user 2's strategy affects user 1's strategy through the expected budget constraint that user 1 must satisfy. This way, user 2 will choose $f_2(x_2) = a$ for $x_2 \in [l_2, \theta_1]$. Intuitively, even if user 2 knows that it will not win a particular time slot, it will still choose to maximize its bid in order to force user 1 to pay more.

Similarly, given user 2's bidding function f_2, we can carry out the same analysis to find that the optimal bidding function for user 1 is $f_1(x_1) = c_1 \cdot x_1$. The value of c_1 and c_2 can be chosen such that the following two constraint are both satisfied:

$$E_{X_1,X_2}[f(X_2)\cdot 1_{f_1(X_1)\ge f_2(X_2)}] = \alpha \tag{5}$$

$$E_{X_1,X_2}[f(X_1)\cdot 1_{f_2(X_2)\ge f_1(X_1)}] = \beta \tag{6}$$

The Nash equilibrium strategy discussed above in general may not be unique. However, under a continuous channel state distribution that starts with zero, such as the uniform distribution over $[0, 1]$ or the exponential distribution, the Nash equilibrium bidding strategies lead to a unique allocation. Next, we will discuss the Nash equilibrium strategy pair of these two distribution.

3.1 Uniform channel distribution

In this section, we examine the two users case with the channel state X_i uniformly distributed over $[0, 1]$. Following the approach discussed in the previous section, we find the unique allocation resulted from the Nash equilibrium strategy. Given a strategy pair (f_1^*, f_2^*) to be in Nash equilibrium, we first investigate the bids that users submit when the channel state X_i is equal to 0 (i.e., the value of $f_1^*(0)$ and $f_2^*(0)$). The result is stated in the following lemma.

Lemma 1 *For a strategy pair (f_1^*, f_2^*) to be a Nash equilibrium strategy pair, we must have $f_1^*(0) = f_2^*(0) = 0$ when the channels are uniformly distributed over $[0, 1]$.*

With the above lemma, we can get the exact form of the Nash equilibrium strategy pair.

Theorem 1 *With the channel states, X_1 and X_2, uniformly and independently distributed over $[0, 1]$, the Nash equilibrium pair (f_1^*, f_2^*) has the following form: $f_1^*(x_1) = c_1 \cdot x_1$ and $f_2^*(x_2) = c_2 \cdot x_2$ where c_1 and c_2 are chosen such that the expected money constraints are satisfied. Furthermore, any other Nash equilibrium strategies, if exist, will lead to the same allocation.*

The proof is omitted for brevity. It can be followed from the analysis in the general distribution section.

We now calculate the exact value of c_1 and c_2. Without loss of generality, we assume that user 2 has more money than user 1 (i.e., $\alpha < \beta$). Since the form of the optimal bidding strategy for both users is known, we need to get the exact value of c_1 and c_2 from the budget constraint that users must satisfy. Thus, from Eq.(5) and Eq. (6), the constraint for user 1 is given by:

$$\int_0^1 \int_0^{f_2^{-1}(f_1(x_1))} f_2(x_2)\, dx_2\, dx_1 = \alpha \Rightarrow \int_0^1 \int_0^{\frac{c_1}{c_2} x_1} c_2 \cdot x_2\, dx_2\, dx_1 = \alpha \tag{7}$$

Note that the function $f_1^{-1}(f_2(x_2))$ is well defined for $f_2(x_2) \in [0, c_1]$. Therefore, the constraint for user 1 is given by:

$$\int_0^{\frac{c_1}{c_2}} \int_0^{f_1^{-1}(f_2(x_2))} f_1(x_1)\, dx_1\, dx_2 + \int_{\frac{c_1}{c_2}}^1 \int_0^1 f_1(x_1)\, dx_1\, dx_2 = \beta \tag{8}$$

$$\int_0^{\frac{c_1}{c_2}} \int_0^{\frac{c_2}{c_1} x_2} c_1 \cdot x_1\, dx_1\, dx_2 + \int_{\frac{c_1}{c_2}}^1 \int_0^1 c_1 \cdot x_1\, dx_1\, dx_2 = \beta \tag{9}$$

