

# Packet Routing over Parallel Time-Varying Queues with Application to Satellite and Wireless Networks

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**Abstract:** We consider the problem of routing packets from an arbitrary input stream  $X$  over a collection of heterogeneous queues in parallel. When the processing rates  $(\mu_1, \dots, \mu_n)$  of the queues are constant, a simple work conserving routing strategy  $\pi_{WC}$  is shown to hold total system backlog within a fixed upper bound from the resulting backlog of any other policy. Similar results apply to systems with time varying processing rates  $(\mu_1(t), \dots, \mu_n(t))$  when routing decisions can be postponed by placing packets in a pre-queue. For the case when routing decisions must be made immediately upon arrival, we demonstrate that the non-predictive policy of routing packets to the shortest queue ensures system stability (and maintains low packet loss rates for finite buffer systems) whenever the system is stabilizable. Finally, we consider a joint problem of routing and power allocation where, for each time varying channel state  $c_i(t)$ , the rate of each queue  $i$  can be varied by adjusting a power parameter  $p_i$  (subject to a total power constraint  $\sum p_i \leq P_{tot}$ ) according to a rate-power curve  $\mu_i(c_i, p_i)$ . A throughput maximizing algorithm is developed for this joint problem.

I. INTRODUCTION -- This paper treats a problem of routing packets from an arbitrary input stream  $X$  over a set of  $n$  heterogeneous queues in parallel. Routing decisions are made based upon the current state of the queues, and the goal is to maintain an acceptably low level of backlog in the queues for all time. Such a scenario occurs, for example, in a satellite network where data packets arrive to a satellite and can reach the ground using one of several downlink channels. Packet transmission rates along different downlinks may vary as different channel codes are used to adjust to fluctuations in channel conditions (e.g., due to weather).

Previous work on queue control policies for systems with particular types of input processes can be found in [1-6]. A related NP-complete problem of scheduling batch packet arrivals over parallel queues is considered in [7,8]. The main contribution in this paper is to treat stochastic queueing systems and to provide tight, worst case bounds on system performance for arbitrary input processes.

II. PERFORMANCE BOUNDS -- Let  $X(t)$  represent an arbitrary packet arrival process, and assume all packets have bounded lengths  $L_{max}$ . Let policy  $\pi$  represent any policy for routing packets to servers in a multi-server system with rates  $\{\mu_1(t), \dots, \mu_n(t)\}$ . Let policy  $\pi_{WC}$  represent the specific work conserving policy of storing packets in a pre-queue until a server becomes available.

Comparing the unfinished works  $U_\pi(t)$  and  $U_{WC}(t)$  for the arbitrary routing policy and the work conserving routing policy to the unfinished work  $U_{single-server}(t)$  in a single server system with time varying processing rate  $\mu(t) = \mu_1(t) + \dots + \mu_n(t)$ , we have:

Lemma 1:  $U_{single-server}(t) \leq U_\pi(t)$  for all time  $t \geq 0$ .

Lemma 2:  $U_{single-server}(t) \leq U_{WC}(t) \leq U_{single-server}(t) + (n-1)L_{max}$ .

The multiplexing inequality of Lemma 1 demonstrates that it is always better to multiplex data streams from individual queues to a single queue whose rate is equal to the sum of the individual processing rates. It is useful to consider such a virtual single-server queue to provide a baseline for measuring the performance of routing policies. Lemma 2 shows that the work conserving routing strategy  $\pi_{WC}$  performs close to this baseline, and hence never allows more than  $(n-1)L_{max}$  bits in the system beyond the amount of unfinished work under any other policy. This bound is the tightest possible for all non-preemptive schemes which do not know

the future, and hence  $\pi_{WC}$  is the minimax optimal routing strategy.

III. REMOVING THE PRE-QUEUE -- In the case when no pre-queue is available and all packets must be routed immediately upon arrival, it is not possible to provide the same type of performance bounds without fully knowing future events. However, here we show for general arrival and linespeed processes that the policy of routing an incoming packet to the queue with the smallest amount of unfinished work (the "Join-the-Shortest-Queue" policy  $\pi_{JSQ}$ ) stabilizes the system whenever the system is stabilizable. We first introduce a new notion of stability, defined in terms of single queue systems with finite buffers. Let a single queue system have an ergodic input stream of bit rate  $\lambda$ , and a time-varying processing rate with time average value  $\mu_{av}$ . Let  $D(M)$  represent the packet drop rate of the single queue system as a function of the finite buffer size  $M$ .

*Definition:* A system is *loss rate stable* if the drop rate can be made arbitrarily small by increasing buffer capacity, i.e., if  $D(M) \rightarrow 0$  as  $M \rightarrow \infty$ .

It can be shown that a necessary condition for loss rate stability is  $\lambda \leq \mu_{av}$ . Furthermore, if the input stream and server rate process evolve according to an underlying finite state Markov chain, a sufficient condition for loss rate stability is  $\lambda < \mu_{av}$ . This notion of stability is closely tied to the standard notion defined in terms of a vanishing complementary occupancy distribution for infinite buffer capacity queues.

Compare a parallel queue system under the  $\pi_{JSQ}$  strategy to a single queue system whose instantaneous processing rate is again equal to the sum of the processing rates of the  $n$  parallel queues. Let  $D_{JSQ}(M)$  represent the drop rate of the multi-queue system operating under  $\pi_{JSQ}$  when all queues have a finite buffer storage of  $M$  bits.

*Theorem:* For all  $M$ ,  $D_{JSQ}(M + nL_{max}) \leq D_{single-queue}(M)$ , and hence the multi-queue system under  $JSQ$  is stable whenever the single queue system is stable.  $\square$

IV. CONCLUSIONS -- These results provide a means of addressing performance bounds and stability issues in systems with arbitrary input and server rate processes. A joint routing and power allocation problem for satellite and wireless systems can be considered using the same techniques, where server rates for each queue  $j$  depend upon the current channel state  $c_j(t)$  and the power  $p_j(t)$  allocated to the transmitter of queue  $j$  according to a rate-power curve  $\mu_j(p_j, c_j)$  (where  $\sum p_j(t) \leq P_{tot}$ ). A decoupled policy which routes using JSQ and allocates power to maximize the instantaneous processing sum rate maximizes total system throughput.

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