Transmission Scheduling Over a Fading Channel with Energy and Deadline Constraints

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Abstract — **We seek to maximize the data throughput of a power, energy, and time constrained transmitter sending data over a fading channel. The transmitter has a fixed amount of energy, a maximum power level, and a limited amount of time to send data. Given that the fade state, which is random, determines the throughput obtained per unit of energy expended, the goal is to obtain a policy for scheduling transmissions that maximizes the expected data throughput. We develop a dynamic programming formulation that leads to an optimal transmission schedule for channels with independently distributed fade states and also for channels whose fade states are Markov. This approach can be extended to the case of a transmitter seeking to minimize the energy required to send a fixed amount of data over a fading channel given power and deadline constraints.**

I. Introduction

The problem of transmission scheduling over a fading wireless channel has received much attention in the recent past [10] [8] [11] [2] [9]. In this paper, we address this problem in the context of an energy and power limited transmitter. Such a situation may arise in a battery-powered mobile device with a wireless network connection. Because the device relies on energy from a battery, it is limited in energy and power for data transmission. In addition, the network protocol may have timeouts or other limits that necessitate transmission of a data packet within a certain period of time.

In a fading channel environment, a poor channel state requires a large amount of energy to send a given amount of data, while a good channel state requires much less energy for the same amount of data. A transmission scheme that takes advantage of changing channel states can significantly improve the use of scarce energy resources.

If the transmitter wishes to maximize throughput in a fixed time period, it should select the timeslots with the best channel and then transmit during those timeslots. Unfortunately, the fade state of the channel is not known ahead of time; here, we assume that the transmitter knows the current state of the channel just before it transmits.

Under such conditions, the transmitter faces a tradeoff between waiting for a good channel and exceeding the time constraint. Without the time constraint, maximizing throughput for a given amount of energy and power simply consists of waiting for the best possible channel state and then transmitting. However, when a deadline constraint is applied, a transmitter that waits for too long may be forced to send data over poor channels as it seeks to expend its energy before the deadline. If a maximum power constraint is added, a transmitter that waits too long may not even be able to use up all of its energy.

To the best of our knowledge, this problem has not been well-studied in the literature. Resource allocation for fading multi-user broadcast channels is a popular topic in information theory. However, the resource being allocated is usually power or bandwidth, and the quantity to be maximized is most often Shannon capacity. Goldsmith and Li [10] [8] and Tse and Hanly [11] have found capacity limits and optimal resource allocation policies for such channels. Biglieri et.al. [2] have examined power allocation schemes for the block-fading Gaussian channel. Tse and Hanly [9] have also studied channel allocations in multi-access fading channels that minimize power consumption.

More recently, as interest in mobile communications has increased, transmission scheduling for fading channels has attracted more interest. El Gamal et.al. have performed extensive research on communication with energy constraints [5]. Ferracioli et.al. [6] propose a scheduling scheme for the third generation cellular air interface standard that takes channel state into account and seeks to balance service priority and energy efficiency. Wong et.al. [12] studied channel allocation algorithms for cellular base stations. Given known channel characteristics, the authors seek to assign channels in such a way as to minimize total power consumed by all the mobile users corresponding with a base station.

In addition, Chiasserini and Rao [3] [4] explored the phenomenon of charge recovery in batteries. They found that the pulsed discharge of a battery yields considerably more energy than discharging at a constant current. By introducing delays in transmission and using batteries on a round-robin basis, the authors are able to significantly enhance battery performance.

In this paper, we seek to maximize throughput for time, energy, and power constrained transmitters faced with a fading channel. It is assumed that the fade state of the channel is random, independently distributed, and known just before transmission. We formulate the problem using a dynamic programming approach, and prove concavity of the dynamic programming value function. This property leads to the formulation of an optimal policy, and also to a numerical algorithm for rapidly obtaining the optimal policy. The approach can be extended to handle a situation where the channel state is Markov and only the fade state of the previous channel is known. Using the same approach, it is also possible to minimize the amount of energy a power-limited transmitter needs in order to send a given amount of data over a fading channel by a certain deadline.

