# Throughput Analysis of Unscheduled Multicast Transmissions in WDM Broadcast-and-Select Networks

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In a WDM network, in order to receive a transmission on a wavelength nodes must tune their receiver to that wavelength. If each node has only a single receiver, a transmission on one wavelength may not be received by nodes that are tuned to receive a message on another wavelength. In the absence of a scheduling algorithm to coordinate the transmissions, multiple simultaneous transmissions to the same receiver are possible, resulting in a reduction in the system throughput. This problem is similar to that of scheduling traffic in an input queued switch, for which it was shown in [1] that with unicast traffic, under uniform traffic conditions, the throughput of an NxN switch is limited to 58%.

Here we consider a slotted system where the size of a slot is equal to the message size. The network consists of N nodes and W wavelengths and each message is addressed to k randomly chosen nodes. During each slot, W of the N nodes are chosen to transmit and each receiver tunes to one of the wavelength that has a message addressed to it.

In this context, the system throughput can be expressed as the average number of multicast message transmissions completed per slot per wavelength. In other words, it is the inverse of the average number of transmissions required per successful multicast. The following lower bound on this number can be easily obtained by observing that even with an optimal scheduling algorithm, during every slot each node can *at most* receive one message transmission.

$$\bar{T} > \max\left(\frac{kW}{N}, 1\right).$$

It is interesting to note that when $kW$ is less than N the system is channel limited; that is, there are not enough channels to keep all of the receivers busy. While when $kW$ is greater than N, the system is receiver limited; that is, the number of receivers is too small to keep all of the channels busy with new transmissions.

We consider two transmission protocols. With the first protocol, which we call persistent, a message is repeatedly transmitted until it has been received by all of its intended recipients. In order to analyze the achievable throughput of this protocol we assume that the system is constantly backlogged with new messages ready for transmission. Using queueing analysis we show that the average number of transmissions per message is given by,

$$\bar{T} = 1 + \frac{1-(1-\bar{A})^k}{2} + \sum_{l=1}^{l=\infty} l \times [\Pr(q_1 \le l)^k - \Pr(q_1 \le l-1)^k].$$

Where $q_l$ is the queue size of an M/D/1 queue with arrival rate $\bar{A} = (Wk)/(N\bar{T})$.

The second protocol we consider introduces a random delay between retransmissions of a message. This random delay is designed to reduce the dependence that exists between the transmissions on the different channels, thereby alleviating the head-of-line blocking problem. Again, using queueing analysis we are able to obtain the average number of transmissions per multicast message, which we omit for brevity. As shown in figure 1, this back-off protocol yields higher throughput than the persistent protocol.

We also consider the algorithm used by receivers to select which packet to receive when they have multiple packets to choose from. We show that a policy which selects the message with the fewest remaining intended receivers performs better than a random selection policy. These findings are also shown in figure 1. Lastly, we extend our results to a system with multiple receiver per node and show that an interesting tradeoff exists between channel utilization and receiver utilization.

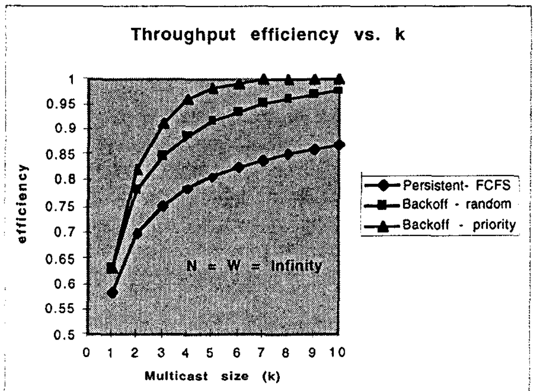


Figure 1. Multicast efficiency for different algorithms with infinite number of nodes and channels.

[1] M.J. Karol, M.G. Hluchyj and S.P. Morgan, "Input Versus Output Queueing in a Space-Division Packet Switch," IEEE Trans. on Comm., December, 1987.

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