

# Delay in Random Access Wireless Networks with Controlled Mobility

Guner Dincer Celik and Eytan Modiano

**Abstract** This paper considers dynamic wireless networks where messages arriving randomly (in time and space) are collected by a mobile receiver. The messages are transmitted to the mobile receiver according to a random access scheme and the receiver dynamically adjusts its position in order to receive these messages in the least amount of time. We study utilizing a combination of *wireless transmission* and *controlled mobility* to improve the delay performance in such networks. In particular, we analytically characterize the tradeoff between wireless transmission and physical movement of the mobile receiver. We derive a fundamental lower bound for the delay in the system and show how its effected by different communication parameters. We show that the combination of mobility and wireless transmission results in a significant improvement in delay as compared to a system where wireless transmission is not used.

## 1 Introduction

Analyzing the performance of mobile wireless networks has received considerable attention in the last decade. Typically throughput and delay performance in wireless networks utilizing some random mobility model were analyzed (e.g., [2], [6], [7], [15]). In this paper we study the use of *controlled mobility* to improve the delay performance of wireless networks. We consider dynamic wireless networks where messages arriving stochastically in time at random locations in the network

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are transmitted to a mobile receiver that uses a combination of physical movement and wireless reception to receive these messages.

We consider simple channel access models where mobile receiver does not have control over message transmissions. In particular the messages are transmitted according to a random access protocol and depart the system upon successful transmission. The mobile receiver adjusts its position in order to receive these messages in the least amount of time under the interference of neighboring transmissions. This model can represent a sensor network where a mobile base station collects data from a large number of sensors that are deployed at random locations inside the network.

We study the tradeoff between the use of *controlled mobility* and *wireless transmission* in this setting. Namely, the receiver can choose between coming close to the location of a message in order to increase the probability of successful reception or staying far away in order to save on travel time at the cost of less reliable transmission. This setup is particularly applicable to networks in which data rate is relatively low so that data transmission time is comparable to the mobile receiver travel time or to networks deployed on a large area so that a mobile element is necessary to provide connectivity. Characterizing this tradeoff analytically, our objective is to first derive fundamental performance bounds for the delay in the system and then to characterize the effect of *reception distance* on delay.

Random mobility models have been studied in communication framework [2], [6], [7], [13], [15] where fundamental results on throughput and delay of wireless networks were presented. Another body of literature focused primarily on mobility [10], [11], [12], [16], [18], [21] where special mobile elements were utilized to collect sensor data in Delay Tolerant Networks (DTN). Usually in these works particular policies were analyzed under simplistic communication models. In a closely related problem where multiple mobile nodes with controlled mobility relay the messages of static nodes, [17] derived a lower bound on delay.

Another related body of literature lies in the area of Vehicle Routing Problems (VRPs) [3], [4], [5], [19]. The most common example of a VRP is the Traveling Salesman Problem (TSP) in which a vehicle is to visit each member of a fixed set of locations such that the total travel cost is minimized. In TSP with neighborhoods (TSPN) problem [1], [14] the vehicle is to visit a neighborhood of each demand location. In this context communication capability does not arise due to the nature of the problem.

Of particular importance to us among the VRPs is the Dynamic Traveling Repairman Problem (DTRP) due to Bertsimas and van Ryzin [3], [4]. DTRP is a stochastic and dynamic VRP in which a vehicle is to serve demands that arrive randomly in time and space. Fundamental lower bounds on delay were established and several vehicle routing policies were analyzed for a single server in [3] and for multiple servers in [4]. We apply this model to wireless networks where the demands are messages to be transmitted to a mobile receiver which is capable of wireless communication. To the best of our knowledge, this is the first attempt to develop fundamental bounds on delay in a system where a mobile receiver is to collect data messages randomly arriving in time and space using wireless communications and controlled mobility.

The main contributions of this work are the following. We analytically characterize the tradeoff between physical movement and wireless transmission in mobile networks where messages arriving stochastically in time at random locations in the network are collected by a mobile receiver. We derive a fundamental bound on the delay in such a system and show how its effected with different communication parameters. We show that compared to a system where wireless transmission is not used, significant improvements in delay can be obtained by utilizing a combination of controlled mobility and wireless transmission.