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II. Maximizing Throughput

A System Model

We consider a transmitter operating over a fading channel. Time is assumed to be discrete, and in each time slot the channel state changes according to a known probability distribution function. The channel state determines the throughput that can be obtained per unit energy expended by the transmitter. The transmitter is assumed to have a battery with a fixed number of discrete energy units available for use. Furthermore, there is a limitation on power: i.e., the number of energy units that can be expended per time slot is limited. The objective is to find a transmission schedule that maximizes expected throughput for a given amount of energy and a deadline by which it must be consumed (or otherwise wasted).

Denote the available energy in the battery at time slot k with the variable a_k. The battery starts with a_0 units of energy and must complete transmission by time slot n. At each time slot the transmitter can consume up to $\min(a_k, P)$ units of energy, where P is the power limit. Each unit of energy expended results in a certain amount of data throughput that is dependent on the channel fade state.

It is assumed that the throughput of the channel is linear with power, or equivalently, expended energy during a time slot. Thus, for a given channel state q_k and consumption c_k at time k, the data throughput is given by $q_k c_k$. Although the relationship between throughput and power is actually dependent on the modulation scheme and the capacity of the channel, assuming linearity is not unreasonable. First, throughput is linear to power for a given modulation scheme. Second, one may obtain a linear relationship with a fixed modulation scheme simply by transmitting only part of the time. Third, channel capacity is linear to energy in a low signal-to-noise ratio or high bandwidth environment.

We examine two possibilities for channel fade state information: first, that the state of the channel is known just before transmission, and second, that only the state of the channel in the previous time slot is known before transmission. In the first case, the state of the channel q_k is modeled as a random variable with an independent probability distribution $p_{q_k}(q_k)$ in each time slot. In the second case, a Markov model can be used. The probability distribution of the state of the channel at time k is dependent on the prior state of the channel at time $k-1$, and the probability distribution for the channel at time k is given by $p_{q_k}(q_k|q_{k-1})$. In both cases it is assumed that the probability distribution functions are known a priori.

In practice, the choice of the model to be used will likely depend on how quickly the channel changes. Obtaining channel state information usually requires that the transmitter send a test sequence and receive feedback from the receiver, or for the receiver to send a test sequence. If the channel does not change significantly in the time necessary for the test sequence to propagate and be analyzed, assuming that the channel is known before transmission may be reasonable. On the other hand, if the channel is changing quickly, the Markov model will be more appropriate.

B Dynamic Programming Formulation

The objective is to find a policy for scheduling transmissions in such a fashion as to maximize expected throughput. To do so, the quantity

$$E\left[\sum_{k=1}^{n} q_k c_k\right]$$

must be maximized, where consumption is subject to power and energy constraints.

The dynamic programming algorithm can be used to find an optimal policy for scheduling transmissions. As usual in dynamic programming, we introduce the value function $J_k(a_k, q_k)$, which provides a measure of the desirability of the transmitter having energy level a_k at time k, given that current channel quality is q_k. The optimal value functions $J_k(a_k, q_k)$ for each stage k are related by the following dynamic programming recursion:

$$J_k(a_k, q_k) = \max_{0 \le c_k \le a_k} \{q_k \min(c_k, P) + \overline{J}_{k+1}(a_k - c_k)\} \quad (1)$$

where $\overline{J}_k(a_k) = E_{q_k}[J_k(a_k, q_k)]$ and

$$\overline{J}_{k+1}(a_k - c_k) = E_{q_{k+1}}[J_{k+1}(a_k - c_k, q_{k+1})]$$

At the last stage, stage n, the value function becomes

$$J_n(a_n, q_n) = q_k \min(a_n, P)$$

The first term in equation (1), $q_k \min(c_k, P)$, represents the data throughput that can be obtained in the current stage by consuming c_k units of energy. The available energy in the next stage is then $a_k - c_k$, and the term $\overline{J}_{k+1}(a_k - c_k)$ represents the expected throughput that can be obtained in the future given $a_k - c_k$ units of energy.

The value function has a number of important properties that should be explored.

