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Delay and Complexity Tradeoffs for Dynamic Routing and Power Allocation in a Wireless Network

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Abstract: We consider the tradeoffs in computation complexity and network delay for a multi-user wireless network with dynamic routing and power allocation. Data arrives to each node of the network randomly, and power allocation and routing decisions are made under the assumption that the computational processing speed at each node is constrained to *C* floating point operations per second. We develop a scheduling algorithm that can be implemented with arbitrarily low complexity, and provides 100% throughput and worst-case delay guarantees. The delay bound is explicitly computed and shown to be inversely proportional to the processing speed *C*.

I. INTRODUCTION -- We consider a multi-node wireless network supporting multiple traffic streams. Nodes communicate with each other via wireless data links, and the instantaneous transmission rate μ_{ij} between nodes *i* and *j* is determined by the power $p_{ij}(t)$ allocated to link *(i,j)* according to a concave rate-power curve $\mu_{ij}(p_{ij})$. All nodes are power constrained so that instantaneous power allocations satisfy $\sum_j p_{ij}(t) \le P_i^{tot}$ for all time and for all nodes *i*.

Previous work in power allocation for wireless systems is found in [1-3], and complexity-aware scheduling algorithms are treated in [4-6]. The contributions in this paper are twofold: (i) We develop a dynamic power allocation algorithm which stabilizes the system with unknown and/or time varying data rates, and (ii) We consider scheduling complexity and show delay is inversely proportional to the processing capability of each node. The algorithm is based on iteratively solving a minimum clearance time problem. Minimum clearance time is also treated in [6], where a polynomial complexity link scheduling algorithm is developed.

II. MINIMUM CLEARANCE TIME -- Assume each node $i \in \{1, \ldots, N\}$ contains unfinished work U_{ij} at time zero, representing the amount of bits in node *i* which are destined for node *j*. No new packets arrive, and a power allocation and routing policy is to be developed that clears all current data in minimum time. Data is assumed to flow as a fluid.

Theorem 1: Optimal power control and routing strategies can be restricted to constant power allocation strategies.

Proof: Consider an optimal strategy which minimizes the clearance time *T*. Let $p_{ij}(t)$ represent the (potentially time-varying) power allocations associated with this optimal strategy, and define $\bar{p}_{ij}$ as the empirical average power allocated for link *(i,j)* during [0, *T*]. Note that the $\bar{p}_{ij}$ values satisfy the power constraint $\sum_j \bar{p}_{ij} \le P_i^{tot}$. Furthermore, by Jensen's inequality and concavity of the rate-power functions:

$$\frac{1}{T}\int_0^T \mu_{ij}(p_{ij}(\tau))d\tau \le \mu_{ij}(\bar{p}_{ij})$$

Thus, using a constant power allocation $\bar{p}_{ij}$ creates a larger average transmission rate for each link *(i,j)*. It can be shown that these resulting rates are sufficient to support a multi-commodity data flow over the network which also clears backlog in the minimum time *T*. ❑

Using this theorem, it is straightforward to develop a power allocation and routing algorithm π_{min} which clears data in minimum time. The policy computes the routes and the constant power allocations as the solution of a standard convex optimization problem, which can be implemented in a distributed fashion.

III. DYNAMIC CONTROL FOR RANDOM TRAFFIC -- Let Λ represent the *capacity region* of the system, i.e., the region of all data rates (λ_{ij}) that the network can stably support. Let $X_{ij}(t)$ represent the amount of bits that arrive to node *i* which are destined for node *j* during the interval [0, *t*]. We assume that traffic satisfies the following time varying leaky bucket constraints over all time intervals:

$$X_{ij}(t+T) - X_{ij}(t) \le \sigma + \int_t^{t+T} \lambda_{ij}(\tau)d\tau$$
$$\text{where } (\lambda_{ij}(t) + \varepsilon) \in \Lambda \text{ for all } t$$

for some positive values σ, ε. Here, $\lambda_{ij}(t)$ represents the instantaneous traffic rate of the $X_{ij}(t)$

stream, σ represents the traffic burst parameter, and ε represents the distance the instantaneous rates are to the boundary of the capacity region. Such leaky bucket constraints are consistent with the *temporary sessions* model for input traffic described in [7]. The values of $\lambda_{ij}(t)$, σ, and ε are unknown to the network controllers.