This paper is organized as follows. In Section 2 we describe the system model. We derive bounds on the travel distance and the reception distance in Section 3 and analyze the system delay in Section 4. We present numerical results in Section 5 and conclude in Section 6.

## 2 Model

Consider a bounded region<sup>1</sup>  $\mathfrak{R}$  of area  $A$  and messages arriving into  $\mathfrak{R}$  according to a Poisson process (in time) of intensity  $\lambda$ . The messages are distributed independently and uniformly in  $\mathfrak{R}$  and they are transmitted to a mobile receiver with wireless communication capability. The receiver's job is to collect these messages in the least amount of time by receiving these messages via wireless communication.

The following *random access* scheme over a slotted channel with a constant rate (where each message takes one-slot time,  $\tau$ , to transmit) is used for message transmissions: Messages are transmitted to the receiver with probability  $p$  in each time slot independently of all other transmissions and until they are successfully received<sup>2</sup>. We utilize the Protocol Model [6], [8], [17] where a transmission from distance  $r$  to the receiver is successful, if there are no simultaneous transmissions within a disk of radius  $(1 + \Delta)r$ , ( $\Delta > 0$ ) around the receiver.

The receiver travels inside the region on straight lines from the current message reception point to the next at a constant speed  $v$ . The receiver knows message locations [1], [3], [17], [19] as typically assumed in vehicle routing problems, and it is a narrowband receiver which can decode at most one transmission at a time. We let  $r$  be the *reception distance* of the receiver. If the location of the next message to be received is within  $r$ , the receiver stops and attempts to receive the message. Otherwise the receiver travels towards the message location until it is within a distance  $r$  away from the message. Note that  $r$  is a design variable which essentially captures the trade off between wireless transmission and controlled mobility. Namely, the receiver can choose between coming close to the location of a message (small  $r$ ) in

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<sup>1</sup> Our results are valid for general network regions of any shape, however, in order to increase the understanding we will assume a disk-shaped network region.

<sup>2</sup> An instant feedback from the receiver is assumed after a successful transmission at which time the message departs from the network.

order to increase the transmission rate or staying far away (large  $r$ ) in order to save on travel time at the cost of less reliable transmission<sup>3</sup>.

We define the stable message arrival rate  $\lambda$  as the throughput in the system. Let  $s_i$  be the reception time for message  $i$  and let  $d_i$  be the receiver travel distance from the receiver's reception location for the message served prior to message  $i$  to receiver's reception location for message  $i$ . Message  $i$ 's total service time consists of the reception time  $s_i$  and the travel time  $d_i/v$ . The average of  $s_i$  and  $d_i$ , denoted by  $\bar{s}$  and  $\bar{d}$  respectively, are defined as their expectation in steady state and are given by  $\bar{s} = \lim_{i \rightarrow \infty} E[s_i]$  and  $\bar{d} = \lim_{i \rightarrow \infty} E[d_i]$  respectively, where the limits are assumed to exist. The fraction of time the server spends in receiving messages is denoted by  $\rho$ , and for stable systems  $\rho = \lambda \bar{s}$ . We define  $T_i$  as the time between the arrival of message  $i$  and its successful reception. The waiting time of message  $i$  is denoted by  $W_i$ , hence  $W_i = T_i - s_i$ . Average system time of messages in steady state is defined by  $\bar{T} = \lim_{i \rightarrow \infty} E[T_i]$  and  $\bar{W} = \bar{T} - \bar{s}$ .  $\bar{T}^*$  is defined as the optimal system time which is given by the policy that minimizes  $\bar{T}$ . We define by  $\bar{N}$  the steady state expected number of messages in the queue.

### 3 The Travel Distance and The Reception Distance

We first derive a lower bound on the average travel distance per message  $\bar{d}$  in terms of the average number of messages in the system  $\bar{N}$  and the reception distance  $r$ . We then obtain an expression for  $r$  in terms of  $\bar{N}$  and the failure probability  $\mu$ .