Theorem 1:

$J_k(a_k, q_k)$ and $\overline{J}_k(a_k)$ are concave in a_k for any fixed value of q_k.

Proof:

Given in appendix A.

The concavity properties of the expected value function $\overline{J}_k(a_k)$ dictate the nature of an optimizing consumption policy. The expected value function $\overline{J}_{k+1}(a_{k+1})$ for time $k+1$ represents the expected throughput for saving energy at time k. Since this function is concave, it translates into a decreasing marginal throughput for saving energy. The marginal throughput for consuming energy is also decreasing: it is q_k and then zero after the power limit is reached. The optimal policy must apportion energy between these two concave functions: at the optimum consumption point, either the marginal rewards for saving and consuming energy are equal, or the optimal point is an extreme point.

More formally, let us assume that the variables a_k, c_k, q_k, and P are all integer. Further, define $\phi_k(q_k)$ to be the smallest x in the range $0 \le x \le a_0$ (where a_0 is the initial energy in the battery) such that

$$\overline{J}_{k+1}(x+1) - \overline{J}_{k+1}(x) < q_k$$

and set $\phi_k(q_k) = a_0$ if such an x does not exist. Then it can be shown [7] that an optimal policy is to let c_k be

$$\begin{array}{lll} 0 & \text{for} & a_k \leq \phi_k(q_k) \\ \min(a_k - \phi_k(q_k), P) & \text{for} & \phi_k(q_k) < a_k \end{array}$$

In effect, $\phi_k(q_k)$ is a threshold beyond which the throughput gained by consuming exceeds the throughput gained by saving. In other words, it is best to save the first $\phi_k(q_k)$ energy units in the battery and to use any extra units to transmit.

C Computation of the Value Function

If one assumes that the variables a_k, c_k, q_k, and P are integer, it is relatively easy to program a computer to perform the dynamic programming recursion. However, execution time can be slow. The major difficulty is computing the expectation $E_{q_{k+1}}[J_{k+1}(a_k - c_k, q_{k+1})]$ for every possible a_k, q_k, and k. Evaluating this expectation requires a four-layer nested loop: the algorithm must maximize over c_k and consider all values of a_k and q_k in each stage, and there are a total of n stages.

Fortunately, this process can be considerably simplified. When $a_k = 0$, it is clear that the value function is

$$\begin{aligned} \overline{J}_k(0) &= \overline{J}_{k+1}(0) \\ &= 0 \end{aligned}$$

When $a_k > 0$ it can be shown that the value function can be reformulated as

$$\begin{aligned} J_k(a_k, q_k) = \overline{J}_{k+1}(a_k) \\ + \sum_{c_k=1}^{\min(a_k,P)} \max(q_k - \overline{J}'_{k+1}(a_k - c_k), 0) \end{aligned} \quad (2)$$

where $\overline{J}'_k(a_k)$ is the first difference of $\overline{J}_k(a_k)$:

$$\overline{J}'_k(a_k) = \overline{J}_k(a_k + 1) - \overline{J}_k(a_k)$$

Using equation (2), it is demonstrated in appendix B that for $a_k > 0$ the expected value function can be written as

$$\begin{aligned} \overline{J}_k(a_k) =& \overline{J}_{k+1}(a_k) + \sum_{c_k=1}^{\min(a_k,P)} \{G_{q_k}(\lceil \overline{J}'_{k+1}(a_k - c_k) \rceil) \\ &- \overline{J}'_{k+1}(a_k - c_k) F_{q_k}(\lceil \overline{J}'_{k+1}(a_k - c_k) \rceil)\} \end{aligned} \quad (3)$$

where

$$F_{q_k}(x) = \sum_{q_k=x}^{\infty} p(q_k)$$

$$G_{q_k}(x) = \sum_{q_k=x}^{\infty} q_k p(q_k)$$

and $\lceil \cdot \rceil$ is the ceiling operator that rounds up.

The above equation, although complicated in appearance, is relatively easy to evaluate numerically. Note that $F_{q_k}(x)$, and $G_{q_k}(x)$ do not change unless the probability distributions for q_k change with time. In any case, the calculations of $F_{q_k}(x)$, $G_{q_k}(x)$, and $\overline{J}'_{k+1}(a_k)$ are extremely simple, there is no maximization over c_k, and the summation over c_k for n stages results only in a two-stage nested loop. This approach leads to a considerably more efficient computation of expected value.