Iterative Minimum Emptying Time strategy (IMET):

1. If the system is empty, wait for new data to enter.
2. Start iteration k (and define this starting time as t_k) by observing the current backlog $U_{ij}[k]$ in the system and computing the solution to the minimum clearance time algorithm π_{min} (which clears all data $U_{ij}[k]$ in time T_k). Hold the routing decisions and power allocations associated with this solution fixed for duration T_k.
3. Repeat for iteration k+1.

Theorem 2: The IMET algorithm guarantees a worst case bit delay of $2\sigma/\varepsilon$.

Proof: Let $\lambda_{ij} = \frac{1}{T_k}\int_{t_k}^{t_k+T_k}\lambda_{ij}(\tau)d\tau$, and notice that $(\lambda_{ij}) \in \Lambda - (\varepsilon)$. Thus, it can be shown that there exists a matrix (λ_{ij}^*) where $\lambda_{ij} + \varepsilon \le \lambda_{ij}^*$, and rate matrix (λ_{ij}^*) can be supported by a multi-commodity flow on a network with link capacities $\mu_{ij}(p_{ij}^*)$ achieved by a constant power allocation p_{ij}^*. To start an inductive proof, assume that $T_k \le \sigma/\varepsilon$. Because T_{k+1} is the minimum time required to clear the backlog seen at the start of the $(k+1)^{th}$ interval, it is less than the clearance time achieved by the constant power allocations p_{ij}^*, and hence:

$$T_{k+1} \le \max_{(i,j)}\{U_{ij}[k+1]/\lambda_{ij}^*\} \le \max_{(i,j)}\{[\sigma + \lambda_{ij}T_k]/\lambda_{ij}^*\} \le \sigma/\varepsilon$$

It follows that $T_k \le \sigma/\varepsilon$ for all k. Delay is held within two consecutive intervals T_k, T_{k+1}. ❑

IV. COMPLEXITY/DELAY TRADEOFFS -- The *IMET* algorithm requires the solution of a convex optimization problem to be computed instantaneously (at the beginning of each iteration). A feasible implementation is to allow the optimization to proceed while the system allocates the fixed powers determined by the computation result of the previous iteration. A modified policy can be developed which uses this principle, allowing more time to compute the solution of the minimum clearance time problem at the expense of using out-of-date backlog information. This idea is similar to our work in [4], where algorithms are designed to stabilize $N \times N$ packet switches with explicit tradeoffs in complexity and average delay guarantees.

Using this simple idea, it can be shown that a modified *IMET* strategy provides bounded worst case delay guarantees even when nodes have arbitrarily small processing capabilities. Let $T_{worst-case}(C)$ represent the worst case delay when all nodes operate with a computation rate of C operations/second. Using the modified *IMET* algorithm, we have:

Theorem 3: $T_{worst-case}(C) \le 3\max(\sigma/\varepsilon, a_N/C)$, where C is the processing speed (in Mega-Flops) and a_N represents the number of operations required to solve the minimum clearance time problem π_{min} for a network of size N. ❑

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APPENDIX:

A. The convex program for the min-clearance time problem π_{min}:

Maximize γ

Subject to: $f_{ij}^{(c)} \geq 0$

$$\sum_{a=1}^{N} f_{ai}^{(c)} - \sum_{b=1}^{N} f_{ib}^{(c)} = -\gamma U_{ic} + \delta_{i-c} \sum_{j=1}^{N} \gamma U_{jc}$$

$$\sum_{c=1}^{N} f_{ij}^{(c)} \leq \mu_{ij}(\bar{p}_{ij})$$

$$\sum_{j=1}^{N} \bar{p}_{ij} \leq P_i^{tot}$$

where $T = 1/\gamma$ represents the corresponding minimum clearance time. (Note that δ_{i-c} takes the value 1 if $i=c$, and 0 else.) The resulting power and routing values (p_{ij}) and ($f_{ij}^{(c)}$) specify the power settings and the flows for the multi-commodity routing (where $f_{ij}^{(c)}$ represents the flow of data destined for node c along the (i,j) link).

B. The modified *IMET* algorithm to achieve a delay guarantee of:

$$T_{worst-case}(C) \leq 3max(\sigma/\varepsilon, a_N/C)$$

Modified *IMET*:

1. Start iteration k by using the fixed power allocation and routing values (p_{ij}) and ($f_{ij}^{(c)}$) computed during the previous time interval T_{k-1}.

2. Hold these allocations fixed for duration $T_k = max(1/\gamma, a_N/C)$, where $1/\gamma$ is the time required to clear the $U_{ij}[k-1]$ backlogs.

3. During interval T_k, compute the solution of the min clearance time problem π_{min} for the $U_{ij}[k]$ backlogs.

4. Repeat for iteration $k+1$.