Solving the two equations, we get

$$c_1 = 2(2\alpha + \beta), \quad c_2 = 2(2\alpha + \beta)^2/(3\alpha) \tag{10}$$

The throughput of each user is then given by

$$G_1 = \alpha/(\beta + 2\alpha), \quad G_2 = 1/2 - 3\alpha^2/(2(\beta + 2\alpha)^2) \tag{11}$$

Note that the linear bidding function leads to the following allocation: Given that the channel states are x_1 and x_2 during a time slot, the transmitter assigns the slot to user 1 if $x_1 \geq c \cdot x_2$, and to user 2 otherwise. As we will see later, this form of allocation leads to the pareto optimality.

3.2 Exponential distribution

When the channel state X_i is exponentially distributed with rate μ_i, the analysis in the general distribution section is still valid with minor modifications. For example, the set F_i is not assumed to be bounded over the support $[0, \infty)$. The unique Nash equilibrium strategy pair has the same form as the uniform case: $f_1^*(x_1) = c_1 \cdot x_1$ and $f_2^*(x_2) = c_2 \cdot x_2$. Using Eq.(5) and Eq.(6), we get a relationship between c_1 and c_2 to be $c_1/c_2 = (\alpha \cdot \mu_1)/(\beta \cdot \mu_2)$. Thus, the optimal allocation is given by:

$$A^*(x_1, x_2) = \begin{cases} 2 & \text{if} \quad x_2 > (c_1/c_2)x_1 \\ 1 & \text{otherwise} \end{cases}$$

Write the decision in another form $\mu_2 X_2 > (\alpha/\beta)\mu_1 X_1$. We see that only the normalized channel state distribution (i.e., $\mu_2 X_2$ and $\mu_1 X_1$) are used in the comparison. The expected throughput for each user is given by:

$$G_1 = \frac{1}{\mu_1}[1 - \frac{\beta^2}{(\alpha+\beta)^2}], \quad G_2 = \frac{1}{\mu_2}[1 - \frac{\alpha^2}{(\alpha+\beta)^2}]. \tag{12}$$

4 Comparison with Other Allocations

To this end, we have a Nash equilibrium strategy pair and the resulting throughput when both players choose to use the Nash equilibrium strategy. Inevitably, due to the fairness constraint, total system throughput will decrease as compared to the maximum throughput attainable without any fairness constraint. Hence we would like to compare the total throughput of the Nash equilibrium strategy to that of an unconstrained strategy. We address this question by first considering an allocation that maximizes total throughput subject to no constraint. Then, we investigate the throughput of another centralized allocation that maximize the total throughput subject to the constraint that the resulting throughput of individual user is kept at certain ratio. For simplicity, we let the channel state be uniformly distributed over $[0, 1]$.

4.1 Maximizing Throughput with No Constraint

To maximize throughput without any constraints, the transmitter serves the user with a better channel state during each time slot. Then the expected throughput is $E[\max\{X_1, X_2\}]$. Since X_1 and X_2 are independent uniformly distributed in $[0, 1]$, we have $E[\max\{X_1, X_2\}] = \frac{2}{3}$. Using the Nash equilibrium strategy, the total expected system throughput, $G_1 + G_2$, is $\frac{1}{2}$ in the worst case (i.e., one users gets all of the time slots while the other user is starving). *Thus, the channel allocation proposed here can achieve at least* 75 *percent of the maximum attainable throughput.* This gives us a lower bound of the throughput performance of the allocation derived from the Nash equilibrium pair.