D Markovian Model of an Unknown Channel

We now eliminate the assumption that the channel is known just before the decision to transmit. Instead, it is assumed that the current channel is unknown, but that the channel in the previous stage is known and that the state of the channel in the current time step is dependent on the state of the channel in the previous time step. We model this channel dependency as a Markov chain and extend the earlier results to this case.

The objective is again to maximize the quantity

$$E\left[\sum_{k=1}^{n} q_k c_k\right]$$

by finding the best policy $c_1 \ldots c_n$. The value function can be formulated by writing

$$\begin{aligned} J_k(a_k, q_{k-1}) = \max_{0 \leq c_k \leq a_k} \{E_{q_k}[q_k \min(c_k, P) \\ + J_{k+1}(a_k - c_k, q_k) | q_{k-1}]\} \end{aligned}$$

At the last stage, stage n, the value function is

$$J_n(a_n, q_{n-1}) = E_{q_n}[q_n \min(a_n, P) | q_{n-1}]$$

The value function may be rewritten as

$$\begin{aligned} J_k(a_k, q_{k-1}) &= \max_{0 \leq c_k \leq a_k} \{E_{q_k}[q_k \min(c_k, P) \\ &\quad + J_{k+1}(a_k - c_k, q_k) | q_{k-1}]\} \\ &= \max_{0 \leq c_k \leq a_k} \{E_{q_k}[q_k | q_{k-1}] \min(c_k, P) \\ &\quad + E_{q_k}[J_{k+1}(a_k - c_k, q_k) | q_{k-1}] \end{aligned}$$

This expression is virtually identical to the non-Markovian expression given in equation (1) and is concave by the same induction arguments used to show the concavity of equation (1) in the appendix. Because the value functions are so similar, the results for the non-Markov case easily extend to the Markov case. Let us define

$$\hat{J}_{k+1}(a_k, q_{k-1}) = E_{q_k}[J_{k+1}(a_k, q_k) | q_{k-1}]$$

and let $\phi_k(q_{k-1})$ be the smallest x in the range $0 \leq x \leq a_0$ such that

$$\hat{J}_{k+1}(x + 1, q_{k-1}) - \hat{J}_{k+1}(x, q_{k-1}) < E[q_k | q_{k-1}]$$

and set $\phi_k(q_{k-1}) = a_0$ if such an x does not exist. Then an optimal policy is to let c_k be

$$\begin{array}{lll} 0 & \text{for} & a_k \leq \phi_k(q_{k-1}) \\ \min(a_k - \phi_k(q_{k-1}), P) & \text{for} & \phi_k(q_{k-1}) < a_k \end{array}$$

It is clear from the above that in the Markovian model, the expectation $E[q_k | q_{k-1}]$ and probability distribution function $p_{q_k}(q_k | q_{k-1})$ take the place of q_k and $p_{q_k}(q_k)$ in the non-Markovian case. The resulting optimal policy is directly analogous to the non-Markov policy.

III. Minimizing Energy

We have analyzed a situation where we have a given amount of energy and wish to maximize the throughput within a fixed time period. These results can be extended to the somewhat more practical situation in which the transmitter has a given amount of data that must be sent within a fixed time period, and wishes to minimize the amount of energy required to do so.

To a limited extent, we can fit this problem into the existing framework by taking what was previously energy to be data. Let the variable d_k denote the number of data units remaining to be sent at time k, and s_k the amount of data that is actually sent at time k. As before, the channel quality is given by the variable q_k and each unit of data requires $\frac{1}{q_k}$ units of energy to transmit. We assume that the channel is not known until just before time of transmission, but that the probability distribution for the channel state $p_{q_k}(q_k)$ is known. The transmission must be completed by time n, and we seek to find best transmission policy $s_1 \dots s_n$ that minimizes total required energy:

$$\min E\left[\sum_{k=1}^{n} s_k \frac{1}{q_k}\right]$$

If there is no limit on the amount of energy that can be consumed in each time step, a simple optimal stopping problem results. However, if we impose a power limit, the problem becomes more interesting. The power limit effectively imposes a limit on the throughput, given by Pq_k where P is the power limit and q_k is the channel quality. If d_k represents the amount of data remaining to be sent, the dynamic programming cost function can be formulated as

$$J_k(d_k, q_k) = \min_{0 \le s_k \le \min(d_k, Pq_k)} \{\frac{s_k}{q_k} + \overline{J}_{k+1}(d_k - s_k)\} \quad (4)$$

where

$$\overline{J}_{k+1}(d_k - s_k) = E_{q_{k+1}}[J_{k+1}(d_k - s_k, q_{k+1})]$$

This cost function for the power-minimization scenario is very similar to the value function for the throughput-maximization scenario, equation (1). Due to this similarity, the earlier analysis holds to a large extent, with one important caveat: When a power limit is imposed, the transmitter is faced with the possibility that if an insufficient amount of data is not transmitted early on, it may not be possible to send all the data even at maximum power before time runs out. It is therefore necessary to assign a cost $J_n(d_n, q_n)$ for not transmitting all the data before time n. One solution is to impose an infinite energy penalty for not sending all the data. In this case, the cost function at the last stage becomes

$$J_n(d_n, q_n) = \left\{ \begin{array}{ll} 0 & \text{for } d_n \le 0 \\ \infty & \text{for } d_n > 0 \end{array} \right\}$$

and we define $\frac{s_n}{q_n} = J_n(d_n, q_n)$.

Another option is to remove the power constraint at time n and impose an extremely low data rate q_n as a penalty function:

$$J_n(d_n, q_n) = \frac{1}{q_n} d_n$$

where q_n is predetermined and very small.

Theorem 2:

Under both definitions of $J_n(d_n, q_n)$, the cost function $J_k(d_k, q_k)$ and the expected cost function $\overline{J}_k(d_k)$ are convex in d_k for any fixed q_k.

The proof of the theorem is extremely similar to the proof given in the appendix showing concavity of the value function for throughput maximization. In fact, as long as J_n at the final stage is a convex function of d_n, $J_k(d_k, q_k)$ can be shown to be convex. As in the earlier case, the convexity of the expected value function means that an optimal policy for minimizing energy can be readily obtained. Furthermore, a Markov model for the case where the channel is not known before transmission can be derived as well.

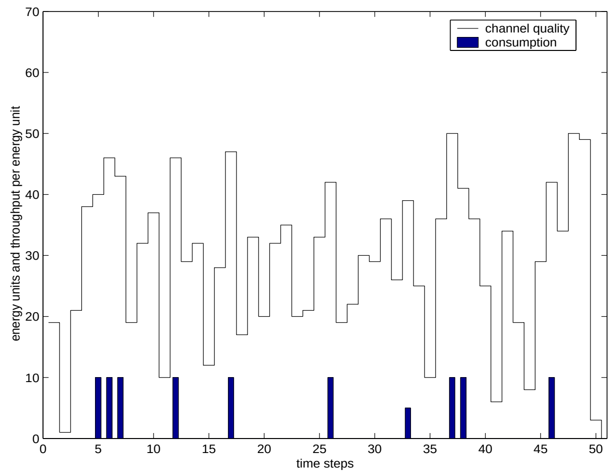

Fig. 1: Channel Quality and Consumption

The optimal policy for integer d_k, s_k, q_k, and P is the following: Define $\phi_k(q_k)$ be the smallest x in the range $0 \le x \le d_0$ such that

$$\overline{J}_{k+1}(x+1) - \overline{J}_{k+1}(x) > q_k$$

and set $\phi_k(q_k) = d_0$ if such an x does not exist. Furthermore, when $\overline{J}_k(x+1)$ and $\overline{J}_k(x)$ are both infinite, we define $\overline{J}_k(x+1) - \overline{J}_k(x)$ to be infinite as well. Then an optimal policy is to let s_k be

$$\begin{array}{cl} 0 & \text{for} \quad d_k \le \phi_k(q_k) \\ \min(d_k - \phi_k(q_k), Pq_k) & \text{for} \quad \phi_k(q_k) < d_k \end{array}$$