**Theorem 1.** *A lower bound on  $\bar{d}$  is given by*

$$\bar{d} \geq \frac{2\sqrt{A}}{3\sqrt{\pi(2\bar{N}+1)}} - 2r\left(1 - \frac{4\pi(2\bar{N}+1)r^2}{3A}\right). \quad (1)$$

*Proof.* See the Appendix.

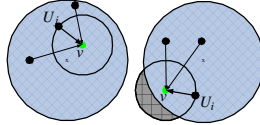
The intuition behind the proof is based on a nearest neighbor argument. Namely, the per-message distance the receiver moves for a given message  $i$  is at least as big as the distance from the closest of all messages (and the location of the receiver at the time of message  $i$ 's arrival) to the location of message  $i$  less  $2r$ . The  $2r$  distance is the maximum possible per-message reduction in travel distance due to wireless transmission. We call the right hand side of this inequality  $f(r)$ . It is easy to see that  $f(r)$  takes its minimum value of 0 at  $\hat{r} \triangleq \sqrt{A}/(2\sqrt{\pi(2\bar{N}+1)})$ . The proof of Theorem 1 is valid for  $0 \leq r \leq \hat{r}$ , therefore we require  $r$  to lie in this range. This implies that  $r$  is limited to the same order of  $\bar{N}$  as the nearest neighbor distance, i.e.,  $O(1/\sqrt{\bar{N}})$ . However as shown in the next section,  $r \propto O(1/\sqrt{\bar{N}})$  is indeed *necessary* to have a

<sup>3</sup> Note that this message reception policy should not be confused with system policies for the mobile receiver that define the order through which the messages are served. An example of a system policy is First Come First Serve policy which serves the messages in the order of their arrival time.

successful transmission under the interference of  $\bar{N}$  concurrent transmissions. Note that  $r = 0$  corresponds to a system without wireless transmission. In this case (1) collapses to the lower bound on  $\bar{d}$  given in [3], namely  $2\sqrt{A}/(3\sqrt{\pi(2\bar{N}+1)})$ . For  $0 < r < \hat{r}$ , the second part of the lower bound in (1) changes between 0 and  $2r$  and it corresponds to the *reduction* in the average travel distance due to wireless transmission. Finally for  $r = \hat{r}$ , the lower bound is zero.

### 3.1 The Failure Probability and The Reception Distance

The new arrivals into the network are distributed according the uniform distribution over the network region, however, the distribution of messages waiting for service is a *perturbed* uniform distribution as it may depend on mobile receiver's policy. However it can be well approximated by a uniform distribution for a stable system. For the analysis of the similar system but without wireless transmission capability (i.e., receiver visiting the message locations) in [3], the delay lower bound with and without the uniformity assumption is fundamentally the *same* (the same expression with a different constant factor, i.e., 1.88 of the actual lower bound). Furthermore, as the load  $\rho$  in the network increases the distribution of message locations in the network approaches a uniform distribution. This is because as the arrival rate increases, the relative impact of the receiver's perturbation on the distribution decreases. Therefore in order to be able to derive analytical results, we assume that the distribution of the message locations are i.i.d. uniform in this section.



**Fig. 1** An example illustrating the reasoning behind (2) and (8). For (2)  $v$  denotes the receiver location and for (8) it denotes the location of a tagged message.  $U_i$  and  $U_j$  are locations of messages  $i$  and  $j$  respectively.

Consider an arbitrary message (designated by message 0) at location  $U_0$  and at a distance  $r'$  from the receiver. Suppose in addition to message 0 there are  $N_s$  other messages waiting for service in the network. Let  $\tilde{N}_s$  be the number of messages that are being transmitted during message 0's reception, enumerated from 1 to  $\tilde{N}_s$ . It is easy to see that  $\tilde{N}_s$  is binomial with parameters  $N_s$  and  $p$ . Let  $U_j, j \in [1, 2, \dots, \tilde{N}_s]$  denote the location of interferer  $j$ . Note that throughout the rest of this proof the probabilities are conditional on the receiver being at position  $v$  and for notational simplicity we will not express this conditioning in the equations. Applying the Protocol Model, message 0 is successfully received if for given  $N_s$  and  $\tilde{N}_s$

$$|U_i - v| \geq (1 + \Delta)|U_0 - v|, \forall i \in [1, 2, \dots, \tilde{N}_s].$$