4.2 Maximizing Throughput with Throughput Ratio Constraint

Now, we investigate an allocation with a fairness constraint that requires the resulting throughput of the users to be kept at a constant ratio. Specifically, let G_1 and G_2 denote the expected throughput for user 1 and user 2 respectively. We have the following optimization problem: for some nonnegative a,

$$\max \; G_1 + G_2, \quad \text{subj.} \; G_1/G_2 = a \tag{13}$$

The optimal allocation is to divide the possible channel state realization, (x_1, x_2), into two regions by the separation line $x_2 = c \cdot x_1$, where c is some positive real number. Above the line (i.e., $x_2 > c \cdot x_1$), the transmitter will assign the slot to user 2. Below the line (i.e., $x_2 < c \cdot x_1$), the transmitter will assign the slot to user 1.

To prove the above, we use a method that is similar to the one in [9]. By using an allocation A, the resulting throughput for user 1 and user 2 are $G_1^A = E[X_1 \cdot 1_{A(X_1,X_2)=1}]$ and $G_2^A = E[X_2 \cdot 1_{A(X_1,X_2)=2}]$ respectively. Now, we define an allocation as follows:

$$A^*(x_1, x_2) = \begin{cases} 1 & \text{if} \quad x_1(1+\lambda^*) \geq x_2(1 - a \cdot \lambda^*) \\ 2 & \text{otherwise} \end{cases}$$

where λ^* is chosen such that $G_1^{A^*}/G_2^{A^*} = a$ is satisfied.

Consider an arbitrary allocation A that satisfies $G_1^A/G_2^A = a$. We have

$$\begin{aligned}
&E[X_1 \cdot 1_{A(X_1,X_2)=1}] + E[X_2 \cdot 1_{A(X_1,X_2)=2}] \\
&= E[X_1 \cdot 1_{A(X_1,X_2)=1}] + E[X_2 \cdot 1_{A(X_1,X_2)=2}] + \lambda^*(E[X_1 \cdot 1_{A(X_1,X_2)=1}] - aE[X_2 \cdot 1_{A(X_1,X_2)=2}]) \\
&= E[(X_1 + \lambda^* X_1) \cdot 1_{A(X_1,X_2)=1}] + E[(X_2 - a\lambda^* X_2) \cdot 1_{A(X_1,X_2)=2}] \\
&\leq E[(X_1 + \lambda^* X_1) \cdot 1_{A^*(X_1,X_2)=1}] + E[(X_2 - a\lambda^* X_2) \cdot 1_{A^*(X_1,X_2)=2}] \\
&= E[X_1 \cdot 1_{A^*(X_1,X_2)=1}] + E[X_2 \cdot 1_{A^*(X_1,X_2)=2}] + \lambda^*(E[X_1 \cdot 1_{A^*(X_1,X_2)=1}] - aE[X_2 \cdot 1_{A^*(X_1,X_2)=2}]) \\
&= E[X_1 \cdot 1_{A^*(X_1,X_2)=1}] + E[X_2 \cdot 1_{A^*(X_1,X_2)=2}]
\end{aligned}$$

The inequality in the middle is from the definition of A^*. Specifically, if we were asked to choose an allocation A to maximize $E[(X_1 + \lambda^* X_1) \cdot 1_{A(X_1,X_2)=1}] + E[(X_2 - a\lambda^* X_2) \cdot 1_{A(X_1,X_2)=2}]$. Then, A^* will be an optimal scheme from its definition. Thus, $A^*(X_1, X_2)$ is an optimal solution to the optimization problem in (13).

So far, we have shown that the optimal allocation for the problem in (13) has the same form as the allocation scheme resulted from the Nash equilibrium strategy of second price auction. Examining the optimization problem in (13), we see that the resulting throughput obtained is pareto optimal, which also implies the pareto optimality of the allocation resulted from equilibrium strategy (i.e., no other allocation performs strictly better).

4.3 Proportional fairness

In this section, we examine the well-known proportional fairness allocation. Let G_1, G_2, A be defined similarly as in the previous section. The objective of proportional fairness is to $\max(\log G_1 + \log G_2)$ [4]. For brevity, we omit the proof and state the optimal allocation below:

$$A^*(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 \geq c \cdot x_2 \\ 2 & \text{otherwise} \end{cases}$$

where the constant $c = G_1^{A^*}/G_2^{A^*}$. Again, we find the allocation with proportional fairness criteria has the same form as the allocation that resulted from the Nash equilibrium strategy. Therefore, by giving each user an appropriate amount of money, the resulting throughput for each user can achieve proportional fairness.

5 Conclusion

We apply an auction algorithm to the problem of fair allocation of a wireless fading channel. Using the second price auction mechanism, we are able to obtain the Nash equilibrium strategies for general channel state distribution. Our strategy allocates bandwidth to the users in accordance with the amount of money that they possess. Hence, this scheme can be viewed as a mechanism for providing quality of service (QoS) differentiation; whereby users are given fictitious money that they can use to bid for the channel. By allocating users different amounts of money, the resulting QoS differentiation can be achieved. We also show that the Nash equilibrium strategy of this auction leads to an allocation at which total throughput is no worse than 3/4 the maximum possible throughput when fairness constraints are not imposed (i.e., slots are allocated to the user with the better channel). Moreover, the equilibrium strategies leads to an allocation that is pareto optimal.

References

[1] A. Fu, E. Modiano, and J. Tsitsiklis, "Optimal energy allocation for delay-constrained data transmission over a time-varying channel," *IEEE INFOCOM 2003*, San Francisco, CA, April 2003.

[2] P. Marbach and R. Berry, "Downlink resource allocation and pricing for wireless networks," *IEEE INFOCOM 2002*, New York, NY, June 2002.

[3] P. Marbach, "Priority service and max-min fairness," *IEEE INFOCOM 2002*, New York, NY, June 2002.

[4] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Tran. on Information Theory*, vol. 48, no. 6, pp. 1277-1294, June 2002.

[5] L. Tassiulas and S. Sarkar, "Maxmin fair scheduling in wireless networks," *IEEE INFOCOM 2002*, New York, NY, June 2002.

[6] F. P. Kelly, A. K. Maulloo, and D. K. H. Tan, "Rate control for communication networks: Shadow prices, proportional fairness and stability," *Journal of Operation Research Society*, 49(1998), 237-252.

[7] A. El Gamal, E. Uysal, and B. Prabhakar, "Energy-efficient transmission over a wireless link via lazy packet scheduling," *IEEE INFOCOM 2001*, Anchorage, April 2001.

[8] B. Collins and R. Cruz, "Transmission policies for time varying channels with average delay constraints," *Proceeding, 1999 Allerton Conf. on Commun., Control, and Comp.*, Monticello, IL, 1999.

[9] X. Liu, E. K. P. Chong, and N. B. Shroff, "Opportunistic transmission scheduling with resource-sharing constraints in wireless networks," *IEEE Journal of Selected Areas in Communications*, vol. 19, no. 10, pp. 2053-2064, October 2001.

[10] D. Bertsekas and R. Gallager, *Data Networks*, Prentice Hall, 1991.

[11] D. Bertsekas, *Nonlinear Programming*, Athena Scientific, 1999.

[12] R. Johari and J. Tsitsiklis, " Network resource allocation and a congestion game," submitted May 2003.

[13] P. Klemperer, "Auction theory: A guide to the literature," *Journal of Economics Surveys*, vol. 13(3), pp. 227-286, July 1999.

[14] Y-K. Che and I. Gale, "Standard auctions with financially constrained bidders," *Review of Economic Studies*, vol 65, pp. 1-21, January 1998.

[15] T. R. Palfrey, "Multiple-object, discriminatory auctions with bidding constraints: A game-theoretic analysis," *Management Scienece*, vol 26, pp. 935-946, September 1980.

[16] D. Famolari, N. Mandayam, and D. Goodman, "A new framework for power control in wireless data networks: games, utility, and pricing," *Allerton Conference on Communication, Control, and Computing*, Monticello, IL, September 1998.

[17] T. Basar and R. Srikant, "Revenue-maximizing pricing and capacity expansion in a many-users regime," *IEEE INFOCOM 2002*, vol. 1, pp. 23-27, June 2002.