In the infinite penalty case, it can be shown that for time $n - i$ and when q_k takes on possible values between q_{min} and q_{max} the expected cost function can be written in the form

$$\overline{J}_{n-i}(d_{n-i}) = \left\{ \begin{array}{ll} f(d_{n-i}) & \text{for } d_{n-i} \le Piq_{min} \\ \infty & \text{for } d_{n-1} > Piq_{min} \end{array} \right\} \quad (5)$$

where $f(x)$ is a finite convex function. When the value function is infinite, there is a finite probability of not completing transmission by the deadline, even if one transmits at full power during every remaining time slot.

The net effect of the cost function having the form given in (5) is that the transmitter avoids missing the data throughput target at all costs, even if the probability of missing the target is vanishingly small. In some sense this is suboptimal; it may be considerably more efficient if the algorithm is allowed to use a policy that results in a miniscule probability of missing the data throughput constraint.

IV. Example: Throughput Maximization

We now apply the throughput maximization procedure to a simple scenario and compare its performance to a threshold heuristic that transmits whenever channel quality is above a fixed threshold. We find that no matter what the threshold

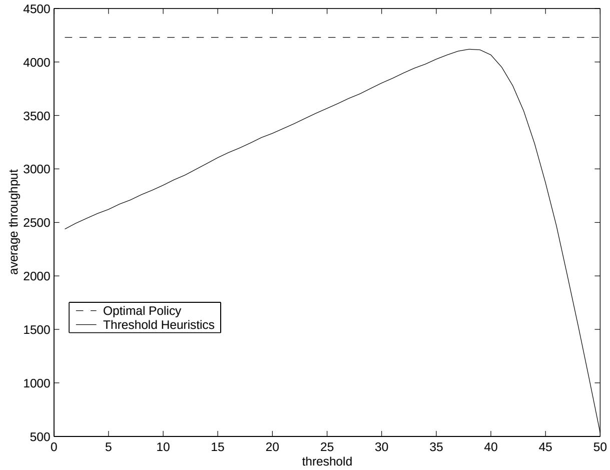

Fig. 2: Average Throughput for Optimal Policy and Heuristics

is, we are able to obtain better average performance by using the optimal algorithm developed above.

The scenario consisted of 50 time steps where channel throughput per unit energy q_k was integer and uniformly distributed between 1 and 50 during each time step. The initial energy was 95 energy units and the power limit for each time step was 10 units of energy.

As an example, we show a single randomly generated channel state trajectory and the resulting optimal policy for throughput maximization in figure 1. The figure also shows the channel qualities and the consumption schedule as determined by the optimal policy.

Figure 2 shows the average throughput obtained by the optimal policy and different threshold policies. The average throughput for each policy was obtained by generating 500 different channel state trajectories and applying the policies to each trajectory. The horizontal dashed line represents the average throughput obtained by the optimal policy, and the solid line plots the throughput obtained by a threshold policy as a function of the threshold. The leftmost point on the curve corresponds to a greedy heuristic that transmits no matter what the channel quality, while the rightmost points correspond to heuristics that transmit only for the very best channel states. As can be seen from the figure, the optimal policy obtained a higher average throughput than any possible threshold policy.

V. Conclusion

This paper developed a dynamic programming formulation for maximizing throughput over a fading channel given constraints on time, energy, and power. Furthermore, a method for efficiently obtaining a solution was presented. In addition, the same approach was used to solve the problem of minimizing the energy required to send a fixed amount of data over a fading channel given constraints on time and power.

VI. Appendix

A Proof of Theorem 1

We wish to show that the dynamic programming recursion given in equation (1) is concave in a_k for every q_k. We say that a function $f : \Re^n \to \Re$ is concave if for $0 \le \lambda \le 1$ and $\lambda + \overline{\lambda} = 1$

$$f[\lambda y + \overline{\lambda} z] \ge \lambda f(y) + \overline{\lambda} f(z)$$

The theorem can be proved by induction. First, note that at time n, the value function $J_n(a_n, q_n)$ is piecewise linear and concave in a_n. The expected value function $\overline{J}_n(a_n) = E_{q_n}[J_n(a_n, q_n)]$ is then also concave to a_n since a linear combination of concave functions is again concave.