We are given that  $|U_0 - v| = r'$ . Hence we have

$$\Pr(|U_i - v| \geq (1 + \Delta)r') \geq 1 - \frac{\pi(1 + \Delta)^2 r'^2}{A}. \quad (2)$$

The last inequality follows since  $\Pr(|U_i - v| < (1 + \Delta)r')$  is upper bounded by the probability that a uniformly distributed random variable lies in a disk of radius  $(1 + \Delta)r'$  centered at  $v$  divided by the total area  $A$ . To see this, as illustrated in Fig. 1, if the receiver location  $v$  is close to the peripheral of the network area, then  $\Pr(|U_i - v| < (1 + \Delta)r')$  is equal to the *portion* of the area of the disk of radius  $(1 + \Delta)r'$  centered at  $v$  that is *inside* the region  $\mathfrak{X}$ , divided by the total network area  $A$ . Note that given that the receiver is at position  $V = v$ , the random variables  $|U_i - v|$ ,  $i \in [1, 2, \dots, \tilde{N}_s]$  are i.i.d. with the tail distribution bounded by (2). Moreover the bound in (2) is valid for all receiver positions. Hence, we obtain the success probability as

$$\Pr(\text{Success} | N_s, \tilde{N}_s) \geq \left(1 - \frac{\pi(1 + \Delta)^2 r'^2}{A}\right)^{\tilde{N}_s} \geq 1 - \frac{\pi(1 + \Delta)^2 r'^2}{A} \tilde{N}_s. \quad (3)$$

The last inequality is due to the fact that  $g(\varepsilon) = (1 - \varepsilon)^{\tilde{N}_s} - 1 + \varepsilon \tilde{N}_s \geq 0$  where  $0 \leq \varepsilon = \pi(1 + \Delta)^2 r'^2 / A < 1$ . To see this,  $g'(\varepsilon) = \tilde{N}_s(1 - (1 - \varepsilon)^{\tilde{N}_s - 1}) > 0$  for  $\varepsilon \in (0, \pi z^2 / A)$ . Therefore  $g(\cdot)$  is an increasing function of  $\varepsilon$ . Finally noting that  $g(\varepsilon = 0) = 0$  proves that  $g(\varepsilon) \geq 0$ . Taking the expectation of both sides with respect to  $\tilde{N}_s$  for a given value of  $N_s$  and noting that  $E[\tilde{N}_s] = N_s p$  we obtain

$$\Pr(\text{Success}) \geq 1 - \frac{\pi(1 + \Delta)^2 r'^2}{A} N_s p. \quad (4)$$

Now taking expectation with respect to  $N_s$  (note that  $E[N_s] = \bar{N}$ ), we see that choosing the reception distance according to  $r' \leq (1 + \Delta)^{-1} \sqrt{\mu A / (\pi p \bar{N})}$  guarantees an average success probability of  $1 - \mu$ .<sup>4</sup> We define the reception distance  $r$  to be used in the message reception policy of the receiver defined in Section 2 to be<sup>5</sup>

$$r = (1 + \Delta)^{-1} \sqrt{\frac{\mu A}{\pi p (2\bar{N} + 1)}}. \quad (5)$$

This implies that with  $O(\bar{N})$  concurrent transmissions in the channel, the success probability of a transmission at a distance  $O(1/\sqrt{\bar{N}})$  from the receiver is independent of  $\bar{N}$ . A similar phenomenon has been observed by Tse *et al.* [7] and by Hajek *et al.* [9]. Note that  $r(\mu)$  is a monotone increasing function of  $\mu$ , we take its inverse to find  $\mu = (1 + \Delta)^2 \pi p (2\bar{N} + 1) r^2 / A$ . Given that  $r$  is confined to the interval  $0 \leq r \leq \hat{r}$ ,  $\mu$  takes values in the interval  $0 \leq \mu \leq \hat{\mu}$  where  $\hat{\mu} = \min(p(1 + \Delta)^2 / 4, 1)$ .

<sup>4</sup> Note that  $1 - \mu$  is the guaranteed success probability when  $r$  is chosen as described. However we assume that the success probability is given by  $1 - \mu$  when  $r$  is chosen as described above.

<sup>5</sup> Note that  $r$  can be chosen as large as  $(1 + \Delta)^{-1} \sqrt{\mu A / (\pi p \bar{N})}$ , however, choosing  $r$  as defined in (5) ensures analytical tractability.

## 4 Lower Bound on Delay

We derive a lower bound on delay by parameterizing the system in terms of the failure probability  $\mu$ . This ensures analytical tractability and we present the numerical results in Section 5 in terms of the reception distance  $r$ . We substitute the  $r$  expression in (5) into (1) to obtain  $\bar{d} \geq \frac{2\sqrt{A}}{3\sqrt{\pi(2\bar{N}+1)}}\kappa(\mu)$  where  $\kappa(\mu) \triangleq 1 - 3\frac{\sqrt{\mu}}{\sqrt{p}}(1+\Delta)^{-1}(1 - \frac{4\mu(1+\Delta)^{-2}}{3p})$ . Each message is transmitted with probability  $p$  to the receiver in each slot until it is successfully received where the average success probability of each attempt is  $1 - \mu$ . Note that the success probabilities of each of these attempts are approximately independent since each node transmits with probability  $p$  in each slot independent of its past attempts and other transmitters<sup>6</sup>. Assuming that the message being received is transmitted with probability 1 certainly yields a lower bound on transmission time. In this case the reception time is a geometric random variable with mean  $\bar{s} = \tau/(1 - \mu)$ .

**Theorem 2.** *The optimal system time of a message,  $T^*$ , is lower bounded by*

$$T^* \geq \kappa(\mu)^2 \frac{2\lambda A}{9\pi v^2(1-\rho)^2} - \frac{1-2\rho}{2\lambda}. \quad (6)$$

*Proof.* The set of stable rates in the system is given by  $\bar{s} + \bar{d}/v \leq 1/\lambda$ . To see this, note that  $\bar{s} + \bar{d}/v$  is the average time the receiver spends per message. Therefore the average interarrival time  $1/\lambda$  has to be greater than this time for the system to be stable. Using the  $\bar{s}$  and  $\bar{d}$  expressions above we can rewrite the stability condition as

$$\lambda \leq \left( \frac{\tau}{1-\mu} + \frac{2\sqrt{A}}{3v\sqrt{\pi(2\bar{N}+1)}}\kappa(\mu) \right)^{-1}. \quad (7)$$

For a stable rate  $\lambda$ , we utilize Little's law to get  $\bar{N} = \lambda W = \lambda(T - \bar{s})$ . Substituting this in (7), recalling that  $\rho = \lambda\bar{s} = \lambda\tau/(1 - \mu)$  and rearranging yields (6).

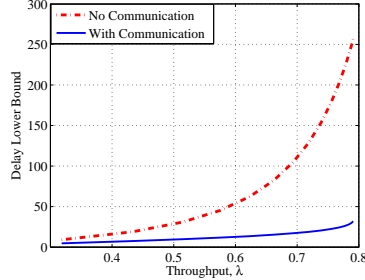
Note that  $\mu = 0$  ( $r = 0$ ) corresponds to a system without wireless transmission. In this case (6) collapses to the delay lower bound given in [3]. The  $\kappa(\mu)$  term in (6) together with  $\rho = \lambda\tau/(1 - \mu)$  represent the effect of communication capability on delay. Note that  $\kappa(\mu)$  takes values in the interval  $[0, 1]$  as  $\mu$  varies in the interval  $[0, \hat{\mu}]$  and hence we obtain smaller message delays by saving on the receiver travel time. Further observing (6), we notice that the communication model parameter  $\Delta$  affects the optimal delay only through the term  $\kappa(\mu)$ .

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<sup>6</sup> They are approximately independent since there is small dependence from slot to slot due to the location of transmitters.

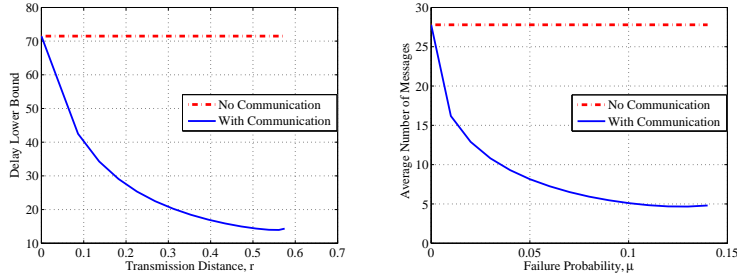
## 5 Results

We investigate the delay bound in (6) for different values of the transmission distance  $r$  and the arrival rate  $\lambda$  in this Section. Fig. 2 shows the minimum delay



**Fig. 2** Delay lower bound vs. throughput. Reception distance  $r = 0.4$ , the network area  $A = 200$ , the transmission probability  $p = 0.85$ , the capture parameter  $\Delta = 0.2$ , the receiver velocity  $v = 1$ , and the slot time  $\tau = 1$ .

versus throughput for a given reception distance  $r$ . The minimum delay in the system is considerably less than the minimum delay in the corresponding system without wireless transmission. The difference is more significant for high arrival rates. This is due to the fact that with an increased message density in the system, small savings on delay at each message reception add up to make a significant impact on the total delay. To see this, note that the average number of messages served during the waiting time of a message is  $\bar{N}$ . Heuristically, if we assume the savings on delay due to wireless transmission is proportional to the reception distance for each transmission, then the total waiting time of a message is decreased by  $\bar{N}r/v = \bar{N}(1 + \Delta)^{-1} \sqrt{\mu A} / \sqrt{v\pi p(2N + 1)}$ , i.e., by  $O(\sqrt{\bar{N}})$ .



**Fig. 3** Delay lower bound vs. transmission distance and the corresponding average number of messages in the system vs. the failure probability. The parameters used are the throughput  $\lambda = 0.4$ , the network area  $A = 100$ , the transmission probability  $p = 0.9$ , the capture parameter  $\Delta = 0.2$ , the receiver velocity  $v = 1$ , and the slot time  $\tau = 2$ .



The plots in Fig. 3 present the improvement in the minimum average delay and the average number of messages as we increase the reception distance for a constant throughput in the system. Initially increasing the reception distance yields lower delay since the mobile receiver saves on travel time (even though the failure probability increases). However, as the reception distance increases beyond some optimal point (0.55 in this case), then the failure probability increases rapidly and therefore the delay starts to increase. Hence we expect an optimal reception distance yielding the minimum delay in the system, and this is confirmed in Fig. 3.

## 6 Conclusion

In this paper we considered the use of controlled mobility in order to improve the delay performance of wireless networks where messages arriving randomly in time and space are collected by a mobile receiver via wireless reception. We characterized the tradeoff between mobility and wireless transmission and derived bounds for the travel time and the total system time. We analyzed the effects of different communication parameters on these metrics. Our results show that combined mobility and wireless transmission improves the delay performance of wireless networks significantly as compared to a system where wireless transmission is not utilized.

This work is our first approach towards utilizing a combination of controlled mobility and wireless transmission for data collection in wireless networks. We intend to develop policies for the mobile receiver and compare their performance to the delay bound. We also plan to extend the results in this paper to the multiple mobile receivers case. Finally, we intend to study more advanced wireless communication models such as modeling the transmission rate as a function of the transmission distance and enabling the mobile receiver to receive messages while moving.

## Appendix-Proof of Theorem 1

The proof is based on a nearest neighbor argument. The methodology of the proof is similar to the proof of the  $\bar{d}$ -lower bound in [3] but with simpler techniques and with the added complexity of communication capability in the system. We consider an arbitrary message (henceforth denoted as the tagged message) and define:

$\Omega_p \equiv$  The set of locations of the messages that are in queue at the time of the tagged message's arrival plus the location of the receiver;

$\Omega_f \equiv$  The set of locations of the messages that arrive during the tagged message's waiting time ordered by their time of arrival;

$Y_0 \equiv$  The tagged message's location;

$N_p \equiv |\Omega_p|$  and  $N_f \equiv |\Omega_f|$ .

We next define  $Z_i \equiv \|Y_i - Y_0\|$  where  $Y_i$  is the location of the  $i$ th message that will arrive after the tagged message, i.e.,  $\Omega_f = \{Y_1, Y_2, \dots, Y_{N_f}\}$ . Note that for a given tagged message location  $Y_0 = y_0$ ,  $\{Z_i; i \geq 1\}$  are i.i.d. with

$$\Pr(Z_i \leq z) \leq \frac{\pi z^2}{A}. \quad (8)$$

To see this, given  $y_0$ ,  $\Pr(Z_i \leq z)$  is upper bounded by the probability that a uniformly distributed random variable lies in a disk of radius  $z$  centered at  $y_0$ . This is illustrated in Fig. 1, namely, if  $y_0$  is close to the peripheral of the network area, then  $\Pr(Z_i \leq z)$  is equal to the *portion* of the area of the disk of radius  $z$  centered at  $v$  that is *inside* the region  $\mathfrak{A}$ , divided by the total area  $A$ .

We define the minimum of the sequence of random variables  $\{Z_i; 1 \leq i \leq N_f\}$  as  $Z_f \equiv \min_{y \in \Omega_f} \|y - Y_0\|$ . Note that  $N_f$  is a random variable with an unknown distribution and expectation  $\bar{N}$ . Similarly we define  $Z_p \equiv \min_{y \in \Omega_p} \|y - Y_0\|$ . The reason we treat the message locations in the set  $\Omega_p$  differently is because the distribution of the locations in  $\Omega_p$  which depends on the receiver policy may not be uniform. Note that in a system *without* communication capability (where the receiver has to visit the message locations)  $\bar{d}$  is lower bounded by  $E[\min\{Z_f, Z_p\}]$ . This is because the tagged message has to be served after the receiver is at one of the locations in the set  $\Omega_p \cup \Omega_f$ . When there is communication capability in the system, we define the random variable  $Z_f^c$  corresponding to  $Z_f$  ( $Z_p^c$  is defined similarly) as

$$Z_f^c = \begin{cases} Z_f - 2r & \text{if } Z_f \geq 2r \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

The receiver message reception policy in Section 2 implies that the receiver attempts to receive the next message when its on the circle of radius  $r$  around the next message location (unless the receiver is already inside that circle when it finishes serving the previous message). Therefore compared to a system without wireless transmission capability, the distance the receiver travels between two message locations is decreased the most when the receiver is on the line connecting the previous and the next message locations. This maximum difference is  $2r$ , i.e., two times the reception distance. Hence in our system a lower bound on  $\bar{d}$  is given as  $\bar{d} \geq E[\min\{Z_f^c, Z_p^c\}]$ . In order to bound this expectation, we have

$$\Pr(\min\{Z_f^c, Z_p^c\} > z) = 1 - \Pr(\min\{Z_f^c, Z_p^c\} \leq z) \geq 1 - \Pr(Z_f^c \leq z) - \Pr(Z_p^c \leq z). \quad (10)$$

Note that given  $y_0$  the random variables  $Z_i$  are independent conditioned on  $N_f$ . Therefore using (8) we obtain

$$\Pr(Z_f > z | N_f) = \Pr(Z_1 > z, Z_2 > z, \dots, Z_{N_f} > z | N_f) \geq \left(1 - \frac{\pi z^2}{A}\right)^{N_f} \geq 1 - \frac{\pi z^2}{A} N_f.$$

The last inequality is due to the fact that  $g(\varepsilon) = (1 - \varepsilon)^{N_f} - 1 + \varepsilon N_f \geq 0$  where  $0 < \varepsilon = \pi z^2 / A < 1$ . To see this,  $g'(\varepsilon) = N_f(1 - (1 - \varepsilon)^{N_f - 1}) > 0$  for  $\varepsilon \in (0, 1)$ . Therefore

$g$  is an increasing function of  $\varepsilon$ . Finally noting that  $g(\varepsilon = 0) = 0$  proves that  $g(\varepsilon) \geq 0$ . Since the above bound is true for all  $Y_0 = y_0$ , and since  $Y_0$  is independent of future arrivals, it holds without the conditioning on  $Y_0$ . Taking its expectation with respect to  $N_f$  we obtain

$$\Pr(Z_f > z) \geq 1 - \frac{\pi z^2}{A} \bar{N}. \quad (11)$$

In order to bound  $\Pr(Z_p > z)$ , we have the following lemma given in [4].

**Lemma 1.**

$$\Pr(Z_p \leq z) \leq \frac{\pi z^2}{A} (\bar{N} + 1). \quad (12)$$

*Proof.* Let  $\Omega$  be any set of  $k$  points in a region  $\mathfrak{X}$  with area  $A$ . Let  $Y$  be a uniformly distributed location in  $\mathfrak{X}$  independent of  $\Omega$  and define  $Z^* \triangleq \min_{y \in \Omega} \|Y - y\|$ . For each point in  $\Omega$ , we construct a circle of radius  $z$  centered at the point. Let  $A_\Omega$  denote the total area in  $\mathfrak{X}$  covered by the union of these circles. Then the event  $\{Z^* \leq z\}$  happens if and only if  $Y$  lies somewhere inside the region  $A_\Omega$ . Hence

$$\Pr(Z^* \leq z) = \frac{A_\Omega}{A} \leq \frac{\pi z^2}{A} k.$$

Now we apply this result to  $Z_p$  by conditioning  $\Pr(Z_p \leq z)$  on  $N_p$ , i.e., the size of  $\Omega_p$ . Note that  $Y_0$  is independent of  $\Omega_p$  under conditioning on  $N_p$ . Therefore we obtain  $\Pr(Z_p \leq z | N_p) \leq \frac{\pi z^2}{A} N_p$ . Averaging over  $N_p$  and observing that  $E[N_p] = (\bar{N} + 1)$  we obtain (12).

It is easy to show for  $z > 0$  that

$$\begin{aligned} \Pr(Z_f^c > z) &= \Pr(Z_f^c > z | Z_f \geq 2r) \Pr(Z_f \geq 2r) + \Pr(Z_f^c > z | Z_f < 2r) \Pr(Z_f < 2r) \\ &= \Pr(Z_f > z + 2r | Z_f \geq 2r) \Pr(Z_f \geq 2r) = \Pr(Z_f > z + 2r). \end{aligned}$$

A similar expression is valid for  $Z_p^c$ . Therefore from (11) and (12) we have

$$\Pr(Z_f^c > z) \geq 1 - \frac{\pi(z+2r)^2}{A} \bar{N} \quad \text{and} \quad \Pr(Z_p^c > z) \geq 1 - \frac{\pi(z+2r)^2}{A} (\bar{N} + 1).$$

Then (10) yields

$$\Pr(\min\{Z_f^c, Z_p^c\} > z) \geq 1 - (2\bar{N} + 1) \frac{\pi(z+2r)^2}{A}. \quad (13)$$

Now  $\Pr(\cdot) \geq 0$  gives  $z \leq \frac{\sqrt{A}}{\sqrt{\pi(2\bar{N}+1)}} - 2r \triangleq \hat{z}$  and since  $Z_f^c$  and  $Z_p^c$  are nonnegative random variables we have<sup>7</sup>  $z \geq 0$ . Hence

<sup>7</sup> These two bounds on  $z$  imply that  $r \leq \frac{\sqrt{A}}{2\sqrt{\pi(2\bar{N}+1)}}$ . This is critical since it gives a natural bound on the maximum reception distance on  $r$  for which the analysis is valid. We will discuss this further in Section 4.

$$E[\min\{Z_f^c, Z_p^c\}] \geq \int_{0^+}^{\hat{z}} Pr(\min\{Z_f^c, Z_p^c\} > z) dz \geq \int_{0^+}^{\hat{z}} 1 - \frac{\pi(z+2r)^2}{A} (2\bar{N} + 1) dz,$$

which yields (1) after a few manipulations.

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