A Model for the Approximation of Interacting Queues that Arise in Multiple Access Schemes

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In this paper we present a new approximate model for the analysis of systems of interacting queues which often arise in multiple access network protocols. This new model is a refinement of an existing model developed in [1] for the ALOHA multiple access protocol. We begin by applying this model to the analysis of a multiple-node broadcast algorithm for a mesh network, which was presented in [2]. We then show how our model can be used to study the performance of the ALOHA multiple access protocol.

A multiple-node broadcast is a common task in the execution of parallel algorithms in a network of processors, where every processor may have a message to be broadcast to all other processors. In [2] an algorithm was developed which performs periodic, synchronized, broadcast cycles, where during each cycle only a small number of nodes are allowed to broadcast their message. Consider an N by N mesh, where each node has exogenous packets arriving (to be broadcast) independently according to a Poisson random process and placed in infinite-capacity queues. Our broadcast algorithm works as follows: We partition the mesh into N vertical rings, such that each node belongs to exactly one ring. At the beginning of every broadcast cycle each ring selects, at random, up to d packets to be broadcast throughout the mesh. The broadcast of the d packets from each ring is performed and has a fixed duration of $(d+1)(N-1)$ time slots. Clearly, the queues at the N nodes on each ring are highly dependent on each other. In fact, the queue sizes of the N nodes on each ring form an N-dimensional infinite Markov chain. Obtaining analytic expressions for the steady-state behavior of such a system is very difficult. Even a numerical evaluation of such systems can be computationally prohibitive. A similar difficulty arises in the analysis of the Aloha multiple access protocol and no exact analysis for packet delay is known, for that case either. Several approximate models have been proposed for the analysis of ALOHA which may be useful in analyzing our system.

In [1], Ephremides and Saadawi developed an approximate model for a system of interacting queues for analyzing the ALOHA protocol. In their model they approximate a system of N infinite queues as a single dimensional infinite Markov chain representing the state of one user together with an N-dimensional finite Markov chain representing the state of the rest of the system. They use parameters from the solution of one chain in analyzing the other and solve the two chains together using an iterative algorithm. This two-chain approach tracks the interaction between the different users in a system model that can be analyzed. We develop a similar approximate model for the system of interacting queues in the mesh broadcast case.

One Markov chain in our model, termed the user chain, represents the queue size for a single user. It is, therefore, an infinite chain. Packets arrive according to a Poisson random process and depart only when this node is chosen for service. We denote the probability that this node is chosen for service by P_s and show that the delay, D, can by expressed as

$$D = \frac{S}{2} + \frac{\lambda S(2-\lambda S)}{2(P_s - \lambda S)}$$

where S is the cycle duration which is equal to (d+1)(N-1). The missing ingredient in this expression, P_s, is the one term that can be obtained from the other chain in our model, termed the system chain.

The system chain represents the number of non-empty nodes on one ring (the ring containing our node of interest). Clearly, this chain consists of N+1 states. The transition probabilities between these states can be expressed in terms of parameters from the user's chain. If S_i denotes the i^{th} state of the system with i non-empty and (N-i) empty nodes and if P_i denotes the steady-state probability of S_i, then P_s can be expressed as

$$P_s = \frac{\sum_{i=1}^{i=d} P_i + d\sum_{i=d+1}^{i=N} \frac{1}{i} P_i}{1 - P_0}$$

Since the system chain equations depend on parameters from the user's chain and visa versa, the two chains are solved together using an iterative algorithm. The results from our approximate model compare very well with simulation, particularly when arrival rates are low.

In order to improve the accuracy of the model, we expanded the system chain to include the identity of the individual queues and their status (empty or non-empty). This adjustment to the system model proved to dramatically improve the performance of our approximation. However, with this change the system chain consists of 2^N states and is difficult to solve for all but very small values of N. To overcome this shortcoming of the expanded model, we limited the system chain so that it merely represents the dentity and state of one user (our user of interest) along with the number of non-empty nodes on the ring. This modification permits a more accurate derivation of the probability of success for the user chain. This is because the probability of success is defined to be the probability that the user is chosen to be served given that it is non-empty. Therefore, when the system chain contains the state of our user, we can compute the probability of success by conditioning on the user state being non-empty. It turns out that this new model is just as accurate as the previous model (containing the identities of all of the users) but since this new chain has only 2(N+1) states it is much easier to analyze.

Since the improved model offers such an improvement to the original model with a minimal additional complexity, we were motivated to develop a similar modification for the ALOHA multiple access protocol. In the ALOHA case we consider a finite number of users, each accepting packets that arrive independently according to a Bernoulli random process, competing for the use of a single channel. If a terminal is empty (has no packets), a newly arrived packet is transmitted immediately. The transmission is successful if and only if no other user attempts transmission during the same slot, otherwise a collision occurs and the terminal enters the blocked state. When in the blocked state, the terminal attempts re-transmission with probability p. In case of success the terminal becomes unblocked. An unblocked terminal can be in one of two states; idle (when its queue is empty), or active (when its queue is not empty). An active terminal transmits a packet with probability one.

The state of any single user can be specified by its queue size and by the indication of whether it is in the blocked or active states. A complete description of a N-terminal system requires the analysis of a 2N-dimensional infinite Markov chain. Again, such chains are known to be very difficult to analyze. We therefore resort to an approximation.

As was stated earlier, in [1] an approximation was developed which modeled an N-dimensional infinite Markov chain as a one-dimensional infinite chain representing the state of a single user together with a N-dimensional finite chain representing the number of blocked and active users in the entire system. In [3] an improvement to the above model was proposed which expanded the system chain to include the identity of all N users. That expanded model was shown to perform far better than the model in [1]; however, the expanded system chain contained 3^N states and was very difficult to analyze for all but very small values of N. We therefore develop a new system chain, similar to the one developed for the multiple-node broadcast algorithm, which includes the state of only one user together with the number of active and blocked users in the entire system.

Our analysis shows that this refined model performs very well at low arrival rates and offers an improvement over the original model in which the system chain contained no information about the individual terminals; however, it does not perform as well as the improved model which contained the identity of all N users. The differences are most noticeable when the arrival rates are high (close to saturation).

References

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