Now assume $\overline{J}_{k+1}(a_{k+1})$ is concave in a_k. We show that $J_k(a_k, q_k)$ is concave in a_k. To complete the induction, note that if $J_k(a_k, q_k)$ is concave in a_k, $\overline{J}_k(a_k)$ is also concave in a_k, since it is a weighted sum of concave functions.

Let us look at $J_k(x, q_k)$ and $J_k(y, q_k)$ for arbitrary x and y.

$$J_k(x, q_k) = \max_{0 \le c_k \le x} \{q_k \min(c_k, P) + \overline{J}_{k+1}(x - c_k)\}$$

Since $\overline{J}_{k+1}(a_{k+1})$ is nondecreasing with a_{k+1}, the expression above can be replaced by

$$J_k(x, q_k) = \max_{0 \le c_k \le \min(x,P)} \{q_k c_k + \overline{J}_{k+1}(x - c_k)\}$$

There must be an optimizing value for c_k. Denote this by c_k^x. Then

$$J_k(x, q_k) = q_k c_k^x + \overline{J}_{k+1}(x - c_k^x)$$

Similarly,

$$J_k(y, q_k) = q_k c_k^y + \overline{J}_{k+1}(y - c_k^y)$$

where c_k^y is the optimizing value for c_k for $J_k(y, q_k)$. Combining the two equations and weighting by λ or $\overline{\lambda}$,

$$\begin{aligned} &\lambda J_k(x, q_k) + \overline{\lambda} J_k(y, q_k) \\ &\quad = \lambda\{q_k c_k^x + \overline{J}_{k+1}(x - c_k^x)\} + \overline{\lambda}\{q_k c_k^y + \overline{J}_{k+1}(y - c_k^y)\} \\ &\quad = q_k[\lambda c_k^x + \overline{\lambda} c_k^y] + \lambda \overline{J}_{k+1}(x - c_k^x) + \overline{\lambda J}_{k+1}(y - c_k^y)\} \end{aligned}$$

By the induction hypothesis, $\overline{J}_{k+1}(x - c_k^x)$ and $\overline{J}_{k+1}(y - c_k^y)$ are concave in c_k. Then

$$\begin{aligned} &\lambda J_k(x, q_k) + \overline{\lambda} J_k(y, q_k) \\ &\quad \le q_k(\lambda c_k^x + \overline{\lambda} c_k^y) + \overline{J}_{k+1}(\lambda x + \overline{\lambda} y - \lambda c_k^x - \overline{\lambda} c_k^y) \end{aligned}$$

Now examine the range of the maximization. Since $c_k^x \le x$ and $c_k^y \le y$, we have that $\lambda c_k^x + \overline{\lambda} c_k^y \le \lambda x + \overline{\lambda} y$ and $\lambda c_k^x + \overline{\lambda} c_k^y \le \lambda P + \overline{\lambda} P$. These expressions can be combined to obtain

$$\lambda c_k^x + \overline{\lambda} c_k^y \le \min(\lambda x + \overline{\lambda} y, P)$$

and

$$\begin{aligned} &\lambda J_k(x, q_k) + \overline{\lambda} J_k(y, q_k) \\ &\quad \le \max_{0 \le c_k \le \min(\lambda x + \overline{\lambda} y, P)} \{q_k c_k + \overline{J}_{k+1}(\lambda x + \overline{\lambda} y - c_k)\} \\ &\quad = J_k(\lambda x + \overline{\lambda} y, q_k) \end{aligned}$$

This shows that $J_k(a_k, q_k)$ is concave in a_k. Since a linear combination of concave functions is again concave, $\overline{J}_k(a_k)$ is also concave in a_k, and the induction is complete.

B Calculation of the Value Function

We show that under the assumption that all variables are integer, the value function for maximizing throughput given in equation (1) can be expressed as equation (3) for $a_k > 0$.

We first reformulate the dynamic programming equation with the following lemma:

Lemma 1:
For $a_k > 0$,

$$J_k(a_k, q_k) = \max_{0 \le c_k \le a_k} \{q_k \min(c_k, P) + \overline{J}_{k+1}(a_k - c_k)\}$$
$$= \overline{J}_{k+1}(a_k) + \sum_{c_k=1}^{\min(a_k,P)} \max(q_k - \overline{J}'_{k+1}(a_k - c_k), 0)$$

where $\overline{J}'_k(a_k)$ is the first difference of $\overline{J}_k(a_k)$:

$$\overline{J}'_k(a_k) = \overline{J}_k(a_k + 1) - \overline{J}_k(a_k)$$

Proof:

This lemma results from the fact that every incremental unit of energy consumed generates q_k units of immediate throughput and each such unit causes $\overline{J}'_{k+1}(\cdot)$ units of future throughput to be lost. Then, if c_k^* is the optimizing value of c_k in equation (1), we have for $c_k^* > 0$

$$q_k \ge \overline{J}_{k+1}(a_k - c_k^* + 1) - \overline{J}_{k+1}(a_k - c_k^*)$$

Since $\overline{J}_{k+1}(a_k)$ is concave in a_k, the above equation holds for $0 < c_k \le c_k^*$ and in that range

$$\max(q_k - \overline{J}'_{k+1}(a_k - c_k), 0) = q_k - \overline{J}'_{k+1}(a_k - c_k)$$

When $c_k > c_k^*$, or $c_k^* = 0$,

$$\max(q_k - \overline{J}'_{k+1}(a_k - c_k), 0) = 0$$

Then we obtain

$$\sum_{c_k=1}^{\min(a_k,P)} \max(q_k - \overline{J}'_{k+1}(a_k - c_k), 0)$$
$$= q_k c_k^* + \overline{J}_{k+1}(a_k - c_k^*) - \overline{J}_{k+1}(a_k)$$

Adding $\overline{J}_{k+1}(a_k)$ to both sides yields the lemma.

By definition, the expected value function is

$$\overline{J}_k(a_k) = E_{q_k}[J_k(a_k, q_k)]$$

Writing out the expectation,

$$\overline{J}_k(a_k) = \sum_{q_k=0}^{\infty} p_{q_k}(q_k) J_k(a_k, q_k)$$

Using lemma 1,

$$\overline{J}_k(a_k) = \sum_{q_k=0}^{\infty} p_{q_k}(q_k) \Big\{ \overline{J}_{k+1}(a_k) + \sum_{c_k=1}^{\min(a_k,P)} \max(q_k - \overline{J}'_{k+1}(a_k - c_k), 0) \Big\}$$
$$= \overline{J}_{k+1}(a_k) + \sum_{c_k=1}^{\min(a_k,P)} \Big\{ \sum_{q_k=0}^{\infty} p_{q_k}(q_k) \max(q_k - \overline{J}'_{k+1}(a_k - c_k), 0) \Big\}$$

Eliminating the maximization by restating the summation limits,

$$\overline{J}_k(a_k) = \overline{J}_{k+1}(a_k) + \sum_{c_k=1}^{\min(a_k,P)} \Big\{ \sum_{q_k=\lceil \overline{J}'_{k+1}(a_k - c_k) \rceil}^{\infty} p_{q_k}(q_k) \big[q_k - \overline{J}'_{k+1}(a_k - c_k) \big] \Big\}$$
$$= \overline{J}_{k+1}(a_k) + \sum_{c_k=1}^{\min(a_k,P)} \Big\{ \big[\sum_{q_k=\lceil \overline{J}'_{k+1}(a_k - c_k) \rceil}^{\infty} p_{q_k}(q_k) q_k \big] - \overline{J}'_{k+1}(a_k - c_k) \big[\sum_{q_k=\lceil \overline{J}'_{k+1}(a_k - c_k) \rceil}^{\infty} p_{q_k}(q_k) \big] \Big\}$$

Equation (3) directly follows